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Market Power, Price Adjustment, and Inflation

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Abstract

Endogenous fluctuations in mark-ups driven by changes in consumers' search intensity are studied in a monetary search economy. These fluctuations in market power determine the extent to which real and nominal prices adjust to shocks to productivity and the money growth rate in the absence of costs or temporal restrictions on sellers' ability to change prices. A calibrated version of the economy is consistent with several empirical regularities documented in the literatures on exchange rate pass-through and the cyclical properties of mark-ups. In particular, both the pass-through of cost movements to real and nominal prices and the adjustment of nominal prices to changes in the money growth rate are incomplete. Also, mark-up fluctuations may be either pro- or counter-cyclical depending on their source. Furthermore, a higher average rate of inflation results in both a lower average mark-up and increasing sensitivity of prices to fluctuations in either productivity or money growth.

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1. Introduction

In this paper, we develop a monetary search economy in which endogenous fluctuations in market power driven by changes in consumers' search intensity determine the extent to which real and nominal prices adjust to random fluctuations in productivity and the money growth rate. A calibrated version of our economy displays incomplete pass-through of movements in production costs to both real and nominal prices, incomplete adjustment of nominal prices to changes in the money growth rate, and mark-up fluctuations that may be either pro- or counter-cyclical depending on their source. The economy also exhibits less market power and increasing price sensitivity to both productivity and money growth fluctuations the higher is the average rate of inflation.

Empirical studies suggest that the responsiveness of nominal prices to various shocks is incomplete, varies over time and is positively related to the average rate of inflation. Taylor (2000), for example, argues that the response of nominal prices to changes in costs has declined with the rate of inflation over time for the U.S. and other developed countries. In addition, Choudhri and Hakura (2006), Campa and Goldberg (2005), Devereux and Yetman (2002) and others present evidence that the pass-through of nominal exchange rate movements (interpreted as exogenous cost shocks) to consumer prices is increasing in the average rate of inflation. Similarly, Gagnon (2007) documents a positive relationship between inflation and the magnitude of price changes for individual goods in Mexico. There is also evidence that the average mark-up is negatively related to the trend rate of inflation (Banerjee, Mizen, and Russell (2007), Gali and Gertler (1999), and others).

A large literature explores the effects of incomplete price adjustment in models with explicit nominal rigidities.[1] For the most part, the source of nominal rigidity is of secondary concern in this literature—price changes are typically assumed to be subject to costs and/or frequency limitations. Rather, the literature focuses on the dynamics of both inflation and real activity that emanate from productivity and monetary policy shocks given both the source and degree of nominal rigidity. In contrast, we focus on the economic forces that determine the extent of both market power and price adjustment to shocks. Similarly, whereas most of this literature places little emphasis on the trend rate of inflation, focusing on dynamics in a neighborhood of a constant (often zero) inflation steady-state, we consider explicitly the effect of the average rate of inflation on the degree of market power in the economy and the resulting responsiveness of prices to shocks.

We embed the price-posting structure of Burdett and Judd (1983) in a general equilibrium

[1] Some important sources that present key results and describe the literature are Goodfriend and King (1997), Clarida, Gali and Gertler (1999), and Woodford (2003).

environment similar to models of Shi (1999) and Head and Shi (2003) in which money functions as a medium of exchange in the tradition of Kiyotaki and Wright (1993). Head and Kumar (2005) study the welfare costs of trend inflation in a similar but non-stochastic environment. In their model the degree of price dispersion in equilibrium depends on the average rate of inflation. In this paper uncertainty is introduced and our focus is on the response of prices to random shocks. Here, both the average degree of price dispersion and its response to shocks are key factors determining both market power and price adjustment in equilibrium.

In our economy, the adjustment of prices to shocks is determined by the combination of two opposing effects. First, for a fixed degree of search intensity by consumers, an increase of either production costs or the money growth rate is passed-through differentially to consumer prices by sellers pricing in different regions of the price distribution, typically resulting in greater dispersion of prices. Second, increased dispersion raises the gains to search, inducing a larger fraction of buyers to observe more than one price. This reduces sellers' market power and limits the extent to which prices rise. The adjustment of prices to either type of shock is incomplete if the response of search intensity and market power is sufficiently strong.

The relative strength of the conflicting effects depends on the average rate of inflation. At low rates of inflation, a relatively large fraction of buyers observes only a single price. In this case, an increase of either costs or money growth generates a large increase in price dispersion, and thus induces a strong increase in search intensity. The resulting reduction in sellers' market power substantially limits the adjustment of prices in response to these shocks, resulting in a relatively small effect of the shock on the average price level. As the rate of trend inflation rises, the share of buyers observing more than one price rises and the average mark-up falls. A given shock has a smaller effect on price dispersion and so the response of search intensity diminishes. As a result, prices are more responsive to either type of shock than at lower rates of inflation. Moreover, at sufficiently high inflation, average prices become closely tied to marginal cost and prices and inflation effectively move one-for-one with changes in costs and the money growth rate.

In a calibrated version of our economy the extent of market power and the responsiveness of prices to shocks are negatively and positively related, respectively, to the average rate of inflation. These predictions are consistent with the empirical studies noted above. The economy also predicts an asymmetry between the effects of productivity and money growth shocks. Movements in productivity induce pro-cyclical fluctuations in the average mark-up as they are positively correlated with output but negatively correlated with price dispersion and search intensity (and hence positively correlated with market power). In contrast, changes in the money growth rate generate counter-cyclical movements in the mark-up at low and moderate trend inflation rates.

The relationship between average inflation and the extent of price adjustment in response to shocks is also studied in the literature on state-contingent pricing. We obtain results similar in several respects to those of this literature in spite of the fact that we impose no exogenous nominal rigidity. For example, our model predicts asymmetric responses of prices to positive and negative cost shocks, as does that of Devereux and Siu (2007). In both our model and theirs, increases in cost may lead to larger price responses than reductions in cost of the same magnitude. Also, state-contingent pricing models with menu costs (*e.g.* Dotsey, King, and Wolman (1999)), predict the price level to be more responsive to shocks at higher inflation, as a larger share of firms will find it profitable to change prices in a given period the higher the rate of inflation.

Our work is also related to research focusing on price adjustments in environments with search frictions. In one such environment with menu costs, a negative relationship between inflation and the degree of market power is also found by Benabou (1988, 1992). Craig and Rocheteau (2005) also consider the implications of menu costs for the welfare costs of inflation in a search model. Their economy is similar to ours in the sense that a search friction makes fiat money essential in equilibrium. They do not, however, consider the adjustment of prices to shocks. Eden (1994) considers the adjustment of prices to monetary shocks in a model of uncertain and sequential trade. The mechanism by which price stickiness is generated in his model differs from ours, and changes in expected inflation have no effect on real prices. Alessandria (2005) uses a non-monetary model with a similar form of price determination to study international price differentials. Search intensity of the type we study, however, is constant in his model.

The remainder of the paper is organized as follows: Section 2 describes the environment and defines a symmetric Markov monetary equilibrium. Section 3 describes qualitatively the mechanisms driving price adjustment to shocks in the economy. Section 4 presents the calibration and illustrates the relationships between the degree of price adjustment to shocks and several parameters including the average rate of inflation. Section 5 discusses implications of the results for future work and concludes.

2. The Economy

2.1. The environment

Time is discrete. There are $H \geq 3$ different types of households, and there are unit measures of households of each type. A type h household produces good h and derives utility from consumption of good $h + 1$, modulo H. Each household is comprised of unit measures of two different types of members: “buyers” and “sellers”. Individual household members do not have independent preferences but rather share equally in household utility.[2]

Members of a representative type h household who are *sellers* produce good h in period t at marginal cost $\phi_t > 0$ utils per unit. Production costs are stochastic; ϕ_t evolves via a discrete Markov chain with

$$\mathrm{Prob}\left\{\phi_{t+1} = \phi' | \phi_t = \phi\right\} \equiv \pi^{\phi}(\phi', \phi) \qquad \forall t, t+1; \quad \phi', \phi \in \mathcal{P}, \tag{2.1}$$

where $\mathcal{P}$ is a finite set. If we let y_t denote the total quantity of good h produced by all sellers from this household in period t, then the household's total period disutility from production is equal to $\phi_t y_t$.

Members of this household who are *buyers* observe random numbers of price quotes and may purchase good $h+1$ at the lowest price that they observe individually. Let q_{kt} denote the measure of the household's buyers who observe $k \in \{0, \ldots, K\}$ price quotes at time t. The household chooses the probabilities with which buyers observe different numbers of quotes. Since the household contains a unit measure of buyers, the probability of an individual buyer observing k prices is equivalent to the measure of a household's buyers who observe k prices.[3] For each price quote observed, the household pays an information or search cost of μ utils. Thus, the household's total disutility of search in period t is equal to $\mu \sum_{k=0}^{K} k q_{kt}$.

A representative household seeks to maximize the expected discounted sum of its period utility over an infinite horizon:

$$U = \mathrm{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left(u(c_t) - \phi_t y_t - \mu \sum_{k=0}^{K} k q_{kt} \right) \right]. \tag{2.2}$$

Here, $u(c_t)$ denotes consumption utility where c_t is the total purchases of good $h+1$ by the household's buyers. We assume that $u(\cdot)$ is strictly increasing and strictly concave with $u'(\cdot) > 0$ and $u''(\cdot) < 0$.

Since a type h household produces good h and consumes good $h+1$, a double coincidence of wants between members of any two households is impossible. Moreover, it is assumed that households of a given type are indistinguishable and that individual household members are anonymous

[2] We denote the economy-wide *per household* level of variable x with a tilde, $\tilde{x}$, and the individual household level without a tilde. Also, we use upper case to denote nominal variables and lower case to denote “real” variables, by which we mean nominal values divided by the per household money stock.

[3] The maximum number of price quotes observed, K, is unimportant, as we will show later. We may think of K as being chosen by the household, or of the household as setting $q_{kt} = 0$ for all $k \geq K$ and for all t.

and cannot be relocated in the future following an exchange. Since consumption goods are non-storable, direct exchanges of goods cannot be mutually beneficial. Instead, exchange is facilitated by the existence of perfectly durable and intrinsically worthless *fiat money*. A type h household may acquire fiat money by having its producers sell output to buyers of type $h-1$ households. This money may then be exchanged for consumption good $h+1$ by the household's own buyers in a future period.

In the initial period ($t=0$) each household is endowed with $\tilde{M}_0$ units of fiat money. The average money stock across households at time t is denoted $\tilde{M}_t$. At the beginning of each period $t \geq 1$ each household receives a lump-sum transfer, $(\gamma_t - 1)\tilde{M}_{t-1}$, of new units of money from a monetary authority with no purpose other than to change the stock of money over time. We assume that the gross growth rate of the average money stock,

$$\gamma_{t+1} = \frac{\tilde{M}_{t+1}}{\tilde{M}_t}, \tag{2.3}$$

evolves stochastically via a discrete Markov chain:

$$\text{Prob}\left\{\gamma_{t+1} = \gamma' | \gamma_t = \gamma\right\} \equiv \pi^{\gamma}(\gamma', \gamma) \qquad \forall t, t+1; \quad \gamma', \gamma \in \mathcal{G}, \tag{2.4}$$

where $\mathcal{G}$ is a finite set.

