A Rent Extraction View of Employee Discounts and Benefits

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Abstract

We offer a novel view of employee discounts and in kind compensation. In our theory, bundling perks and cash compensation allows a firm to extract information rents from employees who have private information about their preferences for the perk and about their outside opportunities. We show that in maximizing profit with heterogeneous workers, the firm creates different bundles of the perk and salary in response to different employee characteristics and marginal costs of the perk. Our key result is that strategic bundling can lead firms to provide perks even in the absence of any cost advantage over the outside market and to deviate from the standard marginal cost pricing rule. We study how this deviation depends upon the set of feasible contracts, upon the perk’s marginal cost, and upon the correlation between the agents’ preferences for the good and their reservation utilities.

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1. Introduction

A typical U.S. worker receives a sizable portion of his or her compensation in the form of employee benefits (throughout the paper, we will use the terms "benefits", "perks", and "in kind compensation" interchangeably). Examples include but are not limited to health insurance, maternity leave, paid time off, subsidized meals and transportation, and on-site fitness facilities. In a 2002 survey by the U.S. Chamber of Commerce, employee benefits constituted 42.3 percent of company payrolls. In addition, many firms offer employee discounts on their products. For example, university employees and their families often pay lower tuition than outsiders, airline employees travel for a nominal fee, instructors at ski resorts use the lifts for free, and so on. According to a recent survey, 75 percent of workers reported that their employers offered discounts, of 29.5% on average (Fram and McCarthy, 2003).

The leading explanation for why firms include in-kind payments in their compensation packages is that they have cost advantages in providing the good or the service.¹ This would be the case mostly because of tax exemptions, but also because of economies of scale, because the firm specializes in producing the good (the case of employee discounts), because it has enough bargaining power to secure a discount from the provider (e.g. health and accident insurance), or because the good is simply not available in the outside market (e.g., a pleasant work environment or paid time off). An additional rationale given in the literature for in-kind payments is that they may enhance the workers’ productivity, as in the case of free lifts for ski instructors. Finally, discounts and benefits could be just a manifestation of an agency problem, especially at the senior executive level.

These explanations are plausible, but do not seem to account for all observed patterns of perk provision. First, not all of the existing non-cash payments are tax free and some predate meaningful

¹See, e.g., Rosen (2000), who provides a discussion of the main existing arguments for the use of non-cash compensation. See also Oyer (2004).
Federal and State taxes. Second, the cost advantage theory predicts that the optimal amount of benefits is the one at which the marginal rate of substitution between the benefit and money is equal to the marginal cost of the benefit. Thus, we should see all benefits and discounted goods that do not improve productivity being sold to employees at their marginal cost. This prediction, though, appears to be often violated in practice. For example, some firms offer company loans to their senior executives, which are frequently at below market interest rates (Weston, 2002). Similarly, universities that offer free tuition to their employees' relatives forego the revenues they could get from regular, tuition-paying students.

In addition, the cost advantage theory cannot explain why different categories of employees are sometimes charged different prices for the goods and services offered by the firm. For instance, in 2000, the employee discount provided by Federated Department Stores (which operates Macy’s, Bloomingdale’s and other specialty retailers) was 20% for most employees, but 38% for senior executives (Strauss, 2001). It is unlikely that the marginal cost of selling the merchandise to an executive is considerably lower than the marginal cost of selling it to a lower ranked employee.\(^2\) Similarly, company contributions for executive versions of 401(k) retirement plans are typically 60% higher than for ordinary employees (Weston, 2002). Again, this does not appear to conform to marginal cost based pricing.

It is possible that some of these deviations from the marginal cost pricing rule are due to agency problems, but in many cases (e.g., free tuition, services and merchandise for rank-and-file employees) this does not seem convincing. Moreover, according to a recent study by Rajan and Wulf (2006), agency considerations do not seem to do a good job explaining the provision of perks

\(^2\)It might be tempting to explain this difference in discounts by the employees' differing marginal tax rates. However, employee discounts can be exempt from taxes only if they are offered in a non-discriminatory way. Moreover, an income tax can be thought of as affecting the employee's marginal willingness to pay for the perk. The standard theory would then state that this effective willingness to pay would equal the firm's marginal cost of providing the perk, which typically does not depend upon the employees' tax rates. The tax advantage would therefore increase the employee's optimal quantity of the perk, but not change the price and marginal cost equality.
even if one concentrates on the companies’ senior executive officers.

In this paper, we offer an alternative theory of employee discounts and in kind compensation, in which a firm may find it optimal to deviate from the marginal cost pricing rule. We argue here that the marginal cost pricing prediction is the correct one if the firm has full information about the workers’ preferences and reservation utilities, but not when workers are heterogeneous and have private information about their preferences for the perk and about their outside opportunities. In the latter case, bundling perks and cash compensation allows firms to extract information rents from the employees and this can be done more effectively by deviating from marginal cost pricing.\(^3\) In particular, we consider a model with two types of workers. The workers’ productivities in the firm are common knowledge, but their preferences for the perk as well as reservation utilities are private information to the employee. The firm offers an employment contract which consists of a salary, possibly a quantity of the perk, and possibly a price for the perk. The perk could be an employee discount or other goods, not produced by the firm, like health insurance or the use of athletic facilities, as well as amenities such as pleasant work environment and other less tangible forms of compensation. To account for the possibility that there may exist real world constraints which prevent the firm from offering a full menu of contracts, we consider two types of compensation bundles. The first type is constrained to be uniform across worker types in either the price or the quantity of the perk, while the second is non-uniform and represented by a menu of bundles where the employees self select into a contract. Our key findings are the following:

(1) The firm may find it optimal to offer a perk even if the perk has no productivity effects and the firm’s cost of providing it is higher than the price at which the employees could obtain the perk

\(^3\)One of the perks enjoyed by the 150 lawyers of a Virginia law firm LeClair Ryan is that they can request funding for personal projects that would "enhance their lives". The firm has approved funding for such things as family trips to the Grand Canyon, the purchase of a piano and piano lessons, cosmetic surgery, a personal fitness trainer, or a week with family in a Zen monastery in France (Bacon, 2005). This example illustrates not only that there is indeed asymmetric information between firms and their employees regarding the employees’ preferences for perks, but also that firms care about these preferences and try to elicit information about them from the employees. Note also that this example does not easily fit any of the existing theories.
in the outside market.

(2) If the firm is constrained to offer a uniform quantity or uniform price contract, then the optimal quantity is above the efficient one for high marginal cost perks and below it for low marginal cost perks. Moreover, in equilibrium, the workers may buy additional quantities of the perk from the outside market.

(3) If the firm is constrained to offer a uniform price contract, it will tend to charge a price which is lower than the efficient price when marginal cost of the perk is high and conversely when the marginal cost of the perk is low.

(4) If the firm can offer a menu of quantity contracts into which worker types can self select, then the perk is provided in a more efficient manner. Nevertheless, high valuation workers are over-supplied when the perk’s marginal cost is high and low valuation workers are under-supplied when the perk’s marginal cost is low.

Result 1 demonstrates that our theory is driven by economic forces that differ from those behind the extant theories of perk provision. The general message of conclusions 2-4 is that firms with heterogeneous workers will typically find it optimal to deviate from efficient provision of perks, and they will do so in a systematic way, depending upon the perk’s marginal cost.

We view our theory as applying primarily to perks offered to workers in high skill jobs, because it seems reasonable that when filling a job that requires a high skilled worker, firms select employees based on their skills rather than on their preferences for perks. Once these employees are sorted on skill, our model then examines perk design in the employment contract. Our theory therefore fits well with the claims found in the popular press that firms provide perks to retain and attract high quality workers. For example, perks have been argued to help firms "attract and retain the most talented employees" (Ryan, 2005), to help "recruit and keep good employees" (Irvine, 1998), and to help "hold sway with a top-tier work force" (Bacon, 2005; italics added). This emphasis on
high skills as a crucial factor behind firms’ decisions to provide perks is not easily reconciled with the standard theory, especially in the case of perks that appear to have no effect on the workers’ productivities.

In its focus on the relationship between non-monetary aspects of a job and the monetary compensation needed to attract workers, our paper is related to the large literature on compensating differences, which dates back to Adam Smith, and where a good starting point is Rosen (1986). Much of this literature is empirical, trying to document a trade-off between the job benefits a worker receives and his monetary compensation (e.g., Brown, 1980; Montgomery, Shaw, and Benedict, 1992). The logic of our results makes our work also related to the literature on price discrimination, for example Oi (1971) and Mussa and Rosen (1978), and to the literature on commodity bundling, where the classic contributions are by Adams and Yellen (1976) and McAfee and McMillan (1989).

The paper is organized as follows. Section 2 introduces the model and describes our main assumptions. In Section 3, we formulate the general bundling problem and narrow down the set of all possible contracts to three types of contracts that we consider to be of interest. Section 4 contains an analysis of the case where the firm is constrained to offer a uniform quantity contract to all worker types. Section 5 studies the case where the firm offers a uniform price contract. In both Section 4 and Section 5 we also discuss the model’s predictions. The case of a full menu of contracts into which heterogenous workers self-select is considered in Section 6. Section 7 concludes.

2. The Model

Preferences. Consider a firm that needs to hire a single employee. The firm offers a contract that consists of cash and in-kind compensation and hires from a pool of potential employees, indexed by $n$, each of them being one of two possible types. Type $H$, whose proportion in the labor pool is $\pi_H$, has a higher total and marginal valuation for the in-kind product offered by the firm than
type $L$, who represent a proportion $\pi_L = 1 - \pi_H$ of potential employees. The utility that a type $i$ worker, $i = H, L$, derives from consuming quantity $q$ of the perk and from a numeraire commodity, denoted $M$, is given by the quasilinear function $u_i(q) + M$, where the functions $u_i(.)$ satisfy

**Assumption 1.** a) $u_i(0) = 0$, $u'_i(0) = \infty$, $u'_i(.) > 0$, and $u''_i(.) < 0$.

b) $u'_H(q) > u'_L(q)$ for each $q$.

Assumption 1a) contains the usual restrictions on utility. Part b) asserts the Spence-Mirrlees sorting condition; it says that the marginal utility from the good is always greater for the type-$H$ agents than for the type-$L$ agents.

