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Contracting in Vague Environments

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Abstract

This paper shows that a new trade-off arises in the optimal contract when contracting takes place with vague information (objective ambiguity), reflecting that real-world contracting often takes place under imprecise information. The choice-theoretic framework captures a decision-maker's attitude towards vagueness by his optimism. The new trade-off is between (a) incentive provision and (b) exploitation of heterogeneity that arises endogenously because of the vague environment. Consequently, the optimal contract may distort effort in order to relax incentive compatibility and fully exploit the endogenously created heterogeneity, even when the agent is risk neutral and there is no insurance need in the relationship.

Keywords: contracts, vagueness, optimism, pessimism, incentives, objective ambiguity
JEL classification: D82, D80, D20, D86

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1 Introduction

Many real-world contracting situations are characterized by imprecise information, and it is therefore natural to directly allow for contracting environments to reflect this. The distinctive feature of the model in the present paper is to introduce vague information into the canonical principal-agent model with hidden information. This is done by assuming that the contracting parties do not know the exact probability distributions, or lotteries, over final outcomes; this is called a vague environment and can be thought of as a situation of objective ambiguity. The paper applies the choice theory for such vague environments introduced in Olszewski (2007).[1] According to this, a decision maker's attitude towards vagueness is captured by a scalar parameter interpreted as his optimism. Optimism (to be made precise later) is a relative measure that reflects the decision maker's view of what the outcome of the vagueness will be. When information is vague, optimism is therefore an important determinant of behavior.[2]

In the standard problem with risk neutral parties and no vagueness, the optimal contract is to "sell the firm to the agent". That is, the optimal contract gives the principal (she) the same utility regardless of the agent's type, interpreted as the agent (he) buying the firm for a fixed price. Such a contract is optimal because there is no insurance need in the contracting relationship when the agent is risk neutral, and selling the firm to the agent completely solves the incentive provision problem. The agent will then, on his own, choose the effort levels that maximize total surplus.

The introduction of vagueness gives rise to a new trade-off in the optimal contract. The trade-off is between, on the one hand, to provide efficient incentives for the agent and, on the other hand, to exploit heterogeneity that the principal can create endogenously because of the vague environment. Such heterogeneity results in the parties having a motive to bet on the resolution of vagueness. Whether or not the betting motive can be exploited depends on the parties' levels of optimism. Whenever the optimism parameters are at levels that do allow for exploitation of the betting motive, the optimal contract distorts effort away from the efficient level in order to give the principal more room for betting. This is

[1] Vierø (2009) provides a representation of preferences in a more general vague environment with multiple states. I use the terminology "vague environment" and "Optimism-Weighted Expected Utility" from Vierø (2009).

[2] Andersen, Fountain, Harrison, and Rutström (2009) provide experimental evidence of the relevance of optimism. The relevance of pessimism among decision makers is well accepted based on Ellsberg (1961).

possible because distortion of effort makes the binding incentive compatibility constraint less restrictive. As a result, with vagueness the optimal contract can have distortion of effort even when the agent is risk neutral and there is no insurance need in the relationship. The force for betting is stronger than the force for incentive provision.

The betting motive arises because in a vague environment the overall weights that the contracting parties assign to different final outcomes become endogenous and, generally, endogenously heterogeneous. Specifically, the endogeneity arises because the principal can affect these weights through the contract offered. The fact that the betting motive arises endogenously is a driving force behind the change in the mechanism that delivers the optimal contract.

The main result (Theorem 2) shows how the optimal contract varies with the parties' optimism parameters. It states that for large parts of the parameter space, "sell the firm to the agent" contracts are dominated by contracts that distort effort. It also shows that there exist principals who will optimally choose to sell the firm to some agents but not to others.

The paper also offers comparative statics results for how the agent's optimism affects the principal's indirect utility. The agent's optimism matters for the compensation needed for him to accept the offered contract. When deciding whether to accept a contract, the agent evaluates the tradeoff between the disutility he will suffer from completing the task and the compensation he will receive. If there is vague information about how difficult it will be to complete the task, the agent's optimism influences his perception of the disutility he will suffer. The more optimistic he is, the easier he will think it is to complete the task, and the smaller is the compensation needed for him to accept the contract.

One possible application of the model and results is to explain patterns observed in franchising. According to Lafontaine (1992) and many others, most franchisors operate through a mix of centrally operated outlets and franchisee-operated outlets, mixing the two types in varying proportions. Scott (1995) furthermore points out that within the same chain, many outlets with apparently identical attributes are operated differently, with centrally owned outlets interspersed among franchisee-owned outlets, often seemingly at random. This indicates that factors beyond measurable attributes determine the form of operation. My results show that optimism could be such a factor.

In the context of franchising, the agent is the local entrepreneur and his type can be interpreted as the entrepreneur's knowledge about local market conditions etc. The principal is the franchisor. The "sell the firm to the agent" contract is then interpreted as choosing to operate through franchising, while the alternative contracts are interpreted as operating through centrally owned outlets. My result that there exist principals who will optimally choose to sell the firm to some agents but not to others, offers a rationalization of why franchisors mix the two ways of operation and why these are interspersed among each other in a seemingly random way. Optimism presumably varies across entrepreneurs, and an economist embarking on an empirical analysis of franchising chains will generally not have entrepreneurial optimism among the variables available for the analysis. On the other hand, it is reasonable to assume that the entrepreneur's optimism will be revealed to the franchisor through the parties' negotiations about their relationship and that the contract therefore can depend on it. Thus, entrepreneurial optimism can be used to explain the observed patterns.

The present paper is related to a group of papers that consider contracting when the parties have heterogeneous beliefs. These include Adrian and Westerfield (2009) who consider a continuous-time dynamic model with moral hazard where the parties have heterogeneous beliefs, and Carlier and Renou (2005, 2006) who consider specific static problems with heterogeneous beliefs. When beliefs are heterogeneous, the parties also want to place side-bets on the resolution of uncertainty, but there is no possibility for the principal to influence the agent's weight on the different final scenarios. With precise information and heterogeneous beliefs, all differences between the contracting parties are exogenous. Vagueness, on the other hand, creates an endogenous difference in the weights the principal and agent assign to the different final scenarios.

Another group of related papers analyze contracting problems in non-standard choice theoretic settings. Mukerji (1998) considers a moral hazard problem with firms in a vertical relationship and a discrete choice set and shows that ambiguity aversion among the parties can rationalize incomplete contracts. His decision makers are characterized by an extreme degree of pessimism, while the present paper considers more general optimism/pessimism levels. Mukerji and Tallon (2004) also consider a contracting problem where agents are characterized by an extreme degree of pessimism. Lopomo, Rigotti, and Shannon (2011) consider a principal-agent model with moral hazard where the agent's beliefs are imprecise due to incomplete preferences, while the principal has precise beliefs that are nested within the agent's set of beliefs. They find that the optimal contract has the form of a flat payment plus a constant bonus for some set of output levels. In Lopomo et al. (2011)

some alternatives may be incomparable, which is not the case in the present paper. Note that these papers consider contracting with moral hazard, while the present paper considers contracting with hidden information. Vierø (2010) considers contracts that are conditional on vague signals when the contracting environment is itself precise. There, the vague signal is an extra instrument that the principal can use when designing the contract, whereas here vagueness is about the contracting environment itself.

A third group of related papers analyze mechanism design problems under uncertainty. Levin and Ozdenoren (2004) consider auctions when there is ambiguity about the number of bidders and individuals have maxmin expected utility (MEU) preferences. Bose, Ozdenoren, and Pape (2006) and Bose and Daripa (2009) study auctions when there is ambiguity about the bidders' valuation, also in the context of MEU; the former paper studies optimal static auctions while the latter allows for the auction mechanism to be dynamic. De Castro and Yanellis (2010) show that when individuals have MEU preferences, then any efficient allocation is incentive compatible. Lopomo, Rigotti, and Shannon (2009) consider mechanism design when preferences are incomplete, which gives rise to multiple relevant notions of incentive compatibility since some alternatives are incomparable. In the present paper's contracting problem, vagueness affects the agent's individual rationality constraint and the principal's objective function, while the incentive compatibility constraints are unaffected.

Note that the notion of optimism in the present paper is different from the notion of overconfidence in people's own abilities that we see in e.g. Benabou and Tirole (2002). In the present paper, optimism is a consistent feature of the individual's personality, which reflects his or her attitude towards the vagueness present in the choice environment.

The paper is organized as follows: Section 2 presents the model with vagueness. Section 3 analyzes the problem with asymmetric information and contains the results. Section 4 concludes. Proofs are given in the appendices. The online Appendix contains analysis of two special cases of the general model, which are not included in the main paper.

2 Model

Consider the canonical principal-agent problem with hidden information. A risk neutral principal wants to hire a risk neutral agent to complete a task. It is assumed that the agent's utility depends on a variable, measuring how well suited to the required task he will find himself, the value of which is realized after the contract is signed. For convenience, I will refer to this variable as the agent's efficiency level, but it could be interpreted in a variety of ways. Suppose the agent's effort can be measured by a one-dimensional variable $e \in [0, \infty)$. The principal's gross profit is a function of the agent's effort, $\pi(e)$, with $\pi(0) = 0$, first-order derivative $\pi_e(e) > 0$ $\forall e$, and second-order derivative $\pi_{ee}(e) < 0$ $\forall e$. Her Bernoulli utility function is given by her net profits,

$$u_P(w, e) = \pi(e) - w,$$

where w denotes the wage she pays to the agent.

The agent's Bernoulli utility function depends on his wage w, how much effort he chooses to exert, and his efficiency x, which affects how much disutility, denoted $g(e, x)$, he experiences from effort. It is assumed that there are only two possible values of x: the agent is either of high-efficiency type x_H or of low-efficiency type x_L. The efficiency level is unobservable for the principal, while effort is assumed to be observable and contractable.

Assume further that the agent is risk neutral with a Bernoulli utility function of the form

$$u_A(w, e, x) = v(w - g(e, x)) = w - g(e, x).$$

The disutility $g(e, x)$ is assumed to satisfy the following standard conditions: the first-order derivative w.r.t. e, $g_e(e, x) > 0$ $\forall e > 0$ and the second-order derivative w.r.t. e, $g_{ee}(e, x) > 0$ $\forall e$, such that his disutility from effort is increasing at an increasing rate, $g(0, x_H) = g(0, x_L) = g_e(0, x_H) = g_e(0, x_L) = 0$, such that the agent suffers no disutility if he does not exert any effort, and $g_e(e, x_L) > g_e(e, x_H)$ $\forall e > 0$, such that his marginal disutility from positive effort is higher if he is of low-efficiency type. Note that these conditions imply that $g(e, x_L) > g(e, x_H)$ $\forall e > 0$, that is, the disutility of any positive effort level is higher for the low-efficiency type. Finally, let $\overline{u}$ denote the agent's reservation utility, which for simplicity (and without loss of generality) is assumed equal to zero.

This paper's departure from the standard canonical principal-agent model with hidden information is to assume that the contracting environment is vague: the contracting parties only know that the probability of the agent being of high-efficiency type belongs to an interval $Q = [a, b] \subseteq [0, 1]$, with $a < b$, which is common to both parties.[3] The parties hence have common but vague knowledge of this probability.

[3]If $a = b$, the model reduces to the standard expected utility model with no vagueness.

Contracting is assumed to take place ex-ante, i.e. before the agent knows his type. Ex-ante contracting has two stages: the agent first agrees to a menu of wage-effort pairs $((w_H, e_H), (w_L, e_L)) \in \Re \times \Re_+ \times \Re \times \Re_+$, one pair intended for each type. Then, once he learns his type, the agent selects one of the wage-effort pairs in the menu by announcing his type. I assume that the principal is unable to observe the agent's efficiency level at any point in time, hence there is asymmetric information at the interim.

For an incentive compatible contract, the agent will truthfully reveal his type.[4] He will thus exercise effort e_H and be paid wage w_H when he is of high-efficiency type x_H and exercise effort e_L and be paid wage w_L when he is of low-efficiency type x_L. Given the interval Q of the probability of high-efficiency type, each incentive compatible contract defines a set $h(w_H, e_H, w_L, e_L)$ of lotteries over final outcomes $z = (w, e, x)$ in the following sense: Since the contract is incentive compatible, the wage-effort pair will be (w_H, e_H) whenever the agent is of type x_H and (w_L, e_L) whenever the agent is of type x_L, and the lotteries in $h(w_H, e_H, w_L, e_L)$ therefore all have support $\{(w_H, e_H, x_H), (w_L, e_L, x_L)\}$. Let $p_H \equiv \Pr(w_H, e_H, x_H)$, $p_L \equiv \Pr(w_L, e_L, x_L)$, and $p = (p_H, p_L)$. Outcome (w_H, e_H, x_H) will obtain if the agent is of type x_H, while (w_L, e_L, x_L) will obtain if the agent is of type x_L. Hence, since the probability of x_H belongs to Q, the set $h(w_H, e_H, w_L, e_L)$ is given by

$$h(w_H, e_H, w_L, e_L) = \{p \mid p_H \in Q \text{ and } p_L = 1 - p_H\}. \tag{1}$$

For ease of notation, the dependency on (w_H, e_H, w_L, e_L) will be suppressed, and the set of lotteries simply written as h.

