The Theory and Practice of Equalization

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Abstract

This paper presents a selective and non-technical survey of the role of intergovernmental equalization transfers as a device for achieving efficiency in the allocation of labor across regions in a decentralized nation, and for achieving fiscal equity among residents of various regions. We discuss some of the issues that arise in attempting to put the principles of equalization into practice.

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1 Introduction

Equalization transfers from central to sub-national governments are a pervasive feature of virtually all nations with multiple levels of government, including federations (Australia, Belgium, Canada, Germany, Spain), multi-sphere governments (South Africa), and unitary-type states (Japan, Scandinavian countries). They are also used between sub-national governments and their municipalities, and to a lesser extent among nations in economic unions (European Union) and worldwide (development aid). In some cases, equalization is a stand-alone program based on an explicit formula (Australia, Canada), while in other cases it may be implicit and embedded in other grant programs or revenue-sharing arrangements (Germany, USA). In all cases, the transfers redistribute from better-off to less well-off jurisdictions. The importance of equalization is highlighted not only by the extent of its use and the fact that it often comprises a substantial share of central government spending, but also from the fact that the requirement for equalization may be found in the relevant national constitution (Canada, Germany, South Africa). It seems important therefore to understand the economic principles underlying equalization transfers.

The economics literature on equalization, which is surprisingly dormant, has developed by and large separately from the practice, and in many cases addresses issues that have not been of primary concern to policy-makers. Two seminal papers by Buchanan (1950, 1952) made the equity and efficiency cases for equalizing transfers, and served as the basis for subsequent work by Buchanan and Goetz (1972), Flatters, Henderson and Mieszkowski (1974), and Boadway and Flatters (1982). Much of the literature is based on concepts and models developed in these papers. Central to this literature is the fact that equalization is a natural complement to decentralization. In a unitary nation, common fiscal programs apply nationwide and equalization among regions is implicit. Decentralization of fiscal authority in a federation results in different regions having differing abilities to provide given levels of public goods and services, so unitary state outcomes cannot be replicated without being accompanied by equalization. Decentralization has increasingly come to be seen as a good thing in established federations, in developing countries and in the new federations of transitional economies, and much literature has been devoted to the pros and cons of decentralization (Oates, 1999). Equalization can be seen as a necessary counterpart to decentralization, offsetting its tendency to create disparities among regions in the ability to provide public goods and services.

The time is overdue to take stock of what we have learned from the theory and practice of equalization. What follows is a selective, non-technical survey of the public economic theory of equalization transfers and their use in practice. Our intent is to indicate what we have and have not learned, what compromises are necessary to bring the theory and principles to practice, and what work remains to be done. The starting point is a broad overview of the theory of equalization as it has evolved in the literature. This can be used as the basis for considering how the principles can inform the practice, and what problems arise in practice that the theory may have difficulty resolving. Before turning to the theory, it is useful to set out some general issues and concepts that pervade both the theory and the practice.
2 Some General Issues

Our discussion will involve two levels of government—one central and more than one regional government.\(^1\) Often it will suffice to think of only two regional governments. We generally suppose that the central government is a first-mover from a policy point of view, followed by the regions and then the private sector, although we briefly consider alternatives later. We assume that governments are benevolent in the sense that their policies are motivated by improving the collective well-being of their residents. Given that redistribution is at the heart of equalization, the interpretation of collective well-being is inherently value-laden. The literature proceeds by positing a social objective that embodies some aversion to income inequality (vertical equity), as well as possibly some other properties, such as horizontal equity.\(^2\)

Since horizontal equity takes on special significance in the federalism literature, it is worth being explicit about its meaning at the outset. The notion of horizontal equity that we adopt is that persons who are equally well-off before government policy should be equally well-off after it: equals should be treated equally. There are many well-known conceptual problems with achieving horizontal equity when the circumstances of persons differ in ways that make it difficult to say when they are equally well-off. For example, they may vary in preferences, in needs, in family status, and so on. We avoid these problems by supposing that persons all have the same utility function so problems of utility comparison do not arise. Instead, the circumstances in which persons can differ is their region of residence. The thrust of horizontal equity in a federalism context is that otherwise identical persons should be treated equally in a nation regardless of where they reside, and this turns out not to be innocuous, as we shall see. The view we take is that the extent of vertical equity and the relevance of horizontal equity are ultimately matters for societal consensus. While we are agnostic on this, it seems clear that most countries reveal in their policies both aversion to income inequality and some desire for horizontal equity.

Equalization consists of a system of unconditional redistributive transfers among governments, and can take two broad forms. A gross scheme involves transfers from the central government to the regions financed from central tax revenues, while a net scheme consists of self-financing region-to-region transfers. In principle, net and gross schemes are equivalent from an equalization perspective: a gross scheme can replicate the equalizing outcome of a net scheme, but to do so it requires a larger share of tax room at the central level relative to its own expenditure requirements, that is, a larger *vertical fiscal gap* (VFG). The VFG plays an important role in a federation in its own right in terms of inducing efficiency in the national economy. It can facilitate the harmonization of taxes, internalize fiscal

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\(^1\) Adding three levels of government gives rise to no further issues as long as the relationship is hierarchical. Complications would arise if the central government dealt directly not only with regional governments but also with municipalities.

\(^2\) In fact, much of the analysis to follow uses for illustration a utilitarian social welfare function as the basis for vertical equity. Such a function involves no aversion to utility inequality, but nonetheless yields aversion to income inequality because of the assumption that individual utilities exhibit diminishing marginal utility of real incomes.
externalities, enable the central government to set the terms of the redistributive interpersonal tax-transfer system, and provide an instrument whereby the central government can influence regional government behavior through conditional grants. It raises issues that go well beyond equalization, but since the latter is our focus we set those aside.\(^3\)

Equalization serves various potential roles. Much of the focus of the theory as mentioned above is its use to correct inefficiencies that are induced by fiscal decentralization. Parallel to that, equalization may be an instrument for achieving horizontal equity among residents of different regions, that is, for ensuring that persons of a given income can obtain comparable public services at comparable tax rates in all regions. Equalization might also serve as a stabilization device, insuring regions against adverse idiosyncratic shocks with which they may not be able to cope themselves. The usual problems of insurance arise, such as moral hazard and adverse selection. As well, time consistency may be an issue, especially if regional budgets are ‘soft’ because of the inability of the central government to commit. Finally, equalization transfers could potentially be used to correct for distorting regional decisions. For reasons of space, our focus will largely be on equalization as a device for addressing inefficiencies and inequities that arise from decentralization.

The case for decentralizing the provision of public goods and services to regional governments is based on two main sorts of considerations. The classic argument emphasized by Musgrave (1959) and Oates (1972), and at the basis of the Tiebout (1956) model, is that regional governments are best able to cater to the preferences and needs of their residents, whereas a central government would tend to provide uniform public programs nationwide. More recently, arguments for decentralization have been based on the notion that regional governments are able to provide given public services (and targeted transfers) at a lower cost than central governments because of informational advantages, lower administrative costs, and the discipline of political competition. As compelling as these arguments are, the decentralization of spending and revenue-raising responsibilities leaves different regions with different fiscal capacities. In the absence of equalization, they would be unable to provide the public services at the tax rates that would otherwise prevail in a centralized setting. Thus, equalization can be seen as an instrument for facilitating effective decentralization by enabling its benefits to be achieved while avoiding its adverse effects.

The literature on equalization typically ignores explicit consideration of the benefits of decentralization, and focuses solely on its role in ensuring that different regions have comparable fiscal capacities to provide given levels of public services. Nonetheless, in designing actual equalization systems, account must necessarily be taken of the fact that different regions exercise their discretion and behave differently. This makes the notion of equalizing fiscal capacities an ambiguous one, somewhat akin to designing interpersonal redistribution systems to achieve equal opportunity among households with different preferences (Roemer, 1998).

The form of equalization will also be affected by the extent to which revenue-raising is

\(^3\) For a general discussion of the role of central-regional transfers as a component of fiscal federalism policies, see Boadway (2001).
decentralized along with the provision of public spending programs. If the VFG is large, as in some federations (Australia), regions will rely heavily on transfers from the central government to finance their spending programs. The preoccupation will be with ensuring that all regions have the revenues to provide adequate levels of program spending, given differences in need that might arise because of, say, differences in regional demographic composition. On the other hand, if regions have substantial revenue-raising authority, whether discretionary (Canada) or via revenue-sharing (Germany), equalization will also focus on the fact that different regions have different abilities to raise revenues using given tax rates.

An important point to emphasize in what follows is the essential difference between the objectives of interpersonal redistribution and interregional redistribution. Broadly speaking, where equity is concerned, the former is preoccupied with vertical equity, while the latter is concerned with horizontal equity in the sense discussed above. That is, equalization is primarily concerned with eliminating differences in the net benefits that the public sector provides otherwise-identical households residing in different regions, so-called net fiscal benefits (NFBs). It is not concerned with reducing differences in real incomes among households. In fact, the need to eliminate NFB differences arises regardless of the amount of interpersonal redistribution that occurs, although the form of equalization will turn out to depend upon the progressivity of regional budgets.

In pursuing this objective of eliminating NFB differentials, some considerations should be born in mind. First, the objective of horizontal equity is not a given. For one thing, as we shall see, horizontal equity may well conflict with social welfare considerations, as Mirrlees (1972) has emphasized in an urban context. That is, social welfare maximization may entail that like persons in different regions are treated differently. For another, horizontal equity involves a value judgment that has some bite in a federation with heterogeneous regions. It requires a consensus that social citizenship or solidarity among all citizens apply with equal force nationwide as opposed to being region-specific. Second, when regions make different fiscal decisions, it will not be possible for an equalization system to ensure that NFB differentials are eliminated for households of all incomes. To do so would involves a much more complicated set of transfers, and would violate the very objective of decentralization, which is to allow regional governments to exercise their discretion freely. Thus, in practice, some compromise must be made, and the one usually aimed for is to equalize the potential for different regions to make fiscal decisions so that NFB differences are eliminated. Third, as argued by Buchanan (1952), efficiency considerations also call for eliminating NFB differences since they provide an incentive for households to choose their locations on the basis of fiscal rather than productivity considerations.

Given these considerations, a convenient conceptual prism through which to view and understand equalization is the unitary state benchmark. Equalization can be thought of as enabling the decentralized nation to have the potential to achieve the outcome of a unitary state, without compelling regions to abide. However, it is only a useful conceptual device if one ignores the benefits of decentralization. The unitary nation benchmark is itself ambiguous if different regions have different preferences and different degrees of consensus for national solidarity. The basic conflict between decentralized decision-making and the
unitary state benchmark implies that equalization must reflect a compromise between the objectives of horizontal equity and respecting regional preferences. Since there are many ways to resolve that compromise, the design of equalization necessarily involves a political policy judgment.