**Workers’ outside opportunities.** The perk’s price in the outside market is denoted as $p^o$ and will be treated as exogenously given. This specification includes the case where the employees cannot obtain the perk in the outside market, for which it is enough to set $p^o$ sufficiently large. If the perk is available on the outside market, the reservation utility of a worker $n$ of type $i$ is given by $\bar{U}_n^i(p^o) = \bar{s}_n^i + u_i(q_o^i) - p^o q_o^i$, where $\bar{s}_n^i$ is the worker’s wage in an alternative employment and $u_i(q_o^i)$ is his utility from consuming the quantity of the perk that he would purchase at the outside price $p^o$.5

**Production technology and workers’ skills.** A worker $n$ of preference type $i$, $i = H, L$, generates a constant revenue $y_i^n$ for the firm. We wish to distinguish between jobs which are easy to fill and therefore the firm might find it worthwhile to screen workers based on their preferences

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4As we show elsewhere (Marino and Zabojnik, 2005), when the perk represents an employee discount on the firm’s product, the firm’s optimal outside price depends upon the employee discount. However, we also show that if the demand by the firm’s employees represents only a small fraction of the overall market for the firm’s product, the optimal outside price can be safely treated as unaffected by considerations of employee discounts.

5Note that when the firm sells the perk in the outside market, our assumption of exogenous outside price does not mean that we assume that the firm does not choose its outside price so as to maximize profit. We could always specify an outside demand function such that $p^o$ is the profit maximizing price.

6Our model is thus in the class of problems with type dependent participation constraints. The countervailing incentives model of Lewis and Sappington (1989) is a primary example of this type of model. See also the paper by Jullien (2000) which examines optimal contracting by an uninformed principal when an informed agent has type dependent reservation utility.
for the perk, and jobs in which screening based on preferences would be too costly because there are few suitable applicants and the position might end up vacant. We will call the former a low skill job and the latter a high skill job.

Formally, the low skill job is not sensitive to workers’ skills and each worker \( n \) has the same productivity \( y \). In the high skill job, the workers’ productivities are sensitive to their skills. Specifically, we assume that worker 1 is the most productive worker regardless of his tastes for the perk: \( y_1 > y_i \), for each \( n \neq 1 \) and \( i = H, L \). An applicant’s reservation utility and his preferences for the perk are his private information, but the firm can observe each worker’s skill level. To simplify the selection process, we will assume that the workers’ productivities are such that a firm seeking to fill a high skill job finds it optimal to hire worker 1 regardless of his preference type. This will be true if \([y_1 - \bar{U}_1(p^o)] - [y_n - \bar{U}_n(p^o)]\) is sufficiently large for all \( n \neq 1 \) and for \( i, j = H, L \).

**The in kind good.** The perk may represent an amount of an endogenously discounted product being produced by the firm or a purchased good. The firm can acquire (either purchase or produce) the perk at a per unit cost \( c \). We will not place any restrictions on the relationship between \( c \) and the perk’s outside price \( p^o \). The case where \( c < p^o \) represents the case where the firm has a cost advantage over the workers in procuring the perk (due to tax exemptions, economies of scale, a complete absence of an outside market for the perk, and so on), as in the standard theories mentioned in the Introduction. When \( c > p^o \), the outside market can supply the perk to the workers at a lower cost than the firm, say, due to transaction costs that the firm has to incur in obtaining the perk in the outside market and distributing it to the workers.

**The contract.** The firm faces a self selection problem where, in the absence of additional constraints, it will have an incentive to offer a menu of contracts \((s_i, p_i, q_i)\). Here, \( s_i \) is the salary

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6Even though in equilibrium one or the other of the preference types might not have a binding participation constraint, this assumption is sufficient because \( y_i \) are exogenous and independent of the cost minimization problem we will analyze.
received by a worker of type $i$ if employed by the firm, $p_i$ is the price at which the firm sells the in-kind good to the employee, and $q_i$ is the quantity of the perk the worker buys from the firm at price $p_i$. To ease our comparison of the profit maximizing price with the price that maximizes total surplus, we will let $p_i$ be independent of $q_i$, so that we can compare numbers rather than functions. This assumption also simplifies parts of the analysis, without being overly restrictive: It will become clear that in two out of the three contracts that we consider the assumption plays no role, while in the third one (analyzed in Section 5) it merely constrains the efficiency of the optimal contract, without much of an effect on our qualitative results.

The worker can choose to purchase an additional amount, $\max\{0, q_i^o - q_i\}$, of the in-kind good in the outside market. The total utility that an agent of type $i$ derives from working for the firm is then $s_i + u_i(q_i^{\max}) - p_i q_i - p^o_i(q_i^o - q_i)$, where $q_i^{\max} \equiv \max\{q_i^o, q_i\}$. To prevent wealth constraints from affecting the optimal contract, we assume that the agents are endowed with a sufficiently large fixed income.

Finally, we assume that the perk provided by the firm cannot be resold by the worker to non-employees of the firm. A large office is a good example of such a perk, secretarial support, services and flexible working hours are three additional examples. In cases where it is physically possible for the employee to resell the perk, the firm could threaten sanctions, as many real world firms do, such as dismissal if an employee is found to be arbitraging the perk. Such sanctions would not be feasible for non-employees.

2.1. A benchmark case with perfect information

Consider a benchmark case of perfect information, in which the firm hires a worker with a known demand for the perk, $q(p)$, implicitly defined by $p = u'(q)$. The firm solves $\max_{\{p, s\}} [pq(p) - s - cq(p)]$ subject to the worker’s participation constraint $s + u(q) - p(q)q \geq \bar{U}(p^o)$, yielding the standard
solution \( u'(q(p)) = c \). It should also be clear that the firm provides the perk only if it can do so more efficiently than the outside market, i.e. only if \( c \leq p^o \). The intuition is that, given that the firm can take back consumer surplus through variations in the worker’s salary, the firm wants to maximize total surplus in its pricing and allocation of the perk. This is analogous to a two part tariff with reduction of the salary as the entry fee.

The above solution obtains in our model when the firm seeks to fill a low skill job. Since all the workers in the labor pool are equally productive in this job, the firm simply designs a separating contract so as to attract the preference type, \( i \in \{L, H\} \), whose participation constraint is cheaper to satisfy. Because the other type does not apply, the firm can offer the full information efficient contract described above, with \( p = c \). Thus, in the case of low skill jobs, we expect the standard hedonic theory to apply.

In contrast, in a high skill job the workers’ skills are more important than their tastes and the firm finds it optimal to hire worker 1 no matter whether he is of type \( H \) or \( L \). Thus, a firm seeking to fill a high skill job is faced with a job candidate who has private information about his preferences for the perk and about his outside opportunities. This setting will be our focus for the rest of the paper.

3. The General Problem and the Set of Possible Contracts

In this section, we set up the firm’s general optimization problem when hiring for a high skill job and characterize the set of possible contracts. For the sake of making our arguments more transparent, we start by assuming that the in-kind good is not available on the outside market. After having presented this clear cut case, we subsequently discuss the effects of an outside market in each of the three distinct settings that we will consider.

Because only the most skilled worker, worker 1, will be hired, we can simplify notation by
suppressing the superscript \( n = 1 \) that denotes this worker. The firm’s general problem is then to offer worker 1 a menu of two contracts, \((s_i, p_i, Q_i)\), \(i = H, L\), where \(s_i \in \mathbb{R}\) and \(p_i \in \mathbb{R}_+\), while \(Q_i \in \mathbb{Q}\) is the set of \(q_i \geq 0\) from which the worker is allowed to choose his preferred quantity of the perk.

The set \(\mathbb{Q}\) that contains the feasible sets \(Q_i\) is exogenously given. It measures the degree of control the firm can exercise in a given specific situation over the quantities consumed by the workers, and it could depend upon things such as the nature of the perk, the firm’s costs of monitoring the workers’ consumption of the perk, and so on. Although this set could be quite general, we will restrict our attention to two specific cases that we find to be empirically most appealing. In the first case, the firm has no direct control over the quantities consumed by the workers and it can only influence these quantities through the prices it sets for the perk. In this case, \(Q_i = \mathbb{R}_+\) and \(\mathbb{Q}\) only has one element: \(\mathbb{Q} = \{\mathbb{R}_+\}\). This case corresponds to complete decentralization of consumption, where the worker is presented with the perk’s price and chooses any quantity he likes. The second case of interest is where the firm has complete control over the quantities consumed by the workers. In this case, \(Q_i\) can be any subset of \(\mathbb{R}_+\), so that \(\mathbb{Q} = \{S : S \subset \mathbb{R}_+\}\). This specification allows the firm to offer each worker a unique quantity he has to consume if he accepts the contract. This would be a natural specification of the firm’s problem when the perk is a public good or it needs to be provided before the job is filled with a particular worker, say an office building.

Using this general formulation, the firm’s problem can be written as follows:

\[
\max_{s_i, p_i, Q_i \in \mathbb{Q}} \sum_{i=H}^{L} \pi_i \left( y_i - s_i + p_i q_i - c q_i \right) \quad \text{(MAX)}
\]

subject to

\[
q_i = \arg \max_{q_i' \in Q_i} \left[ s_i - p_i q_i' + u_i(q_i') \right], \quad i = H, L, \quad \text{(UM}_i)\]

\[
s_i - p_i q_i + u_i(q_i) \geq \max_{q_i' \in Q_i} \left[ s_j - p_j q_i' + u_i(q_i') \right], \quad i, j = H, L, \quad \text{(IC}_i\text{)}
\]

\[
s_i - p_i q_i + u_i(q_i) \geq s_i, \quad i = H, L, \quad \text{(PC}_i\text{)}
\]
The "utility maximization" constraints (UM$_i$) guarantee that the quantity $q_i$ maximizes worker $i$’s utility given the perk’s price $p_i$ and given the worker’s choice set $Q_i$. Constraint (IC$_i$) is an incentive compatibility constraint for type $i$, which ensures that a worker of this type does not choose the contract designed for type $j \neq i$. Finally, the (PC$_i$) constraints ensure that both preference types are willing to accept the employment offer.