Finally, it is useful to define the vector, $\sigma_t \equiv (\phi_t, \gamma_t)$, of exogenous stochastic parameters. Using (2.1) and (2.4) we define a Markov process for σ:

$$\text{Prob}\left\{\sigma_{t+1} = \sigma' | \sigma_t = \sigma\right\} \equiv \pi(\sigma', \sigma) \qquad \forall t, t+1; \quad \sigma', \sigma \in \mathcal{S} \equiv \mathcal{P} \times \mathcal{G}. \tag{2.5}$$

In each period then, the state is given by σ_t and the average money stock, $\tilde{M}_t$.

2.2. The Trading Session

In describing the optimization problem of a representative household (of any type), it is useful to begin with exchange within a period. At the beginning of period t a representative household observes the state of the economy, $(\tilde{M}_t, \sigma_t)$, and has post-transfer individual household money holdings M_t.[4] The household chooses the probabilities with which an individual buyer observes different numbers of price quotes, $\mathbf{q}_t \equiv \{q_{0t}, \ldots, q_{Kt}\}$. Also, because household members do not have independent preferences, we may think of the household as issuing trading instructions to both its buyers and sellers to maximize household utility. Buyers and sellers then divide for a *trading*

[4] In this sub-section, we suppress the economy state vector as it remains fixed throughout the period.

session where fiat money is exchanged for goods. We assume that it is not until this trading session begins that the exact number of quotes observed by individual buyers is known. As a result, households have no incentive to treat their members asymmetrically; they distribute money holdings equally to all buyers and issue the same instructions to all buyers and to all sellers.[5]

In a trading session, sellers post prices and buyers decide whether or not to purchase at the posted price, acting in accordance with household instructions. Since trading begins after $\mathbf{q}_t$ is chosen, we treat the measures of buyers observing particular numbers of price quotes as fixed (as they are throughout the trading session) and return to their determination when we consider households' dynamic optimization problems below. Following trading, buyers and sellers reconvene and the household consumes the goods purchased by its buyers. The sellers' revenue (in fiat money) and any remaining money unspent by the buyers are pooled and carried into the next period, when they are augmented with transfer $(\gamma_{t+1} - 1)\tilde{M}_t$ to become M_{t+1}.

With the measures of buyers observing different numbers of prices fixed, the mechanism by which buyers and sellers are matched is similar to the "noisy search" process of Burdett and Judd (1983). Households know the distribution of prices offered by sellers, but individual buyers may purchase only at a price they are quoted by a specific seller in a particular period.[6] Let the distribution of nominal prices posted by sellers of the appropriate type at time t be described by the cumulative distribution function (*c.d.f.*) $\tilde{F}_t(P_t)$ on support $\tilde{\mathcal{F}}_t$. Given $\tilde{F}_t(P_t)$, the *c.d.f.* of the distribution of the *lowest* price quote received by a buyer at time t is given by

$$J_t(P_t) = \sum_{k=0}^{K} \left(\frac{q_{kt}}{1 - q_{0t}}\right) \left(1 - \left[1 - \tilde{F}_t(P_t)\right]^k\right) \qquad \forall P_t \in \tilde{\mathcal{F}}_t. \tag{2.6}$$

Buyers in a representative household who purchase do so at the lowest price they observe, spending $X_t(P_t)$ when they pay nominal price P_t. Each buyer is constrained to spend no more than the money distributed to them at the beginning of the session by the household. Thus, each buyer faces the following expenditure constraint:

$$X_t(P_t) \leq M_t \qquad \forall P_t. \tag{2.7}$$

Because the household contains a continuum of symmetric buyers, it faces no uncertainty with regard to its overall trading opportunities in the trading session of the current period. Realized

[5] The optimality of equal treatment of symmetric members by the household is addressed by Petersen and Shi (2004). Here we treat it as an assumption.

[6] We assume that buyers cannot return to sellers from whom they have purchased in the past and instead draw new price quotes from the distribution each period.

household consumption purchases in this period are then

$$c_t = (1 - q_{0t}) \int_{\tilde{\mathcal{F}}_t} \frac{X_t(P_t)}{P_t} dJ_t(P_t). \tag{2.8}$$

Individual sellers produce to meet the demand of the buyers who observe their price and wish to purchase. The expected quantity of goods sold in the current period trading session for a representative seller who posts P_t are given by

$$y_t(P_t) = \left[\frac{\hat{X}_t(P_t)}{P_t}\right] \left[\sum_{k=0}^{K} k\hat{q}_{kt} \left[1 - \hat{F}_t(P_t)\right]^{k-1}\right]. \tag{2.9}$$

Here $\hat{X}_t(P_t)$ is the belief of the household regarding the spending rule of its prospective customers, $\hat{q}_{kt}$ is their belief regarding the average measure of those buyers observing k prices, and $\hat{F}_t(P_t)$ is their belief regarding the distribution of prices posted by their competitors. In this expression, the first term represents the quantity per sale. The summation term is the expected number of sales and equals the number of observations of the seller's price multiplied by the probability that it is the lowest price observed. Since the household contains a continuum of sellers, it faces no uncertainty with regard to its total sales in the current trading session. These are given by

$$y_t = \int_{\mathcal{F}_t} y_t(P_t) dF_t(P_t), \tag{2.10}$$

where $F_t(P_t)$ is the distribution of prices posted by an individual seller and $\mathcal{F}_t$ is its support.

Using (2.8)–(2.10) we can write the law of motion for a household's money holdings:

$$M_{t+1} = M_t - \underbrace{(1 - q_{0t}) \int_{\tilde{\mathcal{F}}_t} X_t(P_t) dJ_t(P_t)}_{\text{expenditure}} + \underbrace{\int_{\mathcal{F}_t} P_t y_t(P_t) dF_t(p_t)}_{\text{sales revenue}} + \underbrace{(\gamma_{t+1} - 1)\tilde{M}_t}_{\text{transfer}}. \tag{2.11}$$

A representative household's money holdings going into next period's trading session are M_t minus the amount spent by its buyers this period plus its sellers' receipts of money plus the transfer received at the beginning of the next period.

We now characterize a household's choice of instructions, $X_t(P_t)$ and $F_t(P_t)$, to its buyers and sellers respectively. The household's gain to having a buyer exchange $X_t(P_t)$ units of currency for consumption is given by its marginal utility of current consumption, $u'(c_t)$, times the quantity of consumption good purchased, $X_t(P_t)/P_t$. The household's cost of this exchange is the number of currency units given up, $X_t(P_t)$, times the marginal value of a unit of money in the trading session of the next period, which we denote ω_t. Note that ω_t is the value to the household of relaxing constraint (2.11) marginally. Hence, a household's reservation price equals $u'(c_t)/\omega_t$.

Since individual buyers are small and the household may not reallocate money balances across buyers once the trading session has begun, the optimal spending rule instructs buyers to spend their entire money holdings if the lowest price they observe is below the reservation price and to return with money holdings unspent otherwise:

$$X_t(P_t) = \begin{cases} M_t & \textit{for } P_t \le \frac{u'(c_t)}{\omega_t} \\ 0 & \textit{otherwise,} \end{cases} \tag{2.12}$$

where it is understood that P_t is the lowest observed price. Note that (2.12) is simply an application of Lemma 1 in Head and Kumar (2005).

Next, consider a household's price-posting instructions given to its sellers. The expected return measured in utils to the household from having a seller post price P_t is

$$r_t(P_t) = \underbrace{\left[\omega_t \hat{X}_t(P_t) - \phi_t \frac{\hat{X}_t(P_t)}{P_t}\right]}_{\text{return per sale}} \underbrace{\sum_{k=0}^{K} k\hat{q}_{kt}\left[1 - \hat{F}_t(P_t)\right]^{k-1}}_{\text{expected number of sales}}, \tag{2.13}$$

From (2.13) it can be seen that $r_t(P_t)$ equals the expected return per sale times the expected number of sales. The former term is the utility value of the currency units obtained minus the disutility of production. Here it is clear that the return to posting a price lower than the marginal cost price, $P_t^* \equiv \phi_t/\omega_t$, is negative, and the household will instruct no seller to do so. In addition, the return for posting a price at which no buyer would buy is zero. A household maximizes utility by instructing its sellers to post prices such that

$$P_t \in \underset{P_t}{\operatorname{argmax}}\ r_t(P_t) \equiv \mathcal{F}_t \tag{2.14}$$

The household instructs individual sellers to draw their prices randomly from $F_t(P_t)$ on support $\mathcal{F}_t$ and receives the same expected return from any seller. We discuss the properties of this distribution below.

2.3. *Dynamic optimization*

To this point we have focused on a trading session within a period, holding fixed the probabilities of a representative household's buyers observing different numbers of prices and taking as given the household's marginal value of a unit of money. We now turn to the household's dynamic optimization problem to determine these variables. Throughout, we assume that households employ Markov strategies.

At time t, the state for a representative household is $(M_t, \tilde{M}_t, \sigma_t)$. We represent the dynamic optimization problem of such a household by the following Bellman equation:

$$V_t(M_t, \tilde{M}_t, \sigma_t) = \max_{\mathbf{q}_t, M_{t+1}, X_t(P_t), F_t(P_t), \mathcal{F}_t} \Bigg\{ u(c_t) - \phi_t y_t - \mu \sum_{k=0}^{K} k q_{kt}$$

$$+\beta \sum_{\sigma_{t+1}\in\mathcal{S}} \pi(\sigma_{t+1},\sigma_t) V_{t+1}(M_{t+1},\tilde{M}_{t+1},\sigma_{t+1})\Bigg\}, \tag{2.15}$$

subject to: (2.6)-(2.11),

$q_{kt} \geq 0 \quad \forall \quad k=0,...,K, \quad M_{t+1}\geq 0, \quad X_t(P_t)\geq 0,$

and $\sum_{k=0}^{K} q_{kt} = 1.$

In solving this optimization problem, a household takes as given the actions of other households, $\hat{y}_t(P_t;\tilde{M}_t,\sigma_t)$, $\hat{X}_t(P_t;\tilde{M}_t,\sigma_t)$, and $\hat{\mathbf{q}}_t(\tilde{M}_t,\sigma_t)$; as well as the distributions of prices posted by both its competitors (households of type h) and by producers of its preferred good (type $h+1$). The value function is time varying because it depends on the distributions of nominal prices, which may be expected to change over time as the money stock grows.

The first-order conditions associated with choice of q_{kt} for $k=0,...,K$ are given by

$$u'(c)c_t^k \leq \mu k + \xi_t(M_t,\tilde{M}_t,\sigma_t) \qquad q_{kt}\geq 0 \qquad q_{kt}[u'(c)c_t^k - \mu k - \xi_t(M_t,\tilde{M}_t,\sigma_t)] = 0, \tag{2.16}$$

where $\xi_t(M_t,\tilde{M}_t,\sigma_t)$ is the multiplier associated with the requirement that the q_{kt}'s sum to one. Here c_t^k is the consumption by buyers who observe exactly k prices and is given by

$$c_t^0 = 0, \qquad c_t^k = M_t \int_{\tilde{\mathcal{F}}_t} \frac{1}{P_t} dJ_t^k(P_t) \qquad \forall k\geq 1, \tag{2.17}$$

where $J_t^k(P_t) = 1-[1-\tilde{F}_t(P_t)]^k$. Note that we have made use of the buyers optimal expenditure rule, (2.12), in this derivation.

