Work-Related Perks, Agency Problems, and Optimal Incentive Contracts

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Abstract

This paper examines the effects of work-related perks, such as corporate jets and limousines, nice offices, secretarial staff, etc., on the optimal incentive contract. In a linear contracting framework, perks characterized by complementarities between production and consumption improve the trade-off between incentives and insurance that determines the optimal contract for a risk-averse agent. We show that (i) the perk may be offered even if its direct consumption and productivity benefits are offset by its cost; (ii) the perk will be offered for free; (iii) agents in more uncertain production environments will receive more perks; (iv) senior executives should receive both more perks and stronger explicit incentives; and (v) better corporate governance can lead firms to award their CEOs more perks. Our analysis also offers insights into the firms’ decisions about how much autonomy they should grant to their employees and about optimal perk provision when managers and workers are organized in teams.

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1. Introduction

Work-related perks are ubiquitous. The online betting company Betfair uses performance bonuses to motivate its employees, but, according to David Yu, Betfair's COO, the most important incentive Betfair offers its staff is the working environment: "We... do more trades than the London Stock Exchange... But we are very relaxed: engineers can work from home and have flexible hours. People like working here and with each other." (Vowler, 2005). Electronic Arts, the world's largest independent video game maker, uses perks such as company gym, on-campus masseuse and acupuncturist, and flexible hours instead of overtime pay to induce its employees to provide overtime hours (Richtel, 2005). Similar work-related perks with possible incentive effects complement cash pay in many other companies.

This paper incorporates perks in the principal-agent model, with the aim to examine the relationship between the provision of work-related perks and formal incentives. We focus on perks which, in Rosen’s (2000) terminology, have "productive consumption" attributes, and which we define as non-pecuniary compensation that has productive use and provides intrinsic motivation, as in the above examples. To emphasize this specific nature of work-related perks, we at times refer to them as "technological perks". Our key assumption in modelling this kind of perk is that there are consumption complementarities between the perk and effort (or time working) in the agent's utility function. This is meant to capture what we consider to be an important feature of many work-related perks: an employee is likely to derive a greater utility from a given amount of the perk if he uses it in the production process longer, more frequently, or more intensively. A CEO is more likely to derive utility from a corporate jet if she is fully engaged in the company's operations and goes frequently on business trips; a pleasant working environment is more valued by employees who spend longer hours at work; and so on. Such consumption complementarities between the perk and effort mean that the agent is willing to exert some effort even if he faces no explicit incentives.
The main economic forces that are at play in our model can be explained as follows: The incentive effect of the perk allows the principal to decrease the pay-performance sensitivity of the agent’s explicit incentive contract, which in turn decreases the uncertainty in the agent’s income. Given that the agent is risk-averse, a lower income uncertainty translates into a lower total expected pay that he must get to accept the employment contract. This increases the principal’s expected profit.

Clearly, by focusing on technological perks, we exclude from our analysis some important employee benefits that have no productivity effects, such as dental insurance and pensions. We believe, however, that the dual role of technological perks as a consumption good and as a productivity enhancement tool makes them of special interest, as it is the main source of the controversy surrounding the use of these perks: Following the theoretical arguments of Jensen and Meckling (1976), some authors stress the possibility of agency problems resulting in excessive perk consumption, especially when it comes to the companies’ executive officers (e.g. Yermack (2006)). Others, on the other hand, highlight the legitimate use of the perks as productivity enhancing and incentive tools (Rosen (2000); Rajan and Wulf (2006)). Much of this controversy stems from the fact that the perks’ consumption attributes make them open to misuse by the employees, but their productivity enhancement attributes make the misuse hard to detect.1

Our framework allows us to shed light on this issue by addressing a number of poorly understood questions pertaining to the optimal provision of technological perks: If a firm provides a perk, how much should the perk be subsidized, that is, at what price should it be sold to the agent? How does the optimal perk provision depend upon the model’s exogenous parameters, such as the production

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1 Reflecting this controversy, the very term “perk” is somewhat nebulous. While some authors, for example Yermack (2006), reserve the term for non-productive consumption of in-kind goods and services, others (including Jensen and Meckling, 1976, Rajan and Wulf, 2006, as well as many popular press writers) allow for both productive and non-productive uses. Interestingly, a recent SEC proposal regarding the rules regulating the disclosure of executive perks explicitly avoids providing a definition of perks, citing the elusiveness of the term (SEC, 2006, p. 6553).
technology and production uncertainty? What is the relationship between the amount of the perk and the slope of the agent’s formal incentive contract? How are the optimal price and quantity of the perk and the employee’s formal incentives affected by the possibility that the employee can divert the perk for purely personal use? By offering relatively clear-cut answers to these and similar questions, the model can provide some guidance for future empirical tests attempting to disentangle the agency and the productivity motives behind the observed patterns of managerial perks. Our main results are the following:

(1) In some cases, the firm will find it optimal to provide the perk even if it has no direct effect on the agent’s productivity and the cost of providing the perk is greater than the monetary equivalent of the utility the agent derives from it. This result highlights the fact that the complementarities between the perk and effort that are central to our model make perks valuable for their incentive effects and for the resulting decrease in the salary that the company needs to offer to attract the agent.

(2) The more uncertain is the production process (as measured by the variance of output) and the harder it is to monitor and evaluate the agent’s performance, the more valuable are the perk’s incentive effects and, consequently, the more likely it is that the perk will be provided. This suggests that we should observe more technological perks in larger firms, in privately held firms, in firms in the new economy sector, in firms with many inter-dependent divisions, and in geographically dispersed firms.

(3) The existing informal discussions of the productivity theory of perks, such as the one in Rosen (2000), suggested that productivity enhancing perks should be subsidized by companies, but were not specific about how large the subsidy should be. In our framework, it is always optimal to provide the perk to the employees free of charge.\(^2\) This result does not depend upon how much

\(^2\)Of course, in the end, the agent is always held down to his reservation utility, which means that he pays for the perks up-front, through a lower salary, as in the standard theory of hedonic prices. When we say the perk is provided
or how little the perk increases the agent’s productivity. Neither is it affected by allowing for the possibility that the agent can divert the perk for personal use, such as using a corporate jet for family trips.

(4) In our theory, the problem of optimal provision of technological perks is a part of a more complex problem of designing an optimal incentive package in which explicit incentives are intertwined with intrinsic motivation provided by perks. Consequently, the factors that affect the strength of explicit contracts also play a role in the firm’s decision to provide perks, so that the two variables are correlated. For example, firms with less precise performance measures (e.g. large firms) should provide both weaker explicit incentives and more technological perks. Similarly, all else equal, agents with higher marginal productivities (e.g., the employees with greater skills or more senior managers) will be offered more powerful explicit incentives, accompanied by greater amounts of technological perks.

(5) We introduce agency problems in perk consumption by allowing the employee to divert the perk for purely personal use. Contrary to what one would expect based on Jensen and Meckling’s (1976) analysis, we show that agency problems in our model lead to less equilibrium perk consumption and to greater fractional ownership by the firm’s CEO. Thus, better corporate governance can actually lead firms to award their managers more perks. These results suggest caution in interpreting empirical evidence on CEOs’ perk consumption and on the strength of their incentives as supporting or refuting the agency theory.

(6) Extending our model in a straightforward way, we show that managers and workers organized in teams should receive more technological perks. We also discuss the implications of our model for the optimal degree of employee autonomy.

"Free of charge", we mean that the agent does not pay more for the perk if he uses it more intensively; e.g., the CEO is not asked to share a part of the operation costs incurred when she uses the company aircraft.
1.1. Related literature

For the most part, the literature on employee benefits does not deal with the specific issues considered here. The closest papers are Jensen and Meckling (1976) and Oyer (2006). In their seminal analysis of agency problems in perk consumption, Jensen and Meckling (1976) allow perks to have a productivity use and to increase the firm’s value, but they focus exclusively on the implications of the managers’ ability to misuse the perks. The productivity aspects of the perks appear in their model only in a very reduced form and therefore do not play any interesting role. Oyer (2006) uses a simple model of productivity enhancing benefits to show that a benefit will be provided more frequently the more it lowers an employee’s cost of effort, and he finds support for this prediction using data on company provided meals. However, Oyer does not consider formal incentive contracts, which limits the potential insights from his model.

Our paper is also related to the vast literature on optimal incentive contracts. In particular, although we consider only one explicit performance measure (the agent’s output), the price the agent pays for the use of the perk is akin to a weight put on an additional performance measure. The perk thus plays a similar role in the contract as a second performance measure. This makes the model formally related to the literature on optimal incentive contracts with multiple performance measures, where two recent representative contributions are Baker (2002) and Raith (2005). However, we focus on different issues, not examined elsewhere.

The rest of the paper proceeds as follows. In Section 2, we introduce our basic model, but abstract from agency problems in perk consumption. The analysis of this model follows in Section 3. In Section 4, we introduce agency problems in perk consumption and relate our model to Jensen and Meckling’s analysis. In Section 5 we offer two applications and extensions of our basic framework: We consider here the effects of teamwork on perk provision and discuss the implications of our analysis for the optimal degree of delegation within organizations. Section 6 concludes.
2. The Model

For expositional purposes, we start with a basic model that abstracts from the possibility that the agent could misuse the perk for purely private consumption. We allow for agency problems in perk consumption in Section 4, where we also demonstrate that our main results from this section continue to hold in this richer setting.

The model is based on the linear contract principal-agent framework of Holmström and Milgrom (1987). Consider a firm consisting of a risk neutral principal (e.g., the firm’s owners) and a risk averse agent (e.g., the CEO) with certainty equivalent reservation income \( \bar{w} \). The agent chooses unobservable action, \( a \in \mathbb{R}_+ \), which affects the distribution of the firm’s output. The principal offers the agent a formal linear performance contract, which conditions his monetary pay on his output. In addition, the principal can provide the agent with a technological perk, which the agent views as a consumption good, i.e., he derives utility from its use, but it also can serve as a non-labor input in the sense that it may increase the agent’s productivity (a computer, a quiet office, use of a company aircraft, etc.). The effectiveness of the agent’s action in improving the expected revenue depends upon the amount, \( q \), of the perk provided by the firm, as specified next.

