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The Marginal Cost of Public Funds is the Ratio of Mean Income to median Income

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Abstract

The marginal cost of public funds is the equilibrium price at the intersection of the appropriately-defined demand curve for and the supply curve of public expenditure. In a world with identical people and with no excess burden of taxation, that price would have to be 1. Otherwise the median voter's choice of a demogrant - or of its opposite, a head tax - fixes the marginal cost of public funds at the ratio of the mean income to the median income. A proof of this assertion is presented not for its realism, but because it calls attention to the interaction of the different influences upon the marginal cost of public funds.

JEL Classification H4

Keyword: marginal cost of public funds

[1]In writing this paper, I have benefitted considerably from electronic discussions with Liqun Liu about aspects of the marginal cost of public funds.

The marginal cost of funds is an indicator of the required benefit-cost ratio for public projects, programs and activities, where "benefit" refers to the sum of all benefits to whomsoever they may accrue and"cost" refers to the required public expenditure, excluding deadweight loss or excess burden of taxation to the tax payer. To say that the marginal cost of public funds is, for example, 1.5 is to say that a new hospital that costs $100 million to the Ministry of Health is worth establishing if and only if the combined benefit of the hospital to all users exceeds not $100 million, but $150 million, the extra $50 million being required to cover the excess burden of taxation to the taxpayer when an extra $100 million of public revenue is acquired by raising tax rates. To measure the marginal cost of public funds accurately, one would need to take account of the full response in the private sector to an increase in tax rates: the tax induced shifts in the allocation of time between labour, do-it-yourself activities and leisure, the allocation of purchasing power among goods and between consumption and investment, and tax-payers' incentives to tax avoidance and tax evasion. Simplifications are required if the marginal cost of public funds is to be measured at all.

There are in the literature two principal methods of measuring the marginal cost of public funds. The first is to estimate the extra deadweight loss from identifiable distortions in the tax system. In what is to my knowledge the earliest such measure, Campbell (1972 and 1975) computed the additional deadweight loss per dollar of additional tax revenue associated with the imputed excise taxes on all goods consumed, the assumption being that all such taxes would be increased slightly but proportionately when extra revenue is required. Soon after, Browning (1976) estimated the additional deadweight loss per dollar of additional tax revenue associated with the labour-leisure choice when leisure is exempted from the base of the income tax. The other method, exemplified by Feldstein (1995), is to estimate the elasticity of tax base to tax rate from observations of actual changes in peoples' reported taxable income in response to legislated changes in tax rates. It turns out, as will be shown below, that there is a simple relation between the elasticity of base to rate and the marginal cost of public funds. The marginal cost of public funds would be 1 - implying that the hospital should be built as long as its benefits exceed $100 million - when the elasticity of base to rate is 0, but it would be greater than 1 according to how rapidly the tax base shrinks in response to an increase in the tax rate. Neither of these methods is entirely trustworthy. Both are based upon large and never entirely accurate assumptions about how the economy works. The method proposed in this paper is even less trustworthy, but it is much simpler, and the explanation casts light on otherwise obscure aspects of the problem.

Notwithstanding the accuracy of the measurements, the matter is of the greatest importance because appropriate rates of taxation and the appropriate role of the government in the economy depend critically on what the marginal cost of public funds turns out to be. An estimate of the marginal cost of public funds is especially important for assessing the claim that public expenditure is on the wrong side of the Laffer curve, implying as it does that extra revenue could be acquired by lowering, rather than raising, tax rates. Campbell and Browning estimate the marginal cost of public funds at about 1.25, a level high enough to block some government expenditure but not to drive the economy to the wrong side of the Laffer curve. Feldstein, on the other hand, produces an estimate so high that, if it is right, the United States government could increase tax revenue by lowering tax rates. For a critique of Feldstein's method, see Goolsbee (2002). For a discussion of some recent estimates see Ballard and Fullerton (1992).

One's first thought about the about the marginal cost of public funds is that it should be set equal to 1, that public projects, programs or activities should be undertaken if and only if benefit exceeds cost. When governments engage not in a great variety of activities, but in the production of one type of "public good", the injunction to undertake projects for which benefits exceed cost boil down to the "Samuelson rule": undertake projects if and only if the sum of every person's marginal valuation of public goods in terms of private goods is equal to society's rate of transformation between them, specifically

$$\left\{ \sum_i \left(\frac{\partial u^i}{\partial G} / \frac{\partial u^i}{\partial x} \right) / \left(\frac{P_G}{P_x} \right) \right\} = 1 \qquad (1)$$

where x is a person's consumption of the private good, G is the total supply of the public good, P_x and P_G are their prices, and u^i (x, G, ...) is the utility function is of person i. The formula is valid for an ideal "communist" government that can choose G and each person's x^i to maximize the value of a social welfare function subject only to the constraint of the production possibility frontier for the economy as a whole. The social welfare function boils down to a person's utility function in the special case where everybody is alike.