The first order condition for M_{t+1} is given by

$$\omega_t(M_t,\tilde{M}_t,\sigma_t) = \beta \sum_{\sigma_{t+1}\in\mathcal{S}} \pi(\sigma_{t+1},\sigma_t)\left[\frac{\partial V_{t+1}(M_{t+1},\tilde{M}_{t+1},\sigma_{t+1})}{\partial M_{t+1}}\right]. \tag{2.18}$$

For expenditure, $X_t(P_t)$, assuming the non-negativity constraint is slack, we have:

$$(1-q_{0t})\left(\frac{u'(c_t)}{P_t} - \omega_t(M_t,\tilde{M}_t,\sigma_t)\right) - \lambda_t(P_t;M_t,\tilde{M}_t,\sigma_t) = 0 \qquad \forall P_t, \tag{2.19}$$

where $\lambda_t(P_t;M_t,\tilde{M}_t,\sigma_t)$ is the Lagrange multiplier on a buyers' expenditure constraint, (2.7). Finally, we have the envelope condition

$$\frac{\partial V_t(M_t,\tilde{M}_t,\sigma_t)}{\partial M_t} = \int_{\tilde{\mathcal{F}}_t} \lambda_t(P_t;M_t,\tilde{M}_t,\sigma_t)dJ_t(p_t) + \omega_t(M_t,\tilde{M}_t,\sigma_t). \tag{2.20}$$

Equations (2.16)–(2.20), together with the buyers' expenditure rule, (2.12), and the requirement that $\mathcal{F}_t$ satisfy (2.14) characterize the household's optimal behavior conditional on its money holdings, M_t, the aggregate state, $(\tilde{M}_t,\sigma_t)$, and its beliefs regarding the actions of other households.

2.4 Equilibrium

We consider only equilibria that are symmetric and Markov. By symmetric, we mean that in equilibrium all households choose the same probabilities for their buyers to observe different numbers of price quotes, the same distribution from which sellers draw prices to post, and that all have the same consumption, money holdings, and marginal valuation of money.

The equilibria we consider are Markov in that quantities, price quote probabilities, and the distributions of *real prices*, (measured as nominal prices divided by average money stock), are time invariant functions of the aggregate state. We will denote the Markov equilibrium distributions over real prices and their supports with the same notation as those for nominal prices but without a time subscript, *i.e.* $\tilde{F}(\cdot)$, $F(\cdot)$, $J(\cdot)$, $\tilde{\mathcal{F}}(\cdot)$, and $\mathcal{F}(\cdot)$. In this Markov setting, we drop the time subscript where possible, and use a prime ($'$) to denote the value of a variable in the next period.

We begin by deriving certain properties that such an equilibrium must have, assuming that one exists. In this paper, we do not establish the existence of an equilibrium formally. Rather, we confirm existence by computing directly examples using parameterized versions of the economy.[7] Also, from this point we assume that $\gamma > \beta$ for all $\gamma \in \mathcal{G}$.[8]

If all nominal posted prices at time t are proportional to $\tilde{M}_t$, then there exist distributions of *real* posted prices which depend only on the state. These distributions are characterized by the following supports and conditional *c.d.f.*s:

$$\tilde{\mathcal{F}}(\sigma) \equiv \{p \equiv P_t/\tilde{M}_t \;\; \forall\, P_t \in \tilde{\mathcal{F}}_t;\;\; \forall t \mid \sigma_t = \sigma\}$$

$$\tilde{F}(p \,|\, \sigma) = \tilde{F}_t(P_t) \qquad \forall p \in \tilde{\mathcal{F}}(\sigma), \quad \forall t \mid \sigma_t = \sigma. \tag{2.21}$$

We define $F(p \mid \sigma)$ and $\mathcal{F}(\sigma)$ similarly. If conditional distributions satisfying (2.21) exist, then we may think of buyers as observing *real* price quotes, and define corresponding conditional distribu-

[7] For a similar economy with no aggregate uncertainty (*i.e.* in which both costs and money growth are constant) Head and Kumar (2005) establish formally the existence of an equilibrium of the type considered here. Their arguments may be extended to our stochastic economy by exploiting the continuity of consumption in the parameters governing costs and money growth. We do not do so here, however, because this entails imposing complicated and specific (and economically uninteresting) parameter restrictions governing the degree of variation in these parameters across states. In our numerical experiments, we find that equilibria of the type we consider do indeed exist for a wide range of parameters.

[8] In our economy there can be no equilibrium in which $E_t[\gamma] \leq \beta$ in any state. We are grateful to Shouyong Shi for clarifying this point.

tions of lowest real prices observed in a manner analogous to (2.6):

$$J(p\,|\,\sigma) = \sum_{k=0}^{K} \left(\frac{q_k(\sigma)}{1 - q_0(\sigma)} \right) \left(1 - [1 - \tilde{F}(p\,|\,\sigma)]^k \right). \tag{2.22}$$

Similarly, if the distributions of posted and transactions prices are time-invariant, conditional on σ, then households' nominal money holdings, M_t, expenditure rule for buyers, $X_t(P_t)$, and the support of a household's sellers' posted prices, $\mathcal{F}_t$ may be divided by the average money stock to obtain time-invariant conditional *real* counterparts: $m(\sigma)$, $x(p\,|\,\sigma)$ and $\mathcal{F}(\sigma)$.

We then have the following definition:

Definition: A *symmetric monetary equilibrium* (SME) is a collection of time-invariant, individual household choices, $\mathbf{q}(\sigma)$, $m'(\sigma)$, $x(p\,|\,\sigma)$, $F(p\,|\,\sigma)$; average expenditure rules $\tilde{x}(p\,|\,\sigma)$ and probabilities $\tilde{\mathbf{q}}(\sigma)$; and conditional distributions of posted prices, $\tilde{F}(p\,|\,\sigma)$, such that

1. Taking as given the distributions of posted prices, $\tilde{F}(p\,|\,\sigma)$, the average expenditure rule, $\tilde{x}(p\,|\,\sigma)$, and measures of buyers observing different numbers of price quotes, $\tilde{\mathbf{q}}(\sigma)$; a representative household chooses $\mathbf{q}_t = \mathbf{q}(\sigma)$, $M_{t+1} = m'(\sigma)\tilde{M}_{t+1}$, $X_t(P_t) = x(p\,|\,\sigma)\tilde{M}_t$, and distribution $F_t(P_t) = F(p\,|\,\sigma)$ to satisfy the household Bellman equation, (2.15).
2. Individual choices equal average quantities: $\mathbf{q}(\sigma) = \tilde{\mathbf{q}}(\sigma)$, $x(p\,|\,\sigma) = \tilde{x}(p\,|\,\sigma)$, $F(p\,|\,\sigma) = \tilde{F}(p\,|\,\sigma)$, and individual household money holdings equal the average money stock: $m(\sigma) = 1$.
3. Money has value in all states: For all $\sigma \in \mathcal{S}$, $\tilde{F}(p\,|\,\sigma) > 0$ for some $p < \infty$.

In characterizing an SME for this economy, a key quantity is the sequence of households' marginal valuations of money, $\{\tilde{\omega}_t\}_{t=0}^{\infty}$, as this determines the returns to sellers and buyers from transacting at a particular price at a particular point in time. Returning to the household optimization problem and combining (2.18)-(2.20), we have

$$\begin{aligned} \omega_t(M_t, \tilde{M}_t, \sigma_t) = \beta \sum_{\sigma_{t+1} \in \mathcal{S}} \pi(\sigma_{t+1}, \sigma_t) \Bigg[& [1 - q_0(\sigma_t)] u'(c(\sigma_{t+1})) \int_{\tilde{\mathcal{F}}_t} \frac{1}{P_{t+1}} dJ_{t+1}(P_{t+1}) \\ & + q_0(\sigma_{t+1}) \omega_{t+1}(M_{t+1}, \tilde{M}_{t+1}, \sigma_{t+1}) \Bigg] \end{aligned} \tag{2.23}$$

In an SME, (2.8) and (2.12) imply that average consumption must satisfy

$$\tilde{c}(\sigma_t) = [1 - \tilde{q}_0(\sigma_t)] \tilde{M}_t \int_{\tilde{\mathcal{F}}_t} \frac{1}{P_t} d\tilde{J}_t(P_t) \qquad \forall t. \tag{2.24}$$

Thus, in an SME (2.23) implies, for all t:

$$\tilde{\omega}_t(\tilde{M}_t, \sigma_t) = \beta \sum_{\sigma_{t+1} \in \mathcal{S}} \pi(\sigma_{t+1}, \sigma_t) \left[u'(\tilde{c}(\sigma_{t+1})) \frac{\tilde{c}(\sigma_{t+1})}{\tilde{M}_{t+1}} + \tilde{q}_0(\sigma_{t+1}) \tilde{\omega}_{t+1}(\tilde{M}_{t+1}, \sigma_{t+1}) \right]. \tag{2.25}$$

We define $\tilde{\Omega}(\sigma) \equiv \tilde{\omega}_t(\tilde{M}_t, \sigma_t)\tilde{M}_t$, for $\sigma = \sigma_t$ and derive

$$\tilde{\Omega}(\sigma) = \beta \sum_{\sigma' \in \mathcal{S}} \frac{\pi(\sigma', \sigma)}{\gamma'} \left[u'(\tilde{c}(\sigma'))\tilde{c}(\sigma') + \tilde{q}_0(\sigma')\tilde{\Omega}(\sigma') \right] \qquad \forall \sigma \in \mathcal{S}. \tag{2.26}$$

We thus associate an SME with a collection of state-contingent values, $\tilde{\Omega}(\sigma)$, $\sigma \in \mathcal{S}$, for households' marginal value of fiat money.

