Imperfect Competition and Corporate Governance

Frank Milne
Queen’s University

David Kelsey
University of Exeter

Department of Economics
Queen’s University
94 University Avenue
Kingston, Ontario, Canada
K7L 3N6

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David Kelsey
Department of Economics,
University of Exeter, England.

Frank Milne
Department of Economics,
Queens University, Canada.

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Abstract

This paper studies corporate governance when a firm operates in imperfect markets. We derive firms’ decisions from utility maximisation by individuals. If those involved in decisions are also consumers the usual monopoly distortion is reduced. Corporate governance can effect the equilibrium in the product (or input) markets. This enables us to endogenise the objective function of the firm. If the firm cannot commit not to change its constitution, we find a Coase-like result where all market power is lost in the limit.

We present a more abstract model of governance in the presence of market distortions.

Address for Correspondence
David Kelsey, School of Business and Economics, University of Exeter, Rennes Drive, Exeter, Devon, EX4 4PU, ENGLAND.

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1 INTRODUCTION

1.1 Background

A central problem in corporate governance is to explain why firms are organised in the way they are. As Hansmann (1996) shows there are a wide range of firms in reality. These range from the small single owner/manager firm, through large corporations with separated shareholders, bondholders, boards and managers, to worker cooperatives, professional partnerships and hybrid organizations, which include non-profits such as hospitals, charitable organizations, schools and universities. From his discussion of the firm, he characterizes the myriad organizations that have evolved to deal with a wide range of organizational problems. In particular these arise where agents interact strategically in producing complex commodities or services. Factors such as the degree of competition in product and input markets and the presence of asymmetric information have an influence on the nature of the firm.

Hansmann (1996) cites a number of examples where firms are owned either by those who purchase their outputs or those who supply inputs to the firm. He argues that, in many cases, this is to counter monopoly or monopsony power. This practice is very common among firms, which supply inputs to or buy produce from farms. Refsell (1914) explains in detail how cooperative grain elevators came to dominate the mid-west. Their share of the industry expanded rapidly at the expense of for-profit rivals during the period 1903-1913. It is clear from his account that the main reason for this was a response to monopoly pricing by for-profit grain elevators. In relatively remote rural areas, it is easier to establish a local monopoly. Professional services such as lawyers and accountants usually organise as partnerships. The reason for this is similar. The firm is a monopoly supplier of inputs which these people need to work. Partnerships reduce the monopoly distortion.

In the present paper we consider how imperfect competition interacts with the objective function of the firm. For example, consider the labour market where firms hire specialised labour that is industry- and/or firm-specific. Our model allows a firm to take into account
the strategic effect of hiring the particular type of labour on the reaction of other firms in the industry. In particular a non-profit firm can pursue a more aggressive strategy in the labour market at the expense of profit maximising rivals. Similar considerations apply if there is imperfect competition in the product market.

1.2 Modelling Firm’s Decisions

In this paper, we consider an economy with monopoly or oligopoly. As we shall argue, there is a strong case against assuming profit maximisation when markets are distorted. However, it is not clear what the alternative should be. We model the firm as a collection of individuals, each of whom is maximising his/her utility. Decisions are made by a process of aggregating the preferences of a group of agents within the firm.¹

One approach, which has been used in the past, is to assume decisions are made by a majority vote of shareholders.² However one can object to shareholder voting models by arguing that, in practice, management have more influence than shareholders. To model this, we assume that the firm’s decisions are made by a group of individuals, which we shall refer to as the control group. For example, the control group could consist of the shareholders and senior management. As another example, consider a firm with no shareholders, but is a partnership. This case is common in legal, accounting, finance and professional firms, where the firm produces services that are a function of human capital, individual and team effort. However, to preserve generality, we shall not explicitly describe the criteria for membership of the control group.

At present there is no widely accepted theory of the internal structure of the firm.³ For this reason we use an abstract model. We make, what we believe to be the mild assumption,

¹Examples of such procedures would be the Nash bargaining solution used by Hart and Moore (1990), non-cooperative bargaining, DeMeza and Lockwood (1998), Bolton and Xu (1999) or the voting models used by De Marzo (1993), Kelsey and Milne (1996) and Sadanand and Williamson (1991).


³For recent surveys of the governance literature see Allen and Gale (2000), Becht, Bolton, and Roell (2003), Shleifer and Vishny (1997), and Tirole (2006).
that the firm’s procedures respect unanimous preferences within the control group. Such rules would include, inter alia, those which give a major role for management. Many familiar forms of governance can be seen as special cases, for instance producer cooperatives, consumer cooperatives, including worker representatives on the board (as in Germany) and many types of non-profit organisation. Despite the generality, our model is able to make a number of predictions concerning equilibrium behaviour and to throw some light on policy questions.

In a discussion of firm structures, Hansmann (1996) provides many examples of firms that are cooperatives, partnerships and non-corporate forms. Some are complex non-profit forms, where the services provided appear to require subtle forms of organisation. Hence it is desirable that any model of the firm should be flexible in abstracting from details that are specific to particular situations and should deal instead with the decision-making process in a general way.

Some theories of the firm use bargaining models to determine the relative power of different individuals. By varying the bargaining game, it is possible to induce different outcomes to the management-control mechanism. Although some of these games have some semblance to reality, we feel they are highly stylized. We prefer to abstract from the details of the bargaining process and simply assume that whatever the bargaining or management game, the process leads to an efficient outcome. If one believes that in certain situations, the outcome is inefficient, then it would be important to explain the source of the inefficiency. One could think of our model as the outcome of a process to design an efficient mechanism. If this is infeasible then we are dealing with inefficient mechanisms. As this is an open theoretical question, we simply by-pass it by assuming an efficient mechanism exists and explore the consequences of that assumption.
1.3 Corporate Governance and Imperfect Competition

In oligopolistic markets it is often the case that a firm, which aims to maximise profit, will not necessarily make the highest profit. If a given firm deviates from profit maximisation, this can change the behaviour of rivals in ways which give the original firm a strategic advantage. Vickers (1985), Fershtman and Judd (1987) and Sklivas (1987) have used this to show that owners can increase profits by hiring managers on incentive contracts that reward according to a weighted average of profits and revenues. This makes managers more aggressive, which can raise profits in Cournot oligopoly.

We relate these arguments to the issues of shareholder voting and monopoly. Consider a firm that is the sole producer of a particular good. Assume that there is consumer representation in the control group. We argue that the firm will produce a greater quantity and sell at a lower price than a conventional profit maximising monopolist. A small price reduction will result in a second order loss of profits but a first order gain in their consumer surplus.

In oligopolistic industries there is a similar effect of the firm’s governance on price. In addition, the choice of the firm’s constitution can affect the strategic interaction in markets. Consider a firm in a Cournot oligopoly. Starting at the profit-maximising level, a decrease in price will lower profits but raise consumer surplus. Different individuals will trade-off these effects in different ways depending on their shareholdings and consumption patterns. Suppose a firm gives more weight in its decision procedures to those who have a relatively greater preference for low prices. Then, ceteris paribus, the firm will charge lower prices and produce more output. This will cause rivals to reduce their output thus possibly giving the firm a strategic advantage in the market. Hence increasing influence of consumers on decision-making will, up to a point, increase profits.