**Technology.** If the agent chooses action \( a \) and the firm provides an amount \( q \) of the perk, the firm’s output is given by

\[
y = \beta (1 + qm)(a + \varepsilon).
\]

Here, \( \beta \) captures the agent’s marginal productivity on the job, but it could also be interpreted as the marginal productivity of the firm’s technology, affected by such things as the firm’s market power in its product market, its cost effectiveness, and so on. The parameter \( m \) measures the effect of the perk on the agent’s productivity. We will focus on the parameter values such that \( m \geq 0 \), where \( m = 0 \) allows for the possibility that the perk is a pure consumption good. We would like
to stress, however, that our conclusions remain unchanged even if $m < 0$, as long as $m$ is not too negative. The model thus also applies to the case where the perk distracts the agent from his duties and decreases his productivity. Finally, $\varepsilon$ is a normally distributed noise term, with zero mean and variance $\sigma^2$. Note that both $\beta$ and $q$ enter multiplicatively with $\varepsilon$. This means that the perk does not simply increase the signal to noise ratio of $y$ as it would if $q$ and $\varepsilon$ were additively separable. The same goes for the agent’s marginal productivity $\beta$.

The perk’s acquisition cost is $kq$, $k \geq 0$, and its operating cost is $c(q, a) = \theta qa$, $\theta > 0$. Thus, while the acquisition cost does not depend upon the agent’s work intensity, the operation cost does. In this sense, the acquisition cost can be thought of as a fixed cost of obtaining a given amount of the perk and the operation cost is the variable cost associated with using the perk in production and/or consumption. For example, in the case of a corporate jet, this would be any cost that depends upon the intensity with which the jet is used, such as the costs of fuel, maintenance, perhaps insurance, the plane’s depreciation, and so on.

Preferences. The agent’s utility as a function of his monetary income, $w$, his action, $a$, and his perk consumption, $q$, is given by

$$U(w) = -e^{-r[w+\gamma qa-g(a)]},$$

where $r$ is the agent’s coefficient of absolute risk aversion. The term $\gamma qa$ is his monetary equivalent of utility from consuming $q$ units of the perk, where $\gamma > 0$ is a parameter that allows us to vary

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3 Arguably, the multiplicative specification is more realistic than the additive one, even though the latter has been used more frequently in the literature. If an agent becomes more productive because he is assigned to a more productive technology or receives more perks, it seems reasonable to expect that the variance of his output would increase. However, our main reason for choosing the multiplicative specification is to separate the incentive effects of $\beta$ and $q$ from their effects on the signal-to-noise ratio of the performance measure. In an additive world, $\beta$ and $q$ would increase the signal-to-noise ratio of $y$, which would make the perk even more valuable to the principal than in our model. This effect, though, is well understood.

4 The assumption that the perk’s cost is deterministic does not play any role in our analysis and is adopted purely to simplify exposition.
how much the agent likes the perk. The complementarity between $q$ and $a$ in the agent’s utility, implied by this specification, is a key feature of our model, without which the perk would only affect the optimal incentive contract through the productivity parameter $m$. This would make the perk’s effects indistinguishable from the effects of a standard production capital.$^5$

The term $g(a)$ indicates the agent’s monetary equivalent of disutility from providing action $a$. The function $g$ is differentiable, increasing and convex.

**Contracting.** The firm and the agent sign a formal incentive contract, according to which (i) the agent’s pay, $w$, is a linear function of his output and (ii) the agent is charged a portion $p \geq 0$ of the perk’s operating cost.$^6$ That is,

$$w(y) = s + by - pc(q, a),$$

where $s$ is the agent’s base salary and $b$ is the piece-rate, measuring the strength of the formal incentives. We do not allow for $p < 0$. This restriction is meant to capture the fact that if the agent’s pay increased in $c$, he could game the contract by taking some unobservable action that would increase the costs incurred by the firm without imposing personal costs on himself.$^7$ As will become clear later, an important implication of this assumption will be that, even though the principal can use her knowledge of $q$ and $c(q, a)$ to infer $a$, this will not help her to force the agent to take the optimal action.

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$^5$Oyer (2006) also introduces a complementarity between perk and effort, but it is in the agent’s cost of effort function. Such complementarity does not have the incentive effects present here, which makes his perks hard to differentiate from pure production capital. In particular, our specification guarantees that the agent is willing to use the in-kind good even in the absence of explicit incentives. This is not true if the in-kind good simply lowers the agent’s cost of effort.

$^6$Since we are interested in the effects of $p$ on the agent’s incentives, we ignore the possibility that the agent could also be charged for part of the perk’s acquisition cost $kq$. Such a payment would simply represent a transfer equivalent to a decrease in the agent’s salary, $s$, and hence it would not affect incentives.

$^7$For example, if the agent’s pay increased with the electricity bill that he runs up using his computer, he would simply leave the computer turned on at all times. Formally, this could be easily incorporated in the model by assuming that there is another action, $a'$, that the agent can take, and that $a'$ has a similar effect on $c(q,.)$ as $a$, but it imposes no cost on the agent.
Note that when \( q = 0 \), the problem collapses into a version of the standard principal agent model discussed in Holmström and Milgrom (1987), in which the optimal piece rate is given by

\[
b_0^* = \frac{1}{1 + r\sigma^2g''(a^*)}.
\]

Observe in particular that \( b_0^* \) is independent of the agent’s productivity parameter, \( \beta \). This is because, as we have already explained, \( \beta \) increases not only the agent’s productivity but also the variance of his output, and these two effects cancel out in determining the optimal piece rate. Again, we have chosen this specification purposefully, to highlight that any effect \( \beta \) will have on the optimal incentive contract will be driven by the presence of the perk.

### 3. The Analysis

The agent’s certainty equivalent is

\[
CE(s, b, q, p) = \gamma qa + s + b\beta(1 + qm)a - \frac{1}{2}rb^2\beta^2(1 + qm)^2\sigma^2 - pq\theta a - g(a),
\]

so that his optimal choice of \( a \) is given by the first order condition

\[
g'(a) = \text{max}\{0, b\beta(1 + qm) - pq\theta + \gamma q\}. \tag{1}
\]

Thus, as one would expect, the higher is the price the agent is charged for the perk, the lower is the level of effort he chooses to provide, given any piece rate \( b \).

Because the principal is the residual claimant, her problem is to design for the agent a comprehensive incentive package \( (b, q, p) \), balancing the explicit incentives of the formal contract with the
implicit incentives provided by the perk, so as to maximize the expected total surplus,

\[ TS(b, q, p) \equiv \beta a + qa(\gamma + \beta m - \theta) - \frac{1}{2} \beta^2 (1 + qm)^2 \sigma^2 - g(a) - kq - \bar{w}, \]

subject to the incentive compatibility constraint (1). We will assume that this is a concave problem.\(^8\)

Given that our goal is to focus on the incentive benefits of perks, we restrict our attention to parameter values such that the direct benefits due to increased productivity and consumption utility derived from the perk are not sufficient to offset the perk’s operating costs.

\textbf{Assumption 1:} \( \theta > \beta m + \gamma. \)

Under this assumption, providing the perk is clearly suboptimal, unless the perk can improve the efficiency of the optimal incentive contract. Assumption 1 also guarantees that the optimal quantity of the perk will be finite.\(^9\)

Let \( A(b, q, p) \equiv \frac{\partial TS}{\partial a} \) denote the net marginal benefit to the principal of increased effort. Substituting for \( g'(a) \) from (1), we can write

\[ A(b, q, p) = (1 - b)\beta(1 + qm) - (1 - p)\theta q. \]

Differentiating the total surplus with respect to \( b, p, \) and \( q \) then yields

\[ \frac{\partial TS}{\partial b} = \frac{\partial a}{\partial b} A(b, q, p) - b \beta^2 (1 + qm)^2 r \sigma^2, \]

(2)

\[ \frac{\partial TS}{\partial p} = \frac{\partial a}{\partial p} A(b, q, p), \text{ and} \]

(3)

\[ \frac{\partial TS}{\partial q} = \frac{\partial a}{\partial q} A(b, q, p) - (\theta - \beta m - \gamma) a - b^2 m \beta^2 (1 + qm) r \sigma^2 - k. \]

(4)

\(^8\)A sufficient, but not necessary condition, for this is that \( g''(\cdot) \geq 0. \)

\(^9\)If the inequality in Assumption 1 were reversed, it would be optimal to provide the perk not only for incentive and risk-sharing purposes, but also for its direct consumption and productivity values. Of course, in the comparative statics exercises, the consumption and productivity enhancement motivations are going to play a role even under Assumption 1.
Also from (1), we have
\[
\frac{\partial a}{\partial b} = \frac{\beta (1 + qm)}{g''(a)} > 0, \quad \frac{\partial a}{\partial p} = -\frac{q \theta}{g''(a)} < 0, \quad \text{and} \quad \frac{\partial a}{\partial q} = \frac{b \beta m - p \theta + \gamma}{g''(a)}.
\]

**Lemma 1.** If \( A(b^*, q^*, p^*) \leq 0 \), then it must be \( q^* = 0 \).

The proof for this lemma is in the appendix, as are the proofs of all our subsequent results.