We now show that several properties of the non-stochastic economy examined by Head and Kumar (2005) extend to the stochastic environment studied here. In order to avoid repetition of the results presented in that paper, here we discuss these properties only briefly. The appendix provides a formal presentation of the extensions. First, as described in Proposition 1 of Head and Kumar, if the SME is characterized by some buyers observing one price while others observe more than one price, then the distribution of posted prices will exhibit price dispersion necessarily.[9] Secondly, if an SME exists, then the equilibrium will be characterized by a positive measures of buyers observing one price and all remaining buyers observing two prices, in all states. This is an application of Corollary 2 from Head and Kumar (2005). Thus, we may associate an SME with probability $\tilde{q}(\sigma)$ of a buyer observing a single price in each state. In equilibrium, this will equal the measure of buyers observing one price with the remaining buyers all observing two prices. Taking these two properties together, we see that any SME must exhibit price dispersion in all states.[10]

Head and Kumar (2005) demonstrate that a unique SME exists and derive the c.d.f. of posted prices for that equilibrium. While in this paper we do not prove existence formally, we can show that if an SME exists, the distribution of real posted prices has the same form as in their non-stochastic economy. For all σ, these distributions are characterized by

$$\tilde{F}(p \,|\, \sigma) = \frac{\left[\tilde{\Omega}(\sigma) - \frac{\phi}{p}\right][2 - \tilde{q}(\sigma)] - \left[1 - \frac{\phi}{u'[\tilde{c}(\sigma)]}\right]\tilde{\Omega}(\sigma)\tilde{q}(\sigma)}{\left[\tilde{\Omega}(\sigma) - \frac{\phi}{p}\right] 2[1 - \tilde{q}(\sigma)]} \tag{2.27}$$

with connected supports, $\tilde{\mathcal{F}}(\sigma) = [p_\ell(\sigma), p_u(\sigma)]$, where,

$$p_\ell(\sigma) = \frac{[2 - \tilde{q}(\sigma)]\phi p_u(\sigma)}{2[1 - \tilde{q}(\sigma)]\tilde{\Omega} p_u(\sigma) + \tilde{q}(\sigma)\phi} \qquad \textit{and} \qquad p_u(\sigma) = \frac{u'[\tilde{c}(\sigma)]}{\tilde{\Omega}(\sigma)}. \tag{2.28}$$

9 We exclude the case in which no buyer observes any price (*i.e.* $\tilde{q}_o$=1). In this case, *any* price distribution is consistent with household optimization.

10 This last result differs from those of Burdett and Judd (1983) who find that there is always an equilibrium in which all buyers observe exactly one price and all sellers charge the monopoly price. For a discussion of the reasons for this difference between their economy and ours, see the appendix.

Note that given $\tilde{q}(\sigma)$ and $\tilde{\Omega}(\sigma)$ it is possible to characterize the distribution analytically. Furthermore, using (2.27), it is straightforward to derive expressions for the conditional densities of both posted and transactions prices and to show that they are monotonically decreasing in all states.

Finally, we can show that the optimal choice of $q(\sigma)$ for an individual household is similar to that derived in Head and Kumar (2005):

$$q(\sigma) = \begin{cases} 0 & \text{if } \mu < \mu_L(\sigma) \equiv u'\left(c^2(\sigma)\right)\left[c^2(\sigma) - c^1(\sigma)\right] \\ \frac{u'^{-1}\left(\frac{\mu}{c^2(\sigma)-c^1(\sigma)}\right) - c^2(\sigma)}{c^1(\sigma)-c^2(\sigma)} & \text{if } \mu_L(\sigma) \leq \mu \leq \mu_H(\sigma) \\ 1 & \text{if } \mu > \mu_H(\sigma) \equiv u'\left(c^1(\sigma)\right)\left[c^2(\sigma) - c^1(\sigma)\right] \end{cases}, \tag{2.29}$$

where $c^k(\sigma)$ for $k = 1, 2$ are defined in (2.17). In (2.29) $\mu_L(\sigma)$ and $\mu_H(\sigma)$ are state contingent cut-off levels for search costs. In state σ, if the search cost is *below* $\mu_L(\sigma)$, then the household will choose to have *all* of its buyers observe more than one price (*i.e.* $q(\sigma) = 0$). Similarly, if $\mu > \mu_H(\sigma)$, the household will choose to have *no* buyer observe a second quote (*i.e.* $q(\sigma) = 1$). As discussed above, both of these cases are inconsistent with the existence of a SME so we require $\mu \in [\mu_L(\sigma), \mu_H(\sigma)]$ for all σ.

From (2.29) it is clear in order for an SME to exist, the *constant* search cost parameter must be consistent with an interior value for $q(\sigma)$ in all states. Thus we require that $\mu \in [\bar{\mu}_L, \underline{\mu}^H]$, where $\bar{\mu}_L$ and $\underline{\mu}^H$ denote the maximal lower and minimal upper cut-off levels for search costs across states. This requires that $\bar{\mu}_L < \underline{\mu}^H$ and, hence, restrictions on the range of variation in both ϕ and γ across states. These restrictions are not general as they depend upon the other parameters of the economy and on functional forms. This is the reason why we do not approach the existence of equilibrium formally. Rather, in the next section, we determine search cost parameters for which SME's exist for a given parameterization of the economy. We find that typically the interval $[\bar{\mu}_L, \underline{\mu}_H]$ is non-empty even for substantial variation in σ.

3. Prices and Mark-ups in Equilibrium

We now consider the behavior of equilibrium prices and mark-ups in response to stochastic fluctuations in productivity and the growth rate of the money stock. In this section we describe and provide intuition for the mechanisms by which these exogenous changes are propagated in our economy. We take up the quantitative predictions of a calibrated version of the economy in the next section.

We describe the level of real prices in state σ in an SME by the average real transaction price:[11]

$$\bar{p}(\sigma) \equiv \int_{\tilde{\mathcal{F}}(\sigma)} p(\sigma)\; dJ(p\,|\,\sigma). \tag{3.1}$$

Similarly, we define the *nominal price level* at time t (in state σ_t) as the average nominal transaction price:

$$\bar{P}_t \equiv \int_{\tilde{\mathcal{F}}_t} P_t\; dJ_t(P_t) = \tilde{M}_t \bar{p}(\sigma). \tag{3.2}$$

The nominal price level is not stationary because of money growth and thus is written as a function of time. We define the inflation rate as the net growth rate of the nominal price level:

$$I_t \equiv \frac{\bar{P}_t - \bar{P}_{t-1}}{\bar{P}_{t-1}} \times 100. \tag{3.3}$$

3.1 Stochastic Fluctuations in Marginal Cost

We focus first on the response of real and nominal transaction prices to stochastic movements in the production disutility, ϕ. We define real marginal cost in units of goods in state σ_t as

$$mc(\sigma_t) \equiv \frac{\phi_t}{\Omega(\sigma_t)}. \tag{3.4}$$

We can also define marginal cost in units of money (nominal marginal cost) at time t as

$$MC_t \equiv mc(\sigma_t)\tilde{M}_t. \tag{3.5}$$

The *mark-up* at time t is defined as the ratio of the average nominal transaction price to nominal marginal cost:

$$MU_t \equiv \frac{\bar{P}_t}{MC_t}. \tag{3.6}$$

Note that real and nominal marginal costs fluctuate in response to fluctuations in both ϕ and the money growth rate, γ. (Fluctuations in the latter affect marginal cost through their effects on $\Omega(\sigma)$ and $\tilde{M}$). In this section, our focus is *cost pass-through* to the nominal price level of changes in nominal marginal costs due to changes in ϕ, only.[12] Let

$$\pi_{ik,j\ell} \equiv \text{Prob}\left[\sigma' = (\phi_j, \gamma_\ell) | \sigma = (\phi_i, \gamma_k)\right]. \tag{3.7}$$

[11] For the most part we focus on *transactions* rather than *posted* prices. We do this because changes in the former more accurately signal the quantitative effects of shocks on output, consumption, and welfare. Qualitatively, both transactions and posted prices respond similarly to both production disutility and money growth shocks.

[12] From (3.4) it can be seen that in our economy nominal marginal costs change over time due both to exogenous movements in ϕ and to changes in $\Omega(\sigma)$ which result from the response of households willingness to produce in exchange for money in response to a change in either ϕ or γ. We focus on pass-through of cost changes during state transitions in which *only* ϕ changes because these are comparable to the exogenous cost changes examined in the empirical literature on exchange rate pass-through.

We define the following measure of cost pass-through between state $\sigma_{t-1} = (\phi_i, \gamma_k)$ and state $\sigma_t = (\phi_j, \gamma_k)$ which share the same money creation rate, γ_k:

$$\theta_{ik,jk} \equiv \left[\frac{\frac{\bar{P}_t}{\bar{P}_{t-1}} - 1 - (\gamma_k - 1)}{\frac{MC_t}{MC_{t-1}} - 1 - (\gamma_k - 1)} \right] = \left[\frac{\frac{\bar{p}(\phi_j,\gamma_k)}{\bar{p}(\phi_i,\gamma_k)} - 1}{\frac{mc(\phi_j,\gamma_k)}{mc(\phi_i,\gamma_k)} - 1} \right]. \tag{3.8}$$

Note that when the money growth rate does not change, *nominal* cost pass-through is equal to the ratio of the growth rate of the real average price to that of real marginal cost. Note also that $\theta_{ik,jk}$ is defined *only* when ϕ changes and does not depend on time. When $\theta_{ik,jk} = 1$, we say that cost pass-through is *complete*. Note that this occurs when the mark-up in (3.6) does not change in response to a change from ϕ_i to ϕ_j holding γ_k constant. If $\theta_{ik,jk} < 1$, we say that cost pass-through is *incomplete*. In this case, the mark-up necessarily moves in the opposite direction of the change in ϕ. Finally, pass-through is said to be *more than complete* when $\theta_{ik,jk} > 1$.

Typically we are interested in the average rate of cost pass-through in equilibrium, which we denote $\bar{\theta}$. We measure this by weighting the $\theta_{ik,jk}$'s by the frequencies of possible changes in ϕ, conditional on a change occurring *without* a simultaneous change in the money creation rate:

$$\bar{\theta} = \sum_{k\in\mathcal{G}} \sum_{i\in\mathcal{P}} \bar{\pi}_{ik} \left[\frac{\sum_{j\in\mathcal{P}, j\neq i} \pi_{ik,jk}\theta_{ik,jk}}{\sum_{h\in\mathcal{P}, h\neq i} \pi_{ik,hk}} \right], \tag{3.9}$$

where $\bar{\pi}_{ik}$ is the unconditional probability of state (ϕ_i, γ_k) occurring.

The responses of real and nominal prices to movements in the production disutility parameter are determined by the interaction of two effects. First, holding search intensity fixed, changes in ϕ affect buyers' and sellers' willingness to exchange and produce at a particular price. This induces a change in average transaction prices which we refer as the *direct effect* of a change in production disutility. Secondly, changes in ϕ alter the returns to search on the part of buyers and thus affect average search intensity. This, in turn, induces sellers to change the distribution from which they post prices and, thus, changes the average transaction price. We refer to this latter effect as the *search intensity effect.*

Specifically, consider the case of an increase in ϕ.[13] For simplicity suppose to begin with that ϕ is i.i.d. over time and that the rate of money creation is fixed so that $\tilde{\Omega}(\sigma)$ is constant at $\tilde{\Omega}$ (see (2.26)). An increase in ϕ in this case will raise the real cost of production. We conjecture that in no case will this lower the average real transaction price, thereby increasing consumption.[14] To

[13] The effects of a reduction in ϕ are qualitatively similar.

[14] This conjecture is verified in numerical experiments below.

the extent that this change in costs raises the average real transaction price, consumption falls. On the buyers' side, this raises the household's reservation price by increasing marginal utility. On the sellers' side, we see from (2.27) that an increase in both ϕ and the reservation price induce the household to increase the frequency with which its sellers post higher prices resulting in a rise in the average real posted price, as expected. The degree of pass-through emanating from this direct effect may be more or less than complete depending on the properties of the utility function.[15]

Now consider the search intensity effect. First, note that the utility function determines the response to a cost increase of the ratio $R \equiv p_u/p_l$, which is a measure of price dispersion useful for measuring the returns to search. It can be shown that R will rise in response to an increase in marginal costs if the utility function exhibits a sufficient degree of curvature as measured by $-u''(c)c/u'(c)$.