Consider an entrepreneur who designs the constitution of the firm with a view to selling it to outside investors. Then there is an optimal constitution of the firm, which will maximise its value. This constitution is determined by the nature of the competition which the firm faces in input and product markets. It will only be compatible with profit maximisation in
exceptional circumstances.

Levin and Tadelis (2005) have a related argument. They show that partnerships can be superior to standard firms in the provision of services, when the quality of the service is not observable. As is well known, a partnership will hire less workers than the corresponding for-profit firm. Where worker ability varies, this results in the partnership hiring higher quality workers and hence producing a better service. Assume that customers cannot observe the quality directly, then they will prefer to purchase from partnerships, which can therefore be more profitable. This has a similar structure to the model in the present paper. In both cases the choice of corporate governance affects the beliefs of other agents. This causes them to change their behaviour, which indirectly affects the profits of the firm. Taking into account these indirect effects a conventional firm may not be the most profitable.

We have argued that the firm can improve its market position by strategically choosing its constitution. Suppose that the firm does not just choose its constitution once but is able to revise it at any future time. In this case, we obtain a result similar to the Coase conjecture. Consider a firm which is initially profit maximising. The firm will be tempted to change its constitution to increase sales and profits. However the new control group will wish to further amend the constitution to appear more aggressive than it really is. Hence there could potentially be a whole series of expansions of the control group. The result of this process is that the firm will finish by losing all market power and producing the competitive level of output.

1.4 General Model

The underlying principles behind these results are demonstrated by an abstract model of interaction between firms, which we present in Section 6. In this model, firms play a non-cooperative game where actions are strategic complements or substitutes. In both cases it may be desirable to give influence to an outsider who receives an externality from the firm’s strategic variable.
We apply this to Bertrand competition with differentiated goods. We show that, in this case, it may be desirable to have a degree of overlap between the control groups of two or more firms. To illustrate this point consider two firms competing Bertrand style. Call them firm 1 and firm 2. Suppose that there are shareholders of firm 2 in the control group of firm 1. These individuals will have an incentive to raise the price charged by firm 1 above the profit maximizing level, since by doing so they increase the profits of firm 2. However, up to a point, this can help firm 1. Under Bertrand competition there is a strategic advantage in committing to a high price since it encourages other firms to raise their prices. Each firm has an incentive to raise the price charged by the other firm. If both firms do this, it increases joint profits as it moves the industry closer to a collusive equilibrium. Thus firm 1 has an incentive to encourage representatives of rival firms into its control group.

**Organisation of the Paper** Section 2 explains our model of firm decisions. Its use is illustrated by considering the price and quantity decisions of a uniform pricing monopolist in section 3. The effect of the firm’s objective function on strategic interaction in markets is considered in section 4. The case where the firm is allowed to make multiple revisions to its constitution is modelled in section 5. Section 6 contains the more general model with applications to Bertrand competition. Section 7 summarises our conclusions. The appendix contains proofs of those results not proved in the text.

### 2 FIRMS

#### 2.1 Profit Maximisation

Economists usually assume that firms maximise profits. However the firm’s objective function should be a derived concept. A firm is a collection of individuals, shareholders, managers, workers, customers etc. The firm’s choices come about as a result of maximising behaviour by these individuals. The usual justification for profit maximisation is the Fisher Separation Theorem (see Milne (1974), Milne (1981)), which says that if there are no exter-
nalities, the firm has no market power and financial markets are complete, all shareholders will wish to maximise the value of the firm.

In the presence of market distortions, it is not typically the case that owners will wish firms to maximise profits. The Fisher Separation Theorem does not apply if there is imperfect competition, since in that case, a change in the firm’s production plan will affect prices as well as shareholders’ wealth. Firstly, different shareholders will make different trade-offs between more profits and lower prices. Hence, there will be disagreement between different shareholders about the policy of the firm. Secondly, typically, no shareholder will wish to maximise profits. Indeed the concept of profit maximisation is not well defined. Since the firm’s decisions can change relative prices, there is more than one price system which can be used to measure profits. Other market distortions such as incomplete markets or externalities will create similar problems for the objective function of the firm.4

![Diagram](image)

**Figure 1:**

As argued above, in the presence of market distortions, shareholder unanimity cannot be achieved.

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4Similar issues in the context of incomplete markets and externalities are discussed in Kelsey and Milne (1996) and Kelsey and Milne (2002).
guaranteed. This is illustrated by figure 1, which shows the production set for a monopolist who can produce two goods $X_1$ and $X_2$. Since the firm has monopoly power, the prices will depend on the firm’s trade. The diagram shows two possible production plans for the firm. These will give rise to two different price systems. As can be seen, individuals $A$ and $B$ have opposite preferences over the two production plans.

Despite this, it is still the case that there are decisions on which all members of the control group will agree. For instance, we show that a firm, which has a monopoly, will charge less than the profit maximising price. Thus conventional profit-maximising models may have overstated the size of the distortions due to monopoly.

It has been suggested that in addition to shareholders, other parties affected by a firm’s activities should be given influence in the firm’s decisions. These would include *inter alia* representatives of workers, customers and the local community. This paper is able to throw some light on this proposal. Suppose a firm has monopoly power, which cannot be removed by other means. Our model implies that up to a point, increasing customer influence on decisions will reduce distortions. Moreover it could affect competition in the product market. Similarly increasing worker influence can be beneficial if a firm has monopsony power.

### 2.2 A Model of the Firm

We model the firm as a collection of individuals, shareholders, managers, workers and possibly customers and other stakeholders. Our aim is to relate the firm’s objective function to optimising behaviour by these individuals. The decisions of firm $f$ are assumed to be made by a group of individuals $C^f \subset \{1, ..., H\}$, which we shall refer to as the *control group* of firm $f$. The firm’s preferences are assumed to be a function of the preferences of the control group. We do not assume the firm’s preferences are complete or transitive, thus avoiding social choice problems. Note that we do not exclude the possibility that individuals, who are not shareholders (e.g. managers), are able to influence the firm’s preferences. We shall not model the internal decision making of the control group explicitly but simply assume
that whatever procedure is used, respects unanimity. Hence, our results do not depend very sensitively on the composition of the control group.

**Assumption 2.1** The firm’s decision procedure respects unanimous preferences of the control group in the sense that if all members of the control group prefer policy a to policy b with at least one strict preference, then the firm will not choose policy b.

There is a large literature on the theory of the firm, its objectives and implications for its organization. Some of this assumes a particular objective and explores its implications for product or factor markets when competing with other firms that are profit maximizing.\(^5\) Another related literature tries to derive the firm’s objective as an endogenous implication of a game between players who are either producers or customers of inputs and outputs of the firm. Often the game is described as either a bargaining game or as some non-cooperative game between interested parties to a firm-like organisation. This literature can be characterised as setting up a particular model of the firm that emphasises a particular relationship e.g. human capital acquisition, firm financing or the acquisition of a specific physical asset that gives a player an advantage in acting strategically to determine the actions of the firm.\(^6\)

### 3 MONOPOLY

In this section we study the implications of our model of the firm for monopolies. Recall that a profit-maximising monopolist will price according to the inverse elasticity rule, which says that the mark-up of price over marginal cost is inversely proportional to the elasticity of demand.

#### 3.1 Price Decisions

Consider a firm which is the sole producer of good \(x\). Let \(c(x)\) denote the cost of producing quantity \(x\). Let \(D(p)\) denote the demand when the price of monopoly goods is \(p\). The

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monopolist’s profits are given by $\pi = pD(p) - c(x)$.