As the previous paragraph describes, a choice of a contract is implicitly a choice of a set of lotteries. Hence, the objects of choice are in fact sets of lotteries. Because the decision maker does not know a precise lottery over outcomes, but rather only a set of possible lotteries, the environment is vague. The situation can thus be thought of as one where the parties are faced with objective ambiguity.

To model the contracting problem in this vague environment, I assume that the contracting parties have preferences that are represented by Optimism-Weighted Expected Utility, meaning that both the principal and the agent maximize utility of the following form:

$$OWEU_j(h) = \alpha_j \sum_{i \in \{L,H\}} \overline{p}_j(h)(z_i) u_j(z_i) + (1 - \alpha_j) \sum_{i \in \{L,H\}} \underline{p}_j(h)(z_i) u_j(z_i), \tag{2}$$

[4] See the revelation principle in Theorem 1.

where h is the set (1) of lotteries induced by the contract under evaluation, each sum is over the support of the relevant lottery, $j \in \{P, A\}$, u_j is j's Bernoulli utility function defined over outcomes $z_i = (w_i, e_i, x_i)$, and α_j is a parameter that captures j's degree of optimism. It is assumed that $\alpha_A \in [0, 1]$ and that $\alpha_P \in (0, 1)$. The case of $\alpha_P \in \{0, 1\}$ is analyzed in the online Appendix. Finally, $\overline{p}_j(h)$ and $\underline{p}_j(h)$ are, respectively, the best and worst lotteries from j's point of view given the set h of lotteries. That is,

$$\overline{p}_j(h) = \arg\max_{p \in h} \sum_{i \in \{L,H\}} p(z_i) u_j(z_i) \text{ and } \underline{p}_j(h) = \arg\min_{p \in h} \sum_{i \in \{L,H\}} p(z_i) u_j(z_i).$$

The interpretation of these preferences is that the decision maker (acts as if he/she) computes the von Neumann-Morgenstern utility of the best lottery and of the worst lottery in the set of lotteries induced by the contract. The decision maker then weighs the two together where the weight α_j on the best lottery can be interpreted as a measure of the decision maker's level of optimism, reflecting his or her attitude towards objective ambiguity.

The representation in (2) is axiomatized in Olszewski (2007), see also Vierø (2009) from where the terminology OWEU is taken. Ghirardato, Maccheroni, and Marinacci (2004) provide an axiomatization of similar preferences in the Anscombe-Aumann setting, as do Gilboa and Schmeidler (1989) for the case of $\alpha = 0$. Ahn (2008) provides an alternative representation of preferences in vague environments.

It is important to note that which lotteries are best and worst depend on the contract offered. If, for example, a contract causes the agent to be better off when of high efficiency type than when of low efficiency type, the best lottery in h from his point of view is the one that assigns highest possible probability to z_H and lowest possible probability to z_L. On the contrary, for a contract with which the agent will be better off when of low-efficiency type, the best lottery in h from his point of view is the one that assigns lowest possible probability to z_H and highest possible probability to z_L. Likewise for the principal. This implies that the weights the parties assign to the different final scenarios, i.e. to z_H and z_L, are influenced by the contract and are therefore endogenous and generally heterogeneous. This is the key consequence of vagueness, and this is what causes a new trade-off in the optimal contract.

3 Contracting in vague environments

As is usually the case under asymmetric information, the principal must rely on the agent to reveal his efficiency level. Therefore, the first important step towards analyzing the problem is to show that the revelation principle also holds in a world with vagueness.[5]

Theorem 1 (Revelation principle for vague environments)**.** *In a vague environment, any general incentive compatible contract can be implemented with a truthful revelation mechanism.*

Proof: See Appendix A.

It follows from Theorem 1 that we can restrict our search for optimal incentive compatible contracts to truthful revelation mechanisms.

3.1 The principal's problem

Having established that the revelation principle holds, the principal can find the optimal contract by maximizing her OWEU subject to a participation constraint and two incentive compatibility constraints for the agent:

$$\max_{w_L, e_L \geq 0, w_H, e_H \geq 0} \quad \alpha_P \Big\{ \overline{p}_{H,P}(h)\big(\pi(e_H) - w_H\big) + (1 - \overline{p}_{H,P}(h))\big(\pi(e_L) - w_L\big) \Big\} \\ +(1-\alpha_P)\Big\{ \underline{p}_{H,P}(h)\big(\pi(e_H) - w_H\big) + (1 - \underline{p}_{H,P}(h))\big(\pi(e_L) - w_L\big) \Big\}$$

subject to

$$\begin{aligned} &\alpha_A \Big\{ \overline{p}_{H,A}(h)\big(w_H - g(e_H, x_H)\big) + (1 - \overline{p}_{H,A}(h))\big(w_L - g(e_L, x_L)\big) \Big\} \\ &+(1-\alpha_A)\Big\{ \underline{p}_{H,A}(h)\big(w_H - g(e_H, x_H)\big) + (1 - \underline{p}_{H,A}(h))\big(w_L - g(e_L, x_L)\big) \Big\} \geq 0, \end{aligned} \tag{PC}$$

$$w_H - g(e_H, x_H) \geq w_L - g(e_L, x_H), \tag{IC_H}$$

$$w_L - g(e_L, x_L) \geq w_H - g(e_H, x_L), \tag{IC_L}$$

where $\overline{p}_{H,P}(h)$ and $\underline{p}_{H,P}(h)$ are the high-type probabilities in Q that are best and worst, respectively, from the principal's point of view given the contract $((w_H, e_H), (w_L, e_L))$,

[5] Note that the proof of Theorem 1 does not require that the agent is risk neutral.

and $\overline{p}_{H,A}(h)$ and $\underline{p}_{H,A}(h)$ are the high-type probabilities in Q that are best and worst, respectively, from the agent's point of view given the contract $((w_H, e_H), (w_L, e_L))$. The principal chooses between all the feasible contracts she could offer, while the agent chooses between accepting the offered contract or taking the outside option. Denote the contract that solves the principal's problem $((w_H^*, e_H^*), (w_L^*, e_L^*))$.[6]

Importantly, the best and worst lotteries, $\overline{p}_j(h)$ and $\underline{p}_j(h)$, for both the principal and the agent depend on the contract offered. Hence the contracting problem with vagueness does not reduce to a standard problem with heterogeneous beliefs. Since the best and worst lotteries depend on the contract, so do the weights the parties assign to the different final scenarios. For example, the principal's and the agent's overall weights on high-efficiency type are, respectively,

$$\alpha_P \overline{p}_{H,P}(h) + (1 - \alpha_P)\underline{p}_{H,P}(h)$$

and

$$\alpha_A \overline{p}_{H,A}(h) + (1 - \alpha_A)\underline{p}_{H,A}(h).$$

Thus, vagueness can endogenously create a difference in the parties' weights. With precise information and heterogeneous beliefs, any differences in the weights are, on the contrary, exogenous. This is a fundamental difference between vagueness and a situation with precise information and heterogeneous beliefs, and is the key observation to understand the effects of vagueness and the results that follow.

3.2 Optimal contracts

In the canonical problem with a risk neutral agent and no vagueness, the optimal contract implements the first best. Specifically, since the agent is risk neutral, there is no insurance need in the contracting relationship, so the optimal contract gives the principal a fixed utility, while the agent bears all the risk. This contract completely solves the incentive provision problem. The classical interpretation is that the principal sells the firm for a fixed price to the agent, who then chooses the first-best effort levels that maximize total surplus given his type, i.e. the levels that maximize $\pi(e) - g(e, x)$. The fixed price the

[6] Note that pooling contracts are special cases of separating contracts, since the constraint set for the corresponding pooling problem is a subset of the constraint set for the separating problem. Therefore, pooling contracts need not be considered separately. If the optimal contract is a pooling contract, this will emerge as a solution to the separating problem in which $w_H^* = w_L^*$ and $e_H^* = e_L^*$.

principal sells the firm for equals the value of the firm to the agent. I refer to this contract as the "sell the firm to the agent" contract. Let e_L^o denote the effort level that maximizes total surplus $\pi(e) - g(e, x_L)$ for the low-efficiency type, and let e_H^o denote the effort level that maximizes total surplus $\pi(e) - g(e, x_H)$ for the high-efficiency type.

When there is vagueness, the optimal contract need not be a "sell the firm to the agent" contract. Rather, for many levels of the parties' optimism it can be optimal for the principal to distort effort away from the levels that maximize total surplus given the agent's type. That is, the principal can do better by offering a contract that either has distortion when the agent is of high-efficiency type ("distortion at the top") or has distortion when the agent is of low-efficiency type ("distortion at the bottom").

The reason why distortion of effort is optimal for the principal is that in the presence of vagueness she can create endogenous heterogeneity in the overall weights the parties assign to the two possible final outcomes by requiring a higher price for the firm from one of the types. The incentive compatibility constraint for one of the agent's types, specifically for the type to which the principal assigns relatively higher weight than the agent, will then bind. Distortion of effort for the other type can be used to move the binding incentive compatibility constraint so that it is less restrictive (although still binding).

The intuition can be illustrated via a few examples. Consider first the example where $Q = [0, 1]$, $\alpha_A = 0$, and $\alpha_P = 1$.[7] Since $\alpha_A = 0$, the agent only assigns positive weight to the worst possible scenario for him, while since $\alpha_P = 1$, the principal only assigns positive weight to the best possible scenario for her. The incentive compatibility constraints (IC_H) and (IC_L) together with the assumptions about the function $g(e, x)$ imply that a contract must give the high-efficiency type agent at least as high utility as the low-efficiency type, and strictly higher utility if the contract involves positive effort for type x_L. Thus, with $\alpha_A = 0$, the agent's overall utility will equal his utility when he is of the low-efficiency type.

Suppose that the principal offers the best "sell the firm to the agent" contract. With this contract the price she gets for the firm is equal to the value of the firm to the agent. Denote this value by r. Since effort will be at the first-best level and the agent assigns all his weight to being of low-efficiency type, $r = \pi(e_L^o) - g(e_L^o, x_L)$, which is the total surplus (or the firm's net profits) when the agent is of type x_L.[8] Since the principal gets

[7] The case of $\alpha_P = 1$ is analyzed in detail in the online Appendix.

[8] Recall that the value of the agent's outside option is zero.

the payment r from the agent regardless of his type, her utility is equal to r.

Notice, however, that the agent does not care about how much he has to pay for the firm if he is of the high-efficiency type. Consider therefore an alternative contract that involves a payment higher than r from the agent to the principal when the agent is of type x_H and maintains a payment of r when he is of type x_L. With this contract the principal will be best off when the agent is of the high-efficiency type, and since $\alpha_P = 1$, she assigns all her weight to this scenario. The higher payment required from the high-efficiency type does not affect the agent's utility, so the agent will still participate in the contract. However, the principal is bound by incentive compatibility. If the principal requires too high a payment from the high-efficiency type, this type will pretend to be of type x_L.

Interestingly, the principal can relax this bound by distorting the effort level for the low-efficiency type downwards, in combination with paying this type a lower wage (or, with the alternative interpretation, increase the price this type pays for the firm), where the decrease in wage exactly equals the amount of disutility the type x_L agent saves due to providing lower effort. Since the high-efficiency type would save less disutility by providing lower effort than the low-efficinecy type does, the contract with distorted effort makes pretending to be type x_L less attractive for type x_H. As a result, the principal can raise the payment she requires from the high-efficiency type, and thus her own utility, a little further without violating incentive compatibility. Hence, the optimal contract has distortion at the bottom. As a matter of fact, in this example where the principal assigns zero weight to type x_L and the agent assigns zero weight to type x_H, the optimal contract distorts the effort for the low-efficiency type as far as possible, that is, all the way down to zero, since this allows for the highest payment from the high-efficiency type to the principal.[9]

Next, consider the example where $\alpha_A = \alpha_P = \frac{3}{4}$, still with $Q = [0,1]$. In this example the agent assigns positive weight to both of his potential types and the weighted average of his two types' utility must be at least as high as his reservation utility. With a "sell the firm to the agent" contract, the constant price the agent would pay for the firm would be $\widehat{r} = \frac{3}{4}\big(\pi(e_H^o) - g(e_H^o, x_H)\big) + \frac{1}{4}\big(\pi(e_L^o), g(e_L^o, x_L)\big)$, giving the principal utility of $\widehat{r}$.

Suppose the principal instead offers a contract that involves the low-efficiency agent paying more than $\widehat{r}$ for the firm and the high-efficiency agent paying less than $\widehat{r}$. With this contract the principal will be best off when the agent is of type x_L. The principal thus

[9]The optimal contract has $(w_H^*, e_H^*, w_L^*, e_L^*) = (g(e_H^o, x_H), e_H^o, 0, 0)$.

assigns weight $\alpha_P = \frac{3}{4}$ to the agent being of type x_L, and weight $\frac{1}{4}$ to him being of type x_H. The agent, on the other hand, assigns weight $\frac{1}{4}$ to being of type x_L and $\frac{3}{4}$ to being of type x_H, since (IC_H) and (IC_L) together with the assumptions about the function $g(e, x)$ imply that he will be best off when of the high-efficiency type. Because the principal's weight on type x_L is higher than the agent's, the rate at which she can trade off higher payments from the low-efficiency type for lower payments from the high-efficiency type and still satisfy the agent's participation constraint is favorable to her.