Depending upon the particular economic environment in which they operate, real world firms may face additional constraints on their choice of the sextuple $(s_i, p_i, Q_i)$, $i = H, L$. For example, non-discrimination rules may discourage them from charging different prices for the perk, which would add an additional constraint, $p_H = p_L = p$.\footnote{Such non-discrimination rules are in fact embedded in the U.S. tax code. According to the general rules under IRC Section 132, many benefits (such as discounts) are tax free to employees only if they are nondiscriminatory. Although technically these constraints may not apply if differential prices for the same benefit are offered to all potential workers as a self selection contract, in practice, the firm might find it hard to enforce higher perk prices for workers who chose a higher salary contract.} In the case where the firm can control quantities (i.e., $Q = \{S : S \subset \mathbb{R}_+\}$), the firm may need to purchase the perk before filling the job with a particular worker, which would add an extra constraint $q_H = q_L = q$. Finally, the firm may face an institutional constraint, say due to equity considerations, that prevents it from offering a menu of salaries. This would be captured by a constraint $s_H = s_L = s$. Thus, ideally, we would like to analyze the firm’s problem not only as stated in (MAX), but also subject to all possible variations of additional constraints on $s_i$, $q_i$ and $p_i$ of the form

$$z_H = z_L = z,$$

where $z_i = s_i$, $q_i$, or $p_i$, for $i = H, L$.

For $Q$ such that the firm cannot control the quantity of the perk consumed by the workers ($Q = \{\mathbb{R}_+\}$), so that $q_i$ cannot be constrained by (1), there are three variations of the firm’s problem with additional constraints on $s_i$ and $p_i$ of the form (1), plus the basic problem in (MAX).
The second specification, where the firm has control over the perk’s quantity (\(Q = \{S : S \subset \mathbb{R}_+\}\)), admits a total of eight different variations of the problem. The following lemma shows that, for each \(Q\), all of the possible variations can be mapped (in the sense that they yield the same equilibrium outcomes) to two general formulations of the problem.

**Lemma 1.** Any formulation of the optimization problem (MAX) subject to additional constraints on \(s_i, q_i\) and \(p_i\) of the form expressed in (1) is,

(i) for \(Q = \{S : S \subset \mathbb{R}_+\}\), either equivalent to (a) a problem where the firm chooses \((p_i, s_i, q_i)\), \(i = H, L\), such that \(p_i = p, s_i = s,\) and \(q_i = q, i = L, H\), or to (b) the general problem in (MAX) without additional constraints (1);

(ii) for \(Q = \{\mathbb{R}_+\}\), either equivalent to (c) the problem in (MAX) subject to additional constraints \(p_i = p\) and \(s_i = s\), or to (d) the general problem in (MAX) without additional constraints (1).

Lemma 1 simplifies the analysis by narrowing down the set of the twelve economic environments that can be generated from (MAX) by including additional constraints of the form (1) to four basic problems. In the first one, the firm offers the same contract, \((s, q, p)\), to both types of workers, i.e., \(s_H = s_L = s, q_H = q_L = q,\) and \(p_H = p_L = p\). The second problem allows the firm, if it chooses so, to give each worker an all or none choice of a specific quantity of the perk. In the third problem, the firm offers the same salaries and prices to all workers, but each worker buys any quantity he likes. Finally, in the last setting the firm offers a menu of salaries and prices, and lets each worker buy any quantity he likes.

We first analyze the simpler, and empirically appealing, uniform contracts (a) and (c). Subsequently, we will provide a brief analysis of the menu of quantities contract (b). In order to economize on space, we will omit the menu of prices case (d), which seems to us to be of limited
practical interest.\textsuperscript{8}

4. Case (a): The Firm Offers a Uniform Quantity Contract to all Workers

We start with the uniform quantity problem identified in part (a) of Lemma 1, where the firm devises a single contract offered to all worker types, i.e., $s_i = s$, $q_i = q$, and $p_i = p$, $i = L, H$. Note that this case is equivalent to cases where the firm is constrained to offer $q_i$ and one of either $s_i$ or $p_i$ in a uniform manner. To see this suppose for example that $s_i$ and $p_i$ can vary across the two types of workers, but $q_i$ cannot, that is, it must be that $q_1 = q_2 = q$. Because the workers have quasi-linear utility functions, both the firm and the workers care about $s_i$ and $p_i$ only through the transfer term $t_i = s_i - p_i q_i$. This problem is therefore equivalent to the one where the firm chooses $t_i$ and $q$. However, if, say, $t_1 > t_2$, then both types strictly prefer contract $(t_1, q)$ to contract $(t_2, q)$, which means that no worker will ever select the latter contract. Therefore, this menu of contracts is equivalent to offering a single contract, $(t_1, q) = (t, q)$. Making $q$ and either $p$ or $s$ uniform then makes all three variables uniform. A real world application of this scenario is that the salary is attached to a job position which the firm needs to fill and the perk is provided as a public good ($s_i = s$ and $q_i = q$). For example, all employees are located in a well designed and appointed office building in a desirable location. This assumption is also realistic where the firm has to invest in the perk (say, equip an office) before hiring a particular worker. Finally, in some situations, a firm may choose to give away a particular quantity of the perk to all employees in a uniform way for reasons that are outside of the model, such as simplicity, equity considerations, or impossibility to restrict resale to other employees. For example, every employee of Starbucks Corporation gets a free pound of coffee or box of tea each week ($q_i = q$ and $p_i = p = 0$) (Baltimore Sun, 2004).

\textsuperscript{8}The qualitative characteristics of the optimal contract in case (d) is similar to those obtained for the uniform price contract (c). The only new insight in the menu of prices case is that the perk provision is somewhat more efficient, because the firm has more degrees of freedom in designing the contracts.
4.1. The optimal quantity in the uniform contract

We start by comparing the firm’s optimal amount of the perk with the efficient amount under the assumption that the perk is not available on the outside market, as this comparison allows us to better highlight the intuition behind our results. Conditional on $q_H = q_L$, the ex-ante efficient amount of the perk, $q^e$, maximizes the expected total surplus, $\sum_{i=L}^{H} \pi_i(y_i + u_i(q)) - cq$. The amount $q^e$ is therefore given by the first order condition

$$\pi_H u_H'(q^e) + \pi_L u_L'(q^e) = c. \tag{2}$$

Similarly, let $q^e_i$ be the first-best efficient quantity of the perk good such that

$$u_i'(q^e_i) = c, \ i = H, L.$$

From Assumption 1, it must be both that $q^e_H > q^e_L$ and that the function $\Delta u(q)$, defined by $\Delta u(q) \equiv u_H(q) - u_L(q)$, is strictly increasing in $q$. Also, in order to simplify the statement of our subsequent results, denote as $\Delta \bar{U}(p^o)$ the difference between the two types’ reservation utilities, i.e., $\Delta \bar{U}(p^o) \equiv \bar{U}_H(p^o) - \bar{U}_L(p^o)$ (which simplifies to $\Delta \bar{U}(p^o) = \bar{s}_H - \bar{s}_L$ in the absence of an outside market). Finally, let $\hat{q}$ be implicitly defined by

$$\Delta u(\hat{q}) = \Delta \bar{U}(p^o). \tag{3}$$

The economic meaning of quantity $\hat{q}$ is that it is the quantity at which both participation constraints are binding. Note that, generically, $\hat{q} \neq q^e$.

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9 If the good were available in the outside market at a price $p^o < c$, the efficient quantities $q^e$ and $q^e_i$ would be given by $\pi_H u_H'(q^o) + \pi_L u_L'(q^o) = p^o$ and $u_i'(q^o_i) = p^o$ respectively.
Now, the fact that the firm can control the quantity of the perk consumed by the workers makes the optimal price indeterminate. Thus, in this scenario, the linear pricing assumption imposes no restriction; in fact, we can without loss of generality set \( p = 0 \). Since both the \((\text{UM}_i)\) and the \((\text{IC}_i)\) constraints are trivially satisfied in the present setting (the former because the worker’s choice set contains a single element, \( q_i \); the latter because the firm only offers a single contract), the firm’s optimization problem reduces to choosing the quantity of the perk, \( q^* \), and the salary, \( s^* \), that minimize its cost, \( s + cq \), subject to

\[
s + u_i(q) \geq \bar{U}_i(p^o) = \bar{s}_i, \ i = H, L. \tag{PC_i'}
\]

The solution to the above problem and its allocational efficiency can be described as follows.

**Proposition 1.** Suppose the perk is not available on the outside market. When \( \lim_{q \to \infty} \Delta u(q) > \Delta \bar{U}(p^o) > 0 \), there exist \( c^{**} > c^* > 0 \) such that the allocational efficiency of the contract is characterized by (i)-(iii) below.

(i) If \( c \leq c^* \), then \( q^* = q^e_L < q^e \).

(ii) If \( c \geq c^{**} \), then \( q^* = q^e_H > q^e \).

(iii) If \( c \in (c^*, c^{**}) \), then \( q^* = \hat{q} \in (q^e_L, q^e_H) \).

When \( \Delta \bar{U}(p^o) \leq 0 \), then (i) holds for all \( c \). When \( \lim_{q \to \infty} \Delta u(q) \leq \Delta \bar{U}(p^o) \), then (ii) holds for all \( c \).

Proposition 1 tells us that the firm typically issues the perk in a socially inefficient way. For some parameter values, the amount of the perk issued by the firm is too large, while for other parameter values it is too low. Specifically, when the perk’s marginal cost is low, then both workers optimally consume relatively large quantities of it, so that a type \( H \) worker values the perk substantially
more than a type L worker. If at the same time the difference between the two types’ reservation utilities is relatively small or possibly negative, then the firm needs to worry more about satisfying type L’s than type H’s participation constraint.\footnote{Negative correlation is possible between preference for the perk and the reservation utility when the worker with higher valuation has lower costs of production effort. A person who enjoys contact with people may place a higher value on a desirable downtown location while at the same time have a low cost of doing her job which requires people skills.} In this case, the employment contract is tailored to suit type L workers, offering their surplus maximizing quantity of the perk, $q^e_L$. Because $q^e_L$ is lower than the ex-ante efficient level $q^e$ (which lies between $q^e_L$ and $q^e_H$), in this case the firm under-supplies the perk. An alternative view of this process is that the firm bundles the in-kind good with the worker’s monetary compensation in order to better extract the worker’s surplus. In particular, worker H’s valuation of the perk is so high in this case, that this worker receives an information rent, equal to $[\bar{U}_L(p^o) - u_L(q^*)] - [\bar{U}_H(p^o) - u_H(q^*)] = \Delta u(q^*) - \Delta \bar{U}(p^o) > 0$. Because this information rent increases in $q^*$, the firm minimizes it by under-supplying the perk.