**Notation 3.1** We shall assume, without loss of generality, that the control group of the monopolist is $\{h : 1 \leq h \leq M\}$.

Assume that members of the control group have quasi-linear utility functions

$$u^h = \theta^h \pi + v^h(p),$$

for $1 \leq h \leq M$, where $\theta^h$ denotes the shareholding of individual $h$ and $v^h$ denotes his/her indirect utility function for consumption of the firm’s output. Since the monopolist implements unanimous preferences of the control group, the optimal point can be obtained by maximising a weighted sum, $\sum_{h=1}^{M} \lambda^h u^h$, of the utilities of the control group for some non-negative weights $\lambda^h$. We may normalise the $\lambda$’s by requiring $\sum_{h=1}^{M} \lambda^h \theta^h = 1.7$

A non-profit maximising firm chooses $p$ to maximise:

$$\sum_{h=1}^{M} \lambda^h u^h = \sum_{h=1}^{M} \lambda^h \theta^h pD(p) - \sum_{h=1}^{M} \lambda^h [\theta^h cD(p) - v^h(p)].$$

Differentiating with respect to $p$ we obtain,

$$D(p) + p \frac{dD}{dp} - \frac{dc}{dx} \frac{dD}{dp} + \sum_{h=1}^{M} \lambda^h \frac{dv^h}{dp} = 0.$$  

By Roy’s identity $\frac{dv^h}{dp} = -x^h$, hence, the first order condition may be written as:

$$\frac{p - \frac{dx}{d\pi}}{p} = \frac{1}{\eta} \left( 1 - \sum_{h=1}^{M} \lambda^h \frac{x^h}{x} \right),$$  \hspace{1cm} (1)

where $\eta$ is the elasticity of demand and $x^h$ denotes consumption of good $x$ by individual $h$.

The price is given by a modified version of the inverse elasticity rule. If the firm has a single owner-manager, individual $i$, this can be further simplified to

$$\frac{p - \frac{dx}{d\pi}}{p} = \frac{1}{\eta} \left( 1 - \frac{x^i}{x} \right).$$  \hspace{1cm} (2)

\footnote{This normalisation is possible provided $\sum_{h=1}^{M} \lambda^h \theta^h \neq 0$. If this were not satisfied, the claimants of the firm’s profit stream would be given no influence over the firm’s decisions. We shall not consider this case further, as we believe it to be of little economic interest.}
If the owner consumes *all* of the firm’s output then the price will be equal to marginal cost, while if (s)he consumes none of the output, this reduces to the usual pricing formula. In general, the optimal price is between marginal cost and the profit maximising level. If the elasticity of demand is constant, price is lower the greater the owner’s consumption of the monopoly good.

If the control group has multiple members, price is not necessarily equal to marginal cost, even if they consume all of the firm’s output. The price will also depend on the relative bargaining power of different members of the control group. Those with relatively large shareholdings and lower consumption will want higher prices. Other things equal, the price will be lower, the greater the weight given to members of the control group with higher consumption.8

### 3.2 Stakeholder Representation

It has been argued that firms should not only be run in the interests of shareholders but also other stakeholders, see for instance Tirole (2006). Our model can be used to examine this proposal. We interpret a stakeholder to be an individual who owns no shares but is a consumer. Consider the case where there are two types of individuals in the control group. Type 1 is a representative owner. Type 2 individuals are “stakeholders” who have no ownership share (hence $\theta^2 = 0$) but may nevertheless influence decisions.

Our normalisation of the $\lambda$’s implies that $\lambda^1 = 1$, $0 \leq \lambda^2 < \infty$, hence (1) becomes,

$$\frac{p_m - \frac{\partial c}{\partial x_m}}{p_m} = \frac{1}{\eta} \left[ 1 - \frac{x_1}{x} - \lambda^2 \frac{x_2}{x} \right].$$

(3)

Increasing the influence of stakeholders would correspond to increasing $\lambda^2$. By equation (3) this will lower the price of the monopoly good. Hence if competition is impossible, a firm

8The problem of a monopolist with some consumers in the control group has been previously considered by Farrell (1985), who assumed unanimity as the firm’s decision rule or Hart and Moore (1996) and Renstrom and Yalcin (2003), who used the median voter rule. Our results are more general since we do not restrict attention to a specific decision procedure.
with some stakeholder representation would be preferable to a profit-maximising monopolist. However, if the power of stakeholders is made too great, price could be reduced below marginal cost, which would be inefficient. In this case, stakeholders would be using their influence to make inefficient transfers from the owners to themselves.

Assume \( x_1^m + x_2^m = x_m \) i.e. there are no consumers other than the owners and the stakeholders. A social planner would aim to set price equal to marginal cost, i.e. \( p_m = \frac{\partial c}{\partial x_m} \). By equation (3) this implies \( 1 - \frac{x_1}{x_m} - \lambda^2 \frac{x_2}{x_m} = 0 \), which can be solved to give \( \lambda^2 = 1 \). Hence the firm should maximise the unweighted sum of utility of shareholders and stakeholders.

In practice, this could be implemented by voting over price and giving an equal number of votes to shareholders and stakeholders. In this case, the median voter would choose to set price equal to marginal cost.

3.3 Monopsony

Our theory so far has emphasised imperfect competition in the product market and the involvement of consumers in firms’ decisions. However similar reasoning applies if some input markets are imperfectly competitive. This would provide a justification for giving input suppliers a special role in decisions. A common example is where the firm is owned by suppliers of a particular form of labour. It is not uncommon for firms to face imperfect competition in specialised labour markets, which is often too thin to be competitive. Another example is farm marketing cooperatives, which buy the output of farms on imperfectly competitive markets.

We can apply similar reasoning to that used for monopoly. Hence we can reinterpret the first order conditions replacing monopoly with monopsony and demand elasticity with supply elasticity. If there is more than one owner, the price of the input will be between the competitive level and the monopsony level, depending upon the influence of suppliers in the control group. In other words, the monopsony distortion is moderated by the influence of suppliers, and in turn the inefficiency is moderated by including the supplier of the monop-
sony input. Other things equal, the more elastic the supply the less importance there is in including the supplier in the control group.

4 COURNOT OLIGOPOLY

We now consider oligopolistic markets. Most of our analysis of monopoly can be extended to this case. For instance, if those in charge of the firm are, in part, consumers the price will be below the profit maximising level. In addition the constitution of the firm affects strategic interaction in markets. Giving greater representation to individuals who are relatively high consumers of the product is a means to committing to a large output. This is an advantage in Cournot oligopoly. In effect it makes a Cournot oligopolist more like a Stackelberg, leader.

4.1 Model

Consider a Cournot oligopoly with \( n \) firms, which can produce at constant marginal and average cost \( c \). For simplicity we assume a linear inverse demand curve \( p = 1 - \sum_{i=1}^{n} x_i \), where \( x_i \) denotes the output of firm \( i \).