Before turning to some analysis, it is useful to highlight three important theoretical considerations that affect the role and evaluation of equalization. A first one is whether decentralized decision-making leads to an optimal allocation of resources within each region, that is, whether regions choose the optimal level of public spending and tax structures. If they do, the only inefficiency remaining will be that involving the interregional allocation of resources, and equalization can focus solely on that. In the following section, we take up this important issue of the optimality of decentralized decision-making. Second, regional decision-making may not be optimal in the sense that it may not replicate what would be chosen in a unitary state setting. This can arise because of inter-jurisdictional fiscal externalities on the spending and/or taxing side. Now, equalization alone is not sufficient to obtain an optimum in a federation: other policy instruments must be used. As it turns out, the optimal design of these policies is a complicated matter (Dahlby, 1996; Sato, 2000). In the event that they are not feasible, the design of equalization becomes a second-best exercise in which account must be taken of the effects of equalization on non-optimal decentralized outcomes. The literature on this is relatively limited so far, although there are some specific examples where equalization can have beneficial effects on otherwise non-optimal regional decision-making (Bucovetsky and Smart, 2002). Finally, equalization might introduce inefficiencies in its own right. Like any other redistributive program, it is difficult to avoid introducing adverse incentive effects if the equalization formula must be based on observable outcomes that can be influenced by regional governments. Some of the practical consequences of this for the design of equalization systems will be discussed in a later section.

3 The Optimality of Decentralized Decision-Making

We begin with a consideration of whether decentralized decision-making leads to an optimal allocation of resources within each region, though not necessarily between regions. If so, this will allow equalization to focus on non-optimal allocations between regions.

The theory of equalization has been developed using very simple models. Our approach is to summarize in the text some of the key results that have been derived from these models. Appendix I provides a more formal derivation of some of these results and illustrates the method of analysis. The workhorse model has been a two-region Ricardian model with labor as the only variable input in each region’s aggregate production function. An underlying fixed factor earns rent which may accrue to either the public or private sector. In the base-case model, labor is homogeneous and mobile across regions, possibly with some cost. The cost of migration is typically assumed to vary over households, possibly due to differences in non-pecuniary ‘attachment to home’ (Mansoorian and Myers, 1993). Aggregate output in each region can be divided at will between private goods and public goods, and the latter can have varying degrees of ‘privateness’. Thus, regional public spending $G$ yields per capita services $g = GN^{-\alpha}$, where $\alpha$ is degree of privateness and $N$ is
the region’s population. Following Bewley (1981), we refer to the case of publicly provided private goods \( (\alpha = 1) \) as pure public services. It can be argued that the bulk of regional public spending takes this form (education, health, social services). For simplicity, central spending on goods and services is assumed away: the central government engages only in taxes and transfers, both to persons and to regional governments. All households supply one unit of labor and utility is assumed to be additively separable in private goods \( c \) and regional public services \( g \): \( u(c) + b(g) \). In the base-case model, there are no spillovers from regional public spending, and no mobile capital. These add little to the main arguments for equalization.

In this idealized model, decisions about the level of regional public spending \( G \) and the raising of taxes to finance that spending can be taken centrally or decentralized to the regions. It is convenient to abstract from the possibility that regions may be able to provide regional public goods at a lower cost than a central, fictitious, unitary government. In both cases, we assume that the opportunity cost of a unit of public goods is one unit of private goods. The issue to be explored in this section is under what circumstances decentralized decisions are as efficient as in the unitary state, in the sense of choosing the optimal mix of public and private goods and the optimal tax structure within each region. In the next section, we shall see that even if each region follows the optimal rules for public goods provision and financing, labor might still be misallocated across regions, leading to a case for equalization. It turns out to be sufficient to consider this issue in a first-best world in which lump-sum taxes and transfers are possible. The optimality of decentralized decision-making depends upon whether labor is homogeneous or heterogeneous, that is, upon whether all workers are equally productive or not.

Before considering these two cases, an important feature of overall optimality in federal economies should be recognized. Even if governments can levy lump-sum taxes, the first-best outcomes that can be achieved in a unitary state may be constrained. If labor is mobile between regions, unconstrained social welfare maximization will generally call for different levels of net utility for potential migrants if public goods are not purely private \( (\alpha < 1) \). That is because of the economies of scale in consuming public goods which implies that utility per capita depends upon population in a complicated way. The implications is that migration equilibrium is violated in the unconstrained first best. If it is assumed that the unitary state government cannot violate the migration equilibrium constraint, that is, cannot direct households where to live, only a constrained first-best can be achieved. This will be true whether or not migration is costly.

If labor is immobile, there is no migration equilibrium constraint facing the unitary state

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4 For example, if the objective function is utilitarian and if labor is perfectly mobile, the unconstrained first-best optimum maximizes \( N_1[u(c_1) + b(G_1/N_1^0)] + N_2[u(c_2) + b(G_2/N_2^0)] \) subject to \( N_1c_1 + N_2c_2 + G_1 + G_2 = F_1(N_1) + F_2(N_2) \) and \( N_1 + N_2 = N \), where \( F_i(N_i) \) is the strictly concave production function of region \( i \), whose output can be divided between \( c \) and \( G \). Since utility is additively separable, the solution equalizes consumption \( c_i \) across regions, but the optimality conditions for \( G \) within each region—the Samuelson-type conditions—generally yields \( G_1 \neq G_2 \) if \( \alpha < 1 \). Therefore, per capita utilities differ in the two regions.
planner. However, an analogous constraint will apply if social welfare maximization is constrained by a horizontal equity requirement. In an economy with regional public goods, social welfare maximization will generally require that otherwise identical persons in different regions obtain different utilities, which would violate horizontal equity. While the case for a horizontal equity constraint may not be as compelling as for a migration equilibrium constraint, it is useful to consider it as a possibility since a) it leads to parallel consequences, and b) horizontal equity has played an important role in the equalization literature. The Oates (1972) centralized outcome could be interpreted as being the consequence of such a constraint.

Homogeneous Labor Case

If all workers are equally productive, the circumstances under which decentralized decision-making is optimal are quite broad. The only real decision is the level of spending \( G \) and therefore services \( g \) to provide to local residents. Optimality in region \( i \) requires satisfying Samuelson conditions revised to take account of the degree of privateness of public spending:

\[
\frac{N_i b'(g_i)}{N_i u'(c_i)} = 1 \quad i = 1, 2
\]

With mobile labor, regional governments will use decision rule (1) as long as their objective function is either the per capita utility of their residents or the sum of utilities of either their original or their final residents, and they finance their spending by lump-sum taxes on their residents (Boadway, 1982). Incentive equivalence is said to apply. The equal net utility migration constraint leads to the regional governments having the same objective as the central government. This is the case regardless of the costs of migration, and regardless of the strategic conjecture each region might have about the policies of the other. Indeed, it is even the case with immobile labor. Thus, competition for labor is not the same as for capital or goods. It does not necessarily lead to inefficient regional decisions about public goods provision.

Incentive equivalence will be violated, and thus decision rule (1) not followed, if the assumptions of the basic model do not apply. If regions use source-based taxes on incomes generated for non-residents, tax exporting can occur and the Samuelson conditions (1) will be violated (Boadway, 1982). This would be the case, for example, if the rents to the fixed factor partly accrued to non-resident owners of the fixed factors and could be taxed by the regional government. Alternatively, as Secreri (2003) has shown, if the places of work and residency differ, as in a metropolitan setting, standard tax competition arguments for workers will apply leading to regional governments to deviate from the Samuelson rule, analogous to the case of capital tax competition with mobile capital as in Zodrow and

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5 This is obtained from the first-order conditions to the problem in footnote 4 amended to include a migration equilibrium constraint that requires per capita utility to be equal in the two regions.

6 For example, even if all workers in the nation owned the same share of fixed factors nationwide—a common assumption—tax exporting would occur if source-based taxes were used.
Mieszkowski (1986). In these cases, regional governments violate the Samuelson conditions in order to attract workers or export taxes.

These results generally carry over to settings where the unitary state optimum is second best because of, say, distortionary taxation. Consider the case in which labor supply is variable and the government uses a labor tax, per unit for simplicity. The Samuelson condition (1) must be amended as follows to take account of the distortionary source of finance (Boadway and Keen, 1996):

\[
\frac{N_i b'(g_i)}{N_i^{\alpha} u'(c_i)} = \frac{1}{1 - \tau \eta} \equiv \text{Marginal Cost of Public Funds (MCPF)} \quad i = 1, 2
\]  

(2)

where \( \tau \) is the labor tax rate and \( \eta \) is the elasticity of labor supply. Decentralization now involves an additional consideration. If both the central government and the regional governments use the labor tax to finance their respective spending obligations, there will be a vertical externality imposed by changes in the tax rate at one level of government on the revenues of the other level. The regional government, for instance, will underestimate the incremental welfare cost of increasing its tax rate, thereby perceiving its MCPF to be less than it actually is from a social point of view. It can be shown that the central government can generally restore optimality in regional decision-making by manipulating the VFG.\(^7\) Otherwise, the results of the previous section apply. Decisions of the regional governments are optimal under the same sets of circumstances as in the constrained first-best case.

**Heterogeneous Labor Case**

Once labor becomes heterogeneous, incentive equivalence (the optimality of regional budget decisions) is no longer guaranteed. Both the size of government and the distribution of the tax burden among types—progressivity of the tax structure—are now relevant. It is not trivial to assign functions to the two levels of government to ensure that optimal regional decisions are made. To see this, consider (following Boadway et al, 2002) the case where there are two wage-types, which are perfect substitutes in production. The wage rate per efficiency unit of labor is given, but high-ability workers supply more efficiency units of labor than low-ability workers. As above, the two regions have different production functions, possibly reflecting different endowments of fixed factors. It is useful to distinguish between the costless and the costly migration cases since they generate different results.

**Costless Migration**

With costless migration, the social planning or unitary state optimum is symmetric in the following sense. Optimal populations are the same in both regions, although the proportions of high- and low-ability workers differ so that effective labor supplies differ. Consumption per person is equalized across both ability types and regions. The level of

\(^7\) See Boadway and Keen (1996) for the case of symmetric outcomes and Sato (2000) for the asymmetric case. Alternatively, the central government can use matching grants to influence the incentive of the regional government to spend as suggested by Dahlby (1996).
public goods, determined by the Samuelson conditions, is the same in both regions as is the level of services $g_i$ yielded by the public spending. Moreover, these outcomes imply that in the unitary state optimum, the equal-utility migration equilibrium constraint is not binding.\(^8\) In a decentralized context, decision-making within each region will only be optimal if regions abide by the Samuelson rule and choose their tax structures such that consumption is equalized within their borders. (Cross-regional consumption equality is achieved by an equalization system as discussed in the following section.)