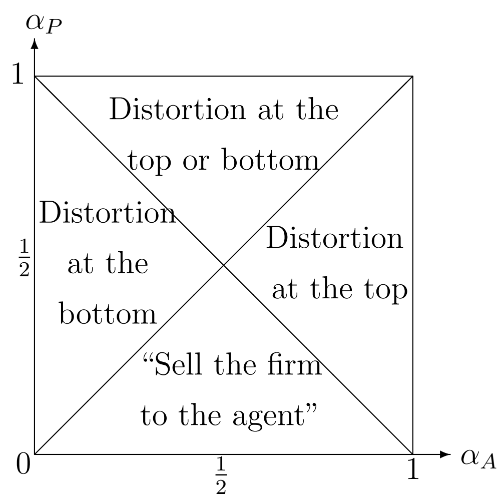
The contract still has to be incentive compatible, so there is a bound for how large a payment for the firm the principal can require from the low-efficiency type relative to that from the high-efficiency type. Beyond that, the low-efficiency type would pretend to be of type x_H. The principal can, however, relax this bound by distorting effort for the high-efficiency type upwards combined with paying him a higher wage to offset the extra amount of disutility he will incur as a result of the higher effort. Since the low-efficiency type has a higher marginal cost of effort for any $e > 0$, the higher wage is not sufficient to offset the extra disutility this type would suffer from the higher effort. Thus, he would not pretend to be of the high-efficiency type. The principal can therefore decrease the wage she pays to the low-efficiency type (or, with the alternative interpretation, increase the price the low-efficiency type pays for the firm) a little further. The optimal contract hence has distortion at the top. Since the principal does assign positive weight to the agent being of high-efficiency type, there is a limit to how much the principal will distort effort for type x_H. Beyond that, the increase in the wage to the x_H agent needed to compensate him for extra effort is too high to make it worthwhile. This pins down the optimal contract.[10]

Finally, consider the example where $\alpha_A = \frac{3}{4}$ and $\alpha_P = 0$, again with $Q = [0, 1]$. Since the principal only cares about the worst possible scenario for her, it is in her interest to keep this worst possible scenario as good as possible. This is done with a "sell the firm to the agent" contract with which the principal will be equally well off regardless of the agent's type and get utility equal to the constant payment $\widehat{r} = \frac{3}{4}\big(\pi(e_H^o) - g(e_H^o, x_H)\big) + \frac{1}{4}\big(\pi(e_L^o) - g(e_L^o, x_L)\big)$.

The intuition behind distortion at the bottom, distortion at the top, and selling the firm to the agent in the three examples considered above, respectively, is also the intuition behind the results in Theorem 2, which characterizes the optimal contracts across (α_A, α_P)-space. The optimal contracts are also depicted in Figure 1.

[10]The details of the optimal contract are given by Case 1 in Appendix B.

Figure 1: Optimal contracts



Note: Characterization of optimal contracts across (α_A, α_P)-space, see Theorem 2.

Theorem 2 (Optimal contracts). *The optimal contract varies across (α_A, α_P)-space as depicted in Figure 1. Specifically:*

If $\alpha_P \leq \alpha_A \leq 1 - \alpha_P$, the optimal contract is a "sell the firm to the agent" contract with $e_L^* = e_L^o$ and $e_H^* = e_H^o$.

If $\alpha_A < \alpha_P$ and $\alpha_A \leq 1 - \alpha_P$, the optimal contract has distortion at the bottom with $e_H^* = e_H^o$ and $e_L^* < e_L^o$.

If $\alpha_A \geq \alpha_P$ and $\alpha_A > 1 - \alpha_P$, the optimal contract has distortion at the top with $e_L^* = e_L^o$ and $e_H^* > e_H^o$.

If $1 - \alpha_P < \alpha_A < \alpha_P$, there exists $\hat{\alpha} \in (1 - \alpha_P, \alpha_P)$ such that the optimal contract has distortion at the bottom for all $\alpha_A < \hat{\alpha}$ and has distortion at the top for all $\alpha_A \geq \hat{\alpha}$.

Proof: See Appendix B.

I will now explain the intuition behind the results in Theorem 2 in more generality. A contract that does not sell the firm to the agent results in endogenous heterogeneity in the overall weights that the parties assign to the different final scenarios. This endogenous heterogeneity is a consequence of the vague environment. It has two possible sources. The first source of heterogeneity is the potential difference in which lotteries are the best and worst for each of the contracting parties, and the second source is the parties' attitudes towards vagueness as captured by the optimism parameters (α_A, α_P). The former depends on the contract offered. For instance, if the contract offered is such that $\pi(e_H) - w_H < \pi(e_L) - w_L$ and $w_H - g(e_H, x_H) > w_L - g(e_L, x_L)$, the best lottery for the principal has $p_H = a$ while the best lottery for the agent has $p_H = b$. Likewise, the worst lottery for the principal has $p_H = b$ while the worst lottery for the agent has $p_H = a$. Therefore, in this example the overall weight on high-efficiency type in the principal's utility function is $\alpha_P a + (1 - \alpha_P)b$, while in the agent's utility function it is $\alpha_A b + (1 - \alpha_A)a$.

The heterogeneity in the parties' overall weights opens up the possibility of betting, or trading, on the resolution of vagueness through the contract. The principal is willing to trade off a lower utility in the scenario to which she assigns low weight relative to the agent, for a higher utility in the scenario to which she assigns high weight relative to the agent. This introduces a new trade-off in the optimal contract: a trade-off between providing incentives for the agent to undertake the desired level of effort and exploiting the betting motive.

Distortion of effort occurs to give the principal more room for betting on the differences in the parties weights. This is possible because distorting effort moves the binding incentive compatibility constraint so that it is less restrictive. A high-efficiency agent saves less disutility from a lower level of effort than does a low-efficiency agent, while a low-efficiency agent suffers more from a higher level of effort than does a high-efficiency agent. Therefore, the principal will distort effort for the type he cares relatively less than the agent about.

With a contract that has distortion at the top, the parties have different best and worst scenarios. The agent is best off and the principal is worst off when the agent is of type x_H, while the agent is worst off and the principal is best off when the agent is of type x_L. Hence, $\overline{p}_{H,A} = \underline{p}_{H,P} = b$ and $\underline{p}_{H,A} = \overline{p}_{H,P} = a$. According to Theorem 2, distortion at the top is optimal when there is sufficient optimism among the parties. The intuition is that joint optimism is large enough that the principal can exploit heterogeneity in the parties' weights on final outcomes and therefore it is worthwhile for her to generate such a difference. That is, the parties' joint optimism ensures that their difference in weights will be large enough that the principal can take advantage of it and offer a contract that makes herself better off than she would be if she were to sell the firm to the agent.

With a contract that has distortion at the bottom, the parties share the same best and worst case scenarios. Both will be best off when the agent is of type x_H and worst off when the agent is of type x_L. Hence, $\overline{p}_{H,A} = \overline{p}_{H,P} = b$ and $\underline{p}_{H,A} = \underline{p}_{H,P} = a$. By Theorem 2, two conditions must hold simultaneously in order for the optimal contract to have distortion at the bottom. First, the agent must be relatively pessimistic such that he assigns lower overall weight than the principal to the best scenario given the contract, which is that he is of type x_H, and second, joint optimism must be too low for it to be worthwhile for the principal to use distortion at the top to get the parties to have different best and worst lotteries. Since the principal's overall weight on type x_H is higher than the agent's the betting motive arises and the principal will distort effort for the low-efficiency type in order to fully exploit it.

Theorem 2 finally states that when $\alpha_P \leq \alpha_A \leq 1 - \alpha_P$, the optimal contract is a "sell the firm to the agent" contract. A necessary condition for optimality of distortion at the top is that

$$\alpha_A \overline{p}_{H,A}(h) + (1 - \alpha_A)\underline{p}_{H,A}(h) > \alpha_P \overline{p}_{H,P}(h) + (1 - \alpha_P)\underline{p}_{H,P}(h) \tag{3}$$

given the contract, that is, the principal's overall weight on high-efficiency type is lower than the agent's *given the contract*. With distortion at the top, $\overline{p}_{H,A} = \underline{p}_{H,P} = b$ and $\underline{p}_{H,A} = \overline{p}_{H,P} = a$. Therefore, condition (3) implies that

$$\alpha_A b + (1 - \alpha_A)a > \alpha_P a + (1 - \alpha_P)b. \tag{4}$$

Likewise, a necessary condition for distortion at the bottom is that

$$\alpha_A \overline{p}_{H,A}(h) + (1 - \alpha_A)\underline{p}_{H,A}(h) < \alpha_P \overline{p}_{H,P}(h) + (1 - \alpha_P)\underline{p}_{H,P}(h) \tag{5}$$

given the contract, i.e. that the principal's overall weight on high-efficiency type is higher than the agent's *given the contract*. With distortion at the bottom, $\overline{p}_{H,A} = \overline{p}_{H,P} = b$ and $\underline{p}_{H,A} = \underline{p}_{H,P} = a$. Thus, condition (5) implies that

$$\alpha_A b + (1 - \alpha_A)a < \alpha_P b + (1 - \alpha_P)a \tag{6}$$

Neither condition (4) nor condition (6) is satisfied when $\alpha_P \leq \alpha_A \leq 1 - \alpha_P$. As a result, the only option for the principal in this case is to sell the firm to the agent. With a "sell the firm to the agent" contract, the principal will be equally well off regardless of the agent's type. Therefore, such a contract enables that the parties' weights are exactly equal, thereby giving no further desire for betting. Since there is no betting, none of the incentive compatibility constraints bind and the optimal contract has $e_L^* = e_L^o$ and $e_H^* = e_H^o$. Hence, there is no need to distort effort to make the incentive compatibility constraints less restrictive.

The most important lesson to learn from the results is that vagueness, or objective ambiguity, introduces a new trade-off in the optimal contract. This trade-off is between, on the one hand, to provide efficient incentives for the agent and, on the other hand, to exploit the betting motive that arises as a consequence of the principal's ability to create endogenous heterogeneity in the vague environment. Therefore, whenever the parties' optimism parameters are at levels that allow for exploitation of the betting motive, the optimal contract distorts effort away from the efficient level for one of the agent's types. Thus, with vagueness there can be distortion of effort even when the agent is risk neutral and there is no insurance need in the contracting relationship. The force for betting is stronger than the force for incentive provision.

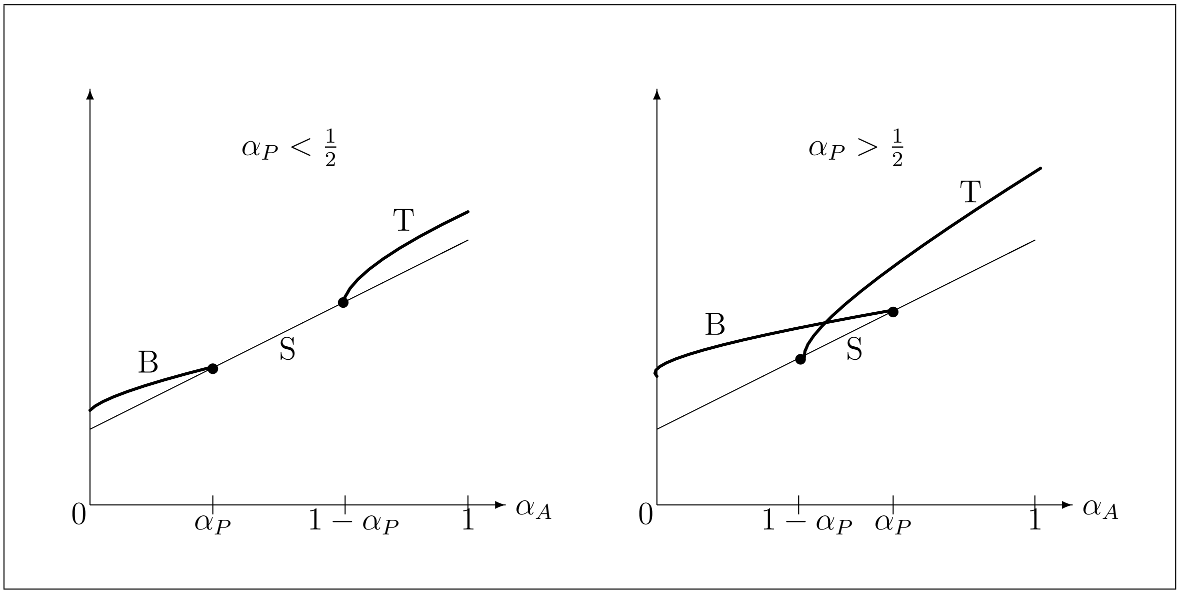
Finally, notice that according to Theorem 2 there exist principals who will choose a "sell the firm to the agent" contract for some agents, but not for other agents. If the principal is interpreted as being a franchisor, the agent as being a local entrepreneur whose type is his knowledge about local market conditions etc., and "sell the firm to the agent" contracts are interpreted as franchising, the results can be used to explain the widespread phenomenon that franchisors use a mix of centrally operated and franchisee-operated outlets and that these are interspersed among each other in a seemingly random way, as documented in Lafontaine (1992) and Scott (1995). The results show that entrepreneurial optimism can be a factor that determines the optimal operational form. Since optimism presumably varies across entrepreneurs and will typically not be among the variables available for empirical analysis, this can explain the observed patterns.