We assume that there are two types of individuals, type \( A \) and type \( B \). Type \( A \) individuals do not consume the industry’s output and \( u^A = \theta^A \pi^A \). Type \( B \) individuals care both about income and consumption of the output. Consequently they have (indirect) utility functions \( u^B = \theta^B \pi^B + v^B (p_x) \), where \( \pi^t \) denotes total profit income accruing to an individual of type \( t \). We consider the case where the firm has a control group which consists of two members. One of type \( A \) and one of type \( B \). The type \( A \) individual is assumed to own all of the equity. Thus the utility of the type \( A \) (resp. \( B \)) individual may be written as \( u^A = \pi^A = [p_x - c] x_i \), (resp. \( u^B = v^B (p_x) \)). The same individual is not represented in the control group of more than one firm. Our normalisation of the \( \lambda \)'s implies \( \lambda^A = 1, 0 \leq \lambda^B \leq \infty \).

As in the previous section, the decisions of the firm may be represented as maximising

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\(^9\)Since there is no uncertainty, there is no serious loss of generality in assuming that the utility of type \( A \) individuals is linear in income.
After normalisation. We write $\lambda_i$ for $\lambda_i^B$. We consider the following 2-stage game. In the first period, the owners choose $\lambda_i$ to maximise the value of the firm. In the second stage, firms compete in quantities Cournot-style. We look for a subgame perfect equilibrium of the 2-stage game.

**Proposition 4.1** In an $n$-firm oligopoly the reaction function of firm $i$ is given by

$$x_i = \frac{1 - c + \lambda_i x^B - \left(\sum_{j \neq i} x_j\right)}{2}.$$  \hspace{1cm} (4)

**Proof.** Firm $i$ maximises: $\psi^i = u^A + \lambda_i u^B = \left[1 - c - \sum_{j=1}^{n} x_j\right] x_i + \lambda_i v^B \left(1 - \sum_{j=1}^{n} x_j\right).$

The first order condition for optimal choice of $x_i$ is: $1 - c - \sum_{j=1}^{n} x_j - x_i - \lambda_i x^B = 0$. By Roy’s identity $v^B = -x^B$, hence, $1 - c - \sum_{j=1}^{n} x_j - x_i - \lambda_i x^B = 0$. The result follows. \medskip

The higher $\lambda_i$ the greater the influence given to individual $B$. The proposition implies that, ceteris paribus, an increase in $\lambda_i$ will increase $x_i$. This makes firm $i$ more aggressive, which is advantageous in a game of strategic substitutes. Firms with a larger value of $\lambda_i$ will produce higher output in equilibrium. A possible example is cooperative grain elevators in the mid west, which expanded their output significantly at the expense of for-profit rivals in the period 1903-1913, see Refsell (1914).

**Proposition 4.2** Let $\lambda_i^* (n)$ denote the value of $\lambda_i$ in a symmetric subgame perfect equilibrium with $n$ firms. Then $\lambda_i^* (n)$ is given by

$$\lambda_i^* (n) = \frac{(n - 1) (1 - c)}{(n^2 + 1) x^B}.$$  

This shows that the optimal value of $\lambda$ tends to 0 as $n$ tends to infinity. The more competitive the market is, the closer firms should stick to profit maximisation. Given that $n$ is restricted to take integer values, the maximum value of $\lambda_i^* (n)$ occurs at $n = 2$ or 3. Thereafter $\lambda_i^* (n)$ is strictly decreasing in $n$. This is intuitive, as $n$ increases the market distortion decreases, thus there is less scope for strategic behaviour.

\footnote{We assume that the firm takes $x^B$ (the consumption of good $x$ by a type-B individual) as given when choosing its output.}
These results generalise. Whenever Cournot oligopoly is a game of strategic substitutes, profit can be raised by giving some influence to consumers. Our analysis does not depend crucially on assumptions about the preferences of the different individuals. Similar results could be obtained if a firm were owned by a number of individuals who have different preferences between consumption and profits. By adjusting the decision weights of these individuals, the firm can commit to a more or less aggressive policy in the product market. This is demonstrated by the results in section 6, in which a more general form of preferences is used.\textsuperscript{11}

Suppose that an entrepreneur designs the constitution of the firm to maximise the value at which he can sell it. If the organisation is sold as a profit maximising firm, the price achieved will only be the Cournot oligopoly profits. Higher profits can be made by selling the firm if it has the optimal degree of consumer representation. In this case the entrepreneur will receive the profits of a Stackelberg, leader.

Equally if the problem is not one of designing a constitution from scratch, in a Cournot duopoly, a firm can increase its profits to the Stackelberg, level by giving representation to consumers. The market for corporate control may have a similar effect. If the firm did not initially have the optimal form of corporate governance then an outsider could profitably buy up the shares and reorganise the firm. Subsequently the firm could be re-sold at a profit.

4.2 Input Markets

Similar arguments can be used if the firm faces imperfectly competitive input markets. If firms compete in quantities Cournot-style in the labour market, then the firm’s strategic po-

\textsuperscript{11}Dierker and Grodal (1996) have a result which is almost the reverse of this. They show that under Bertrand competition owners have higher utility if they delegate the running of the firm to a manager with an incentive to maximise profits than if they directly run the firm themselves. As in our model, increasing consumer influence tends to increase output and decrease prices. They obtain a different result because under price competition rivals will respond to lower prices by reducing their own prices, which hurts the firm. In contrast under Cournot oligopoly, other firms will respond by reducing output, which increases profits.
sition may be improved by giving workers or their representatives influence in decision-making. We believe that imperfect competition may be more important in input markets than in output markets. This is because labour markets are often highly specialised both by skill and by location. In professions such as law, medicine and education, it is common for some or all suppliers of labour to have more influence than in conventional investor-owned firms. These firms typically require highly specialised labour and face thin markets for this labour, hence competition is not possible. In these circumstances, it may be in the interest of the firm’s owners to give shares to workers or other individuals with an interest in increasing labour demand. (Assuming that these individuals could be prevented from re-selling.)

5 EQUILIBRIUM CONSTITUTION OF THE FIRM

As argued in previous sections, if the founder of a firm wishes to maximise profit it is in his/her interest to choose the constitution of the firm strategically. This is equally true when the other firms do not maximise profit. Whenever the rival firms have a downward sloping reaction function, profit can be increased by adopting a constitution, which commits the firm to behaving more aggressively. Likewise, the result does not depend crucially on the original objective of the firm. For instance, suppose a consumer cooperative aims to maximise a weighted average of consumer surplus and profits. Such a firm could better achieve its objective by committing to a more aggressive strategy. This would up to a point raise profits and increase consumer surplus because of the strategic effect on other firms’ output. Hence increasing the cooperative’s objective, provided it gives some weight to profits. More generally as long as the current control group gives positive weight to profits, it is in their interest to adopt a constitution which commits the firm to behaving more aggressively than they would choose themselves.

This suggests an alternative way to endogenise the constitution of the firm. We can define

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12 Roberts and Steen (2000) have made a similar point. It may be in the interest of a firm to give shares to its workers to encourage investment in firm-specific human capital.
the equilibrium constitution of the firm to be such that there is no strategic reason to change
the constitution according to the objective of the firm as defined in the constitution itself.
This is intended as a theory of the objective function of the firm in a long-run equilibrium, in
which all possible adjustments have been made.

We obtain a result similar to the Coase conjecture. Consider a firm which is initially
profit maximising. The firm will be tempted to change its constitution to increase sales and
profits. However if the firm cannot commit to prevent further changes, there could be a series
of expansions of the control group each of which increases the firm’s current objective when
it was implemented. The result of this process is that the firm will finish by producing the
competitive level of output.