On the other hand, if the reservation utility of type H worker is considerably higher than that of type L, while their valuations of the perk do not differ very much (which tends to be the case when $c$ is large so that the optimal consumption levels are low), then attracting worker H is more difficult than attracting worker L. In this case, the contract is optimally designed to suit worker H, offering the quantity of the perk, $q^e_H$, that maximizes this worker’s surplus. Because $q^e_H > q^e$, in this case the firm supplies a greater than constrained-efficient quantity of the perk. Using again the surplus-extraction intuition, when the workers’ reservation utilities and valuations for the perk are correlated but worker H’s valuation of the perk is not too high, then worker L receives an information rent, equal to $[\bar{U}_H(p^o) - u_H(q^*)] - [\bar{U}_L(p^o) - u_L(q^*)] = \Delta \bar{U}(p^o) - \Delta u(q^*) > 0$. Because this information rent decreases in $q^*$, the firm has an incentive to over-supply the perk, which is described by part (ii) in the proposition.

Finally, in the intermediate case where satisfying one type’s participation constraint does not
automatically imply that the other type’s participation constraint is satisfied, the quantity of the perk is intermediate, between \(q_L^e\) and \(q_H^e\). However, constrained efficiency can only be obtained in the knife-edge case in which the workers’ valuations and reservation utilities happen to be such that neither of them receives an information rent, that is, \(\Delta \bar{U}(p^o) - \Delta u(q^e) = 0\).

4.2. The effects of an outside market

The outside market affects the firm’s optimization problem in two ways. First, the firm faces an additional constraint, \(p^* \leq p^o\), reflecting the fact that it has to offer the perk at a price which is no greater than its outside price. Since we can always set \(p^* = 0\), this constraint can be ignored here. The second effect of the outside market is through the workers’ participation constraints. If hired, the worker consumes at least the amount of the perk that he receives from the firm, \(q\), but possibly more, if he chooses to purchase an additional amount, \(q^o - q\), in the outside market. Thus, recalling the notation \(q^\text{max}_i \equiv \max\{q^o_i, q\}\), the participation constraints are now written as

\[
U_i(s, q, p^o) \geq \bar{U}_i(p^o). \tag{PC_i''}
\]

Here, \(\bar{U}_i(p^o) = \bar{s}_i + u_i(q^o_i) - p^o q^o_i\), \(i = L, H\), and

\[
U_i(s, q, p^o) = s + u_i(q^\text{max}_i) - p^o (q^\text{max}_i - q) \geq \bar{s}_i + u_i(q^o_i) - p^o q^o_i
\]

is worker \(i\)’s utility if hired by the firm.

The effect of the outside market on the firm’s decision whether to supply the perk and on the optimal quantity of the perk is characterized in the following proposition.

\[\text{To reduce the number of cases that need to be considered, we are assuming that } p^o \text{ is sufficiently low so that both types buy the perk on the outside market if not employed by the firm.}\]
Proposition 2.

(i) If the firm is constrained to offer a uniform quantity contract, it will provide the perk if and only if \( c \leq p^o \).

(ii) Holding \( c \) constant, \( q^* \) weakly decreases in \( p^o \). For \( c \in (c^*, c^{**}) \), \( q^* \) strictly decreases in \( p^o \).

(iii) If \( \Delta \bar{U}(p^o) \leq 0 \) and \( c \leq p^o \), then in equilibrium the type \( H \) worker receives the perk from the firm but also buys an additional amount on the outside market.

An interesting aspect of the comparative static result of part (ii) in Proposition 2 is that the quantity of the perk that the firm provides declines with the outside price even if the marginal cost to the firm of procuring the perk remains fixed. The economic reasoning is that the higher is the outside price, the more additional utility the workers get from an extra unit of the perk. Hence, all else equal, a smaller quantity of the perk is needed to satisfy the workers’ participation constraints.

4.3. Discussion and implications

Propositions 1 and 2 suggest several implications that distinguish our model from existing theories of employee benefits.

First, an intriguing implication of Proposition 1 (which will be confirmed in the two settings considered subsequently) is that it offers an alternative view of the motives that lead some firms to provide excessive perks to their executives. In both the academic literature and the popular press, executive perks, such as plush offices, lavish retirement packages, corporate jets, and so on, are frequently considered to be excessive compared to the firms’ profit-maximizing levels. Following Jensen and Meckling’s (1976) seminal paper, the academic literature views the excessive level of managerial perks as a demonstration of agency problems that accompany the separation of ownership and control. While we certainly believe that in some cases managers misuse their positions
to extract lavish perks, our model offers an alternative interpretation of the problem. Part (ii) in Proposition 1 implies that it may in fact be in a firm’s best interest to provide a greater than the efficient amount of managerial perks. Thus, in our framework, excessive provision of perks is a deliberate profit-maximizing strategy designed by the firm to minimize the cost of attracting a manager. This meshes well with a recent study by Rajan and Wulf (2006), who test the empirical implications of existing theories of managerial perks and conclude that agency theory alone cannot explain the observed patterns in perk consumption by companies’ executive officers. Moreover, our model offers a potentially testable implication that distinguishes our theory from the standard agency theory: The perks that are apparently over-supplied should have relatively higher marginal costs than those that seem to be provided at efficient (or lower than efficient) levels.

Second, part (ii) in Proposition 2 implies that, controlling for the perk’s marginal cost, we should observe the amount of the perk to be negatively correlated with the perk’s cost in the outside market. For example, we would expect the firm to provide the perk in a smaller amount when the perk is available on the outside market than when it is not.

The last prediction of the uniform quantity model follows from part (iii) in Proposition 2. This result says that it should not be surprising for workers to buy quantities of a perk good provided by their employer on the outside market. For example, free tuition for university employees and their relatives rarely covers both college and graduate studies. The employees and their relatives who pursue both an undergraduate and a graduate degree purchase one or the other from the outside market. This prediction is not readily generated by the standard cost advantage theory.

5. Case (c): The Firm Offers a Contract with a Uniform Price to all Workers

We now consider the setting where the workers are offered the same price for the perk, but they can buy as much of the perk as they wish. The proof of Lemma 1 implies that this case is equivalent
to one in which the firm offers a uniform salary and pricing of the perk to all job candidates. The uniform price contract is very common in real world situations.

When \( s_H = s_L = s, p_H = p_L = p \), and \( Q = \{ \mathbb{R}_+ \} \), the firm’s problem is

\[
\max_{\{p,s\}} \sum_{i=H}^L \pi_i [y_i - s + pq_i(p) - cq_i(p)]
\]

subject to

\[
\begin{align*}
    u_i'(q_i(p)) &= p, \quad i = H, L, \quad (UM_i) \\
    U_i(p, s) &\geq \tilde{U_i}(p^o), \quad (PC_i) \\
    p^* &\leq p^o, \quad (OM)
\end{align*}
\]

where \( U_i(p, s) \equiv s - pq_i(p) + u_i(q_i(p)) \) is worker \( i \)'s utility from accepting a contract \((p, s)\).

As before, we first characterize the optimal contract under the assumption that the perk is not available in the outside market. Thus, for now, \( \tilde{U_i}(p^o) \equiv \bar{s}_i \) and we don’t have to worry about the outside market constraint (OM). In this case, efficiency requires that the price is chosen so as to

\[
\max_p \sum_{i=H}^L \pi_i [y_i + u_i(q_i(p)) - cq_i(p)],
\]

from which, using \( p = u_i'(q) \), the efficient price is \( p^e = c \). This price induces the first-best quantities \( q_i^e, i = H, L \), which represent the appropriate benchmark in this case.

Let \( \Delta U(p) \equiv U_H(p, s) - U_L(p, s) \). We have:

**Proposition 3.** Suppose the firm offers a uniform price contract, where each worker can choose his preferred quantity of the perk. Suppose also the perk is not available on the outside market.

When \( \lim_{c \to 0} \Delta U(p = c) > \Delta \tilde{U}(p^o) \), there exists a \( c^+ \in [0, \infty) \), with \( c^+ > 0 \) if \( \Delta \tilde{U}(p^o) > 0 \),
such that the firm’s optimal pricing and allocation of the perk are given by (i)-(iii) below:

(i) If $c < c^+$, then $p^* > p^e = c$ and $q^*_i < q^e_i$, $i = H, L$, that is, the price is above marginal cost and the firm under-supplies the perk.

(ii) If $c > c^+$, then $p^* < p^e = c$ and $q^*_i > q^e_i$, $i = H, L$, that is, the price is below marginal cost and the firm over-supplies the perk.

(iii) If $c = c^+$, the firm charges the efficient price $p^e = c$ and the workers choose the efficient quantities $q^*_H = q^e_H$ and $q^*_L = q^e_L$.

If $\lim_{c \rightarrow 0} \Delta U(p = c) \leq \Delta \bar{U}(p^o)$, then $c^+ = 0$, so that part (ii) above applies for all $c > 0$.

The results of Proposition 3 have the same flavor, and are driven by similar economic forces, as the standard results on two-part tariff pricing by a monopolist facing heterogeneous consumers. The optimal two-part tariff for heterogeneous consumers sets the per unit price of the good above its marginal cost, which allows the monopolist to better extract consumer surplus from the high demand consumers. This is exactly what drives our result in part (i) of the proposition. When the perk’s marginal cost is low, so that at the efficient price the workers purchase relatively high quantities of the good, the contract gives much more utility to the high valuation type $H$ than to the low valuation type $L$. Thus, if their reservation utilities do not differ that much, so that $\Delta U(p^e) > \Delta \bar{U}(p^o)$, the high type receives an information rent if the perk is offered at the efficient price equal to its marginal cost. By increasing the price, the firm can extract part of this surplus through the profit it makes on the relatively large quantity purchased by the high valuation worker.

The converse logic applies if the high valuation type’s utility from the efficient quantity is not sufficiently high to compensate the worker for his reservation utility, i.e., if $\Delta U(p^e) < \Delta \bar{U}(p^o)$. This case applies when $c$ is large, because then the efficient quantities for both workers are small, and so is the extra surplus the high type derives from efficient perk consumption. In this case,
the firm needs to increase the high type’s utility. The cheapest way of doing so is through slightly
decreasing both the price of the perk and the salaries. Since the high type values the perk more
than the low type, while both value money equally, this adjustment in the contract gives relatively
more utility to the high type, thus decreasing the information rent obtained by the low type.

5.1. The effects of an outside market

In the present setting, the outside market constraint affects the optimal contract in two ways: (i)
through the workers’ reservation utilities and (ii) through the outside market constraint \( p \leq p^o \).
Denote the optimal price in this case as \( p^{**} \).

Proposition 4. If the firm offers a uniform price contract, the effects of the outside market are
categorized as follows:

(a) The cutoff level \( c^+ \) increases in the outside price \( p^o \), i.e. the lower is \( p^o \), the more likely it is
that \( p^* < c \).