If the direct effect of a cost increase raises the price dispersion ratio, R, the returns to search by buyers are normally increased. This leads to an increase in average search intensity (*i.e. it lowers* $\tilde{q}$). From (2.27), we see that sellers respond to this by posting *lower* prices with a higher probability and thus reducing the average real transaction price. This reflects the fact that owing to increased search intensity market power is reduced and the average mark-up falls. The search intensity effect of an increase in ϕ thus to some extent counteracts the direct effect and reduces pass-through. Of course, if R decreases from the direct effect, the resulting fall in search intensity raises mark-ups and magnifies the increase in average prices resulting from the shock. Overall, the degree to which cost increases are passed through to transaction prices is ambiguous,

So far, our discussion has been limited to the case of i.i.d. production disutility shocks. When shocks follow a more general Markov process, the arguments above are somewhat complicated by the response of the marginal value of a unit money. It can be shown, however, that changes in $\tilde{\Omega}$ do not resolve the ambiguity described above with regard to the degree of cost pass-through. In our economy, both incomplete and more than complete pass-through are theoretical possibilities. In the next section we show that pass-through is incomplete in a calibrated version of the economy and this is robust to variation in economy parameters.

3.2 Changes in the Rate of Money Growth

We now consider the price and mark-up effects of stochastic changes to the money growth rate, holding the disutility of production fixed. We focus on the extent to which fluctuations in

[15] For example a monopoly will raise or lower its mark-up in response to a cost increase depending on properties of the demand function it faces.

monetary growth affect real transaction prices. To this end, we introduce the following measure of the *real price elasticity* with respect to the money growth rate from period $t-1$ to period t:

$$\lambda_t \equiv \frac{\left[\frac{\bar{p}(\sigma_t)-\bar{p}(\sigma_{t-1})}{\bar{p}(\sigma_{t-1})}\right]}{\left[\frac{\gamma_t-\gamma_{t-1}}{\gamma_{t-1}}\right]}, \tag{3.10}$$

which is defined only when γ changes. If changes in the money growth rate have no effect on real prices (*i.e.* $\lambda_t = 0$) then nominal prices will change with the rate of money creation so that $I_t = \gamma_t$. If $\lambda_t < 0$, then movements in the inflation rate are smaller than the underlying change in the growth rate of money and we say that nominal price adjustment is incomplete (or nominal prices are "sticky"). In this case a stochastic increase in the money creation rate lowers real prices and raises consumption. In contrast, if $\lambda_t > 0$ inflation changes by more than the change in γ, price adjustment is more than complete, and an increase in the money growth rate raises real prices and lowers consumption. Below, we report the average real price elasticity, denoted $\bar{\lambda}$, which is calculated in an analogous manner as $\bar{\theta}$ in equation (3.9).

Note that if γ is i.i.d. over time, then fluctuations in it have no effect on the marginal value of a unit of money, $\tilde{\Omega}(\sigma)$. With $\tilde{\Omega}(\sigma)$ constant, money growth shocks affect neither buyers' real spending rules nor sellers' real price-posting strategies and and so λ is necessarily equal to zero. Henceforth, we focus on persistent fluctuations in the money growth rate.

As with stochastic changes in the production disutility, we can separate the effects of persistent changes in the money growth rate into two effects. We consider first the effect of a change in γ on buyers and sellers incentive to trade at a given price holding search intensity fixed. We will refer to this as the *inflation tax effect* as it is associated with changes in the expected future value of money. As with shocks to ϕ, changes in the distribution of real prices induce a *search intensity effect* associated with changes in the returns to search.

Consider a persistent increase in the growth rate of money. With search intensity fixed, it can be seen from (2.26) that the effect on $\tilde{\Omega}(\sigma)$ is ambiguous and depends upon the properties of the utility function. We will focus on cases in which in the absence of a change in search intensity, an increase in γ reduces both consumption and the marginal value of money.[16] In this case, from (2.27) it can be seen that sellers post higher prices with higher probability so that the average *real* transaction price will rise. This implies that price adjustment from the inflation tax effect alone

[16] We show in our numerical experiments in the next section that this occurs in all versions of our calibrated economy that we consider.

will be more than complete ($\lambda_t > 0$). This is similar to the effect of a persistent money growth shock in a flexible price real business cycle model with money introduced either directly into the utility function or through a cash-in-advance constraint as in Cooley and Hansen (1989).

Furthermore, from (2.28) it can be seen that if an increase in γ would, in the absence of any change in search intensity, reduce consumption, then $R = p_u/p_l$ will rise. Increased dispersion will again raise search intensity and reduce both mark-ups and the average real transaction price. Thus, in this case the search intensity effect mitigates the increase in real and nominal prices resulting from the inflation tax effect of an increase in money growth. The overall result may be incomplete price adjustment, but this depends on the relative magnitude of the two effects. In the next section we show that incomplete price adjustment does occur in a calibrated version of our economy.

4. Calibration and Numerical Experiments

As demonstrated in the previous section, qualitatively our economy gives ambiguous predictions regarding both the degree of cost pass-through and the responsiveness of nominal prices to fluctuations in the rate of money growth. In this section calibrate the model and study these issues, as well as the cyclical properties of market power. We describe our benchmark calibration, illustrate the effects of fluctuations in productivity and the money growth rate in isolation, and then consider the overall quantitative implications of our calibration. Finally we consider the robustness of our findings to variation in the economy's parameters.

4.1 Calibration

We begin with a benchmark calibration of the economy, starting with preference parameters. We set the discount factor, β, equal to .99, consistent with an annual real interest rate equal to 4% when each period is taken to represent a calendar quarter. We restrict attention to constant relative risk aversion preferences:

$$u(c) = \frac{c^{1-\alpha} - 1}{1 - \alpha} \qquad \alpha \geq 0, \tag{4.1}$$

and set $\alpha = 1.5$, a value within the range typically used in calibrated macroeconomic models.

We assume that γ and ϕ are independent of one another and parameterize the Markov processes governing their evolution separately. This is reasonable given that the growth rates of base money and labour productivity are essentially uncorrelated: Over the period 1960-2003 for the U.S. the two series exhibit a contemporaneous correlation of .01 at the quarterly frequency.

We specify the parameters which govern the process for the growth rate of the money supply to match properties of the quarterly growth rate of the monetary base for the U.S., $\bar{g}_m$, from

1960.1-2003.3. We choose the base because it is a very narrow definition of money and therefore corresponds well to the stock of fiat money in our economy. Over this time period, the average quarterly growth rate of the base was 1.71%, implying an annualized growth rate of 7.02%. The ratio of the standard deviation of the quarterly growth rate relative to its mean is .72%, and its first-order autocorrelation is .45.

Over this time period, the average quarterly growth rate of U.S. real GDP, $\bar{g}_y$, was .77%. Because our economy exhibits both zero growth and velocity equal to one in all periods, we set the average gross growth rate of the money stock to the average gross growth rate of the base net of the gross growth rate of real GDP:

$$\bar{\gamma} = 1 + (g_m - g_y) = 1.0094. \tag{4.2}$$

This implies an annualized average growth rate (and rate of inflation) of 3.8%.[17]

We let the money growth rate take on three values, $\mathcal{G} = \{\gamma_l, \gamma_m, \gamma_h\}$. We assume that the transition matrix, Π^γ, is symmetric and that in each period the probability of transiting to either of the other states is equal and given by $(1-\pi^\gamma)/2$ where π^γ denotes the probability of γ remaining unchanged from this period to the next and is constant across states. Setting $\gamma_m = \bar{\gamma}$, we then choose γ_l, γ_h, and π^γ to match the standard deviation and first-order autocorrelation of the growth rate of the monetary base. This results in:

$$\mathcal{G} = \{1.0004, 1.0094, 1.0184\} \qquad \text{and} \qquad \pi^\gamma = .64. \tag{4.3}$$

Given β, α, and the process for the rate of money creation, we choose the search cost parameter, μ, and the process for ϕ to match the percentage standard deviation and autocorrelation of the deviation of the log of aggregate output from Hodrick-Prescott trend for the period 1960-2003 (1.58% and .86 respectively), while maintaining an average mark-up in equilibrium of 4%.[18] An average mark-up of this magnitude is in line with estimates of the *economy-wide* average mark-up implied by the work of Bowman (2003) and Basu and Fernald (1997). As with the money growth rate, we let ϕ take on three values. We set $\phi_m = .1$, a normalization which has no effect on the responses of prices to shocks given a 4% mark-up. Maintaining again the assumption that the

[17] Average annual inflation based on the CPI over this period was 4.3%.

[18] We choose parameters to match these moments with the sample averages of the standard deviation and first-order autocorrelation of output over 10,000 simulations each 175 periods in length.

diagonal elements of the transition matrix are equal (π^{ϕ}) and that the probabilities of transiting to each of the other two states is equal, our calibrated values are

$$\mathcal{P} = \{.09656, .1, .10344\} \qquad \text{and} \qquad \pi^{\phi} = .97. \tag{4.4}$$

4.2 Fluctuations in Production Costs

In this section we hold the growth rate of the money stock constant at its mean and examine quantitatively the behavior of equilibrium prices and mark-ups in response to shocks to the production disutility parameter, ϕ. For this exercise, we set $\gamma_t = \gamma_m$ for all t and maintain all other aspects of the benchmark calibration. In this section, we will use θ_{ij} for $i, j \in \{l, m, h\}$ to denote cost pass-through (as measured by (3.8)) when ϕ changes from $\phi_{t-1} = \phi_i$ to $\phi_t = \phi_j$.

In Figure 1, the equilibrium densities of real transaction prices for the three values of ϕ in equilibrium are depicted by the dashed and solid lines. The figure also includes the average real transaction price, average mark-up, and the fraction of buyers observing a single price for each value of ϕ. In the figure, it can be seen that as real costs fall and rise, the densities of transaction prices shift to the left and to the right, respectively, and that the distribution of transaction prices stochastically dominates in a first-order sense the distribution of prices in any state with lower costs.

Consider the case in which ϕ increases from ϕ_m to ϕ_h. In this case real marginal cost increases by 2.43% while the average nominal transaction price increases by only 2.07%, implying a pass-through coefficient of $\theta_{mh} = .85$. Similarly, when ϕ falls from ϕ_m to ϕ_l we can calculate $\theta_{ml} = .82$. Note that these coefficients are less than one, indicating incomplete pass-through of cost changes to nominal transaction prices. This incompleteness implies that the average mark-up must be inversely related to costs as indicated in the figure. Secondly, note there is some asymmetry (quantitatively small in this case) in pass-through between cost increases and cost decreases with a larger effect for increases.