According to Lemma 1, the profit-maximizing amount of the perk, \( q^* \), can be positive only if \( A(b^*, q^*, p^*) > 0 \). In such a case, (3) implies that \( \frac{\partial TS}{\partial p} < 0 \) for all \( p > 0 \), so that \( p^* = 0 \) and \( A(b^*, q^*, p^*) = (1 - b^*)/\beta (1 + q^* m) - \theta q^* \). Then \( b^* \) and \( q^* \) solve (2) and (4), which together yield

\[
b^* = \frac{1 - \theta q^*/\beta (1 + q^* m)}{1 + r \sigma^2 g''(a^*)} \quad (5)
\]

and

\[
q^* = \frac{\beta}{\theta - \beta m} - \frac{[(\theta - \beta m - \gamma) a^* + k][1 + r \sigma^2 g''(a^*)]}{(\theta - \beta m) \gamma r \sigma^2}. \quad (6)
\]

Comparing expression (5) with \( b_0^* \), the slope of the optimal contract when no perks are provided, reveals that the effect of the perk on the optimal explicit incentives is captured by the negative term \(-\theta q^*/\beta (1 + q^* m)\) in the numerator of \( b^* \). Because the perk itself has incentive effects, this crowds out formal incentives, which is reflected in a smaller slope of the incentive contract. These results are summarized and the optimal contract is further characterized in the following proposition.

**Proposition 1.** It is always optimal to set \( p^* = 0 \). Also, there exists a \( \gamma^* > 0 \) such that

(i) if \( \gamma > \gamma^* \), then \( b^* \) and \( q^* \) are given by (5) and (6) respectively, where \( q^* > 0 \) and \( b_0^* > b^* > 0 \);

(ii) if \( \gamma \leq \gamma^* \), then \( b^* = b_0^* = \frac{1}{1 + r \sigma^2 g''(a^*)} > 0 \) and \( q^* = 0 \).

Proposition 1 provides two insights into the optimal provision of a technological perk. First,
the firm may find it optimal to provide the perk even if the perk’s cost is greater than its direct benefits represented by the manager’s consumption utility from the perk plus the direct increase in his productivity (i.e., even if \( \theta > \gamma + \beta m \), which is Assumption 1). For these parameter values, the main motivation for providing the perk is that it improves the risk-sharing properties of the optimal contract. Since the incentive effects of the perk increase in \( \gamma \), they can offset the marginal cost of providing the perk (net of the perk’s marginal consumption and productivity improvement values, i.e., \( (\theta - \beta m - \gamma)a + k \)) only if \( \gamma \) is sufficiently high. Hence the condition \( \gamma > \gamma^* \) in the proposition. Notice that this requires that \( \gamma > 0 \), that is, the good indeed needs to be a perk rather than pure production capital. On the other hand, it is not necessary for the argument that the perk improves the agent’s productivity: As long as \( \gamma > \gamma^* \), providing the perk is optimal even if the perk decreases the agent’s productivity, i.e., even if \( m < 0 \).

The second insight offered by Proposition 1 is that the perk should always be provided to the agent for free (i.e., \( p^* = 0 \)). This seems to be an empirically sound prediction, and one that shows that the standard explanation for providing work related perks is incomplete. According to the standard reasoning (found, for example, in Rosen, 2000), a firm needs to subsidize a productivity enhancing perk if the employees do not internalize all the benefits from the productivity increase brought about by the perk. This argument, however, only implies zero price for the perk in the extreme case where the agent’s pay is completely unresponsive to his productivity. Otherwise, if the agent internalizes a part of the productivity increase through an increase in his pay, the logic of the standard argument seems to suggest that he should be charged a positive price for the perk,

\[ k < \frac{\beta r a^2 (\theta - \beta m)}{1 + r a^2 (\theta - \beta m)^2}. \]

However, even when this condition does not hold, it is still true that the firm wants to provide the perk if \( \gamma > \gamma^* \), as claimed in the proposition.

If the agent could choose \( q \), it might be optimal to set \( p > 0 \), to curb his excessive consumption of the perk. Our maintained assumption that the perk is awarded to the agent by the firm may be less realistic in the case of very powerful CEOs. However, if the firm’s board of directors is so weak that it cannot control \( q \), it is not clear why it would be strong enough to control \( p \). Thus, while this is clearly an interesting variation on our analysis, it is outside of the scope of this paper, because it would require a different model – perhaps along the lines of Heremalin and Weisbach (1998) – that would allow us to capture the relative powers of the firm’s CEO and its board of directors.

\[ ^{10} \text{In order to guarantee that } \gamma^* < \theta - \beta m, \text{ so that Assumption 1 is not violated, it must be that the perk’s fixed cost is not too high, } k < \frac{\beta r a^2 (\theta - \beta m)}{1 + r a^2 (\theta - \beta m)^2}. \] However, even when this condition does not hold, it is still true that the firm wants to provide the perk if \( \gamma > \gamma^* \), as claimed in the proposition.

\[ ^{11} \text{If the agent could choose } q, \text{ it might be optimal to set } p > 0, \text{ to curb his excessive consumption of the perk. Our maintained assumption that the perk is awarded to the agent by the firm may be less realistic in the case of very powerful CEOs. However, if the firm’s board of directors is so weak that it cannot control } q, \text{ it is not clear why it would be strong enough to control } p. \text{ Thus, while this is clearly an interesting variation on our analysis, it is outside of the scope of this paper, because it would require a different model – perhaps along the lines of Heremalin and Weisbach (1998) – that would allow us to capture the relative powers of the firm’s CEO and its board of directors.} \]
lest he does not overuse it. In contrast, our analysis shows that once the optimal adjustment in
the incentive contract is taken into account, it is always optimal to offer the perk for free. The
reason is that charging the agent for the use of the perk would discourage him from using it and
hence mute his incentives, as can be seen from the first order condition (1). Therefore, stronger
incentives would have to be provided through an increase in the slope, $b^*$, of the explicit contract.
This, however, would impose additional risk on the agent and hence decrease efficiency.

As a final note on Proposition 1, observe that the result $p^* = 0$ says that the principal will have
no use for her knowledge of $a$ that she infers from her information about $q$ and $c(q, a)$. Intuitively,
this information would only be useful if the agent’s pay could be made an increasing function of
the cost function $c$, which is precluded by our restriction $p \geq 0$.

We now turn our attention to how the optimal incentive contract and the optimal provision of
the perk depend on the economic environment in which the firm operates. In general, the firm’s
decision whether or not to provide a technological perk, how much of the perk to provide, and what
should be the optimal slope of the incentive contract, can all depend on the model’s parameters in
a complicated way, determined by the third derivative of the agent’s cost of effort function $g(.)$. To
avoid these complications, we will from now on assume that $g(a) = a^2/2$, so that $g''(.) = 0$.

**Proposition 2.**

(i) A technological perk is more likely to be provided ($\gamma^*$ is smaller) the greater are $\beta$, $\gamma$, $m$, $r$,
and $\sigma^2$ and the lesser are $k$ and $\theta$.

(ii) The optimal amount of the perk, $q^*$, increases with $\beta$, $m$, $\gamma$, $r$, and $\sigma^2$ and decreases in $k$ and
$\theta$. The optimal pay-performance sensitivity of the incentive contract, $b^*$, increases with $\beta$, $k$,
and $\theta$ and decreases in $\gamma$, $r$, $\sigma^2$, $m$ (and in $q^*$).

Along with several expected predictions, Proposition 2 yields two novel and potentially testable
comparative static results. First, it predicts that the use of technological in-kind compensation should be more prevalent in more uncertain economic environments. The perk allows the firm to improve the incentives-versus-insurance trade-off that determines the optimal explicit incentives and this effect is more valuable in more uncertain environments, in which the inefficiencies caused by the trade-off are greater. Similarly, companies in which monitoring and evaluating the employees’ individual performance is harder should offer more technological perks. These observations suggest the types of organizations that should be more likely to provide top of the line computers, generous secretarial support, nice offices, the use of a company plane, and other technological perks:

(1) Privately held firms. A public company’s stock price provides an informative measure of performance, not available in privately held firms. Privately held firms should therefore find it harder to evaluate their employees, which should make technological perks more valuable to them.

(2) Firms with multiple inter-dependent divisions where coordination is important and where the actions taken by the employees in one division affect the performance of the other divisions.12

(3) Firms that are geographically dispersed and therefore find it harder to monitor employees. Consistent with this interpretation, Rajan and Wulf (2006) find that company planes are more common in geographically dispersed firms. Since this could also be because planes are more useful in geographically dispersed companies (which would be captured by a greater $m$ in our model), more direct support for this prediction would come from technological perks that are not travel related.

(4) Large firms, as these tend to have more noisy measures of individual performance (Schaefer (1998); Baker and Hall (2004)). The existing empirical studies of non-monetary compensation typically examine benefits that are more broadly defined than our technological perks, and therefore can only provide indirect support for our theory. With this caveat in mind, the prediction that

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12 We would like to thank Julie Wulf for suggesting this and the next example.
large firms should provide more perks appears to be consistent with available evidence: numerous studies have documented that large firms offer more non-wage compensation than small firms (e.g., Brown et al (1990), Montgomery and Shaw (1997), Oyer (2006), and Rajan and Wulf (2006)). This prediction is similar to what one would expect in the presence of economies of scale in perk provision (Rosen (2000)). The empirically relevant distinction between our theory and the economies of scale argument is that in our model, the decision whether to provide the perk need not depend upon the actual number of employees within the organization that receive it.

(5) New economy firms. These firms tend to be more R&D intensive, have greater market-to-book ratios, and grow more rapidly than the old economy firms (Ittner, Lambert, and Larcker (2003)). All of these characteristics could make it hard to observe a manager’s marginal contribution. Rajan and Wulf (2006) find that the firm’s growth prospects have a significant positive effect on perk provision (although market-to-book ratio does not). This prediction also seems to be in accord with the popular belief that new economy firms offer more and better perks, especially productivity enhancing ones.