It turns out that in this costless migration case, the optimality of regional decision-making will depend upon the tax instruments that have been decentralized. In particular, regional decision-making is optimal if regional governments can use either lump-sum redistributive taxes or proportional consumption taxes, again regardless of their strategic conjecture with respect to the other region’s policies. In other words, redistribution can either be assigned to the regions (contrary to the standard prescription), or the regions take central redistribution as given and use proportional consumption taxes to finance their spending. In either case, both optimal intra-regional redistribution and the optimal level of public goods are achieved. On the other hand, if the regions are allowed to use general payroll taxes or a general surtax on central government taxes, they will not choose the level of public spending to conform with the Samuelson conditions.

**Costly Migration**

With costly migration, these results on decentralization no longer apply. The optimal outcome now depends upon the initial allocation of workers of the two types between regions. A symmetric equilibrium with equal $c_i$ and $g_i$ is no longer optimal, and the migration equilibrium constraint is generally binding on both types of workers. Decentralization leads to non-optimal regional decisions regardless of the taxes that are assigned locally. Regions must be given lump-sum redistributive taxes if the optimal level of public goods is to be chosen, but they will choose redistribution non-optimally regardless of strategic conjecture.\(^9\) This unhappy outcome complicates matters considerably for the design of inter-governmental transfers. An equalization system alone will not suffice to take the economy to the constrained first-best. Instead, additional incentives must be provided to the regions to induce them to take optimal fiscal decisions. This presumably involves matching transfers based on tax rates and expenditures, but the details have yet to be worked out in the literature. In the absence of these additional policy instruments, the design of an ideal equalization system becomes problematic.

**No Migration**

Now there is no migration equilibrium constraint to contend with. However, horizontal

\(^8\) This characterization assumes that the solution is an interior one in which both types of persons are in both regions in the optimum. If there are more than two regions, the results will only apply if there are at least as many wage-types as regions.

\(^9\) For the details of the argument, see Boadway et al (2002).
equity constraints requiring equal utility for any given type of worker in the two regions play a similar role. If such constraints are imposed, they will generally be binding on both types of households (and with different degrees of tightness). Consumption levels will tend to be lower in the more populous region if $\alpha < 1$ to compensate for the higher level of public services, so full equality of consumption will not be constrained-optimal. Decentralization of taxes and spending will not lead to optimal regional decisions. As in the costly migration case, optimal $G$ requires that regions have redistribution authority, but they will not redistribute optimally because they will not take proper account of the horizontal equity constraint. Decentralization is only optimal if differential matching tax effort grants and expenditure grants used. On the other hand, if horizontal equity is not an objective, decentralization will be optimal if the regions either use a proportional consumption tax or are themselves made responsible for redistribution as in the costless migration case.

Table 1 summarizes the circumstances under which the decentralization of fiscal instruments to regional governments leads to efficient decisions.

All these results generalize to the second-best case where regions use distortionary taxes. In the unitary state optimum with costly migration, the tax system will generally be more progressive in the region with the higher proportion of able persons (Boadway et al, 2003). The migration equilibrium constraint will be binding. It will be optimal to decentralize redistributive taxation to the regions to avoid the vertical fiscal externality and to ensure that regions adopt the optimal decision rule for public spending. But, with migration, non-optimal redistribution will be undertaken, so decentralization will lead to non-optimal regional fiscal decisions. Other policy instruments must be used to restore second-best efficiency.\(^\text{10}\) If there is no migration, regional redistribution will be optimal only if horizontal equity is not a constraint. If it is, it will generally be necessary to have a uniform national redistributive tax system.

Now we turn to the arguments for equalization, taking account of the possible non-optimality of regional decision-making, where necessary.

4 Efficiency Arguments for Equalization

A fundamental insight of the literature on equalization is that even if regional governments use optimal decision rules within their jurisdictions, migration will generally be inefficient. Regional government policies will lead to a fiscal externality associated with migration.\(^\text{11}\) The intuition of this externality is set out in Buchanan and Goetz (1972). A marginal

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\(^{10}\) For example, transfers could be conditional on populations of each type. For an analysis of this for the case of linear progressive taxes, see Boadway et al (1998).

\(^{11}\) There can also be problems of instability and non-existence of migration equilibrium in regional public goods economies, as discussed by Atkinson and Stiglitz (1980) and Bewley (1981). These will be especially problematic if the population of the federation is below the optimal level (Boadway and Flatters, 1982). At the same time, if migration is costly, instability is less likely to be a problem.
in-migrant imposes two offsetting effects on the existing residents of a region. On the one hand, the taxes he pays contributes to the regional budget that finances public spending for all households. On the other, his use of public goods and services may reduce the benefits obtained by existing residents, the so-called congestion effect. We begin by identifying the fiscal externality, and then turn to how it manifests itself in the simple models with homogeneous and heterogeneous labor that we introduced above.

The Fiscal Externality of Migration

In the context of the simple model developed above, the fiscal externality associated with the marginal in-migrant in region $i$ can be written as follows, where $t_i$ is the migrant’s tax payment in $i$: \[ t_i - \frac{\alpha G_i}{N_i} \quad i = 1, 2 \]

If migration is costless, migration will be inefficient if

\[ t_1 - \frac{\alpha G_1}{N_1} \neq t_2 - \frac{\alpha G_2}{N_2} \]

That is, if the size of the fiscal externality differs across regions. This expression applies generally to the case where households are heterogeneous, as long as the terms are defined to refer to the tax paid and the congestion imposed by the marginal migrant of any given type. In the special case in which public goods are Samuelsonian ($\alpha = 0$), the fiscal externality reduces to $t_i$, which is the case considered by Flatters, Henderson and Mieszkowski (1974): no opportunity cost is imposed when an additional resident uses a pure public good. On the other hand, if spending is on public services ($\alpha = 1$), the fiscal externality is $t_i - G_i/N_i$. In this case, a regime of benefit taxation would involve no fiscal externality.

With costly migration, the migration inefficiency expression must be amended to take account of the costs of migration. Let $m(J)$ be the cost of migration of the $J$th migrant. In equilibrium, if migration is from 1 to 2, we have $u(c_1) + b(g_1) = u(c_2) + b(g_2) - m(M)$ for the marginal migrant, where $M$ is the number of migrants. Given the fiscal externality $t_i - \alpha G_i/N_i$, migration inefficiency will now occur if (Boadway et al, 2002):

\[ t_1 - \frac{\alpha G_1}{N_1} \neq t_2 - \frac{\alpha G_2}{N_2} - \frac{\gamma}{\lambda} m'(M) \]

Where $\lambda$ is the shadow price of central government revenue and $\gamma$ is the Lagrange multiplier on the migration equilibrium constraint in the unitary state planner’s problem. In effect,

\[ \frac{\partial G_i}{\partial N_i} = \alpha G_i/N_i. \]

\[ \text{Formally, the congestion effect } \alpha G_i/N_i \text{ is the additional cost required to keep the level of public services } g_i \text{ constant when } N_i \text{ increases. That is, differentiating } G_i = N_i^\alpha g_i \text{ we obtain } \]
fiscally induced migration affects the tightness of the migration equilibrium constraint in the constrained social optimum: by increasing the cost of the marginal migrant, the utility differential is increased between the two regions. However, the sign of $\gamma$ is ambiguous: the migration equilibrium constraint can be binding in either direction depending on the characteristics of the regional economies. This makes it impractical to take migration costs into account in designing equalization systems, even in relatively simple settings. Fortunately, if migration costs are identical for all, $m'(M) = 0,$ the costs of migration can be ignored,\textsuperscript{13} as is typically the case in the equalization literature. For simplicity, we shall follow the same practice.

The extent and direction of the migration externality depends upon the features of the regional economies and their capacity to raise revenues. It is useful to consider separately the cases where households are identical and different.

**Migration Inefficiency with Homogeneous Households**

The extent of migration inefficiency depends on how per capita taxes of migrants $t_i$ are determined. In the simplest case where all regional revenues are obtained from a lump-sum tax on residents, the region-$i$ budget constraint is $G_i = t_i N_i.$ Using this to determine $t_i,$ migration inefficiency will occur if:

\[
\text{Migration inefficiency with head taxes : } \quad \frac{(1 - \alpha)G_1}{N_1} \neq \frac{(1 - \alpha)G_2}{N_2}
\]

Thus, in the case of pure public services ($\alpha = 1$), migration will be efficient and there will be no need for a corrective equalization grant.

Suppose now that the regions can also obtain revenues from income generated at source. For example, let a given proportion $\theta_i$ of regional rents $R_i(N_i)$ accrue to the regional government, so the regional budget constraint becomes $G_i = t_i N_i + \theta_i R_i(N_i).$ In this case, the migration inefficiency condition becomes:

\[
\text{Migration inefficiency with head and rent taxes : } \quad \frac{(1 - \alpha)G_1}{N_1} - \frac{\theta_1 R_1(N_1)}{N_1} \neq \frac{(1 - \alpha)G_2}{N_2} - \frac{\theta_2 R_2(N_2)}{N_2}
\]

Migrants effectively acquire a share of regional rent revenues at the expense of existing residents when they migrate. Other source-based tax revenues will have the same effect. For example, if regions levy a tax rate $\tau_i$ on capital $K_i$ located in their jurisdictions, the migration inefficiency condition can be written:

\textsuperscript{13} This is so because fiscally induced migration does not tighten the migration equilibrium constraint $u(c_1) + b(g_1) = u(c_2) + b(g_2) - m(M)$ if migration costs $m(M)$ are fixed.
Migration inefficiency with head, rent and capital taxes:

\[
\frac{(1 - \alpha)G_1}{N_1} - \frac{\theta_1 R_1}{N_1} - \frac{\tau_1 K_1}{N_1} \neq \frac{(1 - \alpha)G_2}{N_2} - \frac{\theta_2 R_2}{N_2} - \frac{\tau_2 K_2}{N_2}
\]

The interpretation is exactly as in the rent taxation case. Note that in the case of pure public services (\(\alpha = 1\)), the possibility of migration inefficiency arises solely because of source-based regional revenues. As we shall see, this will not be the case when households are heterogeneous.