Things start to go wrong when these extreme assumptions are relaxed. The mix of x and G mandated by the Samuelson rule is not unique. It depends upon the shapes of people's utility functions and on the weighing of people's utilities in the social welfare function. More importantly, the appropriate procedure for a benevolent communist government is not necessarily appropriate for a more limited government that can influence the economy through taxation but cannot choose each person's x^i directly. There are two aspects to this consideration. On the one hand, the cost to the government of public goods - or, more generally, of whatever the government chooses to buy - should be assessed net of tax. If a hammer costs \$10 at the hardware store and if the hardware store pays an excise tax of \$2 on every hammer sold, the cost of the hammer to the government is only \$8 in the sense that the purchase of the hammer can be financed by an tax increase of \$8 rather than \$10. The \$8 is the "shadow price" of a hammer to the government. On the other hand, the raising of an additional \$8 of tax revenue requires an increase, however small, in tax rates, and this in turn induces taxpayers to alter their behaviour to reduce their tax bills switching purchases from more taxed to less taxed goods - in ways that are individually beneficial but harmful to society as a whole. The two problems interact. When tax revenue depends in part on taxpayer's responses to taxation and when taxpayers differ in skill, ownership of property or other dimensions, the government's best choice of public expenditure - and the cut-off for additional public projects - depend critically on the objective function of the government. These considerations are reflected in the direct estimates of the marginal cost of public funds.

The paper begins with a simple derivation of the additional deadweight loss per dollar of additional public revenue for a two-good economy with identical people but where only one good can be taxed. The marginal cost of public funds is then represented as the height of the intersection of appropriately-defined demand and supply curves for public revenue. There follows a brief digression on the double dividend from the taxation of externality-bearing goods. The marginal cost of public funds is pinned down by three extensions to the model: that people differ in pre-tax incomes, that governments can levy lump sum taxes or can supply lump sum transfers, and that governments choose all taxes and expenditures in the interest of the median voter. Together with a few other assumptions, these three extensions to the model are sufficient to establish the proposition in the title of this paper.

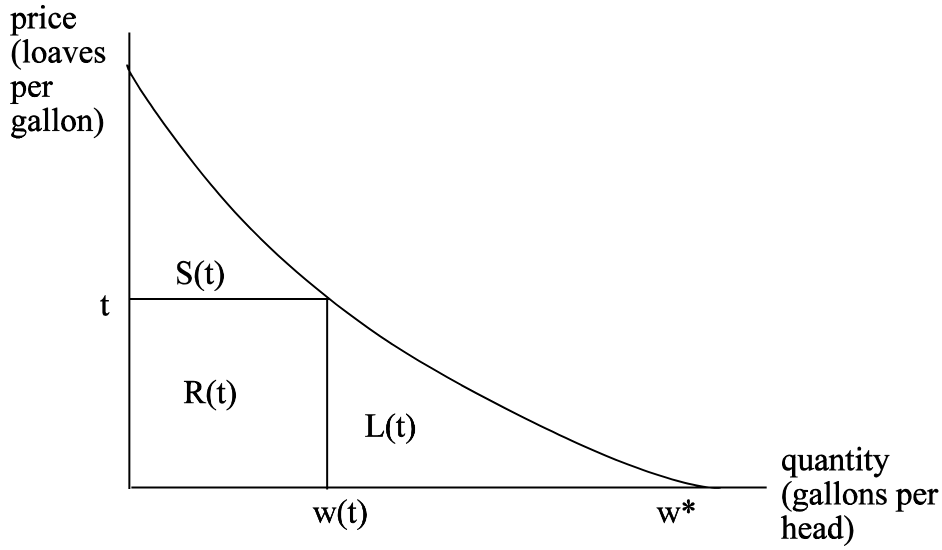
It is often supposed that the possibility of levying lump sum taxation automatically drives the marginal cost of public funds to 1, that a government with the power to levy lump sum taxes undertakes all projects, programs and activities costing less than the sum of the benefits to all citizens. That would be true of a government that could choose each person's lump sum tax or transfer individually and that sought to maximize the value of a social welfare function. It is not true here. The paper closes with a critique of the assumptions, suggesting that the estimate of the marginal cost of public funds is really a lower limit to the true value. The substitution of the median voter for the ubiquitous social welfare function may be of some interest in itself.

A World of Identical Taxpayers

Imagine a society of N identical people who consume only bread and water, where bread is produced and water, unless taxed, is free. Each person produces a fixed amount, b, of bread. Water is available in unlimited quantity and at no alternative cost in terms of bread from a well in the town square. The government hires workers to undertake public projects. There is a shelf of such projects, some more beneficial than others. Government workers are paid in bread which must be acquired by taxation. The "obvious" tax system is a head tax. The government collects a certain number of loaves from each person, enough that workers in the public sector are as well off as workers in the private sector. When public sector workers are taxed at the same rate as private sector workers, the rate must be sufficient to supply public sector workers with pre-tax incomes of b per head. If M public sector workers are hired, the tax, imposed on public sector workers and private sector workers alike, must be bM/N loaves per head. Everybody would then consume as much or as little water as he pleases, for, by assumption, there is plenty of water for everybody from the town well. People would consume water up to the point where the marginal benefit of a glass of water falls to zero.

The key assumption here - an assumption representative of a great variety of constraints in actual tax systems - is that bread cannot be taxed. The government must rely for public revenue upon a tax on water. Perhaps bread production is dispersed in out of the way places that the tax collector cannot find. Regardless, with loaves of bread as the numeraire, a tax of t loaves per gallon is imposed.