As described above, incomplete pass-through is the net result of conflicting direct and search intensity effects. In Figure 1, the dash-dot lines depict the direct effect of a change in ϕ, *i.e.* the response of real transactions prices when search intensity is held fixed at $\tilde{q}(\phi_m) = .649$, equilibrium search intensity at $\phi = \phi_m$. In this case, pass-through coefficients associated with both increases and decreases in costs are given by $\theta_{mh} = \theta_{ml} = 1.04$. Thus, both the incompleteness and asymmetry of cost pass-through result from the search intensity effect; in its absence pass-through is more than complete.

The search intensity effect is present because increases (decreases) in costs raise (lower) the returns to search by affecting price dispersion. In this example, when ϕ increases from ϕ_m to ϕ_h, p_u/p_l rises and this induces households to lower the probability with which buyers observe a single price (as opposed to two prices) by 4.75%. This weakens sellers' market power and pushes all prices closer to the marginal cost price, lowering the average mark-up from 3.55% to 3.18%. Overall, this mitigates the increase in transaction prices that would have taken place if search intensity were fixed and leads to incomplete pass-through. Similarly, cost reductions reduce price dispersion and induce reductions in household search intensity, resulting in an increase in the mark-up. Again, this leads to incomplete pass-through, in this case of a reduction of costs.

In this example (and in any case in which there is incomplete cost pass-through) production costs and the average mark-up are negatively related. Since costs are negatively correlated with aggregate output, this implies that movements in ϕ induce *procyclical* fluctuations in the average mark-up. Note that the mark-ups charged by *individual sellers* are *always* random (even in the absence of changes in either ϕ or γ). Moreover, the fact that mark-ups and output move together in response to cost movements does not necessarily imply procyclical mark-ups in the economy overall. This will depend also on the direction and relative magnitude of mark-up fluctuations associated with shocks to the money growth rate.

We now consider the relationship between cost pass-through and the average rate of inflation when only cost shocks are present. In this exercise we maintain all parameters from the previous experiment (including the process for ϕ), and vary the constant rate of money creation. Figure 2 plots average cost pass-through, $\bar{\theta}$; the average mark-up, $\overline{mu}$; and the fraction of buyers observing a single price, $\bar{q}$ against average annualized inflation.[19]

The figure illustrates that cost pass-through is increasing in the average rate of inflation at a decreasing rate. Moreover, pass-through can be very low; with annual inflation at 1.5%, $\bar{\theta} = .04$. This positive relationship between pass-through and inflation is consistent with a number of empirical studies which have studied the relationship between inflation and exchange rate pass-through to goods prices. Campa and Goldberg (2005), Choudhri and Hakura (2006), Devereux and Yetman (2002), and Gagnon and Ihrig (2004) all use cross-country regressions to demonstrate that countries with lower inflation typically have lower rates of exchange rate pass-through. Bailliu and Fujii (2004) perform structural break tests for several countries and argue that those which experienced a reduction in inflation in the 1990's, also experienced a fall in exchange rate pass-

[19] In this context, the *average* is taken over time or across states, rather than across sellers in a particular state.

through. Devereux and Yetman (2002) also provide evidence that the relationship between inflation and pass-through is non-linear and consistent with our finding that pass-through increases with inflation at a decreasing rate.

Figure 2 also shows that there is a negative relationship between average inflation and the average mark-up implied by our benchmark economy. This too is consistent with empirical evidence provided by several authors. Banerjee, Mizen, and Russell (2007), Banerjee, Cockerell, and Russell (2001), Banerjee and Russell (2001), and Gali and Gertler (1999) all find a negative and significant long-run relationship between inflation and mark-ups using time-series data for the U.S., the U.K., and Australia.

The presence of these relationships in our model has a clear intuitive explanation. First, note that an increase in anticipated inflation reduces the future value of money, raising the real price level and the reservation price directly. Higher real prices lower consumption and thus raise the reservation price further. The marginal cost price, however, is not directly affected and increases only through the effect of a reduction in the value of money on the willingness of households to incur costs in order to acquire money. As a result, price dispersion increases and households optimally choose higher levels of search intensity (as evident in Figure 2). This results in both lower mark-ups and more cost pass-through.

Our results suggest that the magnitude of nominal price changes due to productivity movements will be higher for economies with higher rates of inflation. This result is consistent with the empirical findings of Gagnon (2007) who finds a positive relationship between inflation and the magnitude of price changes in highly disaggregated consumer prices in Mexico from 1994-2004. It also implies that productivity fluctuations of a given magnitude will generate greater volatility of inflation the higher is the average rate of inflation.

4.3 Fluctuations in the Rate of Money Growth

We now explore the behavior of equilibrium prices and mark-ups in response to stochastic changes in the money growth rate, γ, holding the production disutility parameter constant at its mean, $\bar{\phi} = .1$. We maintain all other aspects of the benchmark calibration including the values of $\mathcal{G}$ and π^{γ} given in (4.2). The low, medium, and high rates of money creation, annualized, are .15%, 3.80%, and 7.55%, respectively.

In Figure 3, the equilibrium densities of real transaction prices for these three values of γ are depicted by the dashed and solid lines. The figure also includes the average real transaction price, average mark-up, and the fraction of buyers observing a single price for each value of γ. In the figure, it can be seen that a higher money growth rate is associated with *lower* average real transaction prices and, thus, *higher* output.

In particular, when γ increases from γ_m to γ_h, the average real price falls generating a real price elasticity with respect to the money growth rate equal to $\lambda_{mh} = -.59$. Similarly, when the money growth rate falls from γ_m to γ_l, aggregate output falls and we calculate $\lambda_{ml} = -1.74$. Since these real price elasticities are negative, nominal prices must react incompletely to a change in the growth rate of money.

Note that, as in the case of cost movements, there is an asymmetry (in this case much larger quantitatively) with regard to price adjustment between increases and reductions in the money growth rate. Specifically, reductions in the money growth rate have a larger effect; *i.e.* they reduce output by more than increases in the money growth rate raise it. This result is consistent with the empirical findings of Devereux and Siu (2007) and others who find that positive monetary shocks have smaller effects on output than do negative monetary shocks.

As described above, incomplete price adjustment to a change in the money growth rate is the net result of the combined direct (inflation tax) and search intensity effects. In Figure 3, the dash-dot lines depict the direct effect of a change in γ, *i.e.* the response of real transactions prices when search intensity is held fixed at $\tilde{q}(\gamma_m) = .664$, equilibrium search intensity when $\gamma = \gamma_m$. In this case, the real price elasticities are positive and given by $\lambda_{mh} = 1.86$ and $\lambda_{ml} = 1.76$. Thus, without the search intensity effect, nominal price adjustment to a change in the money growth rate is more than complete and real prices are positively related to changes in the money creation rate while output is negatively related to changes in money growth. In this case, stochastic movements in the money growth rate have a similar effect to the that which they have in the cash-in-advance model of Cooley and Hansen (1989). Thus, it is endogenous search intensity here that results in incomplete price adjustment.

Finally, note that the money growth rate is inversely related to the average mark-up when both effects are taken into account. Since money growth is also inversely related to real prices and, therefore, positively related to output, money growth shocks in this economy generate counter-cyclical fluctuations in the mark-up. This contrasts with the effect of fluctuations in production costs, which are associated with pro-cyclical mark-up fluctuations. Empirical work (see, for example Chirinko and Fazzari (1994)) provides mixed evidence on the cyclicality of mark-ups. Our environment is capable of generating either pro- or counter-cyclical mark-ups in equilibrium depending on whether the effects of cost or money growth shocks dominate. We will consider the overall prediction of our economy for mark-up fluctuations in our full calibration below.

We now consider the relationship between the average rate of inflation and the degree of price adjustment to money growth shocks. Here, we vary the average rate of money creation across experiments, holding the relative standard deviation of γ fixed across experiments. We also keep all other economy parameters fixed at their benchmark levels. Figure 4 plots $\bar{\lambda}$, $\overline{mu}$, and $\bar{q}$ against the average annualized rate of inflation.

The figure indicates that the real price elasticity is increasing at a decreasing rate in the average rate of inflation, and becomes positive when trend inflation is sufficiently high. As in the case of cost fluctuations, there is a negative relationship between average inflation and the average mark-up. The intuition for these relationships is similar to that described above for the case of random fluctuations in ϕ.

4.4 Analysis of the Calibrated Economy

We now analyze the full benchmark calibration with fluctuations in both ϕ and γ to determine the economy's quantitative predictions for cost pass-through, the responsiveness of nominal prices to fluctuations in the money growth rate, and the cyclicality of mark-ups. Beginning with the behavior of prices, cost pass-through to nominal transaction prices in the benchmark economy is incomplete, with $\bar{\theta} = .82$. Similarly, the average elasticity of real prices with respect to money growth is $\bar{\lambda} = -.80$, indicating incomplete response of nominal prices to changes in the money growth rate.

Hence, our calibrated benchmark economy is qualitatively consistent with both observed incomplete cost pass-through and "sticky" nominal prices in response to changes in the growth rate of the money supply. Both of these phenomena are present here in the absence of any restrictions on the ability of sellers to change prices. They are driven instead by endogenous fluctuations in market power which are dominated by fluctuations in buyers' search intensity.

Because in our economy the quantity equation always holds, the average rate of inflation must equal the trend growth rate of the money stock. Thus the average annual rate of inflation is 3.85% in the benchmark economy. Period-by-period changes in inflation, however, are driven by the endogenous responses of prices to stochastic movements in ϕ and γ. The degrees of cost pass-through and the elasticity of real prices to changes in the money growth rate mentioned above generate an average percent standard deviation and autocorrelation of inflation of .86% and .37, respectively. Thus, in equilibrium inflation is slightly more volatile (.86% as opposed to .72%) and slightly less autocorrelated (.37 as opposed to .45) than was the monetary base over the period used to calibrate the stochastic process for money growth.[20]

In the benchmark calibration the average mark-up is 4% with 66% of buyers observing one price with the remaining 34% observing two. Market power is typically counter-cyclical overall, although the correlation between the average mark-up and output is subject to substantial sampling variability. The average correlation is -.10 with a standard deviation of .2o over 10,000 trials each 175 periods in length. This suggests a sense in which the effects of fluctuations in money growth dominate those of fluctuations in production costs for the benchmark economy, as the latter produce pro-cyclical movements in the mark-up.

We now describe qualitatively the effects of varying some of the economy's parameters from their values in the benchmark calibration. For the most part the results of these exercises are not surprising given our description of the basic mechanisms at work in the economy in both this and the last section of the paper. We describe them briefly here to give some idea of the robustness of the predictions of the economy based on the benchmark calibration. To save space, we do not provide tables of results, but they are available on request.

We begin by varying the average rate of money creation, $\bar{\gamma}$, while holding its relative standard deviation and all other economy parameters fixed at their benchmark levels. We find a positive relationship between average inflation and both measures of price responses.[21] We also find a negative relationship between the average mark-up and inflation. Finally, the correlation between output the mark-up and aggregate output rises with inflation, becoming positive at some point. These results are not surprising given our analysis of the fluctuations in ϕ and γ independently and as noted earlier, they are consistent with empirical observations.