If, instead of giving influence away, the original owner sold influence then the process
may even be in the interest of the original owner. Individuals who consume the firm’s output
are always prepared to pay an amount equal to his/her increase in consumer surplus. Up
to the Stackelberg, point, the owner gets an indirect benefit from selling influence via the
strategic effect on profits.

5.1 Model

There are 2 firms, firm 1 and firm 2, which compete Cournot style. For simplicity, we assume
a linear inverse demand curve \( p = 1 - x_1 - x_2 \), where \( x_i \) denotes the output of firm \( i \). Firm
2 is a conventional profit-maximising firm. Firm 1 has two members in the control group,
one type \( A \) individual and one type of \( B \). Recall type \( A \) (resp. \( B \)) individuals have utilities
\( u^A = \pi^1 \), (resp. \( u^B = \nu^B (p) \)).

We impose a non-negative profit condition. There are two reasons for this. Firstly as
price falls below marginal cost, all other firms, which are assumed to be maximising profit,
would exit from the industry. Thus issues of strategic delegation would no longer be relevant.
Secondly since the model is intended as one of long-run equilibrium. The firm would not
be viable in the long-run if it makes losses. The non-negative profit condition can also be
justified since limited liability implies that owners cannot be forced to contribute additional
funds to the enterprise.

**Assumption 5.1** *Firms cannot make losses.*

This implies that, price must be greater than or equal to marginal cost, \( p \geq c \). The
following result demonstrates that, in an equilibrium in which the firm does not wish to
change its constitution, price will equal marginal cost.

**Proposition 5.1** *Under Assumption 5.1, the only equilibrium constitution is where \( \lambda^B = \lambda^*B = \frac{1 - c}{x^*} \), \( x_1 = 1 - c \) and \( p = c \).*

This implies that, in the absence of commitment, the firm will increasingly delegate more
power to consumers’ representatives.

### 5.2 Discussion

We have argued that by a process of successive strategic delegation, a firm can become
taken-over by its customers. At first sight this may appear implausible. However we believe
this story does capture some aspects of reality. Firstly it should be noted that a customer
may be another firm. There are documented cases in which upstream firms have been taken
over by downstream firms, including the well-known takeover of Fisher Body by General
Motors.\(^{13}\) Another example is the purchase by farmers of firms which supply their inputs
(e.g. fertilizer), see Hansmann (1996) and Refsell (1914).

An analogous story could be told in terms of imperfectly competitive input markets. The
conclusion would be that successive rounds of delegation would hand control to suppliers,
who would bid more aggressively in the input market. In this case we would see the suppliers
of inputs eventually take over the firm. If the input is top-level management, there is evidence
that such a takeover has indeed happened, see Roe (1994).

\(^{13}\)This takeover took place in multiple stages, which is compatible with our theoretical model.
An alternative interpretation is that firms will have incentives to adopt devices, which preclude too much strategic delegation to prevent loss of control. Firms do indeed adopt different procedures for different kinds of decisions. Pricing and output decisions are usually made by managers, while mergers and takeovers require the approval of shareholders. The initial controlling group has an interest to commit to no further strategic delegation after the first stage. If such commitment is not possible, then a far-sighted owner may not permit the first round foreseeing that it will trigger a whole series of further delegations, which will ultimately have the effect of reducing his/her profit.\textsuperscript{14}

6 EXTENSIONS

In previous sections we have shown that giving influence to consumer representatives can be a good strategy under Cournot competition. Here we show this can be generalised to a more abstract model of competition, where firms’ actions may be either strategic complements or substitutes. In both cases it may be desirable to give influence to an outsider who receives an externality from the firm’s strategic variable. As an application we show that under Bertrand competition is it desirable to give some influence to representatives of competitor firms. This provides a possible rationale for systems of cross-shareholdings and directorships seen in some industries and in certain countries.

6.1 A More General Model

There are $n$ firms. Firm $i$ chooses a strategic variable $x_i$ from its strategy set $X_i$, which we assume to be a closed interval in $\mathbb{R}$. The profits of firm 1 are given by $\tilde{\pi}^1(x_1, x_{-1}) = \pi^1(x_1, \phi(x_{-1}))$, where $\phi : X_{-1} \to \mathbb{R}$, is increasing. Thus the profits of firm 1 depend on

\textsuperscript{14}A related result can be found in Baye, Crocker, and Ju (1996). They show that firms in Cournot oligopoly have an incentive to divide themselves into competing divisions. The benefit of divisionalisation is that it has a strategic effect on the output of rivals. As the cost of creating new divisions tends to zero, price converges to marginal cost. Again lack of commitment can lead to excessive divisionalisation and a complete loss of market power.
its own action and a 1-dimensional aggregate of the actions of its rivals. We assume that
\( \pi_{11} < 0 \), hence firm 1’s profit is a concave function of its own strategy. Let \( R(x_1) \) denote
the best response of firm 1’s rivals, which is assumed to be unique.\(^{15}\)

The control group of firm 1 potentially consists of \( m + 1 \) individuals, \( 0 \leq i \leq m \).
Individual 0 is only concerned about the profits he receives from firm 0. He/she has utility
function \( u^0(\pi^0) \). For \( 1 \leq i \leq n \), individual \( i \) has utility function \( u^i(\pi^1, x^1) \).
We assume that \( u_{x_2}^i < 0 \). As in previous sections, firm 0 may be represented as maximising,
\[ \sum_{i=0}^{m} \lambda^i u^i, \]
for some weights \( \lambda^i \).

We consider the following 2-stage game. First firm 1 chooses the \( \lambda^i \)’s. In the second
stage, all firms choose their strategic variables simultaneously and independently.

**Proposition 6.1** An optimal value of the \( \lambda \)’s is to set \( \lambda^i = 0, \ i \neq 0, j; \)
\[
\lambda^0 = 1 - \pi_{11} \phi' \left( x_1 \right) \frac{\partial u^i}{\partial x_1} , \lambda^j = \pi_{11} \phi' \left( x_1 \right) \frac{\partial u^j}{\partial x_1}. \quad (5)
\]

This result does not assume that rival firms are maximising profit. Nor indeed is it ne-
necessary that firm 1’s objective is profit maximisation. Incentive compatibility implies that it
will be not be possible to implement a negative value of \( \lambda^j \). However if some individuals
receive positive externalities from the firm’s strategic variable, while others receive negative
externalities there will always be an appropriate kind of outsider who can be given influence.

As an example consider a Cournot duopoly where firms’ quantities are strategic substi-
tutes, hence \( R' < 0 \). In this case an increase in output by one firm reduces profits at all of
its rivals hence \( \pi_{11} < 0 \). Thus provided individual \( j \) gets a positive externality (i.e. \( \frac{\partial u^j}{\partial x_1} > 0 \))
from firm 1’s strategic variable, the firm’s optimum can be implemented with a positive value
of \( \lambda^j \). A consumer is likely to receive a positive externality if a firm in an imperfectly com-
npetitive industry expands output. Thus this confirms our earlier result about the desirability
of consumer representation in Cournot oligopoly.