In addition, because perk provision in our model is intertwined with the problem of designing the optimal incentive contract, all else equal, we would expect the greater amount of perks in the above types of companies to be accompanied by weaker explicit incentives. This is consistent, for example, with the fact that larger firms have been shown to offer weaker formal incentives, at least to their top executives (Schaefer (1998); Baker and Hall (2004)). However, this prediction may

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13 Both R&D expenditures and the market-to-book ratio have been used by empirical researchers to proxy for the degree of difficulty in measuring managerial performance in a firm (e.g., Kole, 1997).

14 For example, Standen (2001) describes how the $700 Aeron office chairs “renowned throughout the office universe for their ergonomically correct luxury” became extremely popular with technology firms. “According to the new-economy ethos, work would be fun; it would be comfortable and ergonomic…”. Similarly, Terry (2001) writes that “Dotcom companies became infamous for pioneering many ... perks, including catered lunches and free car-wash services... At the core of all this ... is anything that focuses on time, because time is such a huge commodity.”

15 This prediction can also be obtained from the standard principal-agent model in the absence of perks, as long as one assumes that the variance of the agent’s output increases faster with the firm’s size than his marginal productivity. See Baker and Hall (2004) for a detailed discussion of this model.
not be robust in environments where greater uncertainty is associated with greater reliance on the 
employee’s specific knowledge, as in such environments the relationship between uncertainty and 
explicit incentives is typically ambiguous (see, e.g., Prendergast (2002), Raith (2005), and Zabojnik 
(1996)).

The second comparative static result worth noting concerns the effects of the agent’s productivity (as measured by $\beta$) on $b^*$ and $q^*$. In contrast to the benchmark case with no perk, if the perk is provided then more productive employees (say, senior managers versus rank-and-file workers) receive stronger explicit incentives.\footnote{This prediction also obtains in the standard model without perks, if one assumes that $\beta$ does not affect the variance, i.e., $y = \beta a + \varepsilon$. In this case, the relationship is driven by the fact that $\beta$ improves the signal to noise ratio of $y$.} More importantly, more productive employees are also more likely to receive technological perks and the amounts of the perks they receive are greater.\footnote{Since $q$ can alternatively be interpreted as the perk’s quality, the model also predicts that senior managers should receive technological perks of higher quality.} The perk becomes more valuable as $\beta$ increases, for two reasons. First, for a given $b$, $\beta$ magnifies the effect of the perk on the agent’s incentives ($\frac{\partial^2 A}{\partial q \partial \beta} > 0$), as well as the net marginal benefit of increased effort ($\frac{\partial A(b,q,p)}{\partial \beta} > 0$). Second, it directly improves the agent’s productivity, through the term $\beta mqa$. This prediction is in accord with the conclusion in Rajan and Wulf (2006) that productivity considerations seem to play an important role in determination of managerial perks in major U.S. public companies. The prediction is also consistent with Krueger and Summers’ (1988) empirical finding that inter-industry wage differentials, with more capital intensive industries typically paying higher wages, are magnified when one accounts for non-wage benefits. This finding suggests that more capital intensive industries tend to provide more fringe benefits (again, more broadly defined than in our paper).

Finally, our conclusions fit well with the common perception that senior managers receive more perks than the average employee.\footnote{Rajan and Wulf (2006) document that CEOs in their sample receive more perks than lower-level managers.} Of course, this relationship could also be driven by a pure
income effect, wherein senior managers demand more perks simply because they have greater wealth. Thus, in testing the productivity theory, one would ideally want to control for the managers’ wealth. Also, the income based explanation applies to all employee benefits, whether they are work-related or not. Hence, a test that would find a stronger relationship between manager seniority and the amount of technological perks than between seniority and non-work related perks could be interpreted as lending support to the productivity theory.

4. Agency problems in perk consumption

As we have argued earlier, the controversial nature of many perks stems partly from the fine line that separates their consumption and production uses. Provision of work-related in kind compensation might induce the agent to take unproductive actions, in cases where such actions would generate personal benefit. For example, the provision of a chauffeured limousine might encourage a CEO to use the car for purely personal purposes. The provision of a computer with high speed internet access might result in the agent wasting time surfing the web. This is the standard agency problem in perk consumption studied by Jensen and Meckling (1976). In this section, we introduce into our basic framework such agency problems and contrast our model with the Jensen-Meckling theory. However, in keeping up with the optimal contracting approach, we depart from Jensen and Meckling by assuming that the agent can only misuse the perk once it is awarded to him by the firm – he cannot unilaterally decide to obtain the perk without the firm’s consent. This is a realistic assumption for most employees, although it may not fit some very powerful top executives in firms with weak boards of directors (see footnote 11). Our approach here is in line with recent empirical studies on executive perks (Rajan and Wulf, 2006; Yermack, 2006) that seem to assume that the perks enjoyed by CEOs and other top executives have been awarded to them.¹⁹

¹⁹For example, in Rajan and Wulf (2006, p.2) the authors state several times that a particular firm has offered or offers an executive a perk. In Yermack (2006) similar language is used (on p.3 and elsewhere).
Let $a_1$ represent a productive action and let $a_2$ represent a non-productive action, where the variable $a_1$ takes the place of $a$ in the basic model. Action $a_2$ only generates a personal benefit $\gamma_2 a_2$ to the agent, while imposing the same marginal cost on the principal as activity $a_1$. The parameter $\gamma_2$ can be thought of as a measure of agency problems in the firm: the greater is $\gamma_2$, the more the employee likes to divert the perk for personal use or the easier it is for him to do so. (In the latter case, $\gamma_2$ captures – in a reduced form – the ease with which the agent’s use of the perk can be monitored by the firm’s owners.)

**Assumption 2:** $\theta > \beta m + \gamma_1 + \gamma_2$.

Assumption 2 is the analogue of Assumption 1; it says that the direct consumption and productivity benefits from the perk are not enough to offset the cost of providing the perk. Thus, it is not efficient to provide the perk unless it helps the principal to design a better incentive contract.

The agent’s personal cost function is $g(a_1, a_2) = \frac{a_1^2}{2} + \frac{a_2^2}{2}$. His certainty equivalent is now written as

$$CE(a_1, a_2, s, b, q, p) = \sum_{i=1}^{2} \gamma_i a_i q + s + b\beta(1 + qm) a_1 - \frac{1}{2} r b^2 \beta^2 (1 + qm)^2 \sigma^2 - pq\theta(a_1 + a_2) - g(a_1, a_2),$$

and his first order conditions for $a_i$ choice are

$$a_1 = \max \{0, \gamma_1 q + b\beta(1 + qm) - pq\theta\},$$

$$a_2 = \max \{0, \gamma_2 q - pq\theta\}.$$  

\[20\] We assume additive separability in the two actions in order to get clean comparative statics results. Our conclusion in Proposition 3 that the perk should again be offered for free easily extends to more general cost functions. For example, the result continues to hold if $g_{12}(a_1, a_2) \neq 0$, as long as $g_{11} = g_{22}$ and $g_{12} \neq g_{11}$ (where the latter is automatically met if $g$ is strictly convex). These assumptions are satisfied, e.g., by the cost function $g(a_1, a_2) = \frac{a_1^2}{2} + \frac{a_2^2}{2} + da_1a_2$, where $d$ is a constant.
The firm’s owners again maximize the total surplus, which in this case is given by

\[ TS(b, q, p) = \beta (1 + qm)a_1 + \sum_{i=1}^{2} \gamma_i a_i q - \frac{1}{2} r b^2 \beta^2 (1 + qm)^2 \sigma^2 - q \theta (a_1 + a_2) - kq - g(a_1, a_2) - \bar{w}. \]

Under what conditions will the firm provide the perk in the presence of agency problems? Also, will the firm now charge for the in kind good in equilibria where it is provided? Let \( x^{**} \) denote an equilibrium variable in the present setting. We have

**Proposition 3.** Suppose the manager can use the perk in a non-productive activity, i.e. \( \gamma_2 > 0 \).

Then \( p^{**} = 0 \), and \( q^{**} > 0 \) if and only if \( q^* > 0 \). Moreover, \( q^{**} < q^* \) and \( b_0^* > b^{**} > b^* \).

Proposition 3 demonstrates that the main conclusions of Proposition 1 remain unchanged when the agent can divert the perk for private consumption. That is, it remains optimal for the firm to offer the perk free of charge and to provide it under the same parameter values (i.e., for \( \gamma_1 \geq \gamma^* \)) as when the purely private consumption was not possible. In particular, this means that the presence of agency problems does not enter the firm’s decision whether to provide the perk. The effect of the agency problems is manifested only through the optimal amount of the perk that the firm provides and through the slope of the optimal incentive contract. Specifically, the firm optimally responds to the possibility that the agent can misuse the perk by providing it in a smaller amount (or in lower quality) and by increasing the slope of the incentive contract. This is demonstrated in the following proposition.

**Proposition 4.** When the perk can be diverted for personal use, \( q^{**} \) decreases and \( b^{**} \) increases in \( \gamma_2 \). With respect to the rest of the parameters, the comparative statics for \( q^{**} \) and \( b^{**} \) are the same as for \( q^* \) and \( b^* \). That is, \( q^{**} \) increases in \( \beta, m, \gamma_1, r, \) and \( \sigma^2 \) and decreases in \( k \) and \( \theta \), while \( b^{**} \) increases in \( \beta, k, \) and \( \theta \) and decreases in \( \gamma, r, \sigma^2, m \) (and in \( q^{**} \)).
Proposition 4 shows that the comparative statics results of Proposition 2 are preserved in the present setting with unproductive effort. Thus, all of our empirical predictions discussed earlier are robust to an extension allowing for agency problems in perk consumption. This allows us to compare our analysis with the predictions of Jensen and Meckling’s (1976) model. First, based on Jensen and Meckling’s analysis, greater agency problems (due to, say, greater difficulties in monitoring the CEO’s actions or due to an increase in the CEO’s taste for perks) should lead to more equilibrium perk consumption, which Jensen and Meckling measure by expenditures on perks. In contrast, Proposition 4 says that, in our model, greater agency problems (a larger $\gamma_2$) lead to fewer perks if their amount is measured by $q^{**}$, and that the relationship is ambiguous if the amount of perk consumption is measured by the total expenditures on perks, $q^{**}k + q^{**}\theta(a_1^{**} + a_2^{**})$.\(^{21}\)

Another prediction that has been attributed to Jensen and Meckling’s theory is that there should be a negative relationship between the CEO’s level of perk consumption and his fractional ownership in the firm (Yermack, 2006). Thus, an increase in the degree of agency problems (i.e., an increase in $\gamma_2$) should lead not only to more perk consumption but also to a smaller fractional ownership (smaller $b^{**}$). Again, we obtain exactly the opposite prediction: in our model, $b^{**}$ increases with $\gamma_2$. Strengthening the CEO’s explicit incentives in response to greater agency problems is optimal in our framework, because this redirects the CEO’s focus from non-productive ($a_2$) to productive ($a_1$) use of the perk.