3.3 Comparative statics

Corollary 1 presents comparative statics with respect to the agent's optimism parameter.

Corollary 1. *The principal's indirect utility is non-decreasing in the agent's level of optimism.*

Figure 2: The principal's indirect OWEU



Note: The figure displays the principal's indirect OWEU as a function of α_A. The left panel is for $\alpha_P < \frac{1}{2}$ and the right is for $\alpha_P > \frac{1}{2}$. The curves T, B, and S are, respectively, the principal's utility with distortion at the top, distortion at the bottom, and "sell the firm to the agent" contracts.

Proof: For contracts that have distortion of the top, the result follows from expression (16) in Appendix B and the discussion in the paragraph following it. For contracts that have distortion of the bottom, the result follows from expression (22) in Appendix B and the discussion immediately below it. For "sell the firm to the agent" contracts, the result follows from expression (25) in Appendix B. Finally, when $\alpha_A = \alpha_P$, (22)=(24) and is equal to (25) in value, while when $\alpha_A = 1 - \alpha_P$, (16)=(24) and is equal to (25) in value. The latter argument shows that the result holds over the entire domain.■

Figure 2 outlines the result in Corollary 1, that is, the principal's indirect OWEU as a function of the agent's level of optimism. The thicker curves labeled T and B show the principal's utility if she offers a contract that has distortion at the top respectively a contract that has distortion at the bottom, while the thinner curve labeled S shows her utility if she sells the firm to the agent. The curves reflect that for each of the three types of contracts, the principal's utility is non-decreasing in the agent's optimism. They also reflect that when $\alpha_A = \alpha_P$ the principal derives the same utility from the best contract with distortion at the bottom as from the best "sell the firm to the agent" contract. Likewise, when $\alpha_A = 1 - \alpha_P$ she derives the same utility from the best contract with distortion at the top as from the best "sell the firm to the agent" contract. For any level of α_A, the principal's utility of the optimal contract is given by the maximal of the curves.

The intuition for why the principal's utility is non-decreasing in the agent's optimism is that optimism affects profits through the compensation of the agent. When deciding whether to accept an offered contract, the agent evaluates the tradeoff between the disutility he will suffer from completing the task and the compensation he will receive. If there is vague information about how difficult it will be to complete the task, the agent's optimism influences his perception of the disutility he will suffer. The more optimistic he is, the easier he will think it is to complete the task, and the smaller is the compensation needed for him to accept the contract offered.

4 Concluding remarks

The analysis in this paper has shown that vagueness, or objective ambiguity, introduces a new trade-off in the optimal contract. The trade-off is between providing efficient incentives for the agent on the one hand and on the other hand exploiting heterogeneity that the principal can create endogenously because of the vague environment. Such heterogeneity results in the parties having a motive to bet on the resolution of vagueness. Therefore, whenever the parties' optimism parameters are at levels that allow for exploitation of the betting motive, the optimal contract distorts effort away from the efficient level for one of the agent's types. Distortion of effort occurs to give the principal more room for betting on the differences in the parties weights. This is possible because distorting effort makes the binding incentive compatibility constraint less restrictive. The result is that with vagueness there can be distortion of effort even when the agent is risk neutral and there is no insurance need in the contracting relationship. The force for betting is stronger than the force for incentive provision.

Appendix A: Proof of Theorem 1

To see that the revelation principle also holds in vague environments, it is sufficient to prove that the best and worst lotteries in each state are the same for the two mechanisms mentioned in the statement of the theorem. The rest of the proof is analogous to the proof when there is no vagueness.

Suppose the optimal incentive compatible contract specifies the set T of strategies for the agent and an outcome $f(t)$, where $f : T \to (W, E)$ and (W, E) is the space of possible wages and effort levels. Denote the agent's optimal strategy with this contract by t^*.

Alternatively, the principal could use a revelation mechanism and let the agent announce his type: $\tilde{x}_H$ denotes an announcement of high-efficiency type, while $\tilde{x}_L$ denotes an announcement of low-efficiency type. Let $\tilde{w}(x)$ and $\tilde{e}(x)$ be defined by

$$\tilde{w}(\tilde{x}_H) = w(t^*(x_H)),\ \tilde{e}(\tilde{x}_H) = e(t^*(x_H)),\ \tilde{w}(\tilde{x}_L) = w(t^*(x_L)),\ \text{and}\ \tilde{e}(\tilde{x}_L) = e(t^*(x_L)).$$

Then, since the original contract $\{(t, f(t)) : t \in T\}$ was incentive compatible it follows that $v(\tilde{w}(\tilde{x}_H), \tilde{e}(\tilde{x}_H), x_H) \geq v(\tilde{w}(\tilde{x}_L), \tilde{e}(\tilde{x}_L), x_H)$ and $v(\tilde{w}(\tilde{x}_L), \tilde{e}(\tilde{x}_L), x_L) \geq v(\tilde{w}(\tilde{x}_H), \tilde{e}(\tilde{x}_H), x_L)$, i.e., that the agent will tell the truth. Also,

$$v(f(t^*(x_H)), x_H) \geq v(f(t^*(x_L)), x_L) \Leftrightarrow v(\tilde{w}(\tilde{x}_H), \tilde{e}(\tilde{x}_H), x_H) \geq v(\tilde{w}(\tilde{x}_L), \tilde{e}(\tilde{x}_L), x_L),$$

which means that the best and worst lotteries for the revelation mechanism are the same as for the original contract.■

Appendix B: Proof of Theorem 2

When $\alpha_P \in (0, 1)$, the principal always assigns non-zero overall weight to both high-efficiency and low-efficiency type, that is, that $\alpha_P \overline{p}_{H,P}(h) + (1 - \alpha_P)\underline{p}_{H,P}(h) \in (0, 1)$. In the online Appendix, I consider separately the two cases $\alpha_P = 0$ and $\alpha_P = 1$, which require special treatment since the principal's overall weight on one scenario may then be zero.

The principal's objective function is non-differentiable at the points (w_H, e_H, w_L, e_L) for which $\pi(e_H) - w_H = \pi(e_L) - w_L$, which is where $\overline{p}_P(h)$ and $\underline{p}_P(h)$ switch. The value of the objective function at any interior candidate for solution therefore has to be compared to its value at these non-differentiability points. Below, I first find solutions to the first-order necessary conditions for interior solutions to the principal's problem, then I do comparison of values.[11]

[11] The other possibilities of non-differentiability of the Lagrangian, i.e. the points (w_H, e_H, w_L, e_L) such that $w_H - g(e_H, x_H) = w_L - g(e_L, x_L)$, are not in the constraint set since they violate (IC_H). Hence, it suffices to consider corner contracts for which $\pi(e_H) - w_H = \pi(e_L) - w_L$.

Note that pooling contracts are special cases of separating contracts, since the constraint set for the corresponding pooling problem is a subset of the constraint set for the separating problem. Therefore, pooling contracts need not be considered separately. If the optimal contract is a pooling contract, this will emerge as a solution to the separating problem in which $w_H^* = w_L^*$ and $e_H^* = e_L^*$.

The Lagrangian for the principal's problem of finding the best contract when she faces a risk neutral agent is

$$\begin{aligned}
\mathscr{L} &= \big(\alpha_P \overline{p}_{H,P}(h) + (1-\alpha_P)\underline{p}_{H,P}(h)\big)\big(\pi(e_H) - w_H\big) \\
&\quad + \big[1 - \big(\alpha_P \overline{p}_{H,P}(h) + (1-\alpha_P)\underline{p}_{H,P}(h)\big)\big]\big(\pi(e_L) - w_L\big) \\
&+ \gamma\Big[\big(\alpha_A \overline{p}_{H,A}(h) + (1-\alpha_A)\underline{p}_{H,A}(h)\big)\big(w_H - g(e_H, x_H)\big) \\
&\quad + \big[1 - \big(\alpha_A \overline{p}_{H,A}(h) + (1-\alpha_A)\underline{p}_{H,A}(h)\big)\big]\big(w_L - g(e_L, x_L)\big)\Big] \\
&+ \lambda_H\big[w_H - g(e_H, x_H) - w_L + g(e_L, x_H)\big] + \lambda_L\big[w_L - g(e_L, x_L) - w_H + g(e_H, x_L)\big].
\end{aligned}$$

The first-order conditions for the problem are:

$$\big(\alpha_P \overline{p}_{H,P}(h) + (1-\alpha_P)\underline{p}_{H,P}(h)\big) - \gamma^*\big(\alpha_A \overline{p}_{H,A}(h) + (1-\alpha_A)\underline{p}_{H,A}(h)\big) = \lambda_H^* - \lambda_L^*, \tag{7}$$

$$\big[1 - \big(\alpha_P \overline{p}_{H,P}(h) + (1-\alpha_P)\underline{p}_{H,P}(h)\big)\big] - \gamma^*\big[1 - \big(\alpha_A \overline{p}_{H,A}(h) + (1-\alpha_A)\underline{p}_{H,A}(h)\big)\big] = \lambda_L^* - \lambda_H^*, \tag{8}$$

$$\begin{gathered}
\big(\alpha_P \overline{p}_{H,P}(h) + (1-\alpha_P)\underline{p}_{H,P}(h)\big)\pi_e(e_H^*) - \gamma^*\big(\alpha_A \overline{p}_{H,A}(h) + (1-\alpha_A)\underline{p}_{H,A}(h)\big)g_e(e_H^*, x_H) \\
-\lambda_H^* g_e(e_H^*, x_H) + \lambda_L^* g_e(e_H^*, x_L) \le 0,
\end{gathered} \tag{9}$$

$$\begin{gathered}
\big[1 - \big(\alpha_P \overline{p}_{H,P}(h) + (1-\alpha_P)\underline{p}_{H,P}(h)\big)\big]\pi_e(e_L^*) - \gamma^*\big[1 - \big(\alpha_A \overline{p}_{H,A}(h) + (1-\alpha_A)\underline{p}_{H,A}(h)\big)\big]g_e(e_L^*, x_L) \\
-\lambda_L^* g_e(e_L^*, x_L) + \lambda_H^* g_e(e_L^*, x_H) \le 0,
\end{gathered} \tag{10}$$

$$\begin{gathered}
\big(\alpha_A \overline{p}_{H,A}(h) + (1-\alpha_A)\underline{p}_{H,A}(h)\big)\big(w_H^* - g(e_H^*, x_H)\big) \\
+\big[1 - \big(\alpha_A \overline{p}_{H,A}(h) + (1-\alpha_A)\underline{p}_{H,A}(h)\big)\big]\big(w_L^* - g(e_L^*, x_L)\big) \ge 0,
\end{gathered} \tag{PC}$$

$$w_H^* - g(e_H^*, x_H) \ge w_L^* - g(e_L^*, x_H), \tag{IC_H}$$

$$w_L^* - g(e_L^*, x_L) \ge w_H^* - g(e_H^*, x_L), \tag{IC_L}$$

where (9), (10), (PC), (IC_H), and (IC_L) hold with equality if, respectively, e_H^*, e_L^*, γ^*, λ_H^*, and λ_L^* are strictly greater than zero.

It follows from (7) and (8) that $\gamma^* = 1$. Furthermore, because $\alpha_P \overline{p}_{H,P}(h) + (1 - \alpha_P)\underline{p}_{H,P}(h) \in (0,1)$, $\pi_e(0) > 0$, and $g_e(0, x_L) = g_e(0, x_H) = 0$, it follows from (9) and (10), respectively, that $e_H^* > 0$ and $e_L^* > 0$. Thus we have equality in (9), (10), and (PC). Equality in the latter means that the agent receives exactly his reservation utility. Note that since $g(e, x_H) < g(e, x_L)$ for all $e > 0$, it follows from (IC_H) and (IC_L) that in any incentive compatible contract involving positive effort for type x_L, the agent is always best off when he turns out to be of the high-efficiency type. Note also that $\alpha_P \overline{p}_{H,P}(h) + (1 - \alpha_P)\underline{p}_{H,P}(h) \in (0,1)$ together with (IC_H) and (IC_L) ensure that the principal cannot exploit differences in emphasis between the parties ad infinitum, thus guaranteeing that the wage payments w_H^* and w_L^* are bounded and that the problem has a solution.

The analysis of the first-order conditions can be broken down into 4 cases.