Next, since empirical estimates of the coefficient of relative risk aversion vary, we consider the extent to which our findings of incomplete cost pass-through and incomplete price adjustment are robust to different values of α. For this exercise, we decrease the cost of search, μ, as we increase α to maintain an average mark-up of 4%. We also vary the parameters governing the process for ϕ so that the variability and persistence of simulated output continues to match the data. Holding all other parameters at their benchmark values, cost pass-through increases α falls, approaching one (complete pass-through) as α approaches one as well. The real price elasticity rises with α, is more responsive to changes in α than is cost pass-through, and is negative for the range of α's for which

[20] The standard deviation and first-order autocorrelation of the CPI over the calibration period were .81% and .73, respectively.

[21] We obtain similar results regardless of whether or not we adjust the process for ϕ to match output moments.

we can compute an equilibrium. Furthermore, the correlation between output and the mark-up is increasing in α, and becomes positive for higher levels of α.

We also consider the relationship between price responses and the average mark-up which we vary by changing the search cost parameter, μ, holding all other parameters at their benchmark levels. Not surprisingly, an increase in search costs decreases average search intensity, and, therefore, raises the average mark-up. This leads to reductions in both cost pass-through and the real price elasticity, implying a negative relationship between price responses and mark-ups. For low enough mark-ups (here, less than approximately 1.2%), the real price elasticity becomes positive indicating more than complete price adjustment to monetary shocks, while cost pass-through approaches one from below as the mark-up approaches 0%. Overall, the stronger is market power on average, the weaker is the adjustment of nominal prices to fluctuations in either production costs or the money growth rate.

We also find that both cost pass-through and the real price elasticity do not change significantly with the variance of production costs. Cost pass-through falls slightly as the persistence of those fluctuations is reduced. The variance of the money growth rate has virtually no effect on the responsiveness of prices by either measure. Increases in the persistence of money growth fluctuations reduce both cost pass-through the real price elasticity.

To summarize, for a wide range of economy parameters centered around our benchmark calibration our economy generates incomplete cost pass-through which is increasing in the trend rate of inflation. The economy also exhibits a form of "sticky prices"; incomplete nominal price response to changes in the money growth rate when market power is not too low (*i.e.* when the average mark-up is above approximately 1.2%). The responsiveness of real prices to money growth changes is increasing in inflation as well. Finally, the average mark-up is negatively related to inflation and the environment is capable of producing a positive or a negative correlation between output and the mark-up, depending on economy parameters.

5. Conclusion

This paper has studied a stochastic monetary economy in which endogenous fluctuations in market power may cause both nominal and real prices to respond incompletely to stochastic fluctuations in productivity and the rate of money creation. Both shocks result in two potentially opposing effects. First, they induce sellers to change prices, with high and low prices adjusting differently, resulting in changes in price dispersion. Second, changes in price dispersion induce adjustments to consumers' search intensity. This effect normally works in the opposite direction from the first, mitigating the response of prices through offsetting movements in the average mark-up. Thus, the adjustment of prices in the economy is driven by endogenous fluctuations in market power emanating from changes in consumers' search intensity without any exogenously imposed constraint on sellers' ability to adjust prices.

In response to productivity shocks, sellers' desire to increase or decrease prices always dominates, and so the average mark-up is pro-cyclical. The search intensity effect may, however, substantially limit pass-through to both real and nominal prices. As the rate of average inflation increases the average mark-up falls. This reduction of market power results in a weakening of the search intensity effect and prices become more responsive to cost changes. Thus, cost pass-through is positively related to average inflation.

In response to a change in the money growth rate, the search intensity effect dominates except at high rates of trend inflation. As a result, mark-up fluctuations that result from money growth shocks may be counter-cyclical. At low rates of average inflation, real prices may fall in response to a money growth shock, generating a form of price stickiness in that nominal prices fail to increase in proportion to stochastic changes in the stock of money. As the trend inflation rate rises, the search intensity effect again weakens and prices become more responsive to shocks.

A calibrated version of the economy can account for several empirical regularities. First, it is well documented that the pass-through of cost movements to real and nominal prices and the adjustment of nominal prices to changes in the money growth rate are incomplete. Second, several studies have presented evidence that a higher average rate of inflation results in both a lower average mark-up and increasing sensitivity of prices to fluctuations in either productivity or money growth. Finally, the literature on the cyclical properties of mark-ups has produced conflicting results with some industries exhibiting pro-cyclical mark-ups and others exhibiting the opposite. Our economy is potentially consistent with these findings as it predicts mark-up fluctuations to be either pro- or counter-cyclical depending on their source.

Our environment can in principle account for incomplete and/or delayed responses of nominal prices to shocks and are consistent with a wide range of possible inflation dynamics. In the present model, however, movements in inflation diverge from movements in the money growth rate only because of fluctuations in the expected future value of money. Incorporating dynamics of this sort entails significant modifications to the environment and is beyond the scope of this paper. In separate research we are currently examining inflation persistence emanating from persistent heterogeneity across households in the spirit of Molico (2006), Molico and Zhang (2005), Berentsen, Camera, and Waller (2005), and Williamson (2005).

Appendix

Extension of Proposition 1 of Head and Kumar (2005):

Let $\hat{q}^{+}(\sigma)$ denote households' beliefs regarding the measure of buyers observing strictly more than one price. Then, given $\hat{\mathbf{q}}(\sigma)$ and $p_u(\sigma)$, we have:

(i.) If $\hat{q}^{+}(\sigma) = 0$, then a household's optimal pricing strategy is to have sellers post $p_u(\sigma)$ with probability one.

(ii.) If $\hat{q}^{+}(\sigma) = 1$, then a household's optimal pricing strategy is to have sellers post the marginal cost price, $p^*(\sigma) = \phi/\tilde{\Omega}(\sigma)$ with probability one.

(iii.) If $\hat{q}^{+}(\sigma) \in (0, 1)$, then the distribution of posted prices in that state is non-degenerate and continuous on a connected support.

Proof: This follows directly from Lemmas 1 and 2 of Burdett and Judd (1983, pp.959-61). To see this, note first that we may define a "firm equilibrium", to use their terminology, as follows. Given beliefs regarding the search behavior of buyers, $\hat{\mathbf{q}}(\sigma)$, and a common reservation price, $p_u(\sigma)$, a *firm equilibrium* is a pair $(F(p\,|\,\sigma), r)$ where $F(\cdot|\,\sigma)$ is a distribution function with support $\mathcal{F}(\sigma)$ and $r = r(p)$ for all $p \in \mathcal{F}$. Next, note that $\bar{p}$, p^*, and r in our notation correspond to $\tilde{p}$, r, and Π respectively, in theirs. Moreover, their probability of observing one price, q_1, is replaced here by $1 - \hat{q}^{+}$, since we have excluded the possibility that $\hat{q}_0(\sigma) = 1$ in any state. ∎

Extension of Corollary 2 of Head and Kumar (2005):

If an SME exists, then $\tilde{q}_1(\sigma) \in (0, 1)$ and $\tilde{q}_2(\sigma) = 1 - \tilde{q}_1(\sigma)$.

Proof: We demonstrate the extension in steps, beginning with three preliminary results.

1. *Extension of Lemma 2 of Head and Kumar (2005):*

   Given $\hat{F}(p\,|\,\sigma)$, $\mathbf{q}(\sigma)$ has $q_k(\sigma) > 0$ for at most two values, k^* and $k^* + 1$.

   Proof: Note first that if the distribution of posted prices is degenerate, then no household has incentive to have buyers observe more than one price. Thus, for price distributions which are degenerate the claim is trivially true, since $q_k > 0$ at most for $k = 0$ and $k = 1$. For the case of a distribution which is non-degenerate and continuous on connected support, we first show that $c^{k+1} - c^k$ is declining in k. It is clear that $J^k(p|\sigma) \equiv 1 - [1 - \tilde{F}(p|\sigma)]^k$ stochastically dominates in a first-order sense $J^{k+1}(p|\sigma)$ so that the expected lowest price observed is declining in the number of price quotes observed, k, and thus that c^k is increasing in k. Moreover, it is straightforward to show that the expected lowest price observed declines at a decreasing rate, and thus that c^k increases at a decreasing rate. The remainder of the proof follows by directly applying the methods of Head and Kumar (2005) in the proof of their Lemma 2.

2. *There can be no SME in which $\tilde{q}^{+}(\sigma) = 1$ in any state.*

   Proof: Suppose that an SME exists with this property in some state. From above, we know that in this case the distribution of real posted prices in this state must be degenerate at the marginal cost price and there can be no gain to households from having a positive measure of its buyers observe a second price quote. Thus, all households will deviate from the conjectured equilibrium search strategy and set $q^{+}(\sigma) = 0$. This contradicts the claim that such an SME exists.

3. *There can be no SME in which $\tilde{q}^{+}(\sigma) = 0$ in any state.*

Proof: Suppose that an SME with this property exists. From above, we have that in this case the distribution of real posted prices must be degenerate at the reservation price. In this case, however, since the household is indifferent between purchasing and holding money over until the next period, it must be the case that the return to the search strategy $q_1(\sigma) = 1$ (recall $q_0(\sigma) = 0$) is negative as search costs are positive and, thus, no household will search. This is inconsistent with a SME.

From these last two results, we have that in any SME a positive measure of buyers must observe one or fewer prices *and* a positive measure must observe two or more prices. Combining this with the first result proves the extension. ∎

Note: This result contrasts with a result of Burdett and Judd (1983) who find that an equilibrium in which all buyers observe exactly one price and all sellers post the "monopoly" price always exists. They obtain this result by assuming that there is a monopoly price sufficiently below buyers' reservation prices so that the surplus from exchanging at the monopoly price more than compensates the buyer for the cost of search. In our monetary economy, if the household expects to be a monopolist with probability one, it will price so as to extract all surplus from the trade. That is, the monopoly price is always equal to the buyers' reservation price. Thus the result of Diamond (1971) obtains: Buyers will not engage in costly search if it means trading at their reservation price with probability one. The result also corresponds to a result obtained in search-theoretic monetary models in which prices are determined by bargaining (see *e.g.* Shi (1995) or Trejos and Wright (1995)). In these models there can be no equilibrium with valued fiat money if sellers make take-it-or-leave-it offers to buyers.