The implication of the above conclusions is that one needs to exercise caution when interpreting empirical evidence on CEO fractional ownership and perk consumption as supporting or refuting the presence of agency problems in perk consumption. First, our model demonstrates that in the case of technological perks (such as the use of company aircraft), greater fractional ownership

\(^{21}\)The effect of $\gamma_2$ on total expenditures is ambiguous because $a_1^{**}$ and $a_2^{**}$ could go up or down with $\gamma_2$. Given that $a_2^{**} = \gamma_2 q^{**}$, an increase in $\gamma_2$ pulls it up, while the decrease in $q^{**}$ pulls it down. As for $a_1^{**}$, an increase in $\gamma_2$ decreases $q^{**}$, which tends to decrease $a_1^{**}$; but it also increases $b^{**}$, which tends to increase $a_1^{**}$.\)
can actually be associated with *more severe* agency problems. Similarly, companies with better corporate governance may be willing to award their CEOs *greater* amounts of perks, because they know that the CEO will use the perks to enhance the firm’s value rather than for personal consumption.\(^2\) Second, the presence of agency problems does not necessarily imply a negative relationship between the CEO’s explicit incentives and his expenditures on perks, because the effect of agency problems on the latter is ambiguous. The reverse argument is also true — if one finds no significant relationship between a CEO’s fractional ownership and his expenditures on perks (as Yermack, 2006, does for the case of corporate jets), this does not imply absence of agency problems in perk consumption. These conclusions reinforce the assessment in Rajan and Wulf (2006, p. 4), who reflect on the lack of empirical support for the agency theory in their data by arguing that "...we need to rethink whether perk consumption should be the canonical example of systematic forms of agency ... as has been suggested in the past."

The last point worth noting regarding the results in Proposition 4 is that they complement the conclusions found in the literature on distorted performance measures (Baker, 1992) and on multitasking (Holmström and Milgrom, 1991). These papers provide a theoretical rationale for the general absence of high-powered explicit incentives within firms, first pointed out by Williamson (1985). The absence of high-powered incentives within organizations is conspicuous both because standard principal-agent models seem to predict that agents should face elaborate incentive contracts and because relationships with independent contractors frequently are governed by such high-powered contracts. Our model also predicts weaker explicit incentives than the standard model (i.e., \(b^* < b_0^\star\)); this is because some incentives are provided indirectly, via work-related fringe benefits. Moreover, it is straightforward to verify that if Assumption 2 is relaxed (which

\(^{22}\)Rajan and Wulf (2006) find that governance does not have a clear-cut impact on perk provision in firms they study. Consistent with our arguments, they recognize that the apparent lack of support in their data for the agency theory could be caused by endogeneity problems, much like the ones we discuss here.
amounts to adding the usual consumption and productivity improvement motives for perk provision), then for $\gamma_2$ small, the optimal amount of the perk completely crowds out explicit incentives. That is, $b^{**} = 0$ and the employee receives a flat wage. Finally, to the extent that firms provide fewer technological perks to independent contractors than to their own employees, and to the extent that, where such perks are awarded, preventing their misuse for personal consumption is easier in the case of employees than in the case of independent contractors, Proposition 4 implies that independent contractors should face stronger explicit incentives.23

5. Applications and extensions

Hayes, Oyer, and Schaefer (2006) suggest that a firm’s top management should be more appropriately viewed as a team rather than a collection of isolated individuals. Also, in the past two decades, many companies have started to implement innovative work practices, most notable among them being probably teamwork and employee autonomy, where the latter denotes granting to the employees flexibility in deciding how to do their job. Furthermore, these two work practices are considered to be complementary, in the sense that the beneficial effects of organizing employees in teams are believed to be greater if teamwork is coupled with greater autonomy (see, e.g., DeVaro, 2006, and the references therein). Although a full treatment of these issues is beyond the scope of this paper, simple extensions of our model allow us to shed some light on optimal perk provision when agents are organized in teams and on the optimal degree of employee autonomy.

23 The idea that firms might want to provide technological perks to independent contractors is not as peculiar as it may sound. For example, independent consultants frequently get the use of a firm’s offices and equipment for the duration of their assignment. Also, the idea makes perfect sense if the perk is interpreted as the degree of employee or contractor autonomy.
5.1. Perks in teams

Consider a team consisting of \( n \geq 2 \) members, where for simplicity all agents are assumed identical. The firm’s output is given by \( y(n) = \beta \sum_{i=1}^{n} (1 + q_i m)(a_i + \varepsilon) \), where \( x_i = q_i \), \( a_i \) denotes the variable pertaining to agent \( i \), and \( \varepsilon \) follows the same normal distribution as before. Each agent’s contract is again a linear function of the firm’s output, i.e., \( w_i(y(n)) = s_i + b_i y - p_i \theta q_i a_i \), where the term \( \theta q_i a_i \) is the operating cost of the perk provided to agent \( i \).

This model differs from the single agent setting only in that the variance term is now given by \( \sigma^2(n) = n^2 \sigma^2 \). Since this does not affect the logic of Proposition 1, the perk is provided to the agents free of charge, \( p^*(n) = 0 \), even if the agents form a team. Moreover, the slope of the optimal contract, \( b^*(n) \), and the optimal amount of the perk, \( q^*(n) \), are given by (5) and (6) respectively, with \( \sigma^2 \) replaced by \( \sigma^2(n) \). Thus, Proposition 2 implies that \( q^*(n) \) increases with the team size, \( n \), i.e., agents in bigger teams should receive more perks.

5.2. Employee autonomy

In recent years, researchers have shown considerable interest in the economics behind the firms’ decisions whether to delegate decision-making authority to their lower level employees and in the incentive effects of this decision. Similarly, one can ask how much decision-making authority should be retained by the firm’s board of directors and how much should be delegated to the company’s CEO. If workers and managers value autonomy, or derive private benefits from having

\[ \text{In this formulation, the firm’s output is additively separable in the agents’ individual outputs and therefore cannot capture complementarities in production that are central to many discussions of teams in the literature. We do this for the reasons of tractability and to economize on space, but the logic of the robust comparative statics analysis based on supermodularity (e.g., Milgrom and Roberts, 1990), leads us to believe that our comparative statics results with respect to } n \text{ would only be reinforced by the introduction of complementarities between individual outputs.} \]

\[ \text{Note that unlike some papers in the literature on teams, we do not impose a balanced budget constraint } b(n) \leq 1/n. \text{ This could be justified by assuming that either the agents or the principal serve as the firm’s budget breakers, as in Holmström (1982). Alternatively, we could restrict our attention to parameter values such that } r \sigma^2 > 1, \text{ in which case the constraint would never bind because } b^*(n) \leq b_0(n) \leq 1/n \text{ for all } n. \]

\[ \text{Papers in this literature include Aghion and Tirole (1997), Marino and Matsusaka (2005), Prendergast (2002), Raith (2005), and Zabojnik (2002).} \]
decision-making authority, as in Aghion and Tirole (1997) and Burkart et al (1997), then "employee autonomy" quite easily fits the description of a technological perk, as formalized in our model.27,28

There are natural complementarities in an agent’s consumption of the perk/autonomy and his work activity, and the perk could be also consumed by the agent in connection with non-productive activities – for example, a manager could use his autonomy to take care of personal errands during work hours. The variable q would then measure the degree of autonomy granted to the agent, the parameter m the direct productivity improvement (or decline) due to the agent’s greater decision-making authority, γ would capture the degree to which the agent values autonomy, and θ would be the principal’s marginal cost of delegating authority (say, the loss of control over which projects the agent pursues). This interpretation of our model is in line with the organizational behavior literature, in which employee autonomy has long been viewed as a motivating benefit. For example, according to Hackman (1987, p. 324),29 team members are motivated when “the task provides group members with substantial autonomy for deciding about how they do the work.”

Our earlier analysis then yields the following four insights into the economic forces that determine the optimal degree of employee autonomy. The first three follow from Proposition 2, the last one from Proposition 3.

(1) First, because q∗ increases in γ, we confirm the finding in Aghion and Tirole (1997) that the degree of autonomy granted to an employee should be greater the greater is the benefit the employee derives from it. In addition, we predict that in this case there should be a negative relationship between the agent’s degree of autonomy and his formal incentives.

(2) More substantively, our second prediction says that the more difficult it is to measure an

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27 For example, Google is known for offering many generous perks, but one of the most important is considered to be that its engineers are allowed to spend 20 percent of their time on pursuing projects of their choice (Lohr, 2005).

28 One can also imagine that in some cases the agent might regard autonomy as a burden or a responsibility which generates net disutility. We consider this case of lesser interest, but our model could formally accommodate it by reversing the sign on the agent’s utility derived from the perk.