Case 1: $\lambda_{\mathbf{L}} > \mathbf{0}$ and $\lambda_{\mathbf{H}} = \mathbf{0}$. With $\lambda_H = 0$, (8) and (10) imply that $e_L^* = e_L^o$, and (IC_L) then implies that

$$w_L^* - g(e_L^o, x_L) = w_H^* - g(e_H^*, x_L). \tag{11}$$

Equations (7) and (9) imply that e_H^* satisfies

$$\big(\alpha_P \overline{p}_{H,P}(h) + (1 - \alpha_P)\underline{p}_{H,P}(h)\big)[\pi_e(e_H^*) - g_e(e_H^*, x_H)] = \lambda_L[g_e(e_H^*, x_H) - g_e(e_H^*, x_L)]$$

and therefore that $e_H^* > e_H^o$, since the right hand side of this expression is negative when $\lambda_L > 0$, and thus e_H^* is on the downward sloping part of the function $\pi(e) - g(e, x_H)$. We therefore have **distortion at the top**. Together (PC) and (11) now imply that

$$w_H^* = \big(\alpha_A \overline{p}_{H,A}(h) + (1 - \alpha_A)\underline{p}_{H,A}(h)\big) g(e_H^*, x_H) + \Big(1 - \big(\alpha_A \overline{p}_{H,A}(h) + (1 - \alpha_A)\underline{p}_{H,A}(h)\big)\Big) g(e_H^*, x_L). \tag{12}$$

By equations (7) and (8),

$$\lambda_L = \big(\alpha_A \overline{p}_{H,A}(h) + (1 - \alpha_A)\underline{p}_{H,A}(h)\big) - \big(\alpha_P \overline{p}_{H,P}(h) + (1 - \alpha_P)\underline{p}_{H,P}(h)\big),$$

so $\lambda_L > 0$ gives that

$$\alpha_A \overline{p}_{H,A}(h) + (1 - \alpha_A)\underline{p}_{H,A}(h) > \alpha_P \overline{p}_{H,P}(h) + (1 - \alpha_P)\underline{p}_{H,P}(h). \tag{13}$$

That is, the principal's overall emphasis (or weight) on high-efficiency type is lower than the agent's in Case 1.

Using (11) and (12), the principal's Bernoulli utility can now be calculated to be

$$\pi(e_H^*) - w_H^* = \pi(e_H^*) - g(e_H^*, x_L) + \big(\alpha_A \overline{p}_{H,A}(h) + (1-\alpha_A)\underline{p}_{H,A}(h)\big)\big(g(e_H^*, x_L) - g(e_H^*, x_H)\big) \tag{14}$$

when the agent is of type x_H and

$$\pi(e_L^o) - w_L^* = \pi(e_L^o) - g(e_L^o, x_L) + \big(\alpha_A \overline{p}_{H,A}(h) + (1-\alpha_A)\underline{p}_{H,A}(h)\big)\big(g(e_H^*, x_L) - g(e_H^*, x_H)\big) \tag{15}$$

when the agent is of type x_L. Since e_L^o is the argmax of the function $\pi(e) - g(e, x_L)$, it follows from (14) and (15) that the principal will eventually be best off if the agent turns out to be of the low-efficiency type x_L. Thus, $\overline{p}_{H,P}(h) = \underline{p}_{H,A}(h) = a < \underline{p}_{H,P}(h) = \overline{p}_{H,A}(h) = b$. Then (13) implies that $\alpha_A > 1 - \alpha_P$, which is therefore a necessary condition for distortion at the top.

Using (14) and (15), the principal's utility in Case 1 can now be written as

$$\begin{aligned} OWEU_P \;=\;& \big(\alpha_P \overline{p}_{H,P}(h) + (1-\alpha_P)\underline{p}_{H,P}(h)\big)\big[\pi(e_H^*) - g(e_H^*, x_H)\big] \\ &+ \big[1 - \big(\alpha_P \overline{p}_{H,P}(h) + (1-\alpha_P)\underline{p}_{H,P}(h)\big)\big]\big[\pi(e_L^o) - g(e_L^o, x_L)\big] \\ &+ \Big[\big(\alpha_A \overline{p}_{H,A}(h) + (1-\alpha_A)\underline{p}_{H,A}(h)\big) \\ &\quad - \big(\alpha_P \overline{p}_{H,P}(h) + (1-\alpha_P)\underline{p}_{H,P}(h)\big)\Big]\Big[g(e_H^*, x_L) - g(e_H^*, x_H)\Big]. \end{aligned} \tag{16}$$

The right-hand side of equation (16) is non-decreasing in the agent's level of optimism for $\alpha_A \in (1-\alpha_P, 1]$. To see why, consider the constraint set. Denote the optimal contract when the agent's optimism is $\hat{\alpha}_A > 1 - \alpha_P$ by $(\hat{w}_H, \hat{e}_H, \hat{w}_L, \hat{e}_L)$. Since this is a solution, it satisfies (IC_H) and (IC_L), and (PC) holds with equality. Consider now an agent with optimism $\tilde{\alpha}_A > \hat{\alpha}_A$. The contract $(\hat{w}_H, \hat{e}_H, \hat{w}_L, \hat{e}_L)$ obviously still satisfies (IC_H) and (IC_L) for the $\tilde{\alpha}_A$-agent, since these constraints do not depend on α_A. To see that (PC) will also be satisfied for the $\tilde{\alpha}_A$-agent, note that (IC_H) and (IC_L) imply that $\hat{w}_H - g(\hat{e}_H, x_H) > \hat{w}_L - g(\hat{e}_L, x_L)$, such that $\overline{p}_{H,A}(h) > \underline{p}_{H,A}(h)$, and therefore $\tilde{\alpha}_A > \hat{\alpha}_A$ implies that

$$\begin{aligned} & \big(\tilde{\alpha}_A \overline{p}_{H,A}(h) + (1-\tilde{\alpha}_A)\underline{p}_{H,A}(h)\big)\big(\hat{w}_H - g(\hat{e}_H, x_H)\big) \\ & + \big[1 - \big(\tilde{\alpha}_A \overline{p}_{H,A}(h) + (1-\tilde{\alpha}_A)\underline{p}_{H,A}(h)\big)\big]\big(\hat{w}_L - g(\hat{e}_L, x_L)\big) \\ > \; & \big(\hat{\alpha}_A \overline{p}_{H,A}(h) + (1-\hat{\alpha}_A)\underline{p}_{H,A}(h)\big)\big(\hat{w}_H - g(\hat{e}_H, x_H)\big) \\ & + \big[1 - \big(\hat{\alpha}_A \overline{p}_{H,A}(h) + (1-\hat{\alpha}_A)\underline{p}_{H,A}(h)\big)\big]\big(\hat{w}_L - g(\hat{e}_L, x_L)\big) = 0. \end{aligned}$$

Hence $(\hat{w}_H, \hat{e}_H, \hat{w}_L, \hat{e}_L)$ also satisfies (PC) for the $\tilde{\alpha}_A$-agent. Since the $(\hat{w}_H, \hat{e}_H, \hat{w}_L, \hat{e}_L)$-contract satisfies all the constraints for the $\tilde{\alpha}_A$-agent, the optimal contract must make the principal at least as well off. It follows that $OWEU_P$ is non-decreasing in the agent's level of optimism for $\alpha_A \in (1-\alpha_P, 1]$.

Case 2: $\lambda_{\mathbf{H}} > \mathbf{0}$ and $\lambda_{\mathbf{L}} = \mathbf{0}$. With $\lambda_L = 0$, (7) and (9) imply that $e_H^* = e_H^o$, and (IC_H) then implies that

$$w_H^* - g(e_H^o, x_H) = w_L^* - g(e_L^*, x_H). \tag{17}$$

Equations (8) and (10) imply that e_L^* satisfies

$$\big(1 - \big(\alpha_P \overline{p}_{H,P}(h) + (1-\alpha_P)\underline{p}_{H,P}(h)\big)\big)[\pi_e(e_L^*) - g_e(e_L^*, x_L)] = \lambda_H [g_e(e_L^*, x_L) - g_e(e_L^*, x_H)]$$

and therefore that $e_L^* < e_L^o$, since the right hand side of this expression is positive when $\lambda_H > 0$, and thus e_L^* is on the upward sloping part of the function $\pi(e) - g(e, x_L)$. We therefore have **distortion at the bottom**. Together, (PC) and (17) now imply that

$$w_L^* = \big(\alpha_A \overline{p}_{H,A}(h) + (1-\alpha_A)\underline{p}_{H,A}(h)\big) g(e_L^*, x_H) + \big(1 - \big(\alpha_A \overline{p}_{H,A}(h) + (1-\alpha_A)\underline{p}_{H,A}(h)\big)\big) g(e_L^*, x_L). \tag{18}$$

By equations (7) and (8),

$$\lambda_H = \big(\alpha_P \overline{p}_{H,P}(h) + (1-\alpha_P)\underline{p}_{H,P}(h)\big) - \big(\alpha_A \overline{p}_{H,A}(h) + (1-\alpha_A)\underline{p}_{H,A}(h)\big),$$

so $\lambda_H > 0$ gives that

$$\alpha_A \overline{p}_{H,A}(h) + (1-\alpha_A)\underline{p}_{H,A}(h) < \alpha_P \overline{p}_{H,P}(h) + (1-\alpha_P)\underline{p}_{H,P}(h). \tag{19}$$

That is, the principal's overall emphasis (or weight) on high-efficiency type is higher than the agent's in Case 2.

Using (17) and (18), the principal's Bernoulli utility can now, similarly to Case 1, be calculated to be

$$\pi(e_H^o) - w_H^* = \pi(e_H^o) - g(e_H^o, x_H) + \big(1 - \big(\alpha_A \overline{p}_{H,A}(h) + (1-\alpha_A)\underline{p}_{H,A}(h)\big)\big)\big(g(e_L^*, x_H) - g(e_L^*, x_L)\big) \tag{20}$$

when the agent is of type x_H and

$$\pi(e_L^*) - w_L^* = \pi(e_L^*) - g(e_L^*, x_H) + \big(1 - \big(\alpha_A \overline{p}_{H,A}(h) + (1-\alpha_A)\underline{p}_{H,A}(h)\big)\big)\big(g(e_L^*, x_H) - g(e_L^*, x_L)\big) \tag{21}$$

when the agent is of type x_L. Since e_H^o is the argmax of the function $\pi(e) - g(e, x_H)$, it follows from (20) and (21) that the principal will eventually be best off if the agent turns out to be of the high-efficiency type x_H (recall that it is always the case that in order for the contract to be incentive compatible, it must eventually make the agent best off if he turns out to be of high-efficiency type). Hence, $\overline{p}_{H,P}(h) = \overline{p}_{H,A}(h) = b > \underline{p}_{H,P}(h) = \underline{p}_{H,A}(h) = a$. Then (19) implies that $\alpha_A < \alpha_P$, which is therefore a necessary condition for distortion at the bottom.

Using (20) and (21), the principal's utility in Case 2 can now be written as

$$\begin{aligned} OWEU_P \;\; = \;\; & \big(\alpha_P \overline{p}_{H,P}(h) + (1-\alpha_P)\underline{p}_{H,P}(h)\big)\big(\pi(e_H^o) - g(e_H^o, x_H)\big) \\ & + \big[1 - \big(\alpha_P \overline{p}_{H,P}(h) + (1-\alpha_P)\underline{p}_{H,P}(h)\big)\big]\big(\pi(e_L^*) - g(e_L^*, x_L)\big) \\ & + \Big[\big(\alpha_A \overline{p}_{H,A}(h) + (1-\alpha_A)\underline{p}_{H,A}(h)\big) \\ & \quad - \big(\alpha_P \overline{p}_{H,P}(h) + (1-\alpha_P)\underline{p}_{H,P}(h)\big)\Big]\big(g(e_L^*, x_L) - g(e_L^*, x_H)\big). \end{aligned} \tag{22}$$

The same argument that was used to show that $OWEU_P$ in (16) is non-decreasing in α_A for $\alpha_A \in (1-\alpha_P, 1]$ can be used to show that $OWEU_P$ in (22) is also non-decreasing in the agent's level of optimism for $\alpha_A \in [0, \alpha_P)$.

Case 3: $\lambda_\mathbf{H} = \mathbf{0}$ and $\lambda_\mathbf{L} = \mathbf{0}$. With $\lambda_L = \lambda_H = 0$, (7) and (9) imply that $e_H^* = e_H^o$ and (8) and (10) imply that $e_L^* = e_L^o$. Hence the contract specifies the first-best levels of effort, which maximize total surplus given the agent's type. Equations (7) and (8) imply that

$$\alpha_A \overline{p}_{H,A}(h) + (1-\alpha_A)\underline{p}_{H,A}(h) = \alpha_P \overline{p}_{H,P}(h) + (1-\alpha_P)\underline{p}_{H,P}(h), \tag{23}$$

i.e. the parties must assign the same weight to the agent being of type x_H in Case 3. Equation (PC) now gives that

$$w_L^* = g(e_L^o, x_L) + \frac{\alpha_A \overline{p}_{H,A}(h) + (1-\alpha_A)\underline{p}_{H,A}(h)}{1 - \big(\alpha_A \overline{p}_{H,A}(h) + (1-\alpha_A)\underline{p}_{H,A}(h)\big)}\big(g(e_H^o, x_H) - w_H^*\big),$$

and (IC_H) and (IC_L), neither of which binds, then give that

$$w_H^* \geq g(e_H^o, x_H) + \Big(1 - \big(\alpha_A \overline{p}_{H,A}(h) + (1-\alpha_A)\underline{p}_{H,A}(h)\big)\Big)\big(g(e_L^o, x_L) - g(e_L^o, x_H)\big)$$

and

$$w_H^* \leq g(e_H^o, x_H) + \Big(1 - \big(\alpha_A \overline{p}_{H,A}(h) + (1-\alpha_A)\underline{p}_{H,A}(h)\big)\Big)\big(g(e_H^o, x_L) - g(e_H^o, x_H)\big).$$

If $\pi(e_H^o) - w_H^* > \pi(e_L^o) - w_L^*$, then $\overline{p}_{H,P}(h) = \overline{p}_{H,A}(h) = b$ and $\underline{p}_{H,P}(h) = \underline{p}_{H,A}(h) = a$, so (23) requires that $\alpha_A = \alpha_P$. If instead $\pi(e_H^o) - w_H^* < \pi(e_L^o) - w_L^*$, then $\overline{p}_{H,P}(h) = \underline{p}_{H,A}(h) = a$ and $\underline{p}_{H,P}(h) = \overline{p}_{H,A}(h) = b$, so (23) requires that $\alpha_A = 1 - \alpha_P$. If $\pi(e_H^o) - w_H^* = \pi(e_L^o) - w_L^*$, we are at the corner where the objective function is non-differentiable. The potential corner solutions will be considered below.