(b) For a given \( c \), there exists a \( \hat{p}^o(c) \) such that the perk is provided if and only if \( p^o \geq \hat{p}^o(c) \). If
\( c > c^+ \), then \( \hat{p}^o(c) = p^* < c \); if \( c \leq c^+ \), then \( \hat{p}^o(c) = c \).

(c) If \( p^o < p^* \) and the perk is provided, then \( p^{**} = p^o \) and hence decreases in \( p^o \).

(d) If \( p^o \geq p^* \), then \( p^{**} = p^* \) and \( p^{**} \) is not affected by the outside market.

Part (b) in Proposition 4 is one of our key results; it demonstrates that firms may have an
incentive to provide perks even if their cost of doing so is strictly higher than the price at which
the workers could obtain the perk in the outside market. This result is driven by the fact that
bundling cash and non-cash compensation allows a firm to save on its total wage bill by decreasing
the workers’ information rents, and this wage bill saving may be high enough to offset the firm’s
relatively high cost of providing the perk.
5.2. Discussion and implications

Proposition 3 shows that when workers have control over the quantities of the perk they consume, the firm will tend to deviate to a price below marginal cost for high marginal cost perks and to a price above marginal cost for low marginal cost perks. This result, together with Proposition 4, suggests three potentially testable implications that are novel to our theory:

The first implication is that, in accord with the analysis of the previous case of uniform quantities, below marginal cost pricing and excessive managerial perks should be more likely observed when the perk’s marginal cost is relatively high than when its marginal cost is relatively low.

Second, part (a) in Proposition 4 says that the larger is the firm’s cost advantage in providing the perk, the more likely is the firm to provide the perk to its employees at an above marginal cost price. Conversely, the smaller is the firm’s cost advantage, the more likely is the firm to provide the perk at a price below the perk’s marginal cost. Thus, a study that would find a positive relationship between $p^{**}$ and $p^o$, while controlling for the firm’s marginal cost $c$, would support our theory.

Finally, we have argued earlier that we view our theory as applying primarily to perks offered to workers in high skill jobs. As mentioned in the Introduction, this fits well with the popular belief, voiced in the business press, that firms provide perks to attract high quality workers. An implication of this view is that deviations from marginal cost pricing should be more likely for perks offered to high skill employees than for perks offered to low skill employees. Moreover, the latter should be less likely offered the perk in the first place. For example, lower level employees should in general get different employee discounts and differently priced company loans and pension plans than senior managers. This is because the relative abundance of low skill workers, combined with the low sensitivity of these workers’ productivities to their skill levels, allows a firm to find workers with relatively homogeneous preferences for the perk. In such a case, we would expect the standard theory to apply, and the perks to be offered to low skilled workers at marginal cost.
(or not at all, if the firm has no cost advantage in providing the perk). In contrast, if high skill workers (say, senior managers) are in relatively short supply, it could be costly for a firm to screen these workers according to their preferences for perks.\(^\text{12}\) It is therefore likely that firms end up with senior managers who differ in their valuations for the perk. In this case, our theory says that deviation from marginal cost pricing may be optimal, as may be providing the perk even in the absence of a cost advantage.

Extending the above argument, our model can also shed some light on the "good jobs/bad jobs" debate. Many researchers have observed that the economy seems to consist of two types of jobs, "good" and "bad", where the good jobs, filled typically with high skilled workers, offer both higher monetary compensation and more benefits (see, e.g. Doeringer and Piore, 1971, for an early formulation of this view). Previous theoretical explanations offered for this phenomenon have built either upon the efficiency wage theory (Bulow and Summers, 1986) or on the search theory (e.g., Acemoglu, 2001). Our theory offers an alternative explanation: The high skill jobs in our model are also good jobs in the above sense; they not only offer high wages (reflecting high worker skills), but also generous benefits, because firms attach to these jobs even some perks that would not be offered under full information. In contrast, low skill jobs are bad jobs, because, in addition to low wages, they offer relatively few benefits—only those benefits are offered for which the firm has a cost advantage over the outside market.

6. Case (b): The Firm Can Offer a Menu of Quantities

In this section, we briefly examine case (b) in Lemma 1, where the firm can offer a full menu of contracts. This last scenario is captured by the optimization problem given by (MAX) under the assumption that the firm has full control over the quantities consumed by the workers, with

\(^{12}\)For example, it could make sense for an airline company to select its flight attendants based on whether they like to travel, but it would be less sensible to put much weight on this criterion if the same company were hiring a CEO.
no additional constraints of the form described by (1). That is, the firm can design a different compensation package \((s_i, q_i, p_i)\) for each type of worker. The contract that solves this problem places an upper bound on how well the firm can do when faced with asymmetric information about the worker’s type. Examples of this type of contract include cases where the firm offers employees the option of, say, less salary and more of a benefit, or conversely. Contract workers, for example, trade off benefits for a higher salary. Professional workers wanting more flexible hours and infrequent travel might self select into a contract in which salary is less as opposed to a career type contract where inflexible hours and frequent travel are requirements.

We again start by describing the firm’s optimal contract when the perk is not available in the outside market. Denote the firm’s profit-maximizing values as \(q_i^*, p_i^*, s_i^*\). As in the case of the uniform quantity contract, the optimal prices \(p_i^*\) are indeterminate here. The following proposition is driven by similar economic forces as Proposition 3.

**Proposition 5.** Suppose that the firm can offer a menu of quantity contracts \((s_i, q_i, p_i)\) and the perk is not available on the outside market. Then \(q_H^* \geq q_H^e\) and \(q_L^* \leq q_L^e\). More specifically, when \(\lim_{q \to \infty} \Delta u(q) > \Delta \bar{U}(p^0) > 0\), there exist \(c'' > c' > 0\) such that:

(i) If \(c < c'\), then \(q_H^* = q_H^e\) and \(q_L^* < q_L^e\).

(ii) If \(c > c''\), then \(q_H^* > q_H^e\) and \(q_L^* = q_L^e\).

(iii) If \(c \in [c', c'']\), then \(q_i^* = q_i^e\), \(i = H, L\).

When \(\Delta \bar{U}(p^0) \leq 0\), then (i) holds for all \(c\). When \(\lim_{q \to \infty} \Delta u(q) < \Delta \bar{U}(p^0)\), then (ii) holds for all \(c\).

As in the two uniform contract scenarios, bundling cash and in-kind compensation allows the firm to increase its profits by extracting information rents from its workers. Not surprisingly, the

\[\text{An example is a "mommy track" position at a professional firm. Here the term mommy is gender neutral.}\]
firm can do this more effectively if it faces fewer restrictions on the set of contracts it can offer. Proposition 5 confirms this intuition by showing that the ability to offer a menu of contracts allows the firm to supply the perk in an efficient manner for a range of parameter values, which was not optimal in the previous two scenarios. Nevertheless, the qualitative results obtained in Propositions 1 and 3 continue to hold in the present setting, as there are still parameter values such that the perk is under-supplied or over-supplied.

Thus, the main new prediction that emerges in the present setting is that the firm will tend to design its menu of contracts so as to induce high valuation workers to self select into a contract that tends to over-supply the perk or to induce low valuation workers to self-select into a contract that tends to under-supply the perk. For example, in firms that allow their employees to choose more flexible hours in exchange for lower salary, we would expect the flexible position to offer too much flexibility (compared to the efficient level) or the regular position to allow for too little flexibility.

6.1. The effects of an outside market

When the firm offers a menu of contracts, the effects of the outside market are described in the following proposition.

Proposition 6.

(a) If the firm offers a menu of quantity and salary contracts, the cutoff levels $c'$ and $c''$ both increase in the outside price $p^o$. That is, a higher $p^o$ makes it more likely that the firm under-supplies the perk to the $L$-type worker and less likely that it over-supplies the perk to the $H$-type worker.

(b) If $c > c^+$, there exists a $\tilde{p}^o(c) < c$ such that the perk is provided to at least one type of worker for all $p^o \geq \tilde{p}^o(c)$.
The above results confirm the main insights of Proposition 4. As before, the smaller is the firm’s cost advantage in providing the perk, the more likely it will over-supply it. Moreover, under some parameter values the perk is provided to at least one type of worker where it would be more socially efficient not to provide it at all.

7. Conclusion

In this paper, we have argued that the standard conclusion that a firm should price a perk at its marginal cost, which appears to be violated in many real world situations, does not apply to a firm that hires workers with private information regarding their heterogeneous preferences for the perk and their heterogeneous outside opportunities. We have developed a theory of perks in which the firm bundles perks with cash compensation in order to extract information rents from workers with private information. This allows the firm to attract workers at a lower cost. The novel implication of our theory is that the firm might find it optimal to provide a perk even if it cannot do so more effectively than the outside market. This never happens when the firm is constrained to offer a uniform quantity contract, but is a distinct possibility both in the case of a menu of quantity contracts and in the case of a uniform price contract.

In addition, our model offers potentially testable implications regarding the prices at which firms will offer the perk to their employees and regarding the quantities of the perk they will provide. In particular, we show that the firm will tend to over-supply the perk when the perk has a relatively high marginal cost and to under-supply it when it has a relatively low marginal cost. Moreover, if the firm is offering a bundle with a uniform price for the perk across workers, it will set the uniform price below the marginal cost price if marginal cost of the perk is high and conversely if the marginal cost is low (unless constrained by the price of the perk in the outside market).

Finally, we have argued that our theory should apply especially to perks associated with high
skill jobs (e.g., senior managers): When a job requires no special skills, the firm might find it
optimal to select its workers based on their preferences for the benefits it offers. On the other
hand, if the required skills are relatively rare, the firm will probably hire the suitable candidate
regardless of his preference for the perk. Low skill positions will then tend to be filled with workers
of homogeneous tastes for the firm’s perk, while high skill positions will likely be populated by
workers with heterogenous preferences for the perk, as in our model. Our theory then predicts
that high skill jobs should be "good jobs", offering not only higher wages but also more benefits,
because the firms might find it optimal to provide even benefits that would be more efficient for the
workers to obtain from the outside market. Since such benefits are not valuable to the firms when
they do not have to worry about skills and can select employees based on their tastes for benefits,
low skill positions should be associated not only with lower wages but also with fewer benefits.