29 As cited in DeVaro (2006).
employee’s performance (i.e., the bigger is $\sigma^2$), the greater should be the degree of autonomy granted to the employee. This result is consistent with the finding in DeVaro (2006), who studies the effects of autonomy on financial performance of teams, and concludes that “the unobserved factors that make autonomy more likely (given that teams are in use) tend to lower financial performance in the presence of teams.”\textsuperscript{30} In our model, an increase in $\sigma^2$ not only makes team autonomy more likely, but also adversely affects the employee’s overall incentives (by decreasing $b^*$) and ultimately the total surplus.\textsuperscript{31}

Also, analogous to our discussion following Proposition 2, all else equal, we would expect the employees in larger organizations, privately held firms, in firms with many inter-dependent divisions, in geographically dispersed firms, and in the new economy firms to enjoy greater autonomy.

(3) More productive employees (those with greater $\beta$), for example the employees higher up in an organization’s hierarchical ladder, should be given more autonomy. This prediction sounds quite intuitive and in line with casual empirical observations.

(4) Finally, our model says that employees organized in teams should be given more autonomy than those engaged in individual production. However, applying the argument discussed in point (2) above, this does not necessarily mean that teams that have more autonomy should perform better. Our model can thus reconcile the popular belief among business practitioners, that teams should be given more autonomy, with the evidence in DeVaro (2006), that there appears to be no significant difference in performance between autonomous and non-autonomous teams.

\textsuperscript{30}DeVaro (2005) reports a similar finding for labor productivity and product quality as alternative measures of team performance.

\textsuperscript{31}This can be seen by differentiating total surplus $TS(b,q,p)$ with respect to $\sigma^2$ and applying the Envelope Theorem.
6. Conclusions

Work-related perks appear to represent an important component of employment contracts, with possible consequences for the structure of observed formal incentives. Moreover, their dual role as consumption goods and productivity enhancement tools makes these perks open to misuse and creates scope for agency problems in perk consumption, studied by economists since the first formalization of the problem by Jensen and Meckling (1976). Yet, a systematic theoretical treatment of the interplay between the productivity and the consumption motives for perk provision has been neglected in the extant literature on optimal incentive contracts. The present paper aims to fill this gap. We point out that a firm’s provision of a work-related perk can be understood by viewing the perk as a component of a complex incentive package, designed to optimally balance its conflicting insurance and incentive roles. This approach yields novel predictions regarding the conditions under which technological perks should be provided, the relationship between the provision of perks and the provision of explicit incentives, the optimal degree of employee autonomy within an organization, and the effects of the firm’s corporate governance on agency problems in managerial perk consumption.

Our framework could be extended in several directions to add more realism. For example, one could incorporate in it multitasking considerations, viewed by many economists to be equally important in practice as the concerns about optimal risk-sharing. In such an augmented framework, we would expect the value of a technological perk to the firm to depend not only upon how easy it is to measure the agent’s performance in the task that the perk is associated with, but also upon the availability of good performance measures for the tasks that are unrelated to the perk but compete for the agent’s attention.
Appendix

Proof of Lemma 1: Suppose $A(b^*, q^*, p^*) < 0$. Then (3) and $\frac{\partial a}{\partial p} < 0$ imply that $\frac{\partial TS}{\partial p} > 0$ for all $q > 0$. Hence, $p^* = \infty$. Similarly, (2) implies that $b^* = 0$, because $\frac{\partial TS}{\partial b} < 0$ for all $b \geq 0$ when $A(b^*, q^*, p^*) < 0$. Using $b^* = 0$ and $p^* = \infty$, we get that $A(b^*, q^*, p^*) < 0$ can only hold if $q^* = 0$.

Now suppose that $A(b^*, q^*, p^*) = 0$. Then (4) implies that $\frac{\partial TS}{\partial q} < 0$ for all $q \geq 0$, which means that $q^* = 0$ in this case, too. ■

Proof of Proposition 1: (i) Because the optimization problem is concave, it must be that $\frac{\partial^2 TS}{\partial q^2} \leq 0$. Consequently, the necessary and sufficient condition for $q^* > 0$ is that $\frac{\partial TS}{\partial q}|_{q=0} > 0$. From (4), this holds if and only if

$$\gamma > \gamma^* \equiv \frac{k + a^*_0(\theta - \beta m)}{a^*_0 + \frac{\beta \sigma^2}{1 + \frac{a^*_0}{g''(a^*_0)}}}$$

(a.1)

where $a^*_0$ is given by (1) evaluated at $q = 0$ (note that $a^*_0$ is independent of $\theta$) and $b = b^*_0 = \frac{1}{1 + \frac{a^*_0}{g''(a^*_0)}}$. Then, from (5), $b^* > 0$ if $q^* < \frac{\beta}{\theta - \beta m}$, which always holds from (6). The analysis in the text proves that $q^* > 0$ implies $p^* = 0$.

(ii) The above argument implies that $q^* = 0$ for $\gamma \leq \gamma^*$. The expression for $b^*$ in part (ii) then follows from $\frac{\partial TS}{\partial b} = 0$ evaluated at $q^* = 0$ and $p^* = 0$ (or from (5) evaluated at $q^* = 0$).

Finally, we prove that the FOC(1) has an interior solution when $q^* > 0$ and $p^* = 0$. To see this, let first $a = 0$. Then $LHS(1) > 0 = RHS(1)$ because $b^* > 0$. On the other hand, if $a \to \infty$, then also $RHS(1) \to \infty$, while $LHS(1) < \infty$ because $b^* < 1$ always and (6) says that $q^* < \frac{\beta}{\theta - \beta m} < \infty$.

The existence of a positive but finite $a^*$ then follows from continuity of all relevant expressions. ■

Proof of Proposition 2: (i) As shown in the proof of Proposition 1, $q^* > 0$ if and only if condition (a.1) holds. This is obviously more likely to hold the higher is $\gamma$. Moreover, because $a^*_0$

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If $TS$ were not concave in $q$, then (a.1) would be a sufficient but not always a necessary condition for $q^* > 0$. □
is independent of $m$ and $k$, $\gamma^*$ increases in $k$ and decreases in $m$. For the rest of the proof, we will use the assumption that $g(a) = a^2/2$. Under this specification, (a.1) becomes

$$\gamma > \gamma^* \equiv \frac{k}{\beta} + \frac{\theta - \beta m}{1 + r \sigma^2},$$

which immediately implies that $\gamma^*$ decreases in $\beta$, $r$, and $\sigma^2$, and increases in $\theta$.

(ii) Using $g(a) = a^2/2$, the first order condition (1) yields $a^* = b\beta(1 + qm) + \gamma q$, which after substituting in $b^*$ from (5) becomes

$$a^* = \frac{\beta + \gamma r \sigma^2 q - q(\theta - \beta m - \gamma)}{1 + r \sigma^2}.$$

Plugging this into (6) and rearranging, we get that $q^*$ is given by $q^* = B/D$, where

$$B \equiv \beta [\gamma r \sigma^2 - (\theta - \beta m - \gamma)] - k(1 + r \sigma^2), \text{ and}$$

$$D \equiv \gamma (2\theta - 2\beta m - \gamma)(1 + r \sigma^2) - (\theta - \beta m)^2.$$

Comparative statics on $q^*$. For any parameter $t$, we have that $\frac{\partial q^*}{\partial t} = \frac{\partial B/\partial t - q^* \partial D/\partial t}{D}$. Since $D > 0$ whenever $q^* > 0$, we have that $q^*$ increases in parameter $t$ if

$$\frac{\partial B}{\partial t} - q^* \frac{\partial D}{\partial t} > 0 \quad (a.3)$$

and decreases in $t$ if the reverse is true. We now investigate for each of the model’s parameters whether (a.3) or its reverse holds.

$\beta$: Differentiating (a.2) with respect to $\beta$, we see that (a.3) holds iff

$$[\gamma r \sigma^2 - (\theta - \beta m - \gamma)] + \beta m > -2mq^*[\gamma r \sigma^2 - (\theta - \beta m - \gamma)],$$
which always holds because the terms in the square brackets must be positive if \( q^* > 0 \) (since \( q^* > 0 \) implies \( B > 0 \)). Therefore, we get \( \frac{\partial q^*}{\partial \beta} > 0 \).

\( m \) : We get \( \partial B/\partial m = \beta^2 > 0 \) and \( \partial D/\partial m = 2\beta[(\theta - \beta m - \gamma) - \gamma r \sigma^2] < 0 \), where the latter inequality follows because \( B > 0 \) for \( q^* > 0 \). Thus, (a.3) always holds for \( t = m \), so that \( \frac{\partial q^*}{\partial m} > 0 \).

\( \theta \) : In this case, \( \partial B/\partial \theta = -\beta < 0 \) and \( \partial D/\partial \theta = 2\beta[\gamma r \sigma^2 - (\theta - \beta m - \gamma)] > 0 \), where the latter inequality again follows from \( B > 0 \). Thus, the reverse of (a.3) holds for \( t = \theta \), so that \( \frac{\partial q^*}{\partial \theta} < 0 \).

\( k \) : We have \( \frac{\partial q^*}{\partial k} = -\frac{(1+r\sigma^2)}{L} < 0 \).

\( r \) and \( \sigma^2 \) : These parameters only enter through \( r\sigma^2 \). Differentiating \( B \) and \( D \) with respect to \( t \equiv r\sigma^2 \), plugging in for \( q^* \) and rearranging, we see that (a.3) holds iff

\[ \gamma \beta (\theta - \beta m - \gamma) > -k(\theta - \beta m). \]

Since this always holds, we have that \( \frac{\partial q^*}{\partial r} > 0 \) and \( \frac{\partial q^*}{\partial \sigma^2} > 0 \).