Figure 1: The Demand Curve for Water



Each
person's demand curve for water is shown on figure 1, with quantity in gallons per head on the horizontal axis and price in loaves per gallon on the vertical axis. Since water is costless to produce, the price of water and the tax on water are one and the same. In the absence of taxation, each person consumes w* gallons, at which the marginal valuation of water is zero, and his benefit from the availability of water - his surplus - is the entire area under the demand curve. The full surplus in the absence of taxation is the sum of the areas indicated by S(t), R(t) and L(t). The dimension of all such areas is loaves of bread per head, the product of "gallons per head" and "loaves per gallon". When a tax of t loaves per gallon is imposed, consumption of water falls from w* to w(t), tax revenue is R(t) equal to tw(t), and the taxpayer's surplus from the availability of water is reduced from the entire area under the demand curve to area S(t) above the tax rate. The difference between the original surplus and the sum of the new surplus and the tax revenue is the deadweight loss, or excess burden of taxation, L(t).

Now suppose that, to acquire additional tax revenue, the government increases the tax rate from t to t + Δt, causing a reduction in consumption of water from w(t) to w(t + Δt) as shown in figure 2 which is a straightforward extension of figure 1. Revenue increases from R(t) to
R(t + Δt), where R(t), equal to tw(t), is represented in the figure as the sum of the areas θ and β, and R(t + Δt), equal to (t + Δt)w(t + Δt), is represented in the figure by the sum of the areas θ and α. Ignoring products of first differences, which are very tiny, the change in tax revenue becomes

$$\Delta R = R(t + \Delta t) - R(t) = \text{area } \alpha - \text{area } \beta = w(t)\Delta t - t\Delta w \qquad (2)$$

where Δw is the absolute value of the difference between $w(t)$ and $w(t + \Delta t)$.

It is immediately evident that the full cost of a tax increase to the taxpayer exceeds the amount of extra tax he pays. The extra tax is ΔR, the increase in tax revenue per head. The full cost to the taxpayer can be thought of equally-well as his loss of surplus, ΔS, from the availability of water where (ignoring products of first differences so that we need not distinguish between $w(t)$ and $w(t + \Delta t)$ in the equations below),

$$\Delta S = \text{area } \alpha = w\Delta t \qquad (3)$$

or as the sum of his extra tax, ΔR as defined in equation (2), and his extra deadweight loss, ΔL, where

$$\Delta L = \text{area } \beta = t\,\Delta w \qquad (4)$$

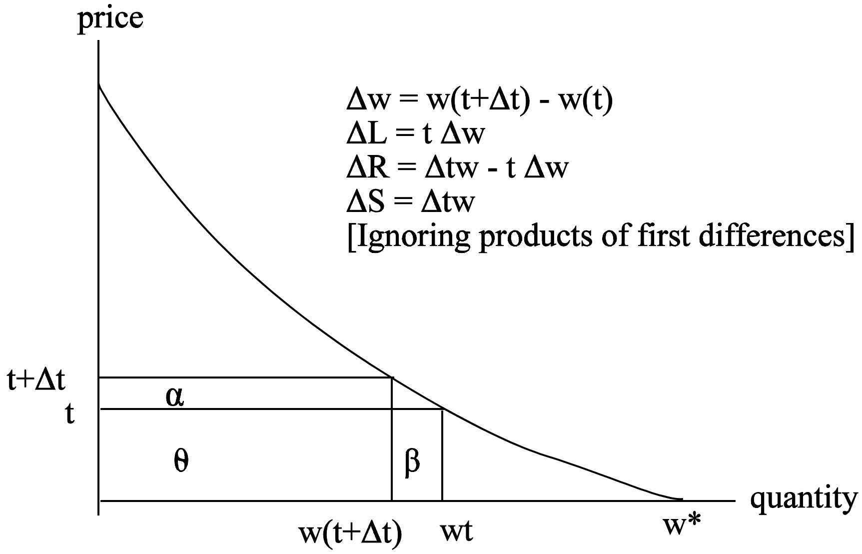


Figure 2: The Effect of a Small Increase in Tax

With an initial tax of t loaves per gallon, the marginal cost of public funds (mcpf) - the full cost to the taxpayer per loaf of bread acquired in taxation - becomes

$$\frac{\Delta R + \Delta L}{\Delta R} = \frac{(w\Delta t - t\Delta w) + t\Delta w}{w\Delta t - t\Delta w} = \frac{1}{1 - \dfrac{t\Delta w}{w\Delta t}} = \frac{1}{1-\eta} \qquad (5)$$

where η, representing the expression (t Δw)/(w Δt), can equally-well be thought of as the absolute value of the elasticity of demand for water or the absolute value of the elasticity of tax base to tax rate. In this context, and in this context only, the two are exactly the same. It is easily shown that if water is not free, and can only be produced at an alternative cost of p loaves per gallon, then the elasticity of tax rate to tax base, η, is connected to the elasticity of demand, ε, and the tax rate, τ, by the formula

$$\eta = \varepsilon\tau \qquad (6)$$

where τ is the tax on water, t/(p + t). expressed a proportion of the demand price.