Appendix

Proof of Lemma 1: (i) Suppose that $Q = \{S : S \subset \mathbb{R}_+\}$. We first show that the firm will find it
optimal to limit $Q_i$ to a single quantity, $q_i$, i.e. $Q_i = \{q_i\}$. Assume to the contrary that $Q_i$ contains
at least two elements, $q_1 \neq q_2$. Suppose further, without loss of generality, that $q_1$ solves the firm’s
optimization problem. Then eliminating $q_2$ from $Q_i$ does not affect the workers’ (PC$_i$) constraints
and makes it easier to satisfy for $q_1$ the (UM$_i$) and (IC$_i$) constraints. Consequently, the firm must
be at least weakly better off if $q_2$ is eliminated from $Q_i$.

Given that $Q_i = \{q_i\}$ in this case, prices are irrelevant (as the firm can force the worker
to consume $q_i$ if he accepts the contract) and we can set $p_i = 0, i = L, H$, so that the firm only
optimizes over $(s_i, q_i)$. We thus need to consider four cases: (1) $s_i$ and $q_i$ are subject to no additional
constraints, (2) $s_i$ are unconstrained but $q_H = q_L = q$, (3) $q_i$ are unconstrained but $s_H = s_L = s$,
and (4) $q_H = q_L = q$ and $s_H = s_L = s$. 

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Case (1) is equivalent to the problem in (MAX). Cases (2) and (3), on the other hand, are both equivalent to case (4). To see this, look at case (2) first and suppose that $s_i > s_j$, $i \neq j$. In this case, the contract $(s_i, q)$ is strictly preferred by both types to the contract $(s_j, q)$. Therefore, it is not possible to satisfy the two incentive compatibility constraints ($IC_H$) and ($IC_L$) simultaneously. Hence, it must be $s_i = s_j$, which makes the firm’s problem equivalent to that in case (iv). The reasoning for the equivalence between cases (3) and (4) is analogous. This proves that the only two relevant cases to consider are case (1), which is equivalent to (MAX) and yields part (a) in the lemma, and case (4), which yields part (b).

(ii) When $Q = \{R_+\}$ then also $Q_i = Q_j = \{R_+\}$, i.e., the firm can only affect $q_i$ through the prices $p_L$ and $p_H$, which uniquely determine the quantities through the workers’ demands, $u'_i(q_i(p)) = p$. Thus, in this case the firm optimizes over $(s_i, p_i)$ and we again need to consider four cases: (1) $s_i$ and $p_i$ are subject to no additional constraints, (2) $s_i$ are unconstrained but $p_H = p_L = p$, (3) $p_i$ are unconstrained but $s_H = s_L = s$, and (4) $p_H = p_L = p$ and $s_H = s_L = s$. The same logic as in part (i) (with $q_i$ being replaced by $p_i$) shows that (2) and (3) are equivalent to (4), so that the only two relevant cases to consider are case (1), which is equivalent to (MAX) and yields part (c) in the lemma, and case (4), which yields part (d).

**Proof of Proposition 1:** Because the firm can control the quantity, $q$, we can, without loss of generality, set $p = 0$ in solving for the optimal contract, as we have already mentioned in the text. To see this, suppose that a contract $(s, q, p)$ is optimal, where $p > 0$. This contract gives the worker utility equal to $s - pq + u_i(q)$. But this is equivalent to a contract $(s', q, p')$, where $p' = 0$ and $s' = s - pq$.

We first prove the following claim:

(i) If $\Delta \bar{U}(p^0) \leq \Delta u(q^0_L)$, then $q^* = q^0_L < q^c$. 

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(ii) If $\Delta \bar{U}(p^o) \geq \Delta u(q^*_H)$, then $q^* = q^*_H > q^e$.

(iii) If $\Delta \bar{U}(p^o) \in (\Delta u(q^*_L), \Delta u(q^*_H))$, then $q^* = \hat{q} \in (q^*_L, q^*_H)$.

Let $\lambda_i$ be the Lagrange multiplier associated with the participation constraint (PC$_i$). The firm’s problem is then given by the Lagrangian

$$
\max_{\{s,q\}} -s - cq + \sum_{i=H}^{L} \lambda_i(s + u_i(q) - \bar{U}_i(p^o)),
$$

which yields the first order conditions:

\begin{align*}
\lambda_H + \lambda_L &= 1, \quad (a.1) \\
\lambda_H u'_H(q) + \lambda_L u'_L(q) &= c. \quad (a.2)
\end{align*}

Suppose first that $\lambda_H = 0$ and $\lambda_L > 0$. Then (a.1) and (a.2) imply $\lambda_L = 1$ and $u'_L(q^*) = c$, so that $q^* = q^*_L$. This is the solution to the firm’s problem if it also satisfies the two (PC$_i$) constraints. Since $\lambda_L > 0$, $s$ must be chosen such that (PC$_L$) binds when $q = q^*_L$. (PC$_H$) then holds if and only if $\Delta u(q^*_L) \geq \Delta \bar{U}(p^o)$, which is the condition in part (i) of the proposition. To conclude the proof of this part, note that $q^*_L < q^e$ because $u'_L(q) < \pi_H u'_H(q) + \pi_L u'_L(q)$ for any $q > 0$ and $\pi_H > 0$.

Next suppose that $\lambda_H > 0$ and $\lambda_L = 0$. Then (1) and (2) imply $\lambda_H = 1$ and $u'_H(q^*) = c$, so that $q^* = q^*_H$. Since $\lambda_H > 0$, $s$ must be chosen such that (PC$_H$) holds with equality when $q = q^*_H$. (PC$_L$) then holds if and only if $\Delta u(q^*_H) \leq \Delta \bar{U}(p^o)$, which is the condition in part (ii) of the proposition. Also, $q^*_H > q^e$ because $u'_H(q) > \pi_H u'_H(q) + \pi_L u'_L(q)$ for any $q > 0$ and $\pi_H < 1$.

Finally, if $\Delta u(q^*_H) > \Delta \bar{U}(p^o) > \Delta u(q^*_L)$, then neither of the above two cases applies, so that it must be $\lambda_H > 0$ and $\lambda_L > 0$. Therefore, both (PC$_i$) constraints must be binding, which implies $\Delta u(q^*) = \Delta \bar{U}(p^o)$, so that $q^* = \hat{q}$. Since $\lambda_H > 0$, $\lambda_L > 0$, and $\lambda_H + \lambda_L = 1$, we have $\lambda_H, \lambda_L \in (0,1)$, so that $u'_H(q) > \lambda_H u'_H(q) + \lambda_L u'_L(q) > u'_L(q)$ for any $q > 0$. This means that $\hat{q} \in (q^*_H, q^*_L)$.
Now, to translate the above claim into the claims in the proposition, note that $\Delta \bar{U}(p^o)$ is unaffected by $c$, but $\Delta u(q^o_L)$ decreases in $c$ (because $q^o_L$ goes down), with $\lim_{c \to \infty} \Delta u(q^o_L) = 0$. Hence, for any $\Delta \bar{U}(p^o) > 0$ there must exist $c^*, c^{**} \in [0, \infty)$, $c^{**} > c^*$, such that $\Delta u(q^o_L) \geq \Delta \bar{U}(p^o)$ iff $c \leq c^*$ and $\Delta u(q^o_H) \leq \Delta \bar{U}(p^o)$ iff $c \geq c^{**}$. This proves (i)-(iii) in the proposition. If $\Delta \bar{U}(p^o) \leq 0$ then it must always be $\Delta \bar{U}(p^o) \leq \Delta u(q^o_L)$, so that (i) in the proposition applies for all $c$. Finally, when $\lim_{q \to \infty} \Delta u(q) \leq \Delta \bar{U}(p^o)$ then $\Delta \bar{U}(p^o) \geq \Delta u(q^o_H)$ always, which proves the last claim in the proposition. ■

**Proof of Proposition 2:** Instead of using the Lagrangian method to characterize the firm’s optimal solution as in the proof of Proposition 1, we offer here a proof that is less formal but more intuitive. Analogous to (3), define $\hat{q}(p^o)$ as the quantity such that $\Delta U(\hat{q}) = \Delta \bar{U}$, where $\Delta U(q) \equiv U_H(s, \hat{q}, p^o) - U_L(s, \hat{q}, p^o)$ and $\Delta \bar{U}(p^o) \equiv \bar{U}_H(\bar{s}_H, p^o) - \bar{U}_L(\bar{s}_L, p^o)$.

The firm’s problem is to satisfy the both workers’ participation constraints ($\text{PC}_i$”) at the lowest possible cost. If a worker $i$’s participation constraint does not hold, the firm can increase his utility $U_i(s, q, p^o)$ in two ways. The first is to increase the workers’ salaries, $s$, which increases their utility dollar by dollar. The second possibility is to increase the quantity of the perk. This costs the firm $c$ per unit and increases the worker’s utility by

$$\frac{\partial U_i(s, q, p^o)}{\partial q} = p^o \quad \text{if } q < q^o_i, \text{ i.e. } q_i^{\text{max}} = q^o_i,$$

and by

$$\frac{\partial U_i(s, q, p^o)}{\partial q} = u'_i(q) \quad \text{if } q \geq q^o_i, \text{ i.e. } q_i^{\text{max}} = q.$$

Thus, if $q < q^o_i$ and $p^o \geq c$, the firm will find it optimal to increase the amount of the perk rather than increasing salaries. If $q < q^o_i$ and $p^o < c$, then the firm wants to decrease $q$ and increase salaries instead. This implies that $q^* \geq q^o_i$ if $p^o \geq c$ and $q^* = 0$ or $q^* \geq q^o_i$ if $p^o < c$. 

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Thus, suppose \( q \geq q_o^c \). Then the firm increases the amount of the perk instead of increasing salaries iff \( u'_i(q) \geq c \). Suppose first that \( p^0 < c \). Since \( q \geq q_o^c \), it must be that \( u'_i(q) \leq p^0 < c \). Hence, the firm wants to decrease \( q \) in this case and we can conclude that \( q^* = 0 \) if \( p^0 < c \).

If \( p^0 \geq c \), then the fact that firm increases the amount of the perk if \( u'_i(q) \geq c \) means that \( q^* \geq q_L^c \), because \( q_L^c \) is given by \( u'_i(q_L^c) = c \) and \( q_L^c < q_H^c \). If both participation constraints are satisfied at \( q = q_L^c \), the firm stops there and \( q^* = q_L^c \). On the other hand, if type \( H \)'s participation constraint does not hold at \( q = q_L^c \), the firm will find it optimal to give this worker additional utility by increasing \( q \) further, because \( u'_i(q_L^c) > c \). The firm will stop increasing \( q \) when \( u'_i(q) \geq c \) stops holding, i.e. when \( q = q_H^c \), or when \( q = \hat{q} \) (in which case both participation constraints are satisfied), whichever comes first.