\( \gamma \) : In this case, after substituting for \( q^* \), we find that (a.3) is equivalent to

\[ \beta(1+r\sigma^2)[2\gamma(\theta-\beta m)-\gamma^2]-\beta(\theta-\beta m)^2-2\beta(\theta-\beta m-\gamma)[\gamma r \sigma^2 - (\theta - \beta m - \gamma)] + 2k(\theta - \beta m - \gamma)(1+r\sigma^2) > 0. \]

Since the left hand side increases in \( k \), the condition holds for all \( k \) if it holds for \( k = 0 \). But when \( k = 0 \), the condition simplifies to \( (\theta - \beta m - \gamma)^2 + \gamma^2 r \sigma^2 > 0 \), which always holds. Hence, \( \frac{\partial q^*}{\partial \gamma} > 0 \).

**Comparative statics on \( b^* \).** From (5), \( b^* \) decreases in \( q^* \). It is then straightforward to see that \( b^* \) decreases in \( \gamma, r \) and \( \sigma^2 \) and increases in \( k \). On the other hand, \( \theta, \beta, \) and \( m \), all have both direct and indirect (through \( q^* \)) effects on \( b^* \) and these work in opposite directions. For example, the direct effect of \( \theta \) on \( b \) is negative, while the indirect effect, through a smaller \( q^* \), tends to increase \( b^* \).
θ : Rewriting (5) to get

\[ b^* = \frac{1 - \theta q^*/\beta(1 + q^*m)}{1 + r\sigma^2}, \]  

we see that \( \frac{\partial b^*}{\partial \theta} > 0 \) iff \( \frac{\partial}{\partial \theta} \left[ \frac{\theta q^*}{1 + q^*m} \right] < 0 \), which holds iff

\[ \frac{\partial q^*}{\partial \theta} < -\frac{q^*(1 + q^*m)}{\theta}. \]  

Using \( q^* = B/D \), (a.4) can be written as

\[ q^* \left[ (1 + q^*m)D - \theta \frac{\partial D}{\partial \theta} \right] < -\theta \frac{\partial B}{\partial \theta}, \]  

which always holds if the term in square brackets is negative, because \( \frac{\partial B}{\partial \theta} = -\beta \). Thus, assume that the bracketed term is positive. Then the condition holds if

\[ q^* < -\frac{\theta \partial B/\partial \theta}{(1 + q^*m)D - \theta \partial D/\partial \theta}, \]  

which can be rewritten as

\[ D + Bm - \theta \frac{\partial D}{\partial \theta} < \theta(\theta - \beta m). \]  

Since \( B \) is the only term in this inequality that depends on \( k \) and \( B \) decreases in \( k \), the inequality holds if it holds for \( k = 0 \). Setting \( k = 0 \), plugging in for \( D \), \( B \), and \( \frac{\partial D}{\partial \theta} \), and performing a few algebraic manipulations, the condition becomes \( \gamma + \beta m > 0 \), which always holds. Therefore, \( \frac{\partial b^*}{\partial \theta} > 0 \).
Form (5'), we see that \( \frac{\partial b^*}{\partial m} < 0 \) iff
\[ \frac{\partial q^*}{\partial m} \left[ \frac{\partial q^*}{\partial (1 + q^* m)} \right] > 0, \]
which holds iff
\[ \frac{\partial q^*}{\partial m} > (q^*)^2. \]  
(a.6)

Using \( q^* = B/D \), we get that (a.6) holds iff
\[ D \frac{\partial B}{\partial m} > B(B + \frac{\partial D}{\partial m}). \]

Plugging in for \( B \) and \( \frac{\partial D}{\partial m} \) from (a.2) and setting \( k = 0 \) (the only effect of \( k \) is to decrease the right hand side of the above inequality), it turns out that \( B + \frac{\partial D}{\partial m} = -B < 0 \). Since \( D \frac{\partial B}{\partial m} > 0 \), this means that the condition always holds. Therefore, \( \frac{\partial q^*}{\partial m} < 0. \)

\[ \beta : \] In this case, (5') implies that \( \frac{\partial q^*}{\partial \beta} > 0 \) iff
\[ \frac{\partial q^*}{\partial \beta} \left[ \frac{\partial q^*}{\beta (1 + q^* m)} \right] < 0, \]
or \( \frac{\partial q^*}{\partial \beta} \beta < 1 + q^* m. \) Using \( q^* = B/D \), this translates into
\[ \frac{\partial B}{\partial \beta} \frac{1}{D} < \frac{\partial B}{\partial \beta} q^* + D(1 + q^* m). \]  
(a.7)

Now, analogous to \( \gamma^* \), define \( k^* \) as the cutoff level such that \( q^* > 0 \) iff \( k < k^* \). From (4), \( k^* \) is given by
\[ k^* = \frac{\beta \gamma r \sigma^2}{1 + r \sigma^2} - (\theta - \beta m - \gamma) a^*_0. \]

Note that \( k^* < \infty \). Next, observe that the left hand side of (a.7) is independent of \( k \), while the right hand side decreases in \( k \) (because \( q^* \) decreases in \( k \)). Thus, (a.7) holds for all \( k \) if it holds for \( k = k^* \). Since by definition of \( k^* \) we have \( q^* = 0 \) when \( k = k^* \), a sufficient condition for (a.7) to hold is that \( \frac{\partial B}{\partial \beta} < D^2 \) when evaluated at \( q^* = 0 \). Differentiating \( B \) with respect to \( \beta \), we get that \( \frac{\partial B}{\partial \beta} = \frac{B}{\beta} - \beta m. \) Because \( q^* = B/D = 0 \) requires that \( B = 0 \), we only need \( -\beta m < D^2 \), which is always satisfied. Hence, \( \frac{\partial q^*}{\partial \beta} > 0. \)
Proof of Proposition 3: Using equations (7) and (8), and assuming \(a_1, a_2 > 0\), we obtain the following results:

\[
\begin{align*}
\frac{\partial a_1}{\partial b} &= \beta(1 +qm), \quad \frac{\partial a_1}{\partial q} = \gamma_1 + \beta bm - p\theta, \quad \frac{\partial a_1}{\partial p} = -q\theta, \\
\frac{\partial a_2}{\partial b} &= 0, \quad \frac{\partial a_2}{\partial q} = \gamma_2 - p\theta, \quad \frac{\partial a_2}{\partial p} = -q\theta.
\end{align*}
\]

Substituting from (7) and (8), we can define

\[
A_1 \equiv \frac{\partial TS}{\partial a_1} = (1 - b)\beta(1 +qm) - (1 - p)q\theta, \\
A_2 \equiv \frac{\partial TS}{\partial a_2} = -(1 - p)q\theta.
\]

The derivatives of \(TS\) in the choice variables \((b, p, q)\) can then be written as

\[
\begin{align*}
\frac{\partial TS}{\partial b} &= A_1 \frac{\partial a_1}{\partial b} + A_2 \frac{\partial a_2}{\partial b} - rb\beta^2(1 +qm)^2\sigma^2, \\
\frac{\partial TS}{\partial p} &= A_1 \frac{\partial a_1}{\partial p} + A_2 \frac{\partial a_2}{\partial p}, \\
\frac{\partial TS}{\partial q} &= A_1 \frac{\partial a_1}{\partial q} + A_2 \frac{\partial a_2}{\partial q} - rb\beta^2(1 +qm)m\sigma^2 - k - (\theta - \beta m - \gamma_1)a_1 - (\theta - \gamma_2)a_2.
\end{align*}
\]

Suppose \(p^{**} > 0\) and \(q^{**} > 0\). Then \(p^{**}\) is determined by \(\frac{\partial TS}{\partial p} = 0\), which, together with \(\frac{\partial a_1}{\partial p} = \frac{\partial a_2}{\partial p}\), yields \(A_1 = -A_2 = \theta(1 - p^{**})q^{**}\), which implies \(A_1(q = 0) = A_2(q = 0) = 0\). Now, the concavity of \(TS\) in \(q\) implies that \(\frac{\partial TS}{\partial q} > 0\) when evaluated at \(p^{**}\) and \(q = 0\). But

\[
\frac{\partial TS}{\partial q} \bigg|_{q=0} = -rb\beta^2m\sigma^2 - k - (\theta - \beta m - \gamma_1)b\beta < 0,
\]

which contradicts the requirement that \(\frac{\partial TS}{\partial q} > 0\). Hence, if \(q^{**} > 0\), then it must be \(p^{**} = 0\).
Given that $p^{**} > 0$, the first order conditions for $b^{**}$ and $q^{**}$ yield

\[
\begin{align*}
    b^{**} &= \frac{1 - \theta q^{**} / \beta (1 + q^{**}m)}{1 + r \sigma^2}, \\
    q^{**} &= \frac{\beta [\gamma_1 (1 + r \sigma^2) + \beta m - \theta] - k (1 + r \sigma^2)}{[(2 \theta - 2 \beta m - \gamma_1) \gamma_1 + (2 \theta - \gamma_2) \gamma_2] (1 + r \sigma^2) - (\theta - \beta m)^2}.
\end{align*}
\]  

(a.8) (a.9)

The corresponding condition for $b^*$ is identical except for the appearance of $q^*$ in the place of $q^{**}$ and the condition for $q^*$ is as in (a.9) with $\gamma_2 = 0$. Thus, equation (a.9) implies that $q^* > q^{**}$ and by the fact that the right side of (a.8) is decreasing in $q$, we have that $b^{**} > b^*$. Because $q^* = q^{**}(\gamma_2 = 0)$ and the numerator of (a.9) does not depend upon $\gamma_2$, it must be that $q^{**} > 0$ if and only if $q^* > 0$. ■