These migration inefficiency conditions have been obtained by combining the regional budget constraints with the expression for the fiscal externality of migration \((t_i - \alpha G_i/N_i)\). It is apparent that in the homogeneous household case, migration inefficiency can be eliminated by manipulating regional budget constraints using an equalization transfer. Let \(E_1\) be the equalization transfer to region 1, and \(E_2(= -E_1)\) the transfer to region 2. The budget constraint for region \(i\) becomes \(G_i = t_i N_i + \theta_i R_i + \tau_i K_i + E_i\). Then, setting \(E_1\) such that migration efficiency is achieved \((t_1 - \alpha G_1/N_1 = t_2 - \alpha G_2/N_2)\), we obtain the optimal equalization entitlement for region 1 as:

**Optimal equalization with head, rent and capital taxes:**

\[
E_1 = \frac{N_1 N_2}{N_1 + N_2} \left[ \left( \frac{(1 - \alpha)G_1}{N_1} - \frac{\theta_1 R_1}{N_1} - \frac{\tau_1 K_1}{N_1} \right) - \left( \frac{(1 - \alpha)G_2}{N_2} - \frac{\theta_2 R_2}{N_2} - \frac{\tau_2 K_2}{N_2} \right) \right]
\]

Several observations should be noted about this characterization of optimal equalization in the homogeneous household case.

1. Under a net equalization scheme, \(E_i\) is the transfer to the regions for equalization purposes. Since it involves a negative transfer to one region, it may not be politically feasible. A gross equalization scheme can be designed that accomplishes the same outcome. Suppose that the transfer to the regions by the central government is financed by a nationwide tax of \(T\) per resident. Then, a gross equalization system would transfer to region \(i\) an amount equal to \(E_i + N_i T\). The central tax \(T\) could always be chosen so that the total transfer is non-negative for all regions. In this case, the VFG is simply \(T\) per person.

2. In a migration-constrained social optimum with homogeneous households, the equal utility constraint is binding with \(\alpha < 1\), as we have mentioned. However, this does not affect the form of the equalization entitlement \(E_i\) as long as migration costs are constant.

3. The equalization formula (3) is based solely on offsetting the fiscal externality of migration. Although this is the only source of inefficiency in our base-case model, in more general settings there may be other sorts of inefficiencies. For example, if source-based tax revenues are obtained from a mobile base (such as capital), this will generally lead to an inefficient choice of regional tax rates with the possibility of production inefficiency and/or an inefficient choice of the size of the public sector. To achieve an optimal outcome, equalization would have to be combined with other measures, including harmonized tax
rates ($\tau_1 = \tau_2$). Failing that, $E_i$ determined by (3) would not be second-best optimal in general.

4. Regional advantages due to rents or source-based tax revenues might be capitalized in local property values mitigating fiscally induced migration. To the extent that this is the case, the role of equalization is lessened. In fact, equalization itself should be capitalized to the same extent so has less of an effect. Of course, to analyze this possibility one would have to introduce real property (e.g., housing and land) into the model.

5. If migration costs exist, the equalization formula (3) can in principle be revised to:

$$E_1 = \frac{N_1 N_2}{N_1 + N_2} \left[ \left( \frac{(1-\alpha)G_1}{N_1} - \frac{\theta_1 R_1}{N_1} - \frac{\tau_1 K_1}{N_1} \right) - \left( \frac{(1-\alpha)G_2}{N_2} - \frac{\theta_2 R_2}{N_2} - \frac{\tau_2 K_2}{N_2} \right) + \frac{\gamma}{\lambda} m' \right]$$

Again, the sign of $\gamma$ is ambiguous, making the last term difficult to implement unless $m' = 0$.

**Migration Inefficiency with Heterogeneous Households**

Matters become more complicated, but at the same time more realistic, if households differ in income-earning capacity. Migration efficiency now applies to each household type, and the corrective mechanism may have to be household specific. As well, with heterogeneous households, there is no longer any presumption that regional decision-making will achieve an optimal allocation of resources within the region. The exception will be the case in which workers can migrate costlessly. As we have seen, the unitary state solution in this case is a symmetric one in which workers migrate until equal total numbers exist in each region, wage rates per efficiency unit of labor are the same, different proportions of high- and low-ability workers reside in each region, the level of public goods is the same in both regions, and the level of consumption of private goods is uniform over regions and ability types. Regional decision-making will lead to optimal intra-regional outcomes if the regions are responsible for redistribution or if they are restricted to proportional consumption taxation. In such a setting, equalization is needed for two reasons. First, if rents are decentralized to the regions, they must be equalized so that fiscally induced migration does not occur. Second, there must be an equalization transfer from the region with the highest proportion of high-ability types to the other region. That is because, the latter region will need more revenues to provide the national average level of consumption. This is considered in more detail below.\(^{14}\)

With costly migration, decentralized decision-making is not optimal unless very complicated transfer schemes are in place, which involve giving regions the right incentive for providing $G$ as well as for redistribution. Such schemes have yet to be worked out except in special cases, even in an otherwise first-best world of lump-sum transfers. Equalization on top of that is very complicated. What the literature tends to do instead is basically ignore non-optimalities in regional public spending and tax policy, and let equalization

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\(^{14}\) Note that the value of $\alpha$ does not matter in this case since $G_i/N_i$ is the same in both regions in equilibrium.
focus on the migration margin alone. In what follows, we not only follow that approach, we also make further simplifying assumptions that roughly reflect the stylized features of real world federations. Thus we are now beginning to move from the theoretical to the practical.

Suppose that expenditures assigned to regional governments take the form of pure public services \((\alpha = 1)\). As well as being a reasonable reflection of reality, this allows us to avoid problems associated with economies of scale in the consumption of public goods. We assume that the regions use proportional taxes, whether on the basis of residence or at source. In fact, allowing the residence taxes to be linear progressive is straightforward since the equal per capita poll subsidy component is similar to the equal per capita provision of public services. And, household incomes are taken to be given without explicitly modeling their source. Nor is migration modeled explicitly: our procedure will be limited to considering the fiscal incentives to migrate and how to mitigate them.

Let the income of a person \(j\) in region \(i\) be \(PI^i_j = W^i_j + P^j\), where \(W^i_j\) is wage income and \(P^j\) is property income. Note that while wage income is contingent on region of residence \(i\), property income is independent of residence and can be earned in any region. For any given value of wage income \(W\), there can be a distribution of property income \(P\). Equivalently, persons at a given income level can have different combinations of wage and property income. The average per capita income in region \(i\) plays an important role in our analysis and is denoted \(\overline{PI}_i\).

Regional policies include a personal income tax rate \(t_i\), a source-based tax on rents and capital at the common rate \(\theta_i\), and the provision of public services \(g_i\). All regional taxes are proportional, and public services \(g_i\) are the same for all residents. The budget constraint of regional government \(i\) in per capita form is \(g_i = t_i\overline{PI}_i + \theta_i(R_i(N_i) + K_i)/N_i\).

Public services and private consumption are assumed to be perfect substitutes in household utility. We can then define person \(j\)'s full income in region \(i\), \(Y^i_j\), as the sum of their consumption of private goods and public services:

\[
Y^i_j = (1 - t_i)PI^i_j + g_i = PI^i_j + NFB^i_j
\]

where \(NFB^i_j = g_i - t_iPI^i_j\) is the net fiscal benefit of person \(j\) in region \(i\). Using the regional government budget constraint, person \(j\)'s NFB can be written:

\[
NFB^i_j = t_i(\overline{PI}_i - PI^i_j) + \theta_i(R_i(K_i) + K_i)/N_i \tag{4}
\]

Finally, we denote the difference in NFBs between the two regions for a type-\(j\) person as \(\Delta NFB^j = NFB^j_1 - NFB^j_2\), or using (4):

\[
\Delta NFB^j = \left(\frac{\theta_1(R_1 + K_1)}{N_1} - \frac{\theta_2(R_2 + K_2)}{N_2}\right) + (t_1\overline{PI}_1 - t_2\overline{PI}_2) + PI^i(t_2 - t_1) \tag{5}
\]

In general, \(\Delta NFB^j\) will differ for households of different incomes \(PI^j\). The exception is the case where both regions choose the same personal income tax rate \((t_1 = t_2)\). In that
case, the last term of (5) disappears, so the NFB difference between regions is the same for households of all income types.

Given that wage income is residence-specific, while property income is not, migration equilibrium (assuming costless migration and an interior solution) for persons of wage $W^k$ and income $PI^j$ satisfies:

$$W^k_1 + NFB^{kj}_1 = W^k_2 + NFB^{kj}_2 \quad \forall \ k, j$$

where $NFB^{kj}_i$ denotes the NFB of such a person in region $i$. However, migration efficiency—fiscal efficiency—requires $W^k_1 = W^k_2$ for all $k$-type workers (regardless of their property income). Thus, migration efficiency—an interior solution—requires $NFB^{kj}_1 = NFB^{kj}_2$ for all persons with wage incomes $W^k$ whatever their incomes $PI^j$. This is obviously not easy to satisfy, given the double distributions of wage and property incomes. However, one special case is notable. If $t_1 = t_2$ in a post-transfer equilibrium, NFB differentials are the same for all persons, and these can be equalized using a single equalization transfer. Using a similar procedure as in the homogeneous household case, the optimal equalization grant to region 1 in the two-region case will be given by:

$$E_1 = \frac{N_1 N_2}{N_1 + N_2} \left[ t_1(P_T_2 - P_T_1) + \frac{\theta_2(R_2 + K_2)}{N_2} - \frac{\theta_1(R_1 + K_1)}{N_1} \right]$$

This says that if $t_1 = t_2$ in equilibrium, differences in per capita tax collections from all revenue sources should be equalized. In equilibrium, both regions provide the same level of public services at the same tax rates. The unitary state allocation would be replicated.

Although (6) is suggestive as a basis for actual equalization, it raises two sorts of issues in practice. The first is that if $t_1 \neq t_2$, NFB$_i^{kj}$ differs across $(k, j)$-types so a single equalization transfer cannot equalize NFBs. In principle, full efficiency could be attained by a complicated system of transfers to households based on individual-specific NFB differentials. However, although that system of transfers would neutralize fiscal incentives for migration, the effect of it would be to undo differences in regional behavior. That would undermine the purpose of decentralized decision-making, which is to enable regional governments to exercise their discretion over fiscal decisions within their realm of assigned responsibilities. The second is that if the equalization system were based on a formula like (6), it would introduce incentives for regions to choose fiscal policies to increase their equalization entitlements. The most obvious of these would be incentives to change regional tax rates (although there are other incentive effects which we discuss further below).