The simultaneous choice of the tax rate on water and the amount of public expenditure can be represented as the intersection of the demand and supply curves on figure 3. The horizontal axis shows public revenue per head on the understanding that revenue and expenditure are the same. The price on the vertical axis is not an ordinary price. It is the "full cost (or benefit) to the taxpayer per dollar of *additional* public revenue. As such, it is dimension-free rather than expressed with reference to a numeraire such as bread or dollars. The height of the supply curve is the ratio (ΔR + ΔL)/ΔR as defined above. The height of the demand curve is the value per dollar of public expenditure of the government's marginal project, program or activity. When public activity is restricted to the provision of a single type of public good, G, the height of the demand curve is the ratio of the terms

$$\sum_i \left(\frac{\partial u^i}{\partial G} / \frac{\partial u^i}{\partial x} \right) \qquad \text{and} \qquad \frac{P_G}{P_x}$$

which is a diminishing function of the amount of G supplied. When public activity consists of many projects, programs and activities, the entire set of projects can be ordered by the ratio of total benefit to cost (exclusive of the excess burden of taxation). The project with the highest ratio is ranked first, the project with the next highest ratio is ranked second, and so on. Then, for any given total expenditure, the government finances projects as far down the list as the available tax revenue allows. At each total expenditure, the height of the demand curve would be the ratio of benefit to cost of the marginal project on the understanding that all projects with higher benefit-cost ratios are undertaken and all projects with lower benefit cost ratios are not.

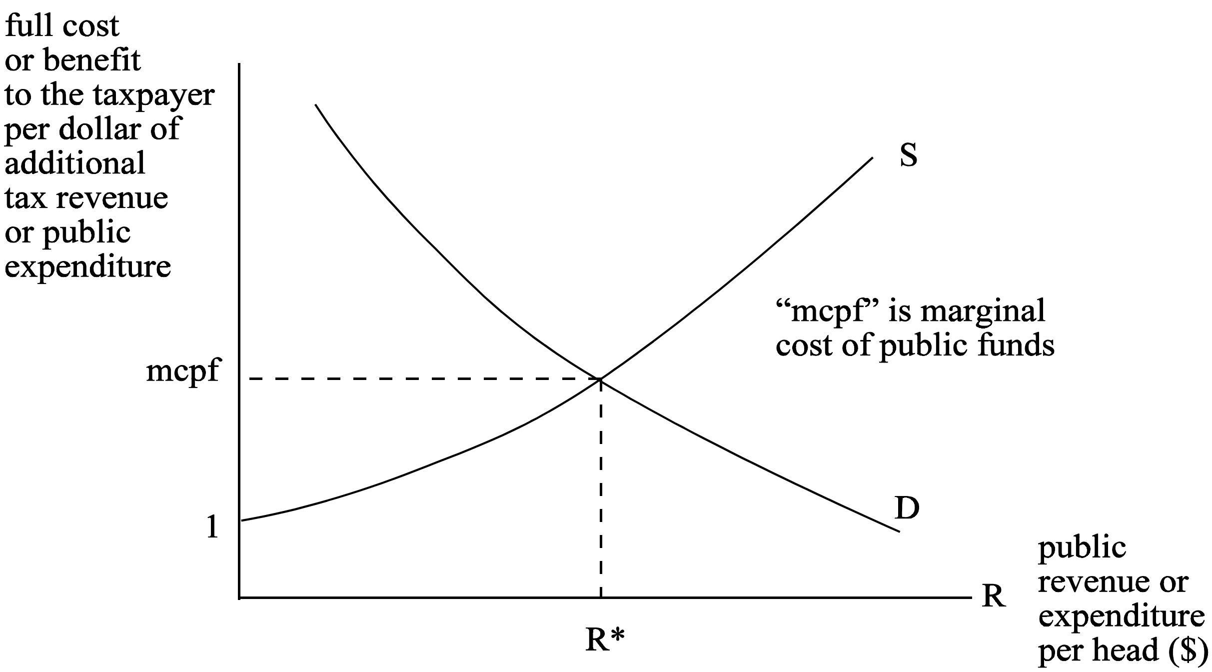


Figure 3: The Derivation of the Marginal Cost of Public Funds

The marginal cost of public funds is defined as the equilibrium “price” of public revenue or expenditure. For our bread and water economy, it is represented in figure 3 as the height of the intersection of the demand and supply curves for public funds. The height of the demand curve for public funds is the left hand side of equation (7). The height of the supply curve is the right hand side of equation (7). The marginal cost of public funds is their common value. The equation itself is a generalization of the Samuelson rule in equation (1). The generalization in equation (7) is only partial, allowing for the impact on the demand for the taxed good of extra tax but not of extra public expenditure.

$$\left\{ \sum_i \left(\frac{\partial u^i}{\partial G} / \frac{\partial u^i}{\partial x} \right) / \left(\frac{P_G}{P_x} \right) \right\} = \frac{1}{1-\eta} \qquad (7)$$

Several observations about this special demand and supply diagram:

1) Equation (7) above (other such equations taking account of various complexities of the tax system) is usually derived by what might be called the double maximization procedure. First, for any given t and G (where t and G might be vectors of taxes on the different private goods and amounts of the different public goods), the taxpayer chooses x (a vector of goods) and h (hours of work) so that x and h become functions of t and G. Then, constrained by these functions, the government chooses t and G to maximize the value of some social welfare. For an economy of identical people the social welfare function and the taxpayer's own utility function are one and the same. Otherwise, the social welfare function is some weighted combination of people's utility functions. The *locus classicus* of this approach is Atkinson and Stern (1972).

2) The horizontal axis can equally-well be represented as total public revenue or as public revenue per person. The resulting marginal cost of public funds is the same. The choice between "total" and "per head" is a matter of convenience.