The principal's utility in Case 3 is

$$\begin{aligned} OWEU_P &= \left(\alpha_P \overline{p}_{H,P}(h) + (1-\alpha_P)\underline{p}_{H,P}(h)\right)\left[\pi(e_H^o) - g(e_H^o, x_H)\right] \\ &\quad + \left(1 - \left(\alpha_P \overline{p}_{H,P}(h) + (1-\alpha_P)\underline{p}_{H,P}(h)\right)\right)\left[\pi(e_L^o) - g(e_L^o, x_L)\right] \\ &= \left(\alpha_A \overline{p}_{H,A}(h) + (1-\alpha_A)\underline{p}_{H,A}(h)\right)\left[\pi(e_H^o) - g(e_H^o, x_H)\right] \\ &\quad + \left(1 - \left(\alpha_A \overline{p}_{H,A}(h) + (1-\alpha_A)\underline{p}_{H,A}(h)\right)\right)\left[\pi(e_L^o) - g(e_L^o, x_L)\right], \end{aligned} \tag{24}$$

where the last equality follows from (23).

Case 4: $\lambda_\mathbf{H} > 0$ and $\lambda_\mathbf{L} > 0$. With $\lambda_H > 0$ and $\lambda_L > 0$, (IC_H) and (IC_L) imply that $e_H^* = e_L^*$. At the same time, (7) and (9) imply that e_H^* satisfies

$$\left(\alpha_P \overline{p}_{H,P}(h) + (1-\alpha_P)\underline{p}_{H,P}(h)\right)[\pi_e(e_H^*) - g_e(e_H^*, x_H)] = \lambda_L[g_e(e_H^*, x_H) - g_e(e_H^*, x_L)]$$

and therefore that $e_H^* > e_H^o$, while (8) and (10) imply that e_L^* satisfies

$$\left(1 - \left(\alpha_P \overline{p}_{H,P}(h) + (1-\alpha_P)\underline{p}_{H,P}(h)\right)\right)[\pi_e(e_L^*) - g_e(e_L^*, x_L)] = \lambda_H[g_e(e_L^*, x_L) - g_e(e_L^*, x_H)]$$

and therefore that $e_L^* < e_L^o$. Taken together (7)-(10) thus imply that $e_H^* > e_L^*$, and hence this case leads to a contradiction.

Corner contracts: $(\mathbf{w_H}, \mathbf{e_H}, \mathbf{w_L}, \mathbf{e_L})$ for which $\pi(\mathbf{e_H}) - \mathbf{w_H} = \pi(\mathbf{e_L}) - \mathbf{w_L}$. Suppose the optimal contract $(w_H^*, e_H^*, w_L^*, e_L^*)$ is a corner contract for which $\pi(e_H^*) - w_H^* = \pi(e_L^*) - w_L^*$. At these points, the principal's objective function is non-differentiable. A necessary condition for optimality of a corner contract is that the principal has no incentive to deviate, which requires that the parties assign the same overall weights to the two final scenarios. Since corner contracts are "sell the firm to the agent" contracts where the principal is equally well off regardless of the agent's type, $\overline{p}_{H,P}(h)$ and $\underline{p}_{H,P}(h)$ can take any value in the interval Q. However, in order for the principal to have no incentive to deviate, they must take the specific values $\overline{p}_{H,P}(h) = \underline{p}_{H,P}(h) = \alpha_A \overline{p}_{H,A}(h) + (1-\alpha_A)\underline{p}_{H,A}(h) = \alpha_A b + (1-\alpha_A)a$, which ensure that $\alpha_P \overline{p}_{H,P}(h) + (1-\alpha_P)\underline{p}_{H,P}(h) = \alpha_A \overline{p}_{H,A}(h) + (1-\alpha_A)\underline{p}_{H,A}(h)$, i.e. that the parties assign equal weight to the two final scenarios.

The best the principal can do at the corner is to set $e_H^* = e_H^o$ and $e_L^* = e_L^o$. The contract still has to satisfy the constraints, so (PC) and $\pi(e_H) - w_H = \pi(e_L) - w_L$ give that

$$\begin{aligned} w_L^* &= \pi(e_L^o) - \big[\big(\alpha_A b + (1-\alpha_A)a\big)\big(\pi(e_H^o) - g(e_H^o, x_H)\big) \\ &\quad + \left(1 - (\alpha_A b + (1-\alpha_A)a)\right)\big(\pi(e_L^o) - g(e_L^o, x_L)\big)\big] \end{aligned}$$

and

$$\begin{aligned} w_H^* &= \pi(e_H^o) - \big[\big(\alpha_A b + (1-\alpha_A)a\big)\big(\pi(e_H^o) - g(e_H^o, x_H)\big) \\ &\quad + \left(1 - (\alpha_A b + (1-\alpha_A)a)\right)\big(\pi(e_L^o) - g(e_L^o, x_L)\big)\big]. \end{aligned}$$

The principal's utility at the corner is therefore

$$\begin{aligned} OWEU_P &= \big(\alpha_A b + (1-\alpha_A)a\big)\Big[\pi(e_H^o) - g(e_H^o, x_H)\Big] \\ &\quad + \Big(1 - \big(\alpha_A b + (1-\alpha_A)a\big)\Big)\Big[\pi(e_L^o) - g(e_L^o, x_L)\Big]. \end{aligned} \tag{25}$$

Note that (25) is also increasing in the agent's optimism.

I now have to establish which type of contract is optimal in the different areas of (α_A, α_P)-space. First note that if $\alpha_A = \alpha_P$, then equation (8) implies that (22)=(24) and is equal to (25) in value. On the other hand, if $\alpha_A = 1 - \alpha_P$, then equation (7) implies that (16)=(24) and is equal to (25) in value.

In general, subtracting (22) from (25) gives the difference between the principal's utility with the corner (sell the firm to the agent) contract and with a contract that has distortion at the bottom. Denote this difference by Δ_P^{SB}, which is given by

$$\begin{aligned} \Delta_P^{SB} &= \big(\alpha_A b + (1-\alpha_A)a - \big(\alpha_P b + (1-\alpha_P)a\big)\Big[\pi(e_H^o) - g(e_H^o, x_H) + g(e_L^*, x_L) - g(e_L^*, x_H)\Big] \\ &\quad + \Big(1 - \big(\alpha_A b + (1-\alpha_A)a\big)\Big)\Big[\pi(e_L^o) - g(e_L^o, x_L)\Big] \\ &\quad + \Big(1 - \big(\alpha_P b + (1-\alpha_P)a\big)\Big)\Big[\pi(e_L^*) - g(e_L^*, x_L)\Big]. \end{aligned} \tag{26}$$

Taking the derivative of (26) with respect to α_A gives

$$\begin{aligned}
\frac{\partial \Delta_P^{SB}}{\partial \alpha_A} &= (b-a)\Big[\pi(e_H^o) - g(e_H^o, x_H) + g(e_L^*, x_L) - g(e_L^*, x_H) - \big[\pi(e_L^o) - g(e_L^o, x_L)\big]\Big] \\
&\quad + \Big[\big(\alpha_A b + (1-\alpha_A)a - (\alpha_P b + (1-\alpha_P)a)\big)\big(g_e(e_L^*, x_L) - g_e(e_L^*, x_H)\big) \\
&\quad\; -\big(1 - (\alpha_P b + (1-\alpha_P)a)\big)\big(\pi_e(e_L^*) - g_e(e_L^*, x_L)\big)\Big]\frac{\partial e_L^*}{\partial \alpha_A} \\
&= (b-a)\Big[\pi(e_H^o) - g(e_H^o, x_H) + g(e_L^*, x_L) - g(e_L^*, x_H) - \big[\pi(e_L^o) - g(e_L^o, x_L)\big]\Big], \quad (27)
\end{aligned}$$

where the last equality follows from the first order conditions. The derivative in (27) is positive since $\pi(e_H^o) - g(e_H^o, x_H) > \pi(e_L^o) - g(e_L^o, x_L)$ and $g(e_L^*, x_H) < g(e_L^*, x_L)$. Therefore, since $\Delta_P^{SB} = 0$ when $\alpha_A = \alpha_P$, (27) gives that (22) is greater than (25) for all $\alpha_A < \alpha_P$.

Subtracting (16) from (25) instead gives the difference between the principal's utility with the corner contract and with a contract that has distortion at the top. Denote this difference by Δ_P^{ST}, which is

$$\begin{aligned}
\Delta_P^{ST} &= \big(\alpha_A b + (1-\alpha_A)a\big)\Big[\pi(e_H^o) - g(e_H^o, x_H)\Big] - \big(\alpha_P a + (1-\alpha_P)b\big)\Big[\pi(e_H^*) - g(e_H^*, x_H)\Big] \quad (28) \\
&\quad -\Big(\alpha_A b + (1-\alpha_A)a - \big(\alpha_P a + (1-\alpha_P)b\big)\Big]\Big[\pi(e_L^o) - g(e_L^o, x_L) + g(e_H^*, x_L) - g(e_H^*, x_H)\Big].
\end{aligned}$$

Taking the derivative of (28) with respect to α_A and using an envelope theorem argument as for $\frac{\partial \Delta_P^{SB}}{\partial \alpha_A}$, it follows that

$$\frac{\partial \Delta_P^{ST}}{\partial \alpha_A} = (b-a)\Big[\pi(e_H^o) - g(e_H^o, x_H) - \big[\pi(e_L^o) - g(e_L^o, x_L)\big] - \big[g(e_H^*, x_L) - g(e_H^*, x_H)\big]\Big], \quad (29)$$

which is negative since $g(e, x_L) - g(e, x_H)$ is strictly increasing in e, $e_H^* > e_H^o$, and $\pi(e_H^o) - g(e_H^o, x_H) - \big[\pi(e_L^o) - g(e_L^o, x_L)\big] < g(e_H^o, x_L) - g(e_H^o, x_H)$. Therefore, since $\Delta_P^{ST} = 0$ when $\alpha_A = 1 - \alpha_P$, (29) gives that (16) is greater than (25) for all $\alpha_A > 1 - \alpha_P$.

Finally, subtracting (22) from (16) gives the difference between the principal's utility with a contract that has distortion at the top and with a contract that has distortion at the bottom. Denote this difference by Δ_P^{TB}, which is given by

$$\begin{aligned}
\Delta_P^{TB} &= \big(\alpha_P a + (1-\alpha_P)b\big)\big[\pi(e_H^*) - g(e_H^*, x_H)\big] + \big[1 - \big(\alpha_P a + (1-\alpha_P)b\big)\big]\big[\pi(e_L^o) - g(e_L^o, x_L)\big] \\
&\quad + \Big[\big(\alpha_A b + (1-\alpha_A)a\big) - \big(\alpha_P a + (1-\alpha_P)b\big)\Big]\Big[g(e_H^*, x_L) - g(e_H^*, x_H)\Big] \\
&\quad - \big(\alpha_P b + (1-\alpha_P)a\big)\big(\pi(e_H^o) - g(e_H^o, x_H)\big) - \big[1 - \big(\alpha_P b + (1-\alpha_P)a\big)\big]\big(\pi(e_L^*) - g(e_L^*, x_L)\big) \\
&\quad - \Big[\big(\alpha_A b + (1-\alpha_A)a\big) - \big(\alpha_P b + (1-\alpha_P)a\big)\Big]\big(g(e_L^*, x_L) - g(e_L^*, x_H)\big). \quad (30)
\end{aligned}$$

Taking the derivative of (30) with respect to α_A and using an envelope theorem argument as for $\frac{\partial \Delta_P^{SB}}{\partial \alpha_A}$ gives that

$$\frac{\partial \Delta_P^{TB}}{\partial \alpha_A} = (b-a)\Big[g(e_H^*, x_L) - g(e_H^*, x_H) - \big(g(e_L^*, x_L) - g(e_L^*, x_H)\big)\Big], \tag{31}$$

which is positive by the assumptions made about the function $g(e, x)$.

Note that Case 3 contracts give the principal the same utility as the corner contracts. It is therefore sufficient to compare the principal's utility in Cases 1 and 2 with that under a corner contract.