\(^{15}\)This could be justified by assuming that the rivals’ objective functions are strictly concave in their own
strategies.
If $m > 1$, the optimal values of the $\lambda$’s are not unique, since the first order conditions are linearly dependent. This result shows that the optimum may be achieved by a relatively simple constitution for the firm. The control group need consist of only two individuals, one of which is only interested in profit. The other has either positive or negative externalities (as appropriate) from the firm’s strategic variable. Actual firms have boards which contain more than two individuals. There are a number of reasons for this, some of which relate to issues such as moral hazard, which are beyond the scope of the present paper. However if we extended our model to consider a firm engaged in a number of strategic interactions then by similar reasoning we could show that a multi-member board would be desirable.

### 6.2 Price Competition

In this section we show that under Bertrand competition it is advantageous to include an individual with a preference for high prices in the control group. One interpretation of this result is that it is desirable to give influence to a representative of a rival firm. To study this we need to modify our assumptions to allow the control groups of two firms to overlap. Hence the same individual has influence over the decisions of two or more firms. We show that if firms produce differentiated goods and compete Bertrand style, it is desirable to have representatives from rival firms in the control group. If firms compete in prices there is a strategic advantage to committing to higher prices. Overlapping control groups provide one way to do this. Another interpretation is that it could be desirable to give influence to individuals who suffer from the sunk cost fallacy, (see Al-Najjar, Baliga, and Besanko (2005)).

#### 6.2.1 Model

Under price competition firms’ actions will usually be strategic complements. In this case $R' > 0$ and $\pi_2 > 0$. Thus equation (5) implies that it is desirable to share influence with an individual who gets a positive externality from the firm’s strategic variable, $\frac{\partial u_j}{\partial x_1} > 0$. 

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To illustrate this consider two firms, $i = 1, 2$, which produce differentiated goods at zero average and marginal cost. We assume that firm $i$ faces a linear demand curve: $D_i(p_i, p_j) = \max\{0, a + p_j - c p_i\}, \ a, c > 0, c > 1$, where $p_i$ denotes the price of firm $i$. There are two types of individuals, type $\alpha$ and type $\beta$, which do not consume the industry output but can own shares in the firms. In particular $u^\alpha = \pi_1$ and $u^\beta = \pi_2$. As before, we may represent the objective of firm 1 (resp. 2) as maximising $u^\alpha + \lambda_1 u^\beta$ (resp. $u^\beta + \lambda_2 u^\alpha$). The interpretation is that by including a type $\beta$ individual in the control group of firm 1, we are giving influence to an individual who is, in some sense, a representative of firm 2.

Firms play a two-stage game, where in the first stage they choose the degree of influence given to a representative of the other firm, $\lambda_i$, and in the second the firms compete Bertrand-style. The following result finds the price in a sub-game perfect equilibrium where both firms use the same $\lambda$.

**Proposition 6.2** In a symmetric equilibrium where both firms choose the degree of outsider representation, the value of $\lambda$ is given by:

$$\lambda = \frac{1}{2c - 1}.$$

As $c$ increases products become more differentiated and hence the equilibrium $\lambda$ decreases. The less products are differentiated, the more representation is given to owners of the other firm. When products are less differentiated competition is more intense. This increases the desirability of softening competition by giving representation to the owners of the other firm. If $c = 1$, the market is a Bertrand oligopoly with homogenous goods. In this case the optimal $\lambda$ is 1. This implies that both firms maximise the sum of their profits. The limiting case where $c \to \infty$ corresponds to two independent monopolists operating on unrelated demand curves. In this case the optimal $\lambda$ is 0, since when the demand curves are independent, there is no advantage to involving owners of the other firm in the control group.
6.2.2 Discussion

There are a number of situations where the same economic agent may have influence in two or more firms pursuing related lines of business. Visa and Mastercard have an effective duopoly over credit cards. They are controlled by the member banks. Many banks are members of both systems hence have some influence over the running of both credit card systems.

Links between control groups are very common in the car and airline industries. Industrial groups, as seen in Japan and Korea, may also be in part motivated by the effects discussed in this section. It is clear from recent empirical work that interlocking ownership plays a major role in oligopolistic industries in many countries. For instance Mork, Stangelland, and Yeung (2000) show that pyramidal control structures are common in a number of countries such as Canada, Japan and Germany. Even in the USA, overlapping directorships are not unknown.

The practices described in this section are clearly collusive. Public policy should try to discourage them. In this paper we assume that there is no regulation. However our results make a good case why regulation is desirable.

7 CONCLUSION

In this paper we have argued that a number of issues in the governance of organisations can be explained as responses to market distortions. In particular, in the presence of imperfect competition appropriate choice of corporate governance can reduce market distortions and/or shift the equilibrium in the favour of a given firm. Moreover the general model shows that similar considerations apply to other market distortions. Our conclusions are supported by the evidence in Hansmann (1996).

According to our arguments when there is monopoly or Cournot-style competition there are incentives for increased consumer involvement in the governance of firms. Clearly this
does not happen in all instances of monopoly or imperfect competition. We conjecture the costs to consumers of organising varies between different firms. In those industries where the cost of organisation is low and there is monopoly power, cooperatives or similar organisations come to dominate. An important factor, which affects the costs of organisation is geographical distance. If customers live close together they can meet and organise more cheaply. Thus the customers of a grain elevator can organise relatively cheaply since they are relatively small in numbers and live close together compared to the customers of Microsoft.

A second, possibly more important, cost of organising is the cost of collective decisions. All systems of collective choice impose costs, both direct costs of operating the mechanism and indirect costs if the outcome is inefficient. Costs of collective decisions are greater the more diverse the preferences of the group of individuals making the decision. This provides a second reason why the customers of a grain elevator are able to organise, while the customers of Microsoft are not. Microsoft customers are much more diverse which means that they are more likely to suffer from the various problems of voting such as the Condorcet paradox. For further discussion see Hansmann (1996) pp. 39-44.

Our arguments imply that, in general, consumer representation increases profits in Cournot competition and representation from rival firms increases profits under Bertrand competition. Thus we would expect to see different patterns of organisation of firms under the two types of competition. This hypothesis can, in principle, be tested.

Brander and Lewis (1986) argue that financial structure can affect the product market equilibrium. Consider an industry consisting of a small number of firms which compete Cournot-style. If demand is uncertain a leveraged firm will only care about profits in more favourable events. Consider a given firm which decides to increase its debt. Under their assumptions, the more favourable events are when demand is relatively high. (Since the firm is bankrupt in the less favourable events.) If we assume that marginal profit is high when total profit is high then a firm with more debt, will behave more aggressively in the
product market.\textsuperscript{16} If Cournot oligopoly is a game with strategic substitutes then this will cause the given firm’s rival to reduce their output and hence increase its profits. Thus upto a point, increasing debt will increase the value of the firm. Effectively taking on debt can be a way for the firm to commit to produce the Stackelberg level of output. The Modigliani-Miller theorem does not hold in this context because the debt-equity ratio affects the product market equilibrium and hence the firm’s returns. Their argument is extended to Bertrand oligopoly by Showalter (1995). The argument in the present paper is distinct, since it is based on the control rights of different methods of organising the firm. It would apply even in the absence of uncertainty. In contrast, the Brander-Lewis model relies on the financial claims of securities. In particular an increase in debt induces risk-seeking behaviour if shareholders are protected by limited liability.