To address both these problems, equalization can be based on standardized (national average) rates, $\bar{t}, \bar{\theta}$, calculated as total regional tax collections divided by nationwide aggregate tax bases for each tax types. Then, the equalization entitlement becomes:

$$E_1 = \frac{N_1 N_2}{N_1 + N_2} \left[ \bar{t}(P_T_2 - P_T_1) + \frac{\bar{\theta}R_2}{N_2} - \frac{\bar{\theta}R_1}{N_1} \right]$$

16
This approach largely avoids the incentive for regions to manipulate their tax rates (except to the extent that a region’s tax rate affects the national average tax rate) and equalizes the potential of regions to provide public services, while tolerating differences in regional fiscal behavior. This approach is called the Representative Tax System (RTS) approach, and is used, for example, in Canada. It can readily be extended to several regions and to several tax types.

The RTS approach (7) implicitly assumes away some issues that might have a bearing on migration efficiency. As mentioned above, if migration costs vary among households, the formula should in principle take account of them. In practice, that would be virtually impossible. As well, the RTS approach equalizes only on the basis of differences in the revenue-raising ability of regions. This is because of the twin assumptions that public spending is on pure public services only, and these services are provided on an equal per capita basis. If public services are targeted to particular groups in the region, differences on the population mix would give rise to differences in the need for public spending. The equalization system should take account of these differences in need, as discussed further below. Finally, if $\alpha < 1$, differences in $(1 - \alpha)G/N$ should also be equalized.

Extensions to the Basic Ricardian Model

The above approach is the standard one used to analyze the efficiency case for equalization. As it indicates, equalization transfers from the central government to the regions can correct for the inefficiencies arising from migration equilibrium in a decentralized setting, albeit imperfectly when jurisdictions do not behave the same. The need for equalization and its efficacy have both been questioned in the literature. First, it has been argued that since there are potential efficiency gains from equalization, these should be exploited by regions through voluntary transfers, analogous to the Coasian bargaining remedies for externalities involving small numbers of parties. Second, the above analysis depends crucially on the assumed timing of events. If this changes, the role of equalization diminishes. Consider these two issues in turn.

Voluntary Transfers

Myers (1990) has shown that in the homogeneous labor case with perfect mobility, regions will make optimal voluntary equalization transfers if their objective is to maximize per capita utility. Through the migration equilibrium constraint, all regions will indirectly be motivated to maximize national per capita utility. Consequently, all will agree on the optimal allocation of labor among regions, and all will agree on the direction and magnitude of optimal transfers. In these circumstances, donor regions will willingly make transfers and recipient regions will willingly accept them, and the social optimum will be achieved without central government intervention. This will apply no matter how many regions there are.

This result potentially calls into question the need for central government equalization transfers, so it is worth knowing how far it generalizes to other settings. In fact, the role of voluntary transfers diminishes even in a world of homogeneous households when two key
assumptions concerning the regional objective function and costless mobility do not apply (Boadway et al., 2002). If, for example, the objective of each region is to maximize the sum of its residents' utilities, voluntary transfers will generally not suffice with or without migration costs. Regions will care about the size of their population as well as the utility per resident, and they will no longer agree on the optimal allocation of population among regions. As a result, they may agree on direction of transfers but not their magnitude; they may disagree on direction of transfers; or regions may refuse to accept transfers offered by other regions.

With migration costs, matters are more complicated because the utility level of households will differ among regions. There is now an equity-efficiency trade-off which implies that there is more than one efficient allocation of labor among regions. Voluntary transfers may succeed in selecting one of them (Mansoorian and Myers, 1993), but it will generally not be the one that a unitary state government would choose.

In any case, central government transfers will fully crowd out regional transfers, and will achieve the national optimum avoiding all the above problems. The central government can also get around the problem of regions refusing to accept transfers by making them directly to individuals. Voluntary transfers are thus redundant.

Timing

There are three actors in the simple Ricardian model: households (potential migrants), regional governments and the central government. In the standard approach, it is implicitly assumed that the central government moves first, followed by the regional governments, and then the private sector. However, as the literature on time consistency shows, changing the order of movement—or the ability of policy-makers to commit—can have a serious effect on equilibrium outcomes. Time-consistent policies are typically inferior to those obtained under full commitment. Consider two options of timing. In the first one, migration occurs before either government chooses its policies. In the second, regional governments choose policies before the central government, followed by migration.

The case in which migration occurs before government policies might be relevant to the extent that migration is a long-run decision relative to government policies, and governments are not able to commit to longer term policies. Mitsui and Sato (2001) have analyzed this case in the context of the simple Ricardian model with perfectly mobile households. Standard analysis of sub-game perfect equilibrium outcomes involves backward induction beginning with the last stage of events, which in this case involves government policy choices, given predetermined household location decisions. If horizontal equity is not imposed, a utilitarian central government makes interregional transfers so as to equalize the marginal utility of private consumption nationwide. In the separable utility case that we are con-

\[\text{\footnotesize 15 The exception to this is when full commitment policies are unconstrained first-best, in which case there is no problem of time inconsistency. See Hillier and Malcomson (1984) and Calvo and Obstfeld (1988). In our context, as we have seen, socially optimal outcomes are only first-best if the migration equilibrium constraint is not binding.}\]
sidering, this translates into equalizing per capita consumption across regions. Regional governments in this context behave optimally and choose the level of public spending that satisfies the Samuelson condition (1). Going back to the first stage, households anticipate this in making their location choices. As long as $\alpha < 1$, households will have an incentive to migrate to more populous regions, resulting in an inefficient allocation of labor across regions. In the limit, if migration is costless, all will move to the same region, as Mitsui and Sato note. Obviously, the costless migration case is an extreme one that yields an extreme outcome, but the intuition carries over to more realistic cases. This seemingly serious problem can be avoided in one of two ways. Either the central government can commit to its equalization transfers $E_i$ before migration occurs, or it abides by a horizontal equity constraint, which it imposes on itself.

The case where regional governments choose their policies before the central government also gives rise to potentially serious consequences. Suppose that the regions choose their public service levels $g_i$ anticipating central government policies—including central-regional transfers to finance $g_i$—and migration (which is here determined in the last stage). Regional governments effectively face a soft budget constraint (Wildasin, 1997), and will choose $g_i$ non-optimally, assuming that the central government cannot commit in advance to a set of transfers. In fact, despite the soft budget constraint, there can be either an over- or an under-supply of $g_i$ in equilibrium (Boadway et al, 2002). Although regions expect to receive funding for any level of $g_i$ they preselect, that can be a mixed blessing since the extent of the equalization transfer that is available for private good spending can otherwise be adversely affected.

5 Equity Arguments for Equalization

In a federation in which labor is immobile—or at least some types of labor are immobile—migration efficiency is no longer a concern. The case for equalization must now be based on an equity rationale. In practice, equity arguments form the basis for the advocacy of equalization by policy-makers and their advisors. As we have mentioned, the notion of equity that is at stake is horizontal equity—the equal treatment of equals by the public sector throughout the nation. This is not an innocuous objective for two main reasons. First, horizontal equity may conflict with social welfare maximization as we have seen. This will arise in models of federalism with regional public goods. To avoid this problem, it is convenient to assume, as is done in the policy debate, that regional public spending is on pure public services ($\alpha = 1$), so the horizontal equity constraint would not be binding in the unitary state social welfare optimum.

The second, and more controversial, problem is that applying horizontal equity on a nationwide basis involves a substantial value judgment. It involves the supposition that there is a consensus for social citizenship or solidarity to be applied on a national, as opposed to a regional, basis. In other words, every citizen counts equally in a national social welfare function. In a unitary state, nationwide horizontal equity might be taken for granted. In a federation, it is not so obvious. Taken literally, horizontal equity implies that all

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16 For a brief discussion of the notion of social citizenship, see Purdy (2001).
regions must not only have the resources to apply national standards of redistribution, but they must actually apply them. This would seem to conflict with the principle of federalism which recognizes that different preferences for redistribution might apply in different regions.

A compromise solution to this second problem is to adopt a modified form of horizontal equity, what we refer to as fiscal equity. In this form, the role of equalization transfers is to provide regions with the resources that would enable them to meet national standards of redistributive equity if they so chose, but does not compel them to do so. That is, regions are able to enact standards of vertical equity within their jurisdictions that best suit their preferences or the consensus of their constituents. The implication is that central government and regional government redistribution will coexist, and strict horizontal equity will be compromised. The extent to which central versus regional vertical equity applies will depend upon the extent of decentralization of revenue and expenditure responsibilities. It will also depend on the extent to which the central government influences regional redistribution policies through the use of instruments like conditional grants and mandates.

Equalization itself remains an unconditional transfer system that is designed to ensure that fiscal equity among regions is satisfied. Even this compromise, which leaves room for regions to exercise discretion over redistribution within their borders, can be controversial. It involves a high degree of solidarity nationwide among rich and poor regions, and that solidarity may not exist in decentralized federations with heterogeneous regions. In some cases, this is overcome by constitutional mandates for equalization. In others, the extent of equalization is less than complete. In the discussion that follows, we consider the implications for equalization of assuming that full fiscal equity applies. The argument turns out to closely parallel that in the fiscal efficiency case.

Full horizontal equity in a federation would require that persons equally well-off in the absence of government be equally well-off in its presence, where government now includes both central and regional levels. Using our earlier notation, equally well-off persons in two regions have the same incomes, that is, \( PI_1^j = PI_2^j \) for type-\( j \) persons in regions 1 and 2. Given our definition of full income \( Y^j_i = PI^j_i + NFB^j_i \), we can see using (4) that full income for these two persons will differ as follows:

\[
Y_1^j - Y_2^j = \Delta NFB^j = \left( \frac{\theta_1(R_1 + K_1)}{N_1} - \frac{\theta_2(R_2 + K_2)}{N_2} \right) + (t_1 PI_1^j - t_2 PI_2^j) + PI^j(t_2 - t_1) \tag{8}
\]

where we are again assuming that the regions levy income taxes \( t_i \) and source-based taxes \( \theta_i \) to finance the provision of equal per capita public services \( g_i \). Full horizontal equity would require eliminating differences in NFBs for all households. As (8) indicates, this would require a different transfer for each income type as long as \( t_2 \neq t_1 \), which would be a difficult task (and one that effectively negates each region’s discretionary choice of income tax rates).\(^{17}\) However, as in the efficiency case, NFB differentials would be identical for persons of all income levels if \( t_1 = t_2 \).