To sum up, when \( c \leq p^0 \) we have the following result, which extends Proposition 1 to the present setting:

(i) \( q^* = q_L^c \) if both participation constraints hold at \( q = q_L^c \) when \( s \) is such that \( (PC_L) \) binds, i.e.,

\[
\Delta U(q_L^c) - \Delta \bar{U}(p^0) \geq 0.
\]

(ii) \( q^* = q_H^c \) if type \( H \)'s participation constraint does not hold at \( q = q_L^c \) when \( s \) is such that \( (PC_L) \) binds, i.e. if \( \Delta U(q_L^c) - \Delta \bar{U}(p^0) \leq 0 \).

(iii) \( q^* = \hat{q} \in (q_L^c, q_H^c) \), otherwise, i.e. if \( \Delta \bar{U}(p^0) \in (\Delta U(q_L^c), \Delta U(q_H^c)) \).

We can now characterize the effect of \( p^0 \) on \( q^* \). For this, we need to determine how \( p^0 \) affects \( \hat{q}(p^0) \) and the conditions \( \Delta U(q_L^c) - \Delta \bar{U}(p^0) \geq 0 \) and \( \Delta U(q_H^c) - \Delta \bar{U}(p^0) \leq 0 \).

We start with the two conditions. Define \( A(q) \equiv \Delta U(q) - \Delta \bar{U}(p^0) \), and note that \( p^0 \geq c \) implies that \( q^* \geq q_L^c \geq q_H^c \), so that \( q_L^{\text{max}} = q \) always. Thus,

\[
A(q) = u_H(q_H^{\text{max}}) - u_L(q) - p^0(q_H^{\text{max}} - q) - (\bar{s}_H - \bar{s}_L) - [u_H(q_H^0) - u_L(q_L^0)] + p^0(q_H^0 - q_L^c).
\]
When \( q = q^e_L \), we have to consider two cases. In the first, \( q^e_L \leq q^o_H \), so that \( q^\text{max}_H = q^o_H \). In this case,

\[
A(q^e_L) = u_L(q^o_L) - u_L(q^e_L) + p^o(q^e_L - q^o_L) - (\bar{s}_H - \bar{s}_L).
\]

Differentiating with respect to \( p^o \), we get

\[
\frac{\partial A(q^e_L)}{\partial p^o} = \frac{\partial q^o_H}{\partial p^o} [u'_L(q^o_L) - p^o] + (q^e_L - q^o_L) = q^e_L - q^o_L > 0.
\]

In the second case, \( q^e_L > q^o_H \), so that \( q^\text{max}_H = q^e_L \) and \( A(q^e_L) \) reduces to

\[
A(q^e_L) = u_H(q^o_H) - u_L(q^e_L) - (\bar{s}_H - \bar{s}_L) - [u_H(q^o_H) - u_L(q^o_L)] + p^o(q^o_H - q^o_L),
\]

so that

\[
\frac{\partial A(q^e_L)}{\partial p^o} = -\frac{\partial q^o_H}{\partial p^o} u'_H(q^o_H) + \frac{\partial q^o_L}{\partial p^o} u'_L(q^o_L) + \frac{\partial q^o_H}{\partial p^o} p^o - \frac{\partial q^o_L}{\partial p^o} p^o + (q^o_H - q^o_L) = q^e_L - q^o_L > 0.
\]

Thus, \( A(q^e_L) \) always increases as \( p^o \) goes up, which means that the condition \( \Delta U(q^e_L) - \Delta \bar{U}(p^o) \geq 0 \) is more likely to hold. Thus, \( q^* \) is more likely to be at its minimum level \( q^* = q^e_L \), which means that an increase in \( p^o \) tends to weakly decrease \( q^* \) in this case.

Next, let \( q = q^e_H \). Since \( p^o \geq c \), it must be \( q^e_H \geq q^o_H \), so that \( q^\text{max}_H = q^e_H \) and \( A(q^e_H) \) becomes

\[
A(q^e_H) = u_H(q^e_H) - u_L(q^e_H) - (\bar{s}_H - \bar{s}_L) - [u_H(q^e_H) - u_L(q^e_H)] + p^o(q^e_H - q^o_L).
\]

Differentiating with respect to \( p^o \) then yields

\[
\frac{\partial A(q^e_H)}{\partial p^o} = q^o_H - q^e_L > 0.
\]
Hence, an increase in \( p^o \) makes the condition \( \Delta U(q_H^e) - \Delta \bar{U}(p^o) \leq 0 \) less likely to hold, which again weakly decreases \( q^* \).

Finally, consider the effect of \( p^o \) on \( \hat{q}(p^o) \), which is given by \( A(\hat{q}) = 0 \). We again have to consider two cases: \( q_H^{\max} = q_H^0 \) and \( q_H^{\max} = \hat{q} \). It is straightforward to check that in both cases \( \frac{\partial A(\hat{q})}{\partial p^o} > 0 \) and \( \frac{\partial A(\hat{q})}{\partial \hat{q}} > 0 \). Thus, it must be that \( \hat{q} \) strictly decreases in \( p^o \), which makes \( q^* \) strictly decreasing in \( p^o \) for this range of parameter values.

**Proof of Proposition 3:** Note first that if \( \Delta U(p) > \Delta \bar{U}(p^o) \) for a given \( p \), then (PC\(_{L} \)) implies (PC\(_{H} \)) and (PC\(_{H} \)) is slack. Similarly, if the reverse inequality holds, then (PC\(_{H} \)) implies (PC\(_{L} \)) and (PC\(_{L} \)) is slack. Note also that \( \Delta U(p) \geq 0 \) and \( \frac{\partial \Delta U(p)}{\partial p} < 0 \), where the latter inequality holds because \( u_H^0 > u_L^0 \). Moreover, we have \( \frac{\partial \Delta U(p=c)}{\partial c} < 0 \).

Suppose that \( \lim_{c \to 0} \Delta U(p = c) > \Delta \bar{U}(p^o) \). Then one can define implicitly a \( c^+ \in (0, \infty) \) by \( \Delta U(p = c^+) = \Delta \bar{U}(p^o) \). Assume first that \( c < c^+ \), so that \( \Delta U(p = c) > \Delta \bar{U}(p^o) \), and suppose that \( p^* \leq c \). Then \( \Delta U(p^*) \geq \Delta U(p = c) > \Delta \bar{U}(p^o) \). Hence, at firm’s optimum, (PC\(_{L} \)) must be binding and (PC\(_{H} \)) is slack. Substituting from (PC\(_{L} \)), the firm’s objective function can therefore be written as

\[
\pi_H[y_H - pq_L(p) - \bar{U}_L(p^o) + pq_H(p) - cq_H(p)] + \pi_L[y_L + u_L(q_L) - \bar{U}_L(p^o) - cq_L(p)].
\]

This means that in the firm’s optimum, the following first order condition must hold:

\[
\pi_H[q_H(p^*) - q_L(p^*) + q_L^0(p^*) (u_L^0(q_L) - p^*)] + \sum_{i=H}^{L} \pi_i[p^* - c'_i(q_i(p^*), c)]q_i'(p^*) = 0. \tag{a.6}
\]
Using \( u'_L(q_L) - p^* = 0 \) and \( q_H(p^*) - q_L(p^*) > 0 \), the above condition implies

\[
\sum_{i=H}^{L} \pi_i(p^* - c)q_i^r(p^*) < 0. \tag{a.7}
\]

This immediately yields \( p^* > c \), which contradicts our initial assumption that \( p^* \leq c \). Hence, when \( c < c^+ \) it must be that \( p^* > c \), which proves the first claim in part (i). The second claim of part (i) follows in a straightforward way.

The proof of the first claim in part (ii) follows the same steps, demonstrating a contradiction in the conjecture that \( p^* \geq p^e \) when \( c > c^+ \) and showing that \( p^* \) does not depend on \( p^o \). Since the reasoning here is essentially the same as above, we omit the details. Note also that if \( \lim_{p^e \to 0} \Delta U(p^e) \leq \Delta \bar{U}(p^o) \), then \( \Delta U(p = c) \leq \Delta \bar{U}(p^o) \) for all \( c \geq 0 \). This means that part (ii) always applies, which proves the last claim in the proposition.

When \( c = c^+ \), so that \( \Delta U(p = c) = \Delta \bar{U}(p^o) \), then both \( (PC_i) \) hold with equality. After substituting into its objective function, the firm’s problem in this case becomes identical to \( (EFF) \), up to a constant \( \sum_{i=H}^{L} \pi_i \bar{U}_i(p^o) \). This proves part (iii). \( \blacksquare \)

**Proof of Proposition 4:** (a) If the (OM) constraint \( p \leq p^o \) does not bind, \( p^o \) only affects the firm’s optimization problem through the workers’ reservation utilities. In the proof of Proposition 3, \( c^+ \) is defined by \( \Delta U(p = c^+) = \Delta \bar{U}(p^o) \). Now, \( \Delta U(p) \) decreases in \( p \) and is independent of \( p^o \), while \( \Delta \bar{U}(p^o) \) decreases in \( p^o \) and is independent of \( p \). Consequently, \( c^+ \) increases in \( p^o \).

(b)-(d) Suppose that \( p^o \geq p^* \). Then the (OM) constraint does not bind, so that \( p^{**} = p^* \). The claim that \( p^* \) does not depend on \( p^o \) follows from inspection of the first order condition (a.6) (and its counterpart for \( c > c^+ \)), which is independent of \( p^o \). This proves part (d). In this case, we have an interior solution, which means that the perk is always provided, even if \( p^o < c \).

Now suppose \( p^o < p^* \). Then concavity of the firm’s problem implies that the firm maximizes
its profit either by setting $p^{**} = p^o$, because its profit must be increasing in $p$ at this point, or by not providing the perk at all. In this case $p^{**}$ clearly decreases with $p^o$, which is claim (c) in the proposition.

If the perk is provided and $p = p^o$, then the (PC$_i$) constraints reduce to $s \geq \bar{s}_i$, $i = L, H$, so that the firm’s expected value (profit) is given by

$$EV(\text{perk}) = \sum_{i=H}^{L} \pi_i y_i + (p^o - c) \sum_{i=H}^{L} \pi_i q_i^o - s^o,$$

where $s^o = \max\{\bar{s}_i\}$. If the perk is not provided, then the workers buy the perk good on the outside market, so that their (PC$_i$) constraints again reduce to $s \geq \bar{s}_i$, $i = L, H$. In this case, the firm’s expected value (profit) is given by

$$EV(\text{no perk}) = \sum_{i=H}^{L} \pi_i y_i - s^o.$$

Thus, when $p^o < p^*$, the firm will provide the perk if and only if $c \leq p^o$.