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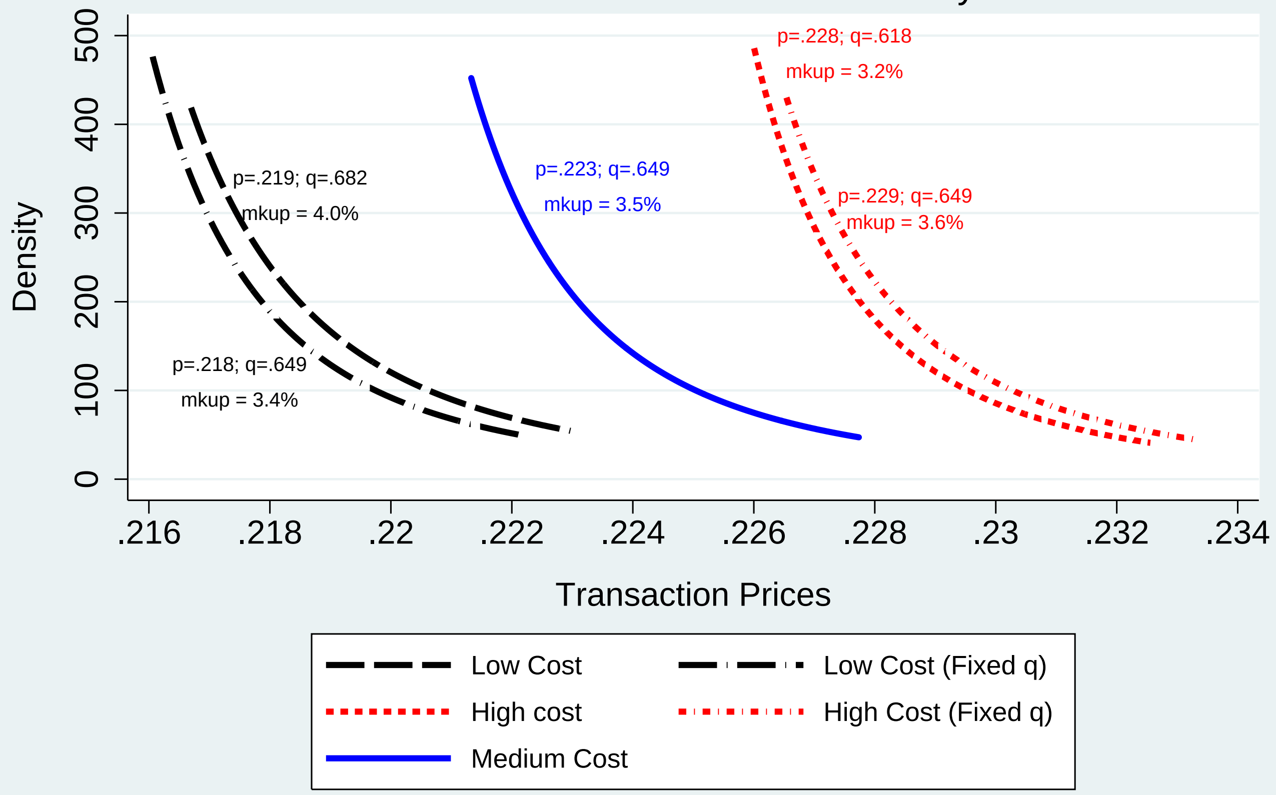
Figure 1

Fixed vs. Variable Search Intensity



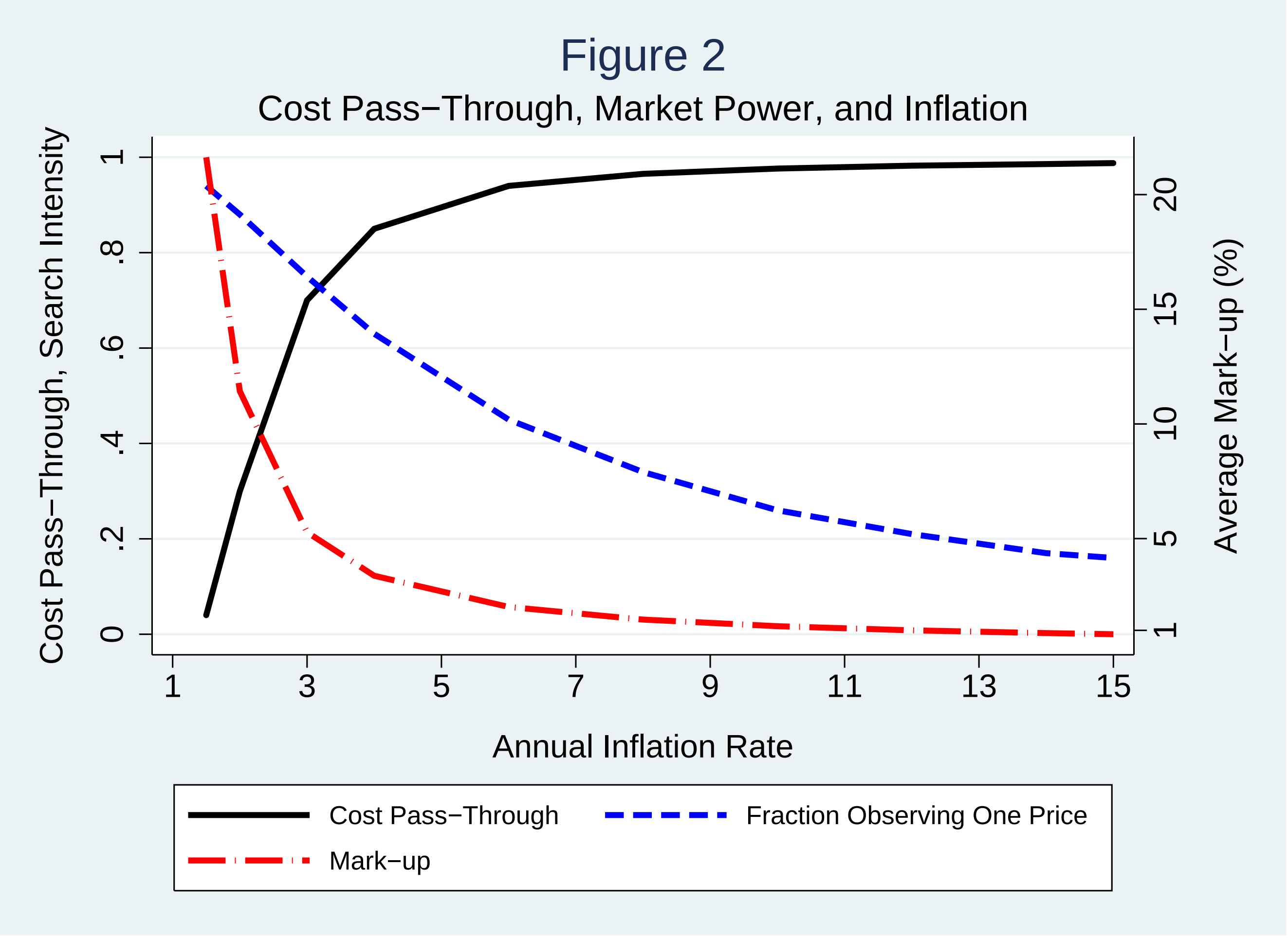


Figure 2
Cost Pass−Through, Market Power, and Inflation
Cost Pass−Through, Search Intensity
0
.2
.4
.6
.8
1
Average Mark−up (%)
1
5
10
15
20
1
3
5
7
9
11
13
15
Annual Inflation Rate
Cost Pass−Through
Fraction Observing One Price
Mark−up

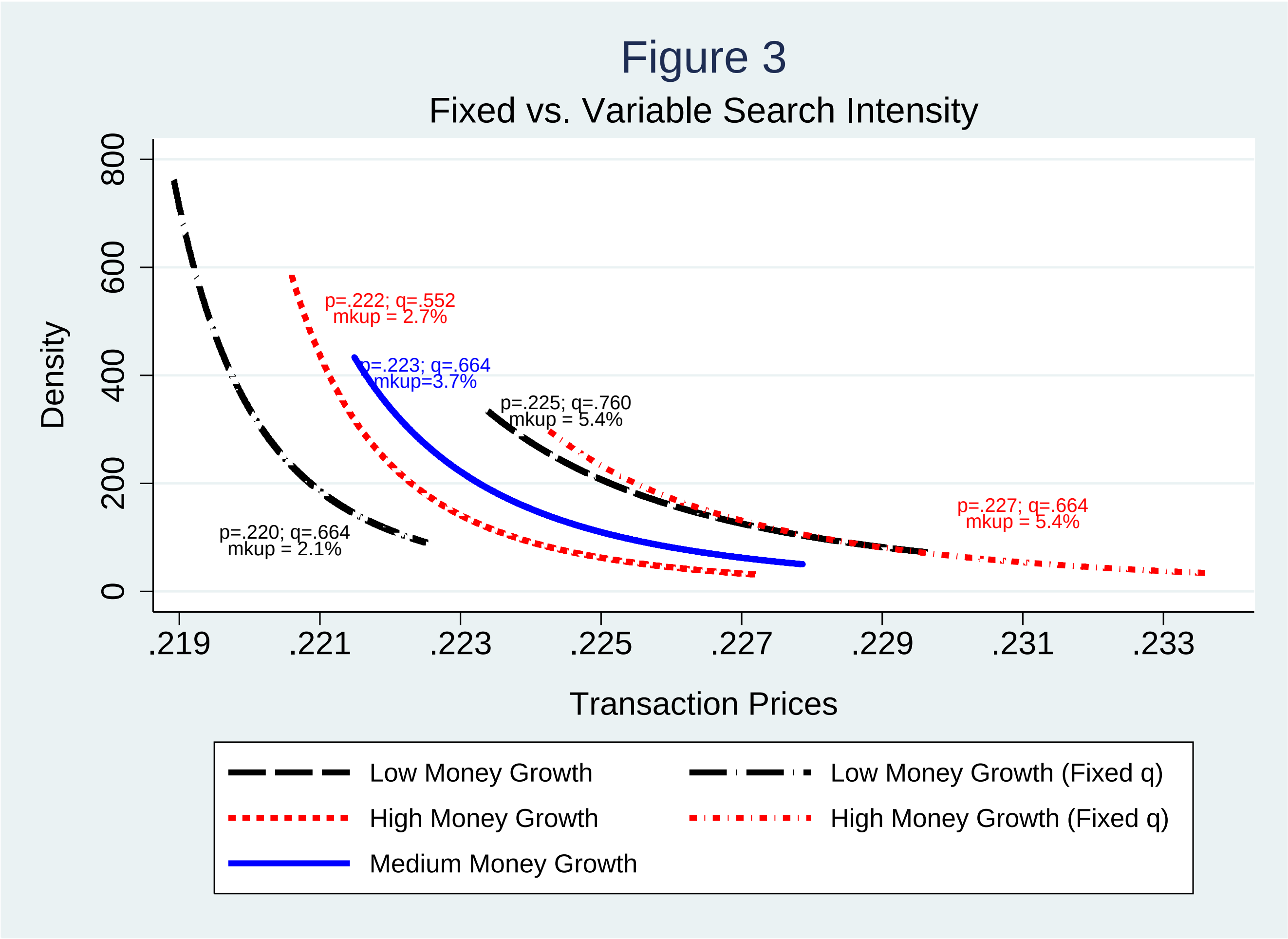


Figure 3
Fixed vs. Variable Search Intensity
Density
0
200
400
600
800
.219
.221
.223
.225
.227
.229
.231
.233
Transaction Prices
p=.222; q=.552
mkup = 2.7%
p=.223; q=.664
mkup=3.7%
p=.225; q=.760
mkup = 5.4%
p=.227; q=.664
mkup = 5.4%
p=.220; q=.664
mkup = 2.1%
Low Money Growth
Low Money Growth (Fixed q)
High Money Growth
High Money Growth (Fixed q)
Medium Money Growth

Figure 4

Real Price Elasticity, Market Power, and Inflation