\(^{17}\) The task would not be as difficult as in the efficiency case. There, NFB differentials would
The same considerations as earlier lead us to the RTS prescription (7) for the equalization system. In this system, equalization transfers enable all regions to raise the same revenues if they applied national average tax rates to their respective tax bases. This has two suitable properties. First, the use of national average tax rates mitigates the incentives that regions would otherwise have to change their tax rates in order to influence the rates at which their transfers are calculated. Second, this system satisfies fiscal equity in the sense that all regions would have the potential to choose fiscal policies that satisfied nationwide horizontal equity if they so choose, without compelling them to do so. In fact, if all regions did behave the same, the unitary state outcome would be achieved. An interesting aspect of this prescription is that efficiency and equity considerations coincide, unlike in the usual economic policy situation where they must be traded off.

6 Caveats, Extensions and Applications

The RTS approach advocates full equalization of all revenue sources at national average tax rates. Although it is grounded in theory and is relatively easy to apply, it is by no means a complete theory of equalization and is subject to some caveats. In this section, we consider some of the caveats and extensions that apply to the RTS framework, with practical policy implications in mind.

Interpersonal versus Interregional Transfers

The theoretical case for equalization is based on undoing potential NFB differentials that arise in a decentralized federation. We have simply assumed that the relevant instrument was a set of intergovernmental equalizing transfers. However, it is clear that in principle the same effect could be achieved by a regionally differentiated interpersonal tax-transfer system. It is rare for central governments to enact region-specific income taxes. There may be administrative reasons for that. Alternatively, there may be advantages to the central government establishing a VFG needed to finance equalization payments. The VFG may serve to facilitate nationwide tax harmonization.

Needs Differences

We have assumed in deriving the RTS formula that regions provide public services on an equal per capita basis to all their residents. In fact, the main sorts of public services that are decentralized, such as education, health and welfare, are those targeted to specific groups. Different regions will have different demographic compositions, and so will have different needs for revenues in order to be able to provide comparable levels of public services to targeted groups in all regions. In this case, fiscal equity and efficiency require that equalization take account both of differences in the ability to raise revenues at national average tax rates, and also differences in the need for regional spending to provide national average levels of public services to targeted groups in all regions.

have to be eliminated for all wage types, each of which has a distribution of NFBs according to their total incomes. In the equity case, one would only have to eliminate NFB differentials by income type, regardless of their underlying wage incomes.
Devising a system of needs-based equalization transfers can be complicated if public services are very heterogeneous in nature. Typically transfers are based on estimates of the cost of providing services of various types to given demographic groups. These can be based on statistical estimation (Australia) or on more ad hoc measures of the average cost of providing broad types of public services (South Africa). In practice, equalization can have both revenue and needs components depending on the degree of decentralization. In countries with large VFGs, equalization tends to be based mainly on needs. Revenue equalization is used where regions have significant revenue-raising authority (Canada) or where they have entitlements to a share of revenue raised centrally (Germany).

Cost Differences

In the standard analysis, the cost of providing services is assumed to be the same across regions. To the extent that some regions have higher costs of provision than others, difficult problems arise for equalization. In a unitary state, if different regions have different costs, different levels of public services will generally be provided. The equity advantages of more equal provision will have to be set against the efficiency costs. Thus, the level of public services is typically lower in remote and scarcely populated areas than in urban areas, but how much lower involves a policy judgment.

The same applies in a decentralized setting in which some regions have more high-cost locales than do other regions. An equalization scheme that mimics the unitary state, at least in potential terms, will only partially equalize for differences in the cost of providing public services. In fact, it is likely that high-cost locations will exist in all regions, but to differing degrees. One option might be to stratify locations in all regions by their costs and equalize among regions within comparable strata.

Heterogeneous Regional Behavior

We have already discussed the requirement to base equalization on the potential of regions to provide comparable levels of public services at comparable tax rates, recognizing that they will use that potential in different ways. However, as regions make more diverse policy choices, the design of an equalization system to achieve this objective becomes more ambiguous. If regions used the same tax bases, and differed only in the choice of tax rates to apply to each base, the RTS method, which applies national average tax rates to the common bases, provides a reasonable approach. However, in highly decentralized federations, regions may choose different tax bases as well.

There may be various responses to this. An attempt might be made to construct a set of tax bases that are broadly representative of the practices of the provinces, although this will involve some arbitrariness. Alternatively, one might instead calculate the revenues that would be raised by applying each region’s tax system to all regions, and calculating equalization entitlements on the basis of some average of these hypothetical revenues. Finally, one might throw up their hands and give up the RTS approach altogether. Instead, a simpler, broader measure of the ability to raise revenues might be used, such as regional output, consumption of personal incomes. This approach—referred to as the macro ap-
proach to equalization—has the advantage of simplicity, but the disadvantage that it no
longer directly reflects the tax policies of the regions that give rise to NFB differentials in
the first place.\textsuperscript{18}

Similar problems will arise on the needs side of equalization. Indeed, heterogeneity in
regional public services is likely to be much greater than on the tax side, not just because
of the multi-dimensional nature of public services but also because spending is generally
more decentralized than taxation. Crude remedies here might include conditioning
a macro indicator on broad demographic or income distribution characteristics. Again,
much arbitrariness is likely to be involved.

**Progressive Regional Taxes**

The standard benchmark model on which policy prescriptions for equalization are based
supposes that taxes are proportional, while spending is equal per capita.\textsuperscript{19} In these cir-
cumstances, full equalization based on representative regional tax rates will fully undo
NFB differences across regions, and will replicate the unitary state outcome if all regions
behave the same way. To the extent that regional tax-spending systems are more or less
progressive than that, the RTS approach to equalization (perhaps augmented by needs-
based equalization) will not adequately offset NFB differentials. Appendix II provides an
illustrative example to show that more than 100 percent equalization is needed if regional
budgets are more progressive, and less than 100 percent if they are less progressive. In the
limit, under a benefit tax system, no NFBs are created and no equalization of residence
taxes is called for. (Source-based taxes will still give rise to NFBs and should be equalized.)

A revision to the RTS approach that will solve this problem is the stratified RTS approach.
This method, illustrated by example in Appendix II, is to stratify the population by income
class, each of which will now face different average tax rates. Each income stratum is
treated as a separate tax base for equalization purposes and is equalized by applying the
RTS approach, using the national average tax rate applicable to that income stratum.
Those regions with a higher proportion of their populations in higher income brackets—
where higher national average tax rates apply—will have lower entitlements than would
be the case if households were not stratified.

**Incentive Effects**

Suppose an ideal equalization system is in place that fully equalizes differences in tax bases
of all types (stratified where necessary) using standard tax rates and differences in needs
using standard needs requirements. Such a system is bound to give rise to inadvertent
incentive effects, some of which we have already mentioned. If these are judged to be
significant, some compromise in equalization might be contemplated. Four different types

\textsuperscript{18} The case for a macro approach may be found in Barro (1986).

\textsuperscript{19} This benchmark case goes back to the seminal contributions of Buchanan (1950, 1952), more
recently discussed in Mieszkowski and Musgrave (1999).
of incentive effects can be identified.

**Base Tax-Back**

As (7) indicates, a region’s equalization entitlement is proportional to the size each of its tax bases that enter the equalization formula. This can include income, payrolls, consumption, capital, resource production, specific commodities or any other items that are revenue sources for the regional governments. Any reduction in the base reduces equalization entitlements by the national average tax rate applied to the change. If the regions can influence the size of their equalized tax bases, they may have an incentive to adopt policies to exploit that. There are two sorts of ways that this might occur. First, an increase in the region’s tax base can cause the size of the base to fall by an amount that depends on the elasticity of the base. For fairly elastic bases, regions will have an incentive to set tax rates too high since the revenue loss from the lower base caused by the high tax rate will be largely offset by increased equalization entitlements. They will perceive their MCPF to be lower than it actually is from a social perspective.

The second, and potentially more powerful, tax-back effect occurs when regions can directly control the size of the tax base. This can occur in the resource sector to the extent that regions manage the rate of resource development, or in the case of gaming where legality is at stake. Equalization using the RTS becomes a powerful disincentive where discretion over the base is possible: revenue gains from increasing resource production will be virtually offset by losses in equalization to the extent that a region’s own resource tax rate is comparable with the national average among all regions.

The disincentive associated with base tax-back effects is a classic incentive-equity trade-off. The appropriate remedy would be to offer less than full equalization for revenue sources that can readily be influenced by regional policies.

**Rate Tax-Back**

The other main element that goes into RTS equalization (7) is the national average tax rate. To the extent that a region’s tax rate influences the national average tax rate, it will have an disincentive to increase rates if its entitlement from that base is negative, and vice versa. In most federations, this is not a serious problem for most tax bases. If each region is relatively small compared with the nation as a whole, a given region’s influence on the national average tax rate will usually be small. An exception will be where a region has a high proportion of a particular tax base. This will likely only occur with narrow and unevenly distributed tax bases, like particular sorts of natural resources. Again, the remedy is to blunt the disincentive effect by reducing the extent of equalization of such bases, even though that also reduces the effectiveness of the equalization.

**Needs Incentives**

If needs differences are equalized along with differences in revenue-raising ability, regions might have an obvious incentive to change their spending policies. Needs equalization is analogous to revenue equalization. The ‘base’ is the number of recipients of a given public
service, and the rate of equalization is a measure of the per capita national average cost of providing the public service. Regions may be able to influence the number of recipients of particular public programs. This might be the case for the number of persons eligible for disability assistance, the number of unemployed or on welfare, and the extent of usage of the health system. They may be less able to influence needs when purely demographic indicators are used.

A needs-based equalization system can be designed in a way that minimizes the possibility of regions influencing their entitlements. Where that is not possible, the extent of equalization can be adjusted.

Offsetting Incentives

Finally, some authors have pointed out that some of the incentive effects of the equalization system might actually counteract inefficiencies that would otherwise exist in the federal fiscal system (Smart, 1998; Bucovetsky and Smart, 2002). For example, it is well-known that one of the effects of tax competition is to induce regions into competing down tax rates on inter-regionally mobile tax bases (e.g. capital, goods) to sub-optimal levels. That is, tax competition induces regions to perceive their MCPFs to be higher than they are from a social point of view. Equalization has the opposite effect, as we have seen above. By offsetting revenue losses from tax base reduction, equalization encourages regions to set their tax rates higher than they otherwise would. In certain circumstances, these two effects can be exactly offsetting.