Now, consider first the situation when $\alpha_P \leq \alpha_A \leq 1-\alpha_P$. Then neither of the necessary conditions for the two distortion-type contracts is satisfied, see (4) and (6). Therefore, the optimal contract is a "sell the firm to the agent" contract. Note that the principal's utility at the corner contract is lower when $\alpha_A = \alpha_P$ than when $\alpha_A = 1 - \alpha_P$, since (25) is an increasing function of α_A.

Second, consider the situation when $\alpha_A < \alpha_P$ and $\alpha_A \leq 1-\alpha_P$. The necessary condition for distortion at the bottom is satisfied while the necessary condition for distortion at the top is not. The discussion below (27) establishes that the principal's utility (22) with a contract that has distortion at the bottom is greater than her utility (25) with a corner contract for all $\alpha_A < \alpha_P$. Hence the optimal contract has distortion at the bottom.

Third, consider the situation when $\alpha_A \geq \alpha_P$ and $\alpha_A > 1-\alpha_P$. The necessary condition for distortion at the top is satisfied while the necessary condition for distortion at the bottom is not. The discussion below (29) establishes that the principal's utility (16) with a contract that has distortion at the top is greater than her utility (25) with a corner contract for all $\alpha_A > 1 - \alpha_P$. Hence the optimal contract has distortion at the top.

Finally, consider the situation when $1 - \alpha_P < \alpha_A < \alpha_P$. Then both of the necessary conditions for the two distortion-type contracts are satisfied. The principal's utility with all three contracts must therefore be compared. When $\alpha_A = \alpha_P$, $\Delta_P^{SB} = 0$ and $\Delta_P^{ST} < 0$, implying that the optimal contract has distortion at the top. Also, when $\alpha_A = 1 - \alpha_P$, $\Delta_P^{SB} < 0$ and $\Delta_P^{ST} = 0$, implying that the optimal contract has distortion at the bottom. Because Δ_P^{TB} is a continuous upward sloping function of α_A on $(1 - \alpha_P, \alpha_P)$, this implies that there exists $\hat{\alpha} \in (1 - \alpha_P, \alpha_P)$ such that the optimal contract has distortion at the top for all $\alpha_A > \hat{\alpha}$ and has distortion at the bottom for all $\alpha_A < \hat{\alpha}$, see also the right panel of Figure 2. For $\alpha_A = \hat{\alpha}$, a contract that has distortion at the top and a contract that has distortion at the bottom are equally good.

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Online Appendix for
"Contracting in Vague Environments"

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Including the case of $\alpha_{\mathbf{P}} \in \{0, 1\}$ in the main result

This online Appendix states the result in Theorem 2 of the main paper for $(\alpha_A, \alpha_P) \in [0,1] \times [0,1]$, and includes the proof of the Theorem when $\alpha_P \in \{0,1\}$. The proof for $\alpha_P \in (0,1)$ is in the main paper.

Theorem 2' (Optimal contracts)**.** *The optimal contract varies across (α_A, α_P)-space as depicted in Figure 1 in the main paper. Specifically:*

If $\alpha_P \leq \alpha_A \leq 1 - \alpha_P$, the optimal contract is a "sell the firm to the agent" contract with $e_L^* = e_L^o$ and $e_H^* = e_H^o$.

If $\alpha_A < \alpha_P$ and $\alpha_A \leq 1 - \alpha_P$, the optimal contract has distortion at the bottom with $e_H^* = e_H^o$ and $e_L^* < e_L^o$.

If $\alpha_A \geq \alpha_P$ and $\alpha_A > 1 - \alpha_P$, the optimal contract has distortion at the top with $e_L^* = e_L^o$ and $e_H^* > e_H^o$.

If $1 - \alpha_P < \alpha_A < \alpha_P$, there exists $\hat{\alpha} \in (1 - \alpha_P, \alpha_P)$ such that the optimal contract has distortion at the bottom for all $\alpha_A < \hat{\alpha}$ and has distortion at the top for all $\alpha_A \geq \hat{\alpha}$.[1]

[1] There is an exception: if $a = 0$, $\alpha_P = 1$, and $\alpha_A > 0$, there is no solution to the principal's problem. The principal would want to offer a contract that has distortion at the top. With that contract, she would assign zero weight to the agent being of type x_H. She would therefore want to distort effort for the high-efficiency type towards infinity, combined with paying this type a correspondingly higher wage and require a correspondingly higher price for the firm from the low-efficiency type. Since $\alpha_A > 0$, the agent assigns positive weight to being of high-efficiency type, thus the contract, with sufficiently high wage to type x_H, would satisfy his participation constraint. Of course, this problem arises only because I have assumed no upper bound on the agent's effort.

Proof of Theorem 2' when $\alpha_{\mathbf{P}} = \mathbf{0}$ or $\alpha_{\mathbf{P}} = \mathbf{1}$

The arguments given above the Lagrangian in Appendix B apply here as well. Equation numbers refer to equations either in the main paper or in this online Appendix.

Proof when $\alpha_{\mathbf{P}} = \mathbf{0}$

The first-order conditions for the principal's problem are given by (7) through (IC_L) with $\alpha_P \overline{p}_{H,P}(h) + (1-\alpha_P)\underline{p}_{H,P}(h) = \underline{p}_{H,P}(h)$, where (9), (10), (PC), (IC_H), and (IC_L) hold with equality if, respectively, e_H^*, e_L^*, γ^*, λ_H^*, and λ_L^* are strictly greater than zero. It follows from (7) and (8) that $\gamma^* = 1$. The analysis of the first-order conditions can again be broken down into 4 cases, which are to be compared to the corner contracts.

Case 1: $\lambda_{\mathbf{L}} > \mathbf{0}$ and $\lambda_{\mathbf{H}} = \mathbf{0}$. In this case, (8) implies that $\underline{p}_{H,P}(h) < 1$ and (10) implies that $e_L^* > 0$. Conditions (8) and (10) then together imply that $e_L^* = e_L^o$. Also, (IC_L) implies that $w_L^* - g(e_L^o, x_L) = w_H^* - g(e_H^*, x_L)$.

Suppose first that $\underline{p}_{H,P}(h) > 0$. Then (9) implies that $e_H^* > 0$ and (9) together with (7) imply that $e_H^* > e_H^o$. We hence have distortion at the top. Condition (8) implies that

$$\alpha_A \overline{p}_{H,A}(h) + (1-\alpha_A)\underline{p}_{H,A}(h) > \alpha_P \overline{p}_{H,P}(h) + (1-\alpha_P)\underline{p}_{H,P}(h). \tag{32}$$

The principal will be best off if the agent is of type x_L, by the same argumentation as used below expressions (14) and (15) in Appendix B. Thus, $\underline{p}_{H,P}(h) = b$. Then (32) is equivalent to $\alpha_A b + (1-\alpha_A)a > b$, which is a contradiction since $\alpha_A \in [0,1]$ and $b > a$. It can therefore be ruled out that $\underline{p}_{H,P}(h) > 0$.

Suppose instead that $\underline{p}_{H,P}(h) = 0$. Then (7) and (9) imply that $g_e(e_H^*, x_H) \geq g_e(e_H^*, x_L)$, which only holds if $e_H^* = 0$. Using this, (IC_H) becomes $w_H^* \geq w_L^* - g(e_L^o, x_H)$ and (IC_L) becomes $w_L^* - g(e_L^o, x_L) = w_H^*$, which together imply that $e_L^o = 0$. This contradicts $e_L^o > 0$.

Since both $\underline{p}_{H,P}(h) > 0$ and $\underline{p}_{H,P}(h) = 0$ lead to contradictions, the conclusion is that Case 1 will never be prevailing when $\alpha_P = 0$.

Case 2: $\lambda_{\mathbf{H}} > \mathbf{0}$ and $\lambda_{\mathbf{L}} = \mathbf{0}$. In this case, (7) implies that $\underline{p}_{H,P}(h) > 0$ and (9) implies that $e_H^* > 0$. Conditions (7) and (9) then together imply that $e_H^* = e_H^o$. Also, (IC_H) implies that $w_H^* - g(e_H^o, x_H) = w_L^* - g(e_L^*, x_H)$.

Suppose first that $\underline{p}_{H,P}(h) < 1$. Then (10) implies that $e_L^* > 0$ and (10) together with (8) imply that $e_L^* < e_L^o$. We hence have distortion at the bottom. Condition (7) implies that

$$\alpha_A \overline{p}_{H,A}(h) + (1-\alpha_A)\underline{p}_{H,A}(h) < \alpha_P \overline{p}_{H,P}(h) + (1-\alpha_P)\underline{p}_{H,P}(h). \tag{33}$$

The principal will be best off if the agent is of type x_H, by the same argumentation as used below expressions (20) and (21) in Appendix B. Thus, $\underline{p}_{H,P}(h) = a$. Then (33) is equivalent to $\alpha_A b + (1-\alpha_A)a < a$, which is a contradiction since $\alpha_A \in [0,1]$ and $b > a$. It can therefore be ruled out that $\underline{p}_{H,P}(h) < 1$.

Suppose instead that $\underline{p}_{H,P}(h) = 1$. Then (8) and (10) imply that

$$g_e(e_L^*, x_H) \leq g_e(e_L^*, x_L). \tag{34}$$

If $e_L^* > 0$ then (34) implies that $g_e(e_L^*, x_H) \leq g_e(e_L^*, x_L)$, which is only satisfied if $e_L^* = 0$. This is a contradiction. If instead $e_L^* = 0$, (IC_H) becomes $w_H^* - g(e_H^o, x_H) = w_L^*$ and (PC) becomes $(\alpha_A b + (1-\alpha_A)a)\left(w_H^* - g(e_H^o, x_H)\right) + (1 - (\alpha_A b + (1-\alpha_A)a))\, w_L^* = 0$, which together imply that $w_H^* = g(e_H^o, x_H)$. This in turn gives that $w_L^* = 0$. But then the principal's Bernoulli utility is $\pi(e_H^o) - g(e_H^o, x_H)$ when the agent is of type x_H and is zero when the agent is of type x_L, which means that the principal is best off when the agent is of type x_H. This contradicts that $\underline{p}_{H,P}(h) = 1$.

Since both $\underline{p}_{H,P}(h) < 1$ and $\underline{p}_{H,P}(h) = 1$ lead to contradictions, the conclusion is that Case 2 will never be prevailing when $\alpha_P = 0$.

Case 3: $\lambda_{\mathbf{H}} = \mathbf{0}$ and $\lambda_{\mathbf{L}} = \mathbf{0}$. In this case, (7) implies that $\underline{p}_{H,P}(h) = \alpha_A \overline{p}_{H,A}(h) + (1-\alpha_A)\underline{p}_{H,A}(h)$. Condition (9) implies that

$$\underline{p}_{H,P}(h)\pi_e(e_H^*) \leq \left(\alpha_A \overline{p}_{H,A}(h) + (1-\alpha_A)\underline{p}_{H,A}(h)\right) g_e(e_H^*, x_H)$$

and condition (10) implies that

$$(1 - \underline{p}_{H,P}(h))\pi_e(e_L^*) \leq \left(1 - \left(\alpha_A \overline{p}_{H,A}(h) + (1-\alpha_A)\underline{p}_{H,A}(h)\right)\right) g_e(e_L^*, x_L).$$

Suppose first that $\alpha_A \overline{p}_{H,A}(h) + (1-\alpha_A)\underline{p}_{H,A}(h) = 0$. This can only hold if $\alpha_A = 0$ and $a = 0$. We then have that $\underline{p}_{H,P}(h) = 0$, which implies that the principal must be best off when the agent is of type x_H. Condition (10) then implies that $e_L^* = e_L^o$, while (PC) implies that $w_L^* = g(e_L^o, x_L)$. Furthermore, (IC_H) gives that $w_H^* - g(e_H^*, x_H) \geq w_L^* - g(e_L^o, x_H) > 0$, while (IC_L) gives that $w_H^* - g(e_H^*, x_L) \leq 0$. These two conditions together imply that $e_H^* > 0$. Because $\alpha_P = 0$ and the principal is best off when the agent is of type x_H, the principal's utility is $OWEU_P = \pi(e_L^o) - g(e_L^o, x_L)$.

Suppose instead that $\alpha_A \overline{p}_{H,A}(h) + (1-\alpha_A)\underline{p}_{H,A}(h) = 1$. This can only hold if $\alpha_A = 1$ and $b = 1$. We then have that $\underline{p}_{H,P}(h) = 1$, which implies that the principal must be best off when the agent is of type x_L. Condition (9) then implies that $e_H^* = e_H^o$, while (PC) implies that $w_H^* = g(e_H^o, x_H)$. Furthermore, (IC_H) gives that $0 \geq w_L^* - g(e_L^o, x_H)$, while (IC_L) gives that $w_L^* - g(e_L^*, x_L) \geq w_H^* - g(e_H^o, x_L)$. Because $\alpha_P = 0$ and the principal is best off when the agent is of type x_L, the principal's utility is $OWEU_P = \pi(e_H^o) - g(e_H^o, x_H)$.

Finally, if $\alpha_A \overline{p}_{H,A}(h) + (1-\alpha_A)\underline{p}_{H,A}(h) \in (0,1)$ we have a corner contract. These are considered in Appendix B.