**Proof of Proposition 4:** From $\theta > \gamma_2$, we have that $(2 \theta - \gamma_2) \gamma_2$ increases in $\gamma_2$. Using (a.9), this means that $q^{**}$ decreases in $\gamma_2$, which in turn implies that $b^{**}$ increases in $\gamma_2$, because $\gamma_2$ only affects $b^{**}$ through $q^{**}$ and $b^{**}$ decreases in $q^{**}$. Also, it is immediate that $q^{**}$ decreases in $k$, which then implies that $b^{**}$ increases in $k$ because $b^{**}$ only depends on $k$ through $q^{**}$. To get the rest of the comparative statics results, write $q^*$ as $q^* = B/D$, as in the proof of Proposition 2, and $q^{**}$ as $q^{**} = B/D'$, where

\[
\begin{align*}
    B & \equiv \beta [\gamma_1 (1 + r \sigma^2) + \beta m - \theta] - k (1 + r \sigma^2), \\
    D & \equiv [(2 \theta - 2 \beta m - \gamma_1) \gamma_1 + (2 \theta - \gamma_2) \gamma_2] (1 + r \sigma^2) - (\theta - \beta m)^2, \text{ and} \\
    D' & \equiv D + \gamma_2 (2 \theta - \gamma_2) (1 + r \sigma^2).
\end{align*}
\]

Comparative statics on $q^{**}$. For any parameter $t$, we have that $\frac{\partial q^{**}}{\partial t} > 0$ if and only if $D' \frac{\partial B}{\partial t} - \frac{\partial D}{\partial t} > 0$.
$B \frac{\partial D'}{\partial t} > 0$, which will be useful to write as

$$D \frac{\partial B}{\partial t} - B \frac{\partial D}{\partial t} + \gamma_2(2\theta - \gamma_2)(1 + r\sigma^2) \frac{\partial B}{\partial t} - B \frac{\partial}{\partial t} (\gamma_2(2\theta - \gamma_2)(1 + r\sigma^2)) > 0.$$ 

If $\frac{\partial q^*}{\partial t} > 0$, then $D \frac{\partial B}{\partial t} - B \frac{\partial D}{\partial t} > 0$ so that to establish $\frac{\partial q^{**}}{\partial t} > 0$ it will be sufficient to show that

$$\gamma_2(2\theta - \gamma_2)(1 + r\sigma^2) \frac{\partial B}{\partial t} - B \frac{\partial}{\partial t} (\gamma_2(2\theta - \gamma_2)(1 + r\sigma^2)) > 0. \quad (a.10)$$

Similarly, if $\frac{\partial q^*}{\partial t} < 0$, then $D \frac{\partial B}{\partial t} - B \frac{\partial D}{\partial t} < 0$ and it is enough to show that

$$\gamma_2(2\theta - \gamma_2)(1 + r\sigma^2) \frac{\partial B}{\partial t} - B \frac{\partial}{\partial t} (\gamma_2(2\theta - \gamma_2)(1 + r\sigma^2)) < 0 \quad (a.11)$$

to establish that $\frac{\partial q^{**}}{\partial t} < 0$.

$\beta$: Since $\frac{\partial q^*}{\partial \beta} > 0$, we only need to show that (a.10) holds for $t = \beta$. In this case, (a.10) becomes

$$\gamma_2(2\theta - \gamma_2)(1 + r\sigma^2)[\gamma_1(1 + r\sigma^2) + 2\beta m - \theta] > 0$$

which is always satisfied because $\theta > \gamma_2$ and because $q^{**} > 0$ requires $\gamma_1(1 + r\sigma^2) + \beta m - \theta > 0$. Therefore, we get $\frac{\partial q^{**}}{\partial \beta} > 0$.

$m$ : Again, $\frac{\partial q^*}{\partial m} > 0$ means that we only need to check (a.10) for $t = m$, which in this case becomes $\gamma_2(2\theta - \gamma_2)(1 + r\sigma^2)\beta > 0$, which again holds. Hence, $\frac{\partial q^{**}}{\partial m} > 0$.

$\theta$ : In this case, $\frac{\partial q^*}{\partial \theta} < 0$, so we need to show that (a.11) holds for $t = \theta$. This condition becomes

$$-\gamma_2(2\theta - \gamma_2)(1 + r\sigma^2)\beta - B \gamma_2 2\theta(1 + r\sigma^2) < 0$$

which holds because $B > 0$. Thus, $\frac{\partial q^{**}}{\partial \theta} < 0$.

$\gamma_1$ : $\frac{\partial q^*}{\partial \gamma_1} > 0$ means that we only need to check that (a.10) is satisfied. For $\gamma_1$, this condition is

$$\gamma_2(2\theta - \gamma_2)(1 + r\sigma^2)\beta \gamma_1(1 + r\sigma^2) > 0$$

which holds. Therefore, $\frac{\partial q^{**}}{\partial \gamma_1} > 0$.

$r\sigma^2$ : Differentiating $B$ and $D'$ with respect to $r\sigma^2$ and rearranging, we see that the condition
\(D' \frac{\partial B}{\partial t} - B \frac{\partial D'}{\partial t} > 0\) holds iff

\[\beta \gamma_1 - k > q^{**}[(2 \theta - 2 \beta m - \gamma_1) \gamma_1 + (2 \theta - \gamma_2) \gamma_2].\]

Plugging in \(q^{**} = B/D'\), rearranging again, and cancelling out terms, this condition simplifies to

\[\theta - \beta m - \gamma_1 > 0,\]

which holds by assumption. This yields \(\frac{\partial q^{**}}{\partial r} > 0\) and \(\frac{\partial q^{**}}{\partial \sigma^2} > 0\).

**Comparative statics on \(b^{**}\).** For convenience, we reproduce here the expression for \(b^{**}\):

\[b^{**} = \frac{1 - \theta q^{**}/\beta(1 + q^{**}m)}{1 + r \sigma^2}.\]  \hspace{1cm} (a.12)

First, notice from (5') and (a.12) that \(b^{**}\) is the same function of \(q^{**}\) as \(b^{*}\) is of \(q^{*}\), and that \(b^{**}\) decreases in \(q^{**}\). It is then immediate that \(b^{*}\) decreases in \(\gamma_1\), \(r\) and \(\sigma^2\).

On the other hand, as with the effects of \(\theta\), \(\beta\) and \(m\) on \(b^{*}\), all of these parameters have both direct and indirect (through \(q^{*}\)) effects on \(b^{**}\) that work in opposite directions.

\(\theta:\) The direct effect of \(\theta\) on \(b^{**}\) is negative, while the indirect effect, through a smaller \(q^{**}\), tends to increase \(b^{**}\). From (a.12), we see that \(\frac{\partial b^{**}}{\partial \theta} > 0\) iff \(\frac{\partial}{\partial \theta} \left[\frac{\theta q^{**}}{1 + q^{**}m}\right] < 0\), which holds iff

\[\frac{\partial q^{**}}{\partial \theta} < -q^{**}(1 + q^{**}m)/\theta.\]  \hspace{1cm} (a.13)

Now, from the proof of Proposition 2, we know that

\[\frac{\partial q^{*}}{\partial \theta} < -q^{*}(1 + q^{*}m)/\theta.\]  \hspace{1cm} (a.14)

Since \(q^{**} < q^{*}\), the right hand side of (a.13) is greater than the right hand side of (a.14). Thus,
(a.13) holds if \( \frac{\partial q^*}{\partial \theta} \leq \frac{\partial q^{**}}{\partial \theta} \), i.e. if

\[
\frac{D' \frac{\partial B}{\partial \theta} - B \frac{\partial D'}{\partial \theta}}{D^2} < \frac{D \frac{\partial B}{\partial \theta} - B \frac{\partial D}{\partial \theta}}{D^2}.
\]

Because the numerators on both sides of the inequality are positive and \( D' > D \), this holds if

\[
D' \frac{\partial B}{\partial \theta} - B \frac{\partial D'}{\partial \theta} < D \frac{\partial B}{\partial \theta} - B \frac{\partial D}{\partial \theta},
\]

which reduces to condition (a.11) for \( t = \theta \). As we have shown above, this condition holds. Hence, (a.14) holds, which means that \( \frac{\partial b^{**}}{\partial \theta} > 0 \).

\( m \): From (a.12), we have that \( \frac{\partial b^{**}}{\partial m} < 0 \) iff \( \frac{\partial}{\partial m} \left[ \frac{\partial q^{**}}{\partial \theta} \frac{1 + q^* m}{1 + q^* m^2} \right] > 0 \), which holds iff

\[
\frac{\partial q^{**}}{\partial m} > (q^{**})^2.
\] (a.15)

Using \( q^{**} = B / D' \), we get that (a.15) holds iff

\[
D' \frac{\partial B}{\partial m} > B \left( B + \frac{\partial D'}{\partial m} \right).
\]

Since from the proof of Proposition 2 we know that \( D \frac{\partial B}{\partial m} > B \left( B + \frac{\partial D}{\partial m} \right) \), and because \( D' > D > 0 \), \( \frac{\partial B}{\partial m} > 0, B > 0, \) and \( \frac{\partial D'}{\partial m} = \frac{\partial D}{\partial m} \), the above condition must hold. Therefore, \( \frac{\partial b^{**}}{\partial m} < 0 \).

\( \beta \): In this case, (a.12) implies that \( \frac{\partial b^{**}}{\partial \beta} > 0 \) iff \( \frac{\partial}{\partial \beta} \left[ \frac{\partial q^{**}}{\partial \theta} \right] < 0 \), or \( \frac{\partial q^{**}}{\partial \theta} \beta < 1 + q^* m \). Using \( q^{**} = B / D' \), this translates into

\[
\frac{\partial B}{\partial \beta} < \frac{\partial B}{\partial \beta} B + D' (D' + Bm).
\] (a.16)

Again, from the proof of Proposition 2 we have that \( \frac{\partial B}{\partial \beta} < \frac{\partial B}{\partial \beta} B + D (D + Bm) \). Thus, (a.16) holds because \( \frac{\partial B}{\partial \beta} > 0, B > 0, \) and \( D' > D > 0 \). Hence, \( \frac{\partial b^{*}}{\partial \beta} > 0 \).
References