3) The story can be extended from the bread and water economy to an economy with many goods and many types of taxes, but the transformed supply curve must be interpreted carefully. By analogy with the construction of an economy-wide supply curve from the supply curves of many firms, one might be that supply curves of different taxes could be added horizontally. That is entirely wrong. Consider a person with a fixed income that he spends on bread and cheese. When cheese alone is taxed there is an upward-sloping supply curve of public revenue like that shown in figure 3. When bread alone is taxed there is also an upward-sloping supply curve of public revenue like that shown in figure 3. When bread and cheese are both taxed, the supply curve becomes flat at a height of 1 because there is no longer any excess burden of taxation.

In general, the supply curve is defined for some constraint on the tax system determining how each and every tax might be increased when more public revenue is required. In our bread-and-water example, the constraint was that water alone could be taxed. In other circumstances, it may be that income can be taxed but leisure cannot. Still other constraints will be discussed below. Whatever the constraints, the height of the supply curve must be interpreted as the corresponding to the mix of taxes yielding the lowest value of $(\Delta R + \Delta L)\Delta L$ for the last dollar of public revenue attained.

4) There are many sources of deadweight loss in the tax system. In the bread-and-water example the source of deadweight loss was that water could be taxed but bread could not. Additional sources are that, for whatever reason, some goods are more heavily taxed than others, that goods can be taxed but leisure cannot, that future consumption is taxed more heavily than present consumption (in the so-called double taxation of saving), that every increase in tax rates induces people to evade more tax legally and illegally, imposing an extra cost of supervision upon the public sector. In principle, all sources of deadweight loss should be recognized and cumulated in computing the marginal cost of public funds. This point is worth emphasizing because much of the literature on the marginal cost of public funds is based, implicitly or explicitly, on the assumption that the labour leisure choice is the only relevant consideration.

Useful as it is in explaining the nature of the marginal cost of public funds, this assumption can be seriously misleading as a basis for computing the marginal cost of public funds at any given time or place.

5) In the bread and water example, the supply curve of public funds begins at a height of 1 on the vertical axis because, as is evident from figures 1 and 2, the ratio of ΔL to ΔR is infinitesimal when public expenditure is very small. That the marginal cost of public funds is 1 for the first dollar of public expenditure is immediately evident from the juxtaposition of equations (6) and (7). When the tax rate, τ, is minute, the elasticity of base to rate, η, must be minute as well, and equation (7) reduces to the original Samuelson rule in equation (1). However the original Samuelson goes considerably further, implying that the supply curve of public funds is flat at a height of 1 above the horizontal axis regardless of the magnitude of public expenditure, for a "communist" government can acquire public revenue without imposing any deadweight loss at all.

There are other possibilities. Though never downward-sloping, the supply curve of public funds can begin below 1 - or even below 0 - in some circumstances. Returning to our bread and water example, continue to suppose that each person can take as much water from the well as he pleases at no cost to himself, but add the assumption that water emerging from the well creates a miasma - analogous to gasoline fumes from an automobile - that is unpleasant and costly to the community as a whole. This is illustrated in figure 4, an extension of figures 1 and 2. The demand curve for water is carried over unchanged from the earlier figures. Total harm to the rest of the community from any person's appropriation of water is x per gallon, an externality of which the person is assumed to take no account in determining his own consumption of water. There is an initial tax on water is t loaves per gallon, where t may be less than or greater than x, but which is shown on figure 4 as less than x.

Suppose the tax is increased from t to t + Δt, causing a reduction, Δw in the consumption of water, and suppose Δt is small enough that the two little triangles between area β and area δ in the figure can be ignored. As before, the additional revenue is ΔR = area α - area β = w Δt - t Δw and the corresponding addition to the deadweight loss is ΔL = area β = t Δw where ΔL must now be interpreted as the value of the loss to the taxpayer himself from the reduction in his consumption of water in response to the increase in the tax. The new ingredient is ΔE, the gain to the rest of the community from the lessening of the miasma as a consequence of this person's reduction in consumption of water. It is

$$\Delta E = x\Delta w = \text{area } \beta + \text{area } \delta \qquad (8)$$

The height of the supply curve of public funds becomes

$$\frac{\Delta R + \Delta L - \Delta E}{\Delta R} = \frac{(w\Delta t - t\Delta w) - x\Delta w}{w\Delta t - t\Delta w} \qquad (9)$$

which is greater than 1 if t > x, equal to 1 if t = x, and less than 1 if t < x as is supposed in the construction of figure 4. If x were large enough and t were small enough and the demand curve were steep enough, the height of the supply curve for public funds could fall below 0 at its intersection with the vertical axis.