Katz (1991) argues that the results of Fershtman and Judd (1987) and Vickers (1985) are not robust if contracts can be secretly be renegotiated. Assume that the owner employs a manager on a contract that requires him/her to maximise a combination of revenue and profit. As a result in the second stage equilibrium the firm will produce the output of a Stackelberg leader and its rival will produce the output of a Stackelberg follower. Then there is scope to secretly renegotiate this contract. The owner and manager can sign a new secret contract whereby output is reduced to the best response to a Stackelberg follower’s output. The resulting increase in profit can be shared between the manager and the owner. Hence the original contract is not an equilibrium. More generally Katz (1991) shows that if contracts are freely renegotiable the manager will always choose the same output that the owner would choose if (s)he were running the firm directly.

We believe that this argument does not apply in the present context because, in equilibrium, individuals are playing best responses to their rivals strategies. However their pay-offs are not profits, which is why they deviate from Cournot behaviour. This can be illustrated by

\textsuperscript{16}Even if this assumption is not satisfied debt levels will affect the product market equilibrium. However it could be more difficult to predict their effect.
a simple example. Consider a given firm in a Cournot oligopoly, which is initially owned by an outsider who is only interested in profits. Then the owner’s value of the firm will be equal to the Cournot oligopoly profits. Suppose a consumer (or group of consumers) purchase the firm and choose to produce the output of a Stackelberg leader and hence lower prices. They will value the firm at the sum of the profits of a Stackelberg leader and the increase in consumer surplus due to the price reduction. Decreasing output to the best response would reduce their consumer surplus by more than it would increase profit. Thus there is no scope for renegotiation. Giving (or better selling) some control rights to consumers commits the firm to high output in a much stronger way than a contract with a manager, which rewards revenue.17

Our framework provides an endogenous theory of corporate governance. This can be done in three ways, the firm’s constitution could be chosen by an entrepreneur to maximise the value at which the firm can be sold; the firm’s constitution could be chosen by those currently controlling the firm to maximise their objectives or the system of corporate governance could be chosen by a social planner to maximise social welfare. Although a number of suggestions have been made, at present economics lacks a well established theory of corporate governance. All three proposals have validity beyond the present context.

More generally we believe that there needs to be a rethinking of many results from industrial organisation to allow for more detailed modelling of the internal organisation of the firm. As an example consider management buy-outs (MBO’s). Much of the existing literature has used an agency theoretic approach. It is argued that their main benefit is improved incentives for management. In the present paper we argue that the changes in corporate governance can affect a firm’s position in the product and/or labour markets. The main effect of an MBO is to transfer control of the firm from investors to suppliers of managerial labour. If the managerial labour market is imperfectly competitive this could have the effect of improving the

17Katz (1991) explicitly rules out differences in preferences between the principal and the agent see page. 310. However such differences are crucial to our argument.
A possible direction for the future is that skilled labour will become more important relative to capital. This would shift the bargaining power within organisations. In the firm of the future it is possible that capital will be hired by a coalition of skilled workers. The model in the present paper may help us to understand such changes.

**A Appendix**

This appendix contains the proofs of those results not proved in the text.

**Proof of Proposition 4.2** We shall look for a symmetric equilibrium where all firms use the same value of $\lambda = \lambda_i^*(n)$ and produce the same output, $\hat{x}$. Using equation (4) we find, 

$$\hat{x} = \frac{1-c-(n-1)\hat{x}+\lambda_i^*(n)x^B}{2}.$$ 

Solving, $\frac{n+1}{2}\hat{x} = \frac{1-c+\lambda_i^*(n)x^B}{2}$. Hence equilibrium output is given by, 

$$\hat{x} = \frac{1-c+\lambda_i^*(n)x^B}{n+1}.$$

We shall now look for the Stackelberg equilibrium, in which firm $i$ is the leader and the outputs of the other firms lie on the equilibrium reaction function:

$$x_{-i} = \frac{n-1}{n} \left(1 - c - x_i + \lambda_i^*(n)x^B\right),$$

where $x_{-i} = \sum_{j \neq i} x_j$ denotes the total output of all firms other than $i$. Firm $i$'s profit is given by, $\pi_i = (1 - c - x_i - x_{-i}) x_i$. Substituting from the reaction function, $\pi_i = \left(\frac{1}{n} (1 - c) - \frac{n-1}{n} \lambda_i^*(n) x^B - \frac{1}{n} x_i\right) x_i$. The first order condition is, $(1-c) - (n-1) \lambda_i^* x^B - 2x_i = 0$. Hence firm $i$'s Stackelberg output is, $x_i = \frac{(1-c)-(n-1)\lambda_i^*(n)x^B}{2}$.

The level of $\lambda_i$, which maximises firm $i$'s profit, is achieved where the equilibrium output is equal to the Stackelberg output. Hence $\frac{1-c+\lambda_i^*(n)x^B}{n+1} = \frac{(1-c)-(n-1)\lambda_i^*(n)x^B}{2}$. Solving for $\lambda_i^*(n) : 2(1-c)+2\lambda_i^*(n)x^B = (1-c)(n+1)-(n^2-1) \lambda_i^*(n)x^B$ or $(n^2+1) \lambda_i^*(n)x^B = (1-c)(n-1)$, from which the result follows.

**Lemma A.1** In a Cournot duopoly where firm 2 maximises profit, the market equilibrium is given by,

$$\bar{x}_1 = \frac{1-c+2\lambda^B x^B}{3}, \quad \bar{x}_2 = \frac{1-c-\lambda^B x^B}{3}, \quad \bar{p} = \frac{1+2c-\lambda^B x^B}{3}. \quad (7)$$
Proof. By Proposition 4.1, the reaction function of firm 1 (resp. 2) is given by \( x_1 = \frac{(1-c-x_2) + \bar{\lambda} B x B}{2}, \) (resp. \( x_2 = \frac{(1-c-x_1)}{2} \)). Solving in the usual way we obtain, \( \bar{x}_1 = \frac{1-c+\bar{\lambda} B x B}{4} \) and \( \bar{x}_2 = \frac{1-c-\bar{\lambda} B x B}{4} \). The equilibrium price is given by, \( \bar{p} = \frac{1+2c-\bar{\lambda} B x B}{3} \).

Proof of Proposition 5.1 To check \( \lambda^B = \lambda^* B \) is indeed an equilibrium. If \( \lambda^B = \lambda^* B \), then from (7) the equilibrium quantity and price will be \( x_1 = 1 - c \) and \( p = c \).

As before, we may represent the firm’s objective as: \( \psi_1 = \pi_1 + \lambda B x B = \frac{1}{9} (1-c)^2 + \frac{1}{9} (1-c) \lambda^B x B^2 - 2 \lambda^B x B x B^2 + \frac{1}{9} \bar{\lambda} B x B \left( \frac{1+2c-\bar{\lambda} B x B}{3} \right) \) for \( \lambda^B, \lambda^* B \leq \frac{1-c}{x^B} \). From the point of view of the equilibrium constitution, the effect of a change in \( \lambda^B \) on the firm’s objective is given by \( \frac{d\psi_1}{dx^B} = \frac{1}{9} (1-c) x B^2 - 4 \lambda^B x B^2 + \frac{1}{9} \bar{\lambda} B x B^2 \). Note that the initial value of \( \lambda^B, \lambda^* B \) is treated as constant for this differentiation. The second order condition is satisfied. Evaluating at \( \lambda^B = \lambda^* B \), \( \frac{d\psi_1}{dx^B} = \frac{1}{9} (1-c) x B^2 - 4 \lambda^B x B^2 + \frac{1}{9} \bar{\lambda} B x B^2 = \frac{1}{9} ((1-c) x B^2 - 4 (1-c) x B^2) + \frac{1}{9} (1-c) x B^2 = 0 \), which establishes that \( \lambda^B = \lambda^* B \), is an optimum and hence that the firm would not want to decrease \( \lambda^B \) below \( \lambda^* B \).