Case 4: $\lambda_{\mathbf{H}} > \mathbf{0}$ and $\lambda_{\mathbf{L}} > \mathbf{0}$. In this case, (IC_H) and (IC_L) imply that $e_H^* = e_L^*$. Conditions (7), (8), (9), and 10 then imply that $e_H^* = e_L^* = 0$. But (9) and (10) imply that $e_H^* > 0$ or $e_L^* > 0$. Hence, Case 4 leads to a contradiction.

Corner contracts: $(\mathbf{w_H}, \mathbf{e_H}, \mathbf{w_L}, \mathbf{e_L})$ for which $\pi(\mathbf{e_H}) - \mathbf{w_H} = \pi(\mathbf{e_L}) - \mathbf{w_L}$. The analysis of the corner contracts is as in Appendix B, and the principal's utility with a corner contract is given by (25).

I now have to establish which type of contract is optimal. Cases 1, 2, and 4 are ruled out. Comparing the principal's utility in Case 3 with her utility under a corner contract, the conclusion is that a corner contract is always optimal when $\alpha_P = 0$.

Proof when $\alpha_{\mathbf{P}} = \mathbf{1}$

The first-order conditions for the principal's problem are again given by (7) through (IC_L), this time with $\alpha_P \overline{p}_{H,P}(h) + (1-\alpha_P)\underline{p}_{H,P}(h) = \overline{p}_{H,P}(h)$, where (9), (10), (PC), (IC_H), and (IC_L) hold with equality if, respectively, e_H^*, e_L^*, γ^*, λ_H^*, and λ_L^* are strictly greater than zero. It follows from (7) and (8) that $\gamma^* = 1$. The analysis of the first-order conditions can be broken down into the same 4 cases as above, which are to be compared with the corner contracts.

Case 1: $\lambda_{\mathbf{L}} > \mathbf{0}$ and $\lambda_{\mathbf{H}} = \mathbf{0}$. In this case, (8) implies that $\overline{p}_{H,P}(h) < 1$ and (10) implies that $e_L^* > 0$. Conditions (8) and (10) then together imply that $e_L^* = e_L^o$. Also, (IC_L) implies that $w_L^* - g(e_L^o, x_L) = w_H^* - g(e_H^*, x_L)$. Condition (8) also implies that

$$\alpha_A \overline{p}_{H,A}(h) + (1-\alpha_A)\underline{p}_{H,A}(h) > \overline{p}_{H,P}(h). \tag{35}$$

Given incentive compatibility, this is equivalent to $\alpha_A b + (1-\alpha_A)a > \overline{p}_{H,P}(h)$, which necessitates that $\alpha_A > 0$ and $\overline{p}_{H,P}(h) = a$. The principal will therefore be best off when the agent is of type x_L.

Suppose first that $\overline{p}_{H,P}(h) = 0$. Then (9) together with (7) imply that $g_e(e_H^*, x_L) \leq g_e(e_H^o, x_H)$, which is only satisfied if $e_H^* = 0$. Then (IC_H) and (IC_L) together imply that $e_L^o = 0$, which is a contradiction. Therefore, it must be that $\overline{p}_{H,P}(h) > 0$, that is, $a > 0$.

When $\overline{p}_{H,P}(h) > 0$, condition (9) implies that $e_H^* > 0$, and (9) together with (7) imply that $e_H^* > e_H^o$. We hence have distortion at the top. Conditions (PC) and (IC_L) imply that

$$w_H^* = \big(\alpha_A \overline{p}_{H,A}(h) + (1-\alpha_A)\underline{p}_{H,A}(h)\big) g(e_H^*, x_H) + \Big(1 - \big(\alpha_A \overline{p}_{H,A}(h) + (1-\alpha_A)\underline{p}_{H,A}(h)\big)\Big)\, g(e_H^*, x_L)$$

and

$$w_L^* = g(e_L^o, x_L) + \big(\alpha_A \overline{p}_{H,A}(h) + (1-\alpha_A)\underline{p}_{H,A}(h)\big)\left(g(e_H^*, x_H) - g(e_H^*, x_L)\right).$$

The principal's utility is given by (16).

Case 2: $\lambda_{\mathbf{H}} > \mathbf{0}$ and $\lambda_{\mathbf{L}} = \mathbf{0}$. In this case, (7) implies that $\overline{p}_{H,P}(h) > 0$ and (9) implies that $e_H^* > 0$. Conditions (7) and (9) then together imply that $e_H^* = e_H^o$. Also, (IC_H) implies that $w_H^* - g(e_H^o, x_H) = w_L^* - g(e_L^*, x_H)$. Condition (7) also implies that

$$\alpha_A \overline{p}_{H,A}(h) + (1-\alpha_A)\underline{p}_{H,A}(h) < \overline{p}_{H,P}(h). \tag{36}$$

Given incentive compatibility, this is equivalent to $\alpha_A b + (1-\alpha_A)a < \overline{p}_{H,P}(h)$, which necessitates that $\alpha_A < 1$ and $\overline{p}_{H,P}(h) = b$. The principal is will therefore be best off when the agent is of type x_H.

Suppose first that $\overline{p}_{H,P}(h) < 1$. Then (10) implies that $e_L^* > 0$ and (10) together with (8) imply that $e_L^* < e_L^o$. The contract hence has distortion at the bottom. Conditions (PC) and (IC_H) imply that

$$w_L^* = \big(\alpha_A \overline{p}_{H,A}(h) + (1-\alpha_A)\underline{p}_{H,A}(h)\big) g(e_L^*, x_H) + \Big(1 - \big(\alpha_A \overline{p}_{H,A}(h) + (1-\alpha_A)\underline{p}_{H,A}(h)\big)\Big)\, g(e_L^*, x_L)$$

and

$$w_H^* = g(e_H^o, x_H) + \Big(1 - \big(\alpha_A \overline{p}_{H,A}(h) + (1-\alpha_A)\underline{p}_{H,A}(h)\big)\Big)\left(g(e_L^*, x_L) - g(e_L^*, x_H)\right).$$

The principal's utility is given by (22).

Suppose instead that $\overline{p}_{H,P}(h) = 1$. Then (10) together with (8) imply that $g_e(e_L^*, x_H) \leq g_e(e_L^*, x_L)$. If $e_L^* > 0$, this implies that $e_L^* = 0$, a contradiction. Hence, it must be that $e_L^* = 0$. Then (PC) and (IC_H) imply that $w_L^* = 0$ and $w_H^* = g(e_H^o, x_H)$. The principal's utility is $\pi(e_H^o) - g(e_H^o, x_H)$. Note that $\overline{p}_{H,P}(h) = 1$ only when $b = 1$.

Case 3: $\lambda_\mathbf{H} = \mathbf{0}$ and $\lambda_\mathbf{L} = \mathbf{0}$. In this case, (7) implies that $\overline{p}_{H,P}(h) = \alpha_A \overline{p}_{H,A}(h) + (1 - \alpha_A)\underline{p}_{H,A}(h)$. Condition (9) implies that

$$\overline{p}_{H,P}(h)\pi_e(e_H^*) \leq \Big(\alpha_A \overline{p}_{H,A}(h) + (1 - \alpha_A)\underline{p}_{H,A}(h)\Big)\, g_e(e_H^*, x_H)$$

and condition (10) implies that

$$(1 - \overline{p}_{H,P}(h))\pi_e(e_L^*) \leq \Big(1 - \Big(\alpha_A \overline{p}_{H,A}(h) + (1 - \alpha_A)\underline{p}_{H,A}(h)\Big)\Big)\, g_e(e_L^*, x_L).$$

Suppose first that $\alpha_A \overline{p}_{H,A}(h) + (1 - \alpha_A)\underline{p}_{H,A}(h) = 0$. This can only hold if $\alpha_A = 0$ and $a = 0$. We then have that $\overline{p}_{H,P}(h) = 0$, and thus principal must be best off when the agent is of type x_L. Condition (10) then implies that $e_L^* = e_L^o$, while (PC) implies that $w_L^* = g(e_L^o, x_L)$. Furthermore, (IC_H) gives that $w_H^* - g(e_H^*, x_H) \geq w_L^* - g(e_L^o, x_H) > 0$, while (IC_L) gives that $w_H^* - g(e_H^*, x_L) \leq 0$. These two conditions together imply that $e_H^* \geq e_L^o > 0$. Because $\alpha_P = 1$ and the principal will be best off when the agent is of type x_L, her utility is $OWEU_P = \pi(e_L^o) - g(e_L^o, x_L)$.

Suppose instead that $\alpha_A \overline{p}_{H,A}(h) + (1 - \alpha_A)\underline{p}_{H,A}(h) = 1$. This can only hold if $\alpha_A = 1$ and $b = 1$. We then have that $\overline{p}_{H,P}(h) = 1$, which implies that the principal must be best off when the agent is of type x_H. Condition (9) then implies that $e_H^* = e_H^o$, while (PC) implies that $w_H^* = g(e_H^o, x_H)$. Furthermore, (IC_H) gives that $0 \geq w_L^* - g(e_L^o, x_H)$, while (IC_L) gives that $w_L^* - g(e_L^*, x_L) \geq w_H^* - g(e_H^o, x_L)$. Together these imply that $e_L^* < e_H^o$. Because $\alpha_P = 1$ and the principal is best off when the agent is of type x_H, the principal's utility is $OWEU_P = \pi(e_H^o) - g(e_H^o, x_H)$.

Finally, if $\alpha_A \overline{p}_{H,A}(h) + (1 - \alpha_A)\underline{p}_{H,A}(h) \in (0, 1)$, we must have a corner contract. These are considered in Appendix B.

Case 4: $\lambda_\mathbf{H} > \mathbf{0}$ and $\lambda_\mathbf{L} > \mathbf{0}$. This case leads to a contradiction, by an argument similar to the one for Case 4 when $\alpha_P = 0$.

Corner contracts: $(\mathbf{w_H}, \mathbf{e_H}, \mathbf{w_L}, \mathbf{e_L})$ for which $\pi(\mathbf{e_H}) - \mathbf{w_H} = \pi(\mathbf{e_L}) - \mathbf{w_L}$. These are analyzed in Appendix B and give the principal the utility in (25).

I now have to establish which type of contract is optimal. Suppose that $a > 0$. The situation when $a = 0$ is considered below. The contracts in Cases 1 and 2 both dominate the corner contracts. Case 3 gives the principal the same utility as the corner contracts. The same argument as was used in Appendix B for $\alpha_P \in (0, 1)$ works to show that there exists $\hat{\alpha} \in (0, 1)$ such that the optimal contract has distortion at the bottom for all $\alpha_A < \hat{\alpha}$, has distortion at the top for all $\alpha_A > \hat{\alpha}$, and has either distortion at the top or distortion at the bottom when $\alpha_A = \alpha_P$.

Suppose now that $a = 0$. If $\alpha_A = 0$, Case 1 is ruled out, and Case 2 dominates Case 3 and the corner contracts. The optimal contract therefore has distortion at the bottom.

If, on the other hand, $\alpha_A > 0$, there does not exist a solution to the principal's problem when $a = 0$ and $\alpha_P = 1$. To see this, suppose first that the principal offers the contract $(\tilde{w}_H, e_H^o, \tilde{w}_L, e_L^o)$, where $\tilde{w}_H$ and $\tilde{w}_L$ are such that (PC) and (IC_L) both hold with equality. With this contract, she pays the type x_L agent a lower wage and the type x_H agent a higher wage than with a "sell the firm to the agent" contract. Thus, the principal is best off when the agent is of type x_L, and her utility equals $K + \alpha_A b[\pi(e_L^o) - g(e_L^o, x_L) - \pi(e_H^o) + g(e_H^o, x_L)]$, where K is her utility with a "sell the firm to the agent" contract.

Suppose now that the principal instead offers the contract $(\widehat{w}_H, \widehat{e}_H, \widehat{w}_L, e_L^o)$, where $\widehat{w}_L \equiv \tilde{w}_L - \gamma$ with $\gamma > 0$, and that the contract satisfies both of (PC) and (IC_L) with equality. That is,

$$\alpha_A b(\widehat{w}_H - g(\widehat{e}_H, x_H)) + (1 - \alpha_A b)(\widehat{w}_L - g(e_L^o, x_L)) = 0 \tag{37}$$

and

$$\widehat{w}_L - g(e_L^o, x_L) = \widehat{w}_H - g(\widehat{e}_H, x_L). \tag{38}$$

This contract has distortion at the top. Solving (37) and (38) for $\widehat{w}_L$ gives that $\widehat{w}_L = g(e_L^o, x_L) + \alpha_A b(g(\widehat{e}_H, x_H) - g(\widehat{e}_H, x_L))$. With distortion at the top the principal is best off when the agent is of type x_L, hence, since $\alpha_P = 1$, her utility with this contract is

$$OWEU_P = \pi(e_L^o) - \widehat{w}_L = \pi(e_L^o) - g(e_L^o, x_L) - \alpha_A b\left(g(\widehat{e}_H, x_H) - g(\widehat{e}_H, x_L)\right).$$

This is strictly increasing in $\widehat{e}_H$ for all $\widehat{e}_H > 0$. The principal therefore want to distort effort for the high-efficiency type towards infinity. It follows that no solution exists to the principal's problem when $\alpha_P = 1$, $a = 0$, and $\alpha_A > 0$.■