To sum up, the above arguments imply that when $c \leq c^+$, so that $p^* \geq c$ by Proposition 3, then the perk is provided if and only if $p^o \geq c$. That is, $\hat{p}^o(c) = c$ in this case. When $c > c^+$, so that $p^* < c$ by Proposition 3, the perk is provided if and only if $p^o \geq p^*$. In this case, $\hat{p}^o(c) = p^* < c$.

This concludes the proof of part (b).

**Proof of Proposition 5:** As we have argued in the text, this problem is equivalent to a formulation where the firm chooses arbitrary $s_i$ and $q_i$ but sets the prices $p_H = p_L = 0$. To see this, notice that $p_i$ affects the agent’s overall utility and the firm’s profit only through the term $t_i = s_i - p_i q_i$. Thus, for any given $p_i$ one can find an $s_i$ such that $t_i$ remains the same. The equilibrium contract in this setting, $(s^*_i, q^*_i)$, would yield the same outcome as a contract of the form $(s_i, q_i, p_i)$. 

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The firm’s problem is thus

$$\max_{\{s_H, s_L, q_H, q_L\}} \pi_H(y_H - s_H - c q_H) + \pi_L(y_L - s_L - c q_L)$$

subject to

$$s_i + u_i(q_i) - \bar{U}_i(p^o) \geq 0, \quad i = H, L, \quad (PC_i)$$

$$s_i + u_i(q_i) \geq s_j + u_i(q_j), \quad i, j = H, L. \quad (IC_i)$$

The relevant Lagrangian function for the principal is

$$L = \pi_H(y_i - s_i - c q_i) + \sum_{i=H}^L \sum_{j=H, j\neq i} \lambda_i[s_i + u_i(q_i) - s_j - u_i(q_j)]$$

After some rearranging, the first order conditions for $q_i^*$ and $s_i^*$ are given by

$$u_H'(q_H^*) = c - \frac{\mu_L}{\pi_H} \Delta u'(q_H^*), \quad (a.8)$$

$$u_L'(q_L^*) = c + \frac{\mu_H}{\pi_L} \Delta u'(q_L^*), \quad (a.9)$$

$$\pi_H = \lambda_H + \mu_H - \mu_L, \quad (a.10)$$

$$\pi_L = \lambda_L + \mu_L - \mu_H. \quad (a.11)$$

By $\mu_i \geq 0$ and Assumption 1a), (a.8) and (a.9) immediately imply $q_H^* \geq q_H^o = c$ and $q_L^* \leq q_L^o = c$, which proves the first claim in the proposition.

We now proceed in two steps. First, we characterize the efficiency properties of the optimal quantities and prices as functions of the difference in the workers’ reservation utilities, $\Delta \bar{U}(p^o)$. In the second step, we find the cutoff levels $c'$ and $c''$.

**Step 1.** In this step, we prove the following claim:

(a) If $\Delta u(q_L^o) > \Delta \bar{U}(p^o)$, then $q_H^* = q_H^o$ and $q_L^* < q_L^o$. 

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(b) If $\Delta u(q_H^c) < \Delta \bar{U}(p^o)$, then $q_H^* > q_H^c$ and $q_L^* = q_L^c$.

(c) If $\Delta \bar{U}(p^o) \in [\Delta u(q_L^c), \Delta u(q_H^c)]$, then $q_i^* = q_i^c$, $i = H, L$.

(a) Suppose that $\Delta u(q_L^c) > \Delta \bar{U}(p^o)$. First note that both IC$_i$ cannot be binding, because together they would imply $\Delta u(q_H^c) = \Delta u(q_L^c)$, which cannot hold, as $\Delta u()$ is a strictly increasing function. Next, we show that $q_L^c < q_L^*=u_0(q_L^c))$. Assume to the contrary that $q_L^c = q_L^*$, in which case $\mu_H = 0$. From (a.10), $\lambda_H - \mu_L = \pi_H$ so that $\lambda_H > 0$ and PC$_H$ is binding. From the PC$_i$ and IC$_i$ we have $U_H(p^o) = s_H + u_H(q_H^c) \geq s_L + u_H(q_L^c) \geq \bar{U}_L(p^o)$ and $s_H + u_H(q_H^c) \geq \bar{U}_L(p^o)$ and $s_L \geq \bar{U}_L(p^o) - u_L(q_L^c)$, so that $\bar{U}_i(p^o) \geq \Delta u(q_L^c)$ and we have a contradiction. Therefore, it must be that $q_L^c < q_L^*$. This implies $\mu_H > 0$, so that IC$_H$ is binding. Since both IC$_i$ cannot be binding, IC$_L$ must be nonbinding. Whence, $\mu_L = 0$ and, from (a.8), $q_H^c = q_H^c$.

(b) Consider $q_H^*$ and suppose to the contrary that $q_H^c = q_H^*$, in which case $\mu_L = 0$, from (a.8). From (a.11), $\lambda_L - \mu_H = \pi_L$, so that $\lambda_L > 0$ and PC$_L$ is binding. From the PC$_i$ and IC$_i$ we have $U_L(p^o) = s_L + u_L(q_L^c) \geq s_H + u_H(q_L^c)$ and $s_H + u_H(q_L^c) \geq \bar{U}_L(p^o)$. Therefore, $s_H \geq \bar{U}_H(p^o) - u_H(q_L^c)$ and $s_L \leq \bar{U}_L(p^o) - u_L(q_L^c)$, so that $\Delta \bar{U}(p^o) \leq \Delta u(q_L^c)$ and we have a contradiction. From (a.11), $q_H^c > q_H^*$ implies that $\mu_L > 0$, which means that IC$_L$ is binding. From the proof of (a), both IC$_i$ cannot be binding, so that IC$_H$ is nonbinding and $\mu_H = 0$. The latter implies that $q_L^* = q_L^c$.

(c) Begin by setting $q_i^* = q_i^c$ through $u_i^c(q_i^c) = c$, $i = H, L$. If we can show that, under the assumed parametric specifications, this fully efficient solution satisfies the PC$_i$ and IC$_i$ constraints, then it is optimal. To this purpose, set $s_i^*$ such that $\bar{U}_i(p^o) = s_i^* + u_i^c(q_i^c)$. First check IC$_L$. We have $s_L^* + u_L(q_L^c) \geq s_H^* + u_H(q_H^c)$ if and only if $\bar{U}_L(p^o) \geq \bar{U}_H(p^o) - u_H(q_H^c)$ which in turn is true if and only if $\Delta \bar{U}(p^o) \leq \Delta u(q_H^c)$. Second, check IC$_H$. We have $s_H^* + u_H(q_H^c) \geq s_L^* + u_H(q_L^c)$ if and only if $\bar{U}_H(p^o) \geq \bar{U}_L(p^o) - u_L(q_L^c) + u_H(q_H^c)$ which in turn is true if and only if $\Delta \bar{U}(p^o) \geq \Delta u(q_L^c)$.

Step 2. In this step, we translate the above claim into the claims in the proposition.

(i) Fix $\Delta \bar{U}(p^o) > 0$ and define $q_i^c$ from $u_i^c(q_i^c) = c$. It is clear that $q_i^c(c) < 0$ and that
\( \phi(c) \equiv \Delta u(q^e_L(c)) \) satisfies \( \phi'(c) < 0 \). Further, we have \( \lim_{c \to \infty} q^e_L(c) = 0 \), \( \lim_{c \to 0} q^e_L(c) = +\infty \), \( \lim_{c \to \infty} \phi(c) = 0 \), and \( \lim_{c \to 0} \phi(c) > \Delta U(p^o) \) (by \( \lim_{c \to 0} \phi(c) = \lim_{q \to \infty} \Delta u(q) > \Delta \bar{U}(p^o) \)). Thus, there is a finite and positive \( c' \) such that \( \Delta u(q^e_L(c)) > \Delta \bar{U}(p^o) \) for all \( c < c' \). By Step 1, in this case \( q^* = q^e_H \) and \( q^* < q^e_L \), which means that type \( H \) must be charged a price no larger than the perk’s marginal cost, while type \( L \) could pay an above marginal cost price.

For part (ii), fix a \( \Delta \bar{U}(p^o) > 0 \) and define \( q^e_i(c) \) as in Part (i). Define \( \Phi(c) \equiv \Delta u(q^e_H(c)) \). We have \( \lim_{c \to \infty} q^e_H(c) = 0 \), \( \lim_{c \to 0} q^e_H(c) = +\infty \), \( \lim_{c \to \infty} \Phi(c) = 0 \), with \( q^e_H(c) < 0 \) and \( \Phi'(c) < 0 \). Thus, there exists a finite and positive \( c'' > c' \) such that \( \Delta \bar{U}(p^o) > \Delta u(q^e_H(c)) \) for all \( c > c'' \). By Step 1, in this case \( q^* > q^e_H \) and \( q^* = q^e_L \), which means that type \( H \) must be charged a below marginal cost price, while type \( L \) pays a price no larger than the perk’s marginal cost. Part (iii) follows from the proofs of parts (i) and (ii) and from claim (c) in Step 1.

Finally if \( \lim_{q \to \infty} \Delta u(q) \leq \Delta \bar{U}(p^o) \), then because \( \Delta u(q) \) is increasing in \( q \), we have that \( \Delta u(q) \leq \Delta \bar{U}(p^o) \) for all \( q > 0 \). Claim (c) in Step 1 applies and, thus, part (ii) of Proposition 5 holds.

**Proof of Proposition 6 (sketch):** (a) The reasoning here is similar to that in the proof of part (a) in Proposition 4.

(b) The firm always has the option of setting \( s_L = s_H = s \) and \( q_i = p_i(p) \), \( i = L, H \), for some \( p \), which would yield the same profit as the uniform price contract \( (s, p) \). Since Proposition 4 says that when \( c > c^+ \) the firm will find it optimal to offer the perk under the uniform price contract even for some \( p^o < c \), this must also be true in the present case of menu of quantity contracts.

Hence, there must exist a \( \bar{p}^o(c) < c \) such that under the menu of contracts the perk is provided to at least one type for all \( p^o \geq \bar{p}^o(c) \). ■
References