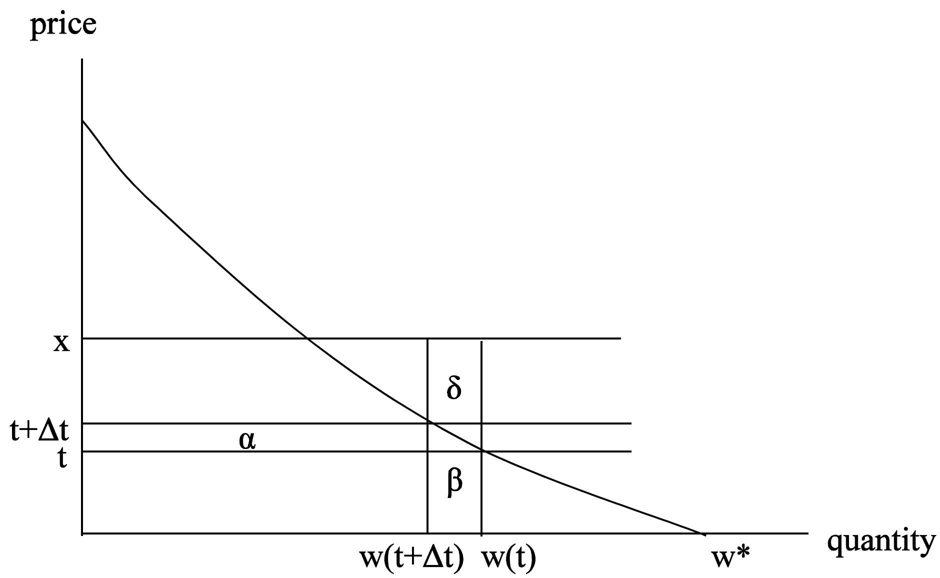


Figure 4: The Double Dividend

Corrective taxation provides what is commonly called a "double dividend ", reducing the output of the externality-bearing good while, at the same time, lowering the supply curve of public funds. Corrective taxation - such as taxation of gasoline to reduce pollution from automobile fumes - comes first on the list of taxes ordered by the additional cost to the taxpayer per dollar of additional tax revenue in equation (9) because the marginal deadweight loss - represented in figure 4 by the difference between ΔL and ΔE - is small or even negative. Comparing public finance with or without corrective taxation, the supply curve of public funds is lower when corrective taxation is imposed, the ultimate marginal cost of public funds is lower too, total public expenditure is correspondingly larger, and the full gain from corrective taxation exceeds the gain to society from the reduction in the externality itself. The first dividend is the reduction in the output of the externality-bearing good. The second dividend is a consequence of the reduction in the marginal cost of public funds; it is the value of the extra public expenditure weighted by the gap between the demand curve and the supply curve of public funds (as the supply curve would be without the corrective taxation). Nevertheless, though the supply curve of public funds may begin well below 1, it would normally rise well above 1 at the point of intersection with the demand curve because the list of desirable public sector activities is long enough to warrant ordinary income taxation and excise taxation for which the excess burden of taxation is considerable.

6) There is a disconnect between the two key assumptions in the derivation of the marginal cost of public funds in figure 3. On the one hand, it was supposed that people are identical. On the other hand it was supposed that water could be taxed, but not bread. Both assumptions can be defended as useful simplifications pointing to phenomena that might otherwise be obscured in a maze of detail. Yet the assumptions fit poorly together because, if people really were identical, it is hard to see why the government could not levy a universal lump sum tax of bM/N loaves of bread, eliminating all excess burden of taxation and leaving people free to consume as much or as little water as they please. The value of ΔL in equation (5) would be 0, the supply curve of public revenue in figure 3 would be flat, regardless of how much of the national income is absorbed by the public sector, and the simple Samuelson rule in equation (1) would be the appropriate indicator of the utility-maximizing provision of the public good.

The juxtaposition of these assumptions can be defended by the argument that diversity among taxpayers renders lump sum taxation infeasible without reversing the implications of the simple model illustrated in figure 3, but such a defense is less persuasive than it might be if grounded in a model of what happens when taxpayers differ.

A World of Diverse Taxpayers

The picture of the formation of the marginal cost of public funds in figure 3 becomes somewhat blurry and incoherent when taxpayers differ. Projects convey different benefits on different people. People differ in their elasticities of tax base to tax rate. People prefer different amounts of public expenditure. Diversity changes the picture in another respect as well. When some people are rich and others are poor, the government as the representative of voters may redistribute income from rich to poor. A government able to levy lump sum taxes may choose not to do so, or may levy lump sum transfers instead. Redistribution may be arranged by means of a a demogrant, or lump sum subsidy, pushing the economy farther along the supply curve of public funds than if people were identical.

The precision of figure 3 can be retained by replacing the sweeping assumption that all taxpayers are alike in every possible respect with three strong but less comprehensive assumptions.

Assumption 1: The allocation among tax payers of the full cost of taxation - the sum of the tax actually paid and the excess burden of taxation - is proportional to pre-tax incomes.

Assumption 2: The allocation among tax payers of the benefits of all public projects, programs and activities is also in proportion to pre-tax incomes.

Assumption 3: Taxation and public expenditure are arranged in accordance with the preferences of the median voter.

Assumption (1) is that a project with total benefits of \$B in an economy of N people would yield an average benefit per person of \$(B/N), but its benefit to person i with income y_i would be \$$(y_i/y_{av})$(B/N) where y_{av} is the average income per person. Similarly, assumption (2) is that a project with a total cost of \$C would impose an average cost per person of \$(C/N), but its cost to person i would be \$$(y_i/y_{av})$(B/N). This is obviously so for proportional income taxation. Assume this allocation holds for excess burden as well. Wealthier people are assumed to place higher values on the benefits of public undertakings, but to bear proportionally more of the tax and of the associated deadweight loss. Consider an intra-marginal project yielding an average benefit of \$10 per person, costing \$4 per person to the government, but costing an average of \$6 per person to the taxpayer because the marginal cost of public funds is 1.5. Such a project must convey a benefit of \$10 and a cost of \$6 to the taxpayer whose pre-tax income is equal to the average pre-tax income in the economy as a whole, but our assumptions require it to yield a benefit of \$20 and to impose a cost of \$12 upon a taxpayer whose income is twice the average, and to yield a benefit of \$5 and to impose a cost of \$3 upon a taxpayer whose income is half the average.