Things are more complicated when both the central and the regional governments impose taxes on overlapping bases. In this case, vertical fiscal externalities are present that, like equalization, induce the regions to underestimate their MCPFs from a social point of view. Indeed, it is possible that the vertical externalities will themselves offset the tax competition effect even in the absence of equalization. Thus, the consequences of these counteracting effects for equalization are not clear.

7 Further Issues

Only some of the main issues that arise in the design of an equalization system have been addressed so far. Space precludes a detailed consideration of all of the other issues, so we can do no more than indicate the relevance of some of them here.

Equalization and Stabilization

Our analysis has focused on equalization in a stationary state setting in which one region is systematically better-off than another, and in which economic variables are fixed. In fact, regional tax bases are likely to vary over time unpredictably. In these circumstances, equalization can play a stabilization role alongside its redistributive role (e.g., von Hagen and Hammond, 1998; Konrad and Seitz, 2003). The argument for equalization as stabilization is essentially an insurance one. Regions are considered to be less able to insure themselves against economic shocks than is the central government, assuming the shock is an insurable one. The implicit—and untested—assumption is that the central government
both has better access to capital markets if necessary and can pool risks across regions in
the event that shocks are idiosyncratic.

In principle, the ideal structure of equalization for stabilization purposes is similar to
that for redistribution. However, as with any insurance scheme, the use of formula-based
equalization as an insurance device gives rise to special incentive problems. In particular,
there may be moral hazard-type problems whereby regions are induced to engage in policies
that affect the chances of an adverse outcome or the size of a loss in the event of an adverse
outcome (Persson and Tabellini, 1996). Or there may be adverse selection problems if the
central government is not fully informed about the relevant riskiness of regions (Lockwood,
1999; Bordignon, et al, 2001). Such asymmetric information problems lead to arguments
for less than full equalization.

If shocks are not idiosyncratic with respect to the regions, further problems arise. Standard
approaches to equalization that are based on the behavior of the representative region, such
as the RTS approach, tend to lead to pro-cyclical equalization entitlements, especially if
regional behavior is itself pro-cyclical. In these circumstances, equalization may not be
stabilizing at all. Indeed, it may be destabilizing, as evidence for Canada shows (Boadway
and Hayashi, 2002; Boothe, 2002). A way around this problem is to base equalization
on longer-term norms or moving averages, although this will tend to undermine insurance
effects.

**Agglomeration Effects**

In the basic Ricardian model, differential fiscal externalities across regions are the source
of labor migration inefficiency (and possibly instability). Another source of migration in-
efficiency might be externalities associated with agglomeration effects in the labor market.
These can take various forms. The endogenous growth literature has emphasized local ex-
ternalities associated with the accumulation of human and physical capital (Romer, 1986).
The new geography literature (Krugman, 1993) focuses on the advantages of product di-
versity that can come with local agglomeration of firms. As well, job matching can improve
with the size of the labor market (Pissarides, 2000). In a federalism context, these various
externality effects imply that the migration of skilled labor and entrepreneurs brings with
it benefits that are not captured by the migrants, and to the extent that these differ across
regions, there may be inefficiencies that can be corrected by equalization.

As in the basic Ricardian model, agglomeration effects can give rise to multiple optima. A
challenge for any equalization system is to be sure that it does not support a purely local
optimum. That involves focusing on more than just the first-order efficiency conditions
that is characteristic of the standard approach. Moreover, to the extent that agglomeration
effects differ for workers of different skills, equalization alone would not be sufficient for
corrective purposes.

**Infrastructure**

Similar considerations arise in relation to public infrastructure. By its very nature, in-
frastucture embodies economies of scale. As with public goods, this can result in fiscal
externalities of migration, and possibly multiple local optima. To the extent that the provision of infrastructure is decentralized to regional governments—which is typically the case—it too may be prone to induce inefficiency in the allocation of labor across regions. In principle, this source of inefficiency ought to be taken account of in the equalization system.

As further concern with infrastructure is its capital nature, and the associated need for financing and ongoing operation and maintenance expenditures. The intertemporal nature of these decisions imply that they are likely to be prone to bailout problems unless the central government is able to commit to a transfer system.

**Political Economy Issues**

Finally, much of the theory of equalization has assumed that governments at all levels are benevolent. To the extent that political policy-makers are unduly influenced by self-interest or by the interest of bureaucrats or lobbyists, both regional and central decision-making are prone to be non-optimal from a social welfare perspective, however that is defined. In the extreme case where political outcomes are the deterministic outcome of the political process, the normative analysis of equalization is essentially redundant unless it can influence political outcomes. The theory instead becomes a positive one in which observed equalization is explained. In less deterministic environments, policies may be constrained by political feasibility factors. It might be the case, for example, that political constraints are more binding at the regional level of government than at the center. In this case, the extent of decentralization and the form that fiscal transfers might take can be used as devices to influence political outcomes at the regional level. However, the agenda involved in taking political considerations into account in the design of an equalization system is a daunting one, and has yet to be exploited.
References


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Table 1. Circumstances under which Decentralization is Optimal
Appendix I: Decentralizing the Unitary State Optimum

Homogeneous Labor Case

Two regions $i = 1, 2$ have $O_i$ original residents and $N_i$ final residents (and labor supply), where $O_i + O_2 = N_1 + N_2 = N$. Regional production functions are: $F_i(N_i)$, where $F_i' > 0 > F_i''$. Workers in $i$ obtain labor income $F_i'(N_i)$, leaving rents of $R_i(N_i) = F_i(N_i) - N_iF_i'(N_i)$, which are assumed to accrue to the government (i.e., $\theta_i = 1$ in the text). The migration cost of the marginal household is given by the function $m(M_i)$, $m' \geq 0$, where $M_i = O_i - N_i$ is the number of out-migrants from $i$. We assume that migration goes from region 1 to region 2, so migration equilibrium requires $u(c_1) + b(g_1) = u(c_2) + b(g_2) - m(M_1)$.

Consider first the unitary state optimum under a utilitarian objective. It is useful to distinguish notionally between a central budget and regional budgets. Central policy instruments include a set of transfers $E_i$ to regions $i = 1, 2$. Regional policies include public services $g_i$ and a tax $t_i$ on region-$i$ residents. Optimality is characterized by the solution to the problem in which a unitary state chooses all central and regional policies subject to the central and regional budgets and migration equilibrium:

$$\max_{\{t_i, E_i, g_i, N_1\}} \quad N_1 [u(c_1) + b(g_1)] + (N - N_1) [u(c_2) + b(g_2)] - \int_{0}^{O_1 - N_1} m(x)dx$$

subject to

$$E_1 + E_2 = 0 \quad (\lambda)$$

$$E_1 + N_1 t_1 + R_1(N_1) - N_1^\alpha g_1 = 0 \quad (\lambda_1)$$

$$E_2 + (N - N_1) t_2 + R_2(N - N_1) - (N - N_1)^\alpha g_2 = 0 \quad (\lambda_2)$$

$$u(c_2) + b(g_2) - m(O_1 - N_1) - u(c_1) - b(g_1) = 0 \quad (\gamma)$$

where $c_i = (F_i'(N_i) - t_i)$. The first-order conditions on $t_i$ and $g_i$ yields the Samuelson conditions (1) in the text, while those on $N_1$ yield the migration efficiency condition

$$t_1 - \frac{\alpha G_1}{N_1} = t_2 - \frac{\alpha G_2}{N_2} - \frac{\gamma}{\lambda} m'(M_1)$$

Combining the three budget constraints, the transfer $E_1$ can be expressed as:

$$E_1 = \frac{N_1 N_2}{N_1 + N_2} \left[ t_1 - t_2 + \frac{G_1}{N_1} - \frac{G_2}{N_2} + \frac{R_2(N_2)}{N_2} - \frac{R_1(N_1)}{N_1} \right]$$

and equivalently for region 2. Thus, equalization compensates for differences in regional per capita revenues less expenditures. Using this condition and the migration efficiency condition, the optimal transfer $E_1$ is given by:

$$E_1 = \frac{N_1 N_2}{N_1 + N_2} \left[ \left( \frac{(1 - \alpha)G_1}{N_1} - \frac{(1 - \alpha)G_2}{N_2} \right) - \left( \frac{R_1(N_1)}{N_1} - \frac{R_2(N_2)}{N_2} \right) + \frac{\gamma}{\lambda} m'(M_1) \right]$$
as in the text. The migration equilibrium constraint is binding in the optimum, so $\gamma \neq 0$, but $\gamma$ can be positive or negative.

In a decentralized setting, the regions choose $g_i$ and a lump-sum tax on their own resident, say, $t_i$. Suppose also that the center is able to levy a tax $\tau_i$ on individuals in region $i$. The central government now chooses $\tau_i$ and $E_i$. The problem of region 1, given central taxes and transfers and some conjecture about the response of the other region’s policies, solves the following problem:

$$\max \{t_1, g_1, N_1\} \quad N_1 [u(F_1'(N_1) - t_1 - \tau_1) + b(g_1)]$$

subject to

$$E_1 + t_1 N_1 + R_1(N_1) - N_1^\alpha g_1 \geq 0$$

$$u(F_1'(N_1) - t_2 - \tau_2) + b(g_2) - m(O_1 - N_1) - u(F_1'(N_1) - t_1 - \tau_1) - b(g_1) = 0$$

The optimality conditions on $t_1$ and $g_1$ yield (1) regardless of the conjecture made about the other region’s policies. A similar result applies for region 2, whose objective function is

$$N_2 [u(F_2'(N_2) - t_2 - \tau_2) + b(g_2)] - \int_{0}^{O_1 - N_1} m(x)dx$$

**Heterogeneous Labor Case**

High-ability and low-ability households supply $a^h$ and $a^\ell$ efficiency units of labor ($a^h > a^\ell$). Production region $i$ is $F_i(A_i)$, where $A_i = a^h N_i^h + a^\ell N_i^\ell$. The wage rates are $a^h F'_i(A_i)$ and $a^\ell F'_i(A_i)$, and regional rents are $R_i(A_i) = F_i(A_i) - A_i F'_i(A_i)$.