The firm has no incentive to increase \( \lambda^B \), since it is already supplying the entire market. Firm 2’s output is zero, hence there is no strategic effect of further reductions in \( \lambda^B \). The only effect of reducing \( \lambda^B \) further would be to cause the firm to increase its output beyond the current level. This is not desirable as the current level is already optimal according to the current objective function.

Uniqueness. Assume \( \bar{\lambda} B < \frac{1-c}{x^B} \). Then \( \frac{d\psi_1}{dx^B} \bigg|_{\lambda^B=\bar{\lambda} B} = \frac{1}{9} ((1-c) x B^2 - 4 \bar{\lambda} x B^2) + \frac{1}{9} \bar{\lambda} B x B^2 = \frac{x B}{9} \left( (1-c) + \bar{\lambda} B (3 - 4 x B) \right) \). If we assume \( x B < \frac{1}{2} \) then it is clear that \( \frac{d\psi_1}{dx^B} \bigg|_{\lambda^B=\bar{\lambda} B} > 0 \). If \( x B > \frac{3}{4} \), \( \frac{d\psi_1}{dx^B} \bigg|_{\lambda^B=\bar{\lambda} B} > \frac{x B}{9} \left( (1-c) + \frac{1-c}{x^B} (3 - 4 x B) \right) = \frac{(1-c)}{9} (1 - x B) > 0 \), since \( x B < 1 \). This implies that if \( \bar{\lambda} B < \frac{1-c}{x^B} \) there is a strategic advantage to increasing \( \lambda^B \). Hence \( \lambda^B < \frac{1-c}{x^B} \) is not compatible with equilibrium.

If \( \bar{\lambda} B > \frac{1-c}{x^B} \), price would be below marginal cost and hence would not satisfy the non-negative profit constraint. Consequently such values of \( \bar{\lambda} B \) would not be sustainable in long run equilibrium. ■

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\(^{18}\)This is the point at which the rival exits the market, which causes the profit function to be non-differentiable.
Proof of Proposition 6.1 If firm 1 can act as a Stackelberg, leader, its profit will be given by, $\pi^1 (x_1, \phi (R (x_1)))$. The first order condition for maximising this is:

$$\pi^1_1 \phi' (R (x_1)) R' (x_1) = 0. \quad (8)$$

Firm 1’s actual first order condition is, $\sum_{i=0}^{m} \lambda^i \frac{\partial u^i}{\partial x_1} \pi^1_1 + \lambda^j \frac{\partial u^j}{\partial x_1} = 0$. Assume that $\lambda^i = 0$, unless $i = 0$ or $i = j$. This simplifies to $\lambda^0 \frac{\partial u^0}{\partial x_1} \pi^1_1 + \lambda^j \frac{\partial u^j}{\partial x_1} = 0$. The $\lambda$’s are only unique up to positive scalar multiple. Hence we may normalise them by requiring $\lambda^0 \frac{\partial u^0}{\partial x_1} + \lambda^j \frac{\partial u^j}{\partial x_1} = 1$. This simplifies the first order condition to

$$\pi^1_1 + \lambda^j \frac{\partial u^j}{\partial x_1} = 0. \quad (9)$$

Comparing (8) and (9) we see that if $\lambda^j \frac{\partial u^j}{\partial x_1} = \pi^1_2 \phi' (R (x_1))$, the firm can obtain profits as if it were a Stackelberg, leader. Since this sets an upper bound to the profits firm 1 can make in the second stage, it follows that this is an optimal value for $\lambda^j$. Solving for $\lambda^0$, we obtain $\lambda^0 = 1 - \lambda^j \frac{\partial u^j}{\partial x_1} = 1 - \pi^1_2 \phi' (x_1) \frac{\partial u^j}{\partial x_1}$. \hfill \blacksquare

The following results apply to the model of price competition from Section 6.2.

**Lemma A.2** In a symmetric Bertrand equilibrium where $\lambda^a = \lambda^b = \lambda$, prices are given by $\hat{p} = \frac{a}{2c - (\lambda + 1)}$.

**Proof.** Firm 1’s objective is to maximise: $u^a + \lambda^\phi u^\phi = p_1 (a + p_2 - cp_1) + \lambda^\phi p_2 (a + p_1 - cp_2)$. The first order condition is: $a + p_2 - 2cp_1 + \lambda^\phi bp_2 = 0$. Thus firm 1’s reaction curve is given by,

$$p_1 = \frac{a + \left( \lambda^\phi + 1 \right) p_2}{2c}. \quad (11)$$

Let $\hat{p}$ denote the symmetric equilibrium price, where both firms use the same $\lambda$. This satisfies $\left[ \frac{2c - (\lambda + 1)}{2c} \right] \hat{p} = \frac{a}{2c}$, from which the result follows. \hfill \blacksquare
Lemma A.3 Assume that firm 1 maximises profit, while firm 2 gives weight $\lambda_2^n$ to type $\alpha$ individuals in its decisions. Suppose that firm 1 is a price leader, then its optimal price is given by:

$$\tilde{p}_1 = \frac{a(2c + 1)}{2[2c^2 - (\lambda_2^n + 1)]}.$$ 

Proof. From equation (11) and symmetry, firm 2’s reaction function is given by: $p_2 = \frac{a}{2c} + \frac{(\lambda_2^n + 1)p_1}{2c}$. Firm 1’s profits are given by: $\pi_1 = p_1 \left( a + \frac{a}{2c} + \frac{(\lambda_2^n + 1)p_1}{2c} - cp_1 \right)$. The first order condition for profit maximisation is:

$$\frac{d\pi_1}{dp_1} = \left( a + \frac{a}{2c} + \frac{(\lambda_2^n + 1)p_1}{c} - 2cp_1 \right) = 0. \tag{12}$$

Hence $2ca + a + 2(\lambda_2^n + 1 - 2c^2)p_1 = 0$, from which the result follows.\(^{19}\)

Proof of Proposition 6.2 At the optimum $\lambda$ the price leader’s price is equal to the equilibrium price: $\frac{a(2c+1)}{2[2c^2-(\lambda+1)]} = \frac{a}{2c-(\lambda+1)}$. Cross-multiplying: $(2c + 1)[2c - (\lambda + 1)] = 2[2c^2 - (\lambda + 1)]$. Expanding, $4c^2 + 2c - 2c\lambda - 2c - (\lambda + 1) = 4c^2 - 2(\lambda + 1)$, hence $2c\lambda = \lambda + 1$ or $1 = \lambda(2c - 1)$, from which the result follows.

References


\(^{19}\)From (12), $\frac{d^2\pi_1}{dp_1^2} = \frac{1}{c} \left[ \lambda_2^n + 1 - 2c^2 \right] < 0$, provided $(\lambda_2^n + 1) < 2c^2$. Since $c > 1$, the second order condition is satisfied provided $\lambda_2^n < 1$. This says that type-\(\alpha\) individuals do not get more weight than type-\(\beta\) individuals in firm 2’s decisions. In other words outsiders get less than half of the control rights. It does not seem unreasonable to view this as the normal case.