Together, assumptions (1) and (2) eliminate all conflict of interest in the choice of public projects, programs and activities, but, in doing so, require a reinterpretation of the scale on the vertical axis of figure 3. Now, the height of the demand curve is not just the benefit per dollar of additional public expenditure. It is the benefit per dollar of additional public expenditure on the understanding that benefit is allocated among people in proportion to their incomes. Now, the height of the supply curve is not just the full cost per dollar of additional public expenditure. It is the full cost per dollar of additional public expenditure on the understanding that cost is allocated among people in proportion to their incomes.

Assumptions (1) and (2) rule out all lump sum taxation or subsidies, but these can be introduced through a distinction between i) "ordinary" taxation and public expenditure for which equations (1) and (2) hold, and ii) a lump sum tax or demorgant (one or the other, but never both for they would cancel out). The distinction is that ordinary tax and expenditure convey costs and benefits proportional to pre-tax income, while the lump sum tax or demogrant convey costs or benefits that are exactly the same for everybody.

Consider the median voter's preferences over ordinary public expenditure, ordinary taxation and the lump sum tax or transfer as the case may be. To be content with the existing mix of taxes and transfers, median voter must be indifferent to all small feasible changes among them. If there is a demogrant, he must be indifferent to a one dollar increase in the demogrant coupled with a one dollar increase in ordinary taxation or a one dollar decrease in ordinary expenditure. If there is a lump sum tax, he must be indifferent to a one dollar increase in the lump sum tax coupled with a one dollar increase in ordinary public expenditure or a one dollar decrease in ordinary taxation.

Suppose the government is currently supplying a demogrant, and consider the impact on the median voter of a one dollar increase (per head) in ordinary public expenditure, financed by a one dollar decrease in the demogrant. The one dollar decrease in the demogrant costs one dollar to the median voter and to everybody else as well. The corresponding benefit from the increase in ordinary public expenditure depends on the marginal cost of public funds. Suppose the marginal cost of public funds is r, meaning that the taxpayer bears a full cost of \$r per dollar of additional public revenue, or, equivalently in view of the crossing of the demand and supply curve of public funds, that an additional dollar of public expenditure yields and additional \$r of benefits. This is true for the average person, but the effect on the median voter depends on the ratio of his income to the average income in the economy as a whole. Specifically, a marginal project with an average benefit of \$r per dollar of expenditure conveys a benefit to the median voter not of \$r, but of ry_{med}/y_{av}, where his income is y_{med} and the average income is y_{av}. To the median voter, the loss of a dollar of demogrant costs \$1, while the gain from an extra dollar of public expenditure provides a benefit of $r\, y_{med}/y_{av}$. The median voter is content with the existing proportion between ordinary expenditure and demogrant if and only if he is indifferent to a small switch from one to the other, that is if