**Costless Migration**

Migration equilibrium with costless migration requires:

$$u(c_i^j) + b(g_1) = u(c_i^2) + b(g_2) \quad j = h, \ell$$

In the unitary state optimum, taxes on households by type and region $t_i^j$ ($j = h, \ell; i = 1, 2$) will suffice. Assuming (arbitrarily) the rents accrue to regional budgets, the problem of the unitary state government is:

$$\max \{E_i, t_i^h, t_i^\ell, g_i, N_i^h, N_i^\ell\}$$

$$\quad N_1^h [u(a^h F'_1(A_1) - t_1^h) + b(g_1)] + N_1^\ell [u(a^\ell F'_1(A_1) - t_1^\ell) + b(g_1)]$$

$$+ N_2^h [u(a^h F'_2(A_2) - t_2^h) + b(g_2)] + N_2^\ell [u(a^\ell F'_2(A_2) - t_2^\ell) + b(g_2)]$$

subject to:

$$E_1 + E_2 = 0 \quad (\lambda)$$

$$E_i + t_i^h N_i^h + t_i^\ell N_i^\ell + R_i(A_i) - (N_i^h + N_i^\ell)^\alpha g_i = 0 \quad i = 1, 2 \quad (\lambda_i)$$
\[ u \left( a^j F'_2(A_2) - t^j_2 \right) + b(g_2) - u \left( a^j F'_1(A_1) - t^j_1 \right) - b(g_1) = 0 \quad j = h, \ell \quad (\gamma^j) \]

where \( N^j_2 = N^j - N^j_1 \) for \( j = h, \ell \). The FOCs on \( t_i^j \) yield the analog of (1):

\[ \frac{N^j_h b'(g_1)}{N^\alpha_1 u'(c^j_1)} + \frac{N^j_1 b'(g_1)}{N^\alpha_1 u'(c^j_1)} = 1 \quad \frac{N^j_h b'(g_2)}{N^\alpha_2 u'(c^j_2)} + \frac{N^j_2 b'(g_2)}{N^\alpha_2 u'(c^j_2)} = 1 \]

Moreover, assuming an interior solution, consumption is equalized \( c^h_1 = c^h_1 = c^h_2 = c \), population is equalized \( N_1 = N_2 \), and public goods provision is equalized \( G_1 = G_2 \). \( g_1 = g_2 \), but effective labor supplies differ \( A_1 \neq A_2 \). And, the migration equilibrium constraints are not binding. Since the proportions of high- and low-productivity persons differ in the two regions, transfers \( E_i \) to the regions will be equalizing.

The FOCs on \( N^j_1 \) yield migration efficiency conditions for the two types of labor:

\[ t^j_2 - \frac{\alpha G_2}{N_2} = t^j_1 - \frac{\alpha G_1}{N_1} \quad j = h, \ell \]

To determine the optimal equalization transfers \( E_1, E_2 \), combine the three budget constraints \((\lambda), (\lambda_1) \) and \((\lambda_2) \) to yield:

\[ E_1 = \frac{N_1 N_2}{N_1 + N_2} \left[ t^h_1 - t^h_2 + \frac{G_1}{N_1} - \frac{G_2}{N_2} + \frac{R_2(N_2)}{N_2} - \frac{R_1(N_1)}{N_1} \right] \]

where \( \overline{t}_i \equiv (t^h_i N^h_i + t^\ell_i N^\ell_i)/N_1 \) is the average tax rate in region \( i \). A similar expression holds for \( E_2 \). It indicates that, as in the homogeneous labor case, differences in per capita revenues less expenditures are equalized. Next, using the migration efficiency conditions, the expression for \( E_1 \) becomes:

\[ E_1 = \frac{N_1 N_2}{N_1 + N_2} \left[ t^h_1 \left( \frac{N^h_2}{N_2} - \frac{N^h_1}{N_1} \right) + t^\ell_1 \left( \frac{N^\ell_2}{N_2} - \frac{N^\ell_1}{N_1} \right) + \frac{G_1}{N_1} - \frac{G_2}{N_2} + \frac{R_2(N_2)}{N_2} - \frac{R_1(N_1)}{N_1} \right] \]

Note that the first two terms involve equalizing for differences in the per capita tax bases of the high- and low-productivity workers in the two regions. In the special case where all public spending is on pure public services \((\alpha = 1)\), the migration efficiency conditions become \( t^h_1 = t^h_2 \equiv t^h, \ t^\ell_1 = t^\ell_2 \equiv t^\ell \). The equalization transfer \( E_1 \) becomes:

\[ E_1 = \frac{N_1 N_2}{N_1 + N_2} \left[ t^h \left( \frac{N^h_2}{N_2} - \frac{N^h_1}{N_1} \right) + t^\ell \left( \frac{N^\ell_2}{N_2} - \frac{N^\ell_1}{N_1} \right) + \frac{G_1}{N_1} - \frac{G_2}{N_2} + \frac{R_2(N_2)}{N_2} - \frac{R_1(N_1)}{N_1} \right] \]

This is precisely the RTS formula, analogous to (7) in the text.

**Costly Migration**

Suppose high- and low- productivity migrants have migration costs functions \( m^j(O_1^j - N^j_1) \ j = h, \ell \), and assume that migration of both types is from region 1 to region 2. The unitary state optimum solves:

\[ \max_{\{E_i, t_i^j, g_i, N_i^j\}} \ N^h_1 \left[ u \left( a^h F'_1(A_1) - t^h_1 \right) + b(g_1) \right] + N^\ell_1 \left[ u \left( a^\ell F'_1(A_1) - t^\ell_1 \right) + b(g_1) \right] \]
\[
+ N_h^h \left[ u \left( a^h F_2'(A_2) - t_h^h \right) + b(g_2) \right] + N_1^\ell \left[ u \left( a^\ell F_2'(A_2) - t_1^\ell \right) + b(g_2) \right] \\
- \int_{O_1^h - N_1^h}^{O_{11}^h} m^h(x)dx - \int_{O_1^\ell - N_1^\ell}^{O_{11}^\ell} m^\ell(x)dx
\]

subject to:

\[
E_1 + E_2 = 0 \\
E_i + t^h_i N^h_i + t^\ell_i N^\ell_i + R_i(A_i) - \left( N^h_i + N^\ell_i \right)^\alpha g_i = 0 \quad i = 1, 2 \quad (\lambda_i)
\]

\[
u (a^j F_2'(A_2) - t^j_2) + b(g_2) - m^j (O^j_1 - N^j_1) - u \left( a^j F_1'(A_1) - t^j_1 \right) - b(g_1) = 0 \quad j = h, \ell \quad (\gamma_j)
\]

The FOCs on \( g_i \) and \( t^j_i \) yield the Samuelson conditions, while those on \( N^j_1 \) yield the migration efficiency conditions:

\[
t^j_2 - \frac{\alpha G_2}{N_2} = t^j_1 - \frac{\alpha G_1}{N_1} + \frac{\gamma_j}{\lambda} m^j (O^j_1 - N^j_1) \quad j = h, \ell
\]

Proceeding as above, the equalization transfer to region 1 is:

\[
E_1 = \frac{N_1 N_2}{N_1 + N_2} \left[ t^h_1 \left( \frac{N^h_2}{N_2} - \frac{N^h_1}{N_1} \right) + t^\ell_1 \left( \frac{N^\ell_2}{N_2} - \frac{N^\ell_1}{N_1} \right) + \frac{G_1}{N_1} - \frac{G_2}{N_2} \\
+ \frac{R_2(N_2)}{N_2} - \frac{R_1(N_1)}{N_1} + \frac{\gamma_j}{\lambda} m^j (O^j_1 - N^j_1) \right]
\]

Unfortunately, this unitary state optimum cannot be decentralized by simply assigning tax instruments to the regions and using a lump-sum equalization transfer.
Appendix II: Illustration of The RTS Approach

Suppose there are two regions, A and B, and two income classes, H and L, with incomes of 40,000 and 20,000 respectively. Region A contains 1 low-income and 2 high-income persons, while region B contains 1 high-income and 2 low-income persons. Regional public services are available on an equal per capita basis. Consider the cases of proportional and progressive tax financing in sequence.

Proportional Taxation

Suppose each region levies a tax of 20%, so taxes are 8,000 for type-H persons and 4,000 for type-L persons in both regions. Public services per capita are then 6,667 in region A and 5,334 in region B. In the absence of equalization, the following NFBs (benefits less tax liabilities) apply:

\[
\begin{align*}
NFB^H_A &= -1,333 \\
NFB^H_B &= -2,666 \\
NFB^L_A &= 2,666 \\
NFB^L_B &= 1,333
\end{align*}
\]

Therefore, the common differences in NFBs between regions \((NFB^j_A - NFB^j_B)\) for the two types are:

\[
\Delta NFB^H = \Delta NFB^L = 1,333
\]

These NFB differences will be eliminated by transferring 667 per capita from region A to region B.

An RTS system will accomplish this as follows. The national average tax rate is 20%, and the per capita tax bases in regions A and B are 33,333 and 26,667. The equalization transfer to B, using equation (7), will be

\[
E_B = 1.5(.2)(33,333 - 26,667) = 1,999
\]

which is 667 per person, as required. When this equalization is in place, both regions can provide the same level of public services at the common tax rate of 20%.

Progressive Taxation

Suppose now that the first 20,000 of income is taxed at 10%, and the second 20,000 is taxed at 30% in both regions. Now, the taxes paid are 8,000 by the type H’s and 2,000 by the type L’s. Per capita public service benefits are 6,000 in region A and 4,000 in region B. NFB’s are as follows:

\[
\begin{align*}
NFB^H_A &= -2,000 \\
NFB^H_B &= -4,000 \\
NFB^L_A &= 4,000 \\
NFB^L_B &= 2,000
\end{align*}
\]

leading to NFB differentials of

\[
\Delta NFB^H = \Delta NFB^L = 2,000
\]

Again, the NFB differential is the same for both income classes. Optimal equalization should transfer 1000 per capita from province A to province B.
Equalization under the simple RTS will be too little. The national average tax rate is calculated to be 16.67% (total national revenues as a proportion of total national income). By (7), the equalization transfer to $B$ would be:

$$E_B = 1.5(0.1667)(33,333 - 26,667) = 1,667$$

which is 556 per person, well below the amount required to eliminate NFB differentials.

The stratified RTS effectively treats different income classes as separate revenue sources. For income class 40,000, the national average tax rate is 20%, and the per capita tax bases are 26,667 in region $A$ and 13,333 in region $B$. The equalization entitlement to region $B$ from this stratum is:

$$E^H_B = 1.5(0.2)(26,667 - 13,333) = 4,000$$

For income class 20,000, the national average tax rate is 10%, and the per capita tax bases are 6,667 in region $A$ and 13,333 in region $B$. The equalization entitlement to region $B$ from this stratum is:

$$E^L_B = 1.5(0.1)(6,667 - 13,333) = -1,000$$

Therefore the total entitlement for region $B$ is 3,000, which is 1,000 per capita, just sufficient to offset NFB differences. It is straightforward to show that the stratified approach will also yield the correct equalization levels when the tax system is regressive.