$$r(y_{med}/y_{av}) = 1 \tag{10}$$

or, equivalently, if

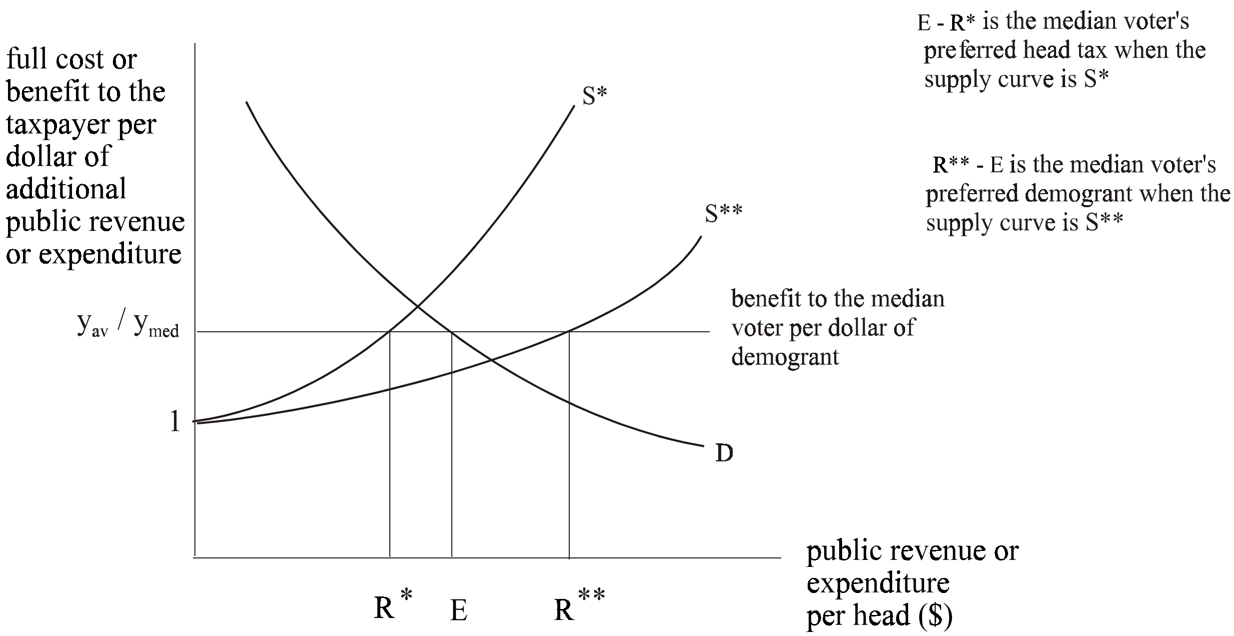
$$r = y_{av}/y_{med} \tag{11}$$

But to say this is to say that the equilibrium marginal cost of public funds is equal to y_{av}/y_{med}, which is precisely the claim in the title of this paper.

This proposition is illustrated with on figure 5, an extension of figure 3 with a the vertical axis reinterpreted in accordance with assumption (1) and (2) as discussed above, with two alternative supply curves of public funds, and with the addition of a horizontal line a distance y_{av}/y_{med} above the horizontal axis. The alternative supply curves of public funds are labelled S* and S**. Both curves start at a height of 1 on the horizontal axis, but the curve S* is relatively steep, while the curve S** is relatively flat. In the absence of lump sum taxes or transfers, everybody would agree that public revenue and expenditure should be as indicated by the intersection of the demand and supply curve of public funds, either the combination D and S* generating a low public expenditure and a high marginal cost of public funds, or the combination D and S** generating a relatively high public expenditure and a relatively low marginal cost of public funds.

The option of lump sum taxes or transfers draws the marginal cost of public funds, up or down as the case may be, to the ratio, y_{av}/y_{med}, of average income to median income. If the supply curve of public funds were S* so that, in the absence of lump sum taxes or transfers, the marginal cost of public funds would be larger than y_{av}/y_{med}, then the median voter would automatically favour the introduction of a lump sum tax, reducing ordinary taxation to R*, increasing total expenditure to E, and reducing the marginal cost of public funds to y^{av}/y^{med}. Similarly, if the supply curve of public funds were S** so that, in the absence of lump sum taxes or transfers, the marginal cost of public funds would be smaller than y_{av}/y_{med}, then the median voter would

Figure 5: Lump Sum Taxation, the Demogrant and the Marginal Cost of Public Funds



automatically favour the introduction of a lump sum subsidy, increasing ordinary taxation to R**, reducing total expenditure to E, and raising the marginal cost of public funds to y_{av}/y_{med}. In either case, the marginal cost of public funds becomes y_{av}/y_{med}, the only value of the marginal cost of public funds for which the median voter is content with the mix of tax and expenditure. The marginal cost of public funds is the same regardless. It settles down at y_{av}/y_{med} - no matter what the shape of the demand and supply curves for public funds - because any higher or lower value of the marginal cost of funds would have induced the median voter to favour a larger lump sum tax or a larger demogrant. Q.E.D.

For the Canada in the year 1998, the ratio of average family income to median family income was 1.202125. If there is any redistribution at all and provided redistribution is arranged by demogrant, then the marginal cost of public funds cannot be other than that. Under no circumstances can the marginal cost of public funds be less than 1 because, were that so, the median voter would have every incentive to increase the demogrant more and more until the entire national income has been distributed. It remains to reconsider the assumptions from which this proposition was established.

Comments on the Assumptions

The Scope for Lump Sum Taxation: The explanation of how lump sum taxes or transfers constrain the marginal cost of public funds depends critically on the assumption that each and every person, without exception, can be subsidized or taxed alike. Drop that assumption and our clean result begins to disintegrate. There is an asymmetry here between lump sum transfers and lump sum taxation. The government could easily supply equal subsidies per person, but some people may be deemed too poor to pay a head tax. Their pre-tax incomes may be less than the head tax or society may not tolerate the publicly-induced reduction in the after-tax incomes of the very poor. This constraint may raise the marginal cost of public funds considerably. For example, if 25% of the population were deemed too poor to pay the lump sum tax, then the cost to the median voter per dollar of tax revenue (per person) acquired by lump sum taxation must rise from $1 (which is what the median voter actually pays) to $1.33 (equal to 1/(.75) which is what the median voter must pay to increase the revenue from the lump sum tax by $1 per person in the population as a whole). Furthermore, since the median voter pays only 83¢ (1/1.2) per dollar of ordinary tax revenue, the full cost per dollar of additional public revenue acquired by ordinary taxation must rise to $1.60 (1.33 x 1.2) before it becomes in the median voter's interest to raise additional revenue by a head tax instead. The exemption of 25% of the population raises the marginal cost of public funds from 1.2 to 1.6. Generalizing, it may be said that the median voter is only content with the mix of ordinary taxation and lump sum taxation when the marginal cost of public funds has risen not to y^{av}/y^{med}, but to $(y^{av}/y^{med})/s$ where s is the proportion of the population on whom the head tax is levied.

That is not the end of the difficulty. An exemption from the head tax must be based on some minimum income. Suppose the minimum income is $20,000 and the head tax is $2,000 per person. In these circumstances, anybody who, in the absence of the head tax, would have a taxable income of between $20,000 and $22,000 is provided by the tax with an incentive to reduce his declared taxable income - working less or evading more - below the critical threshold at which the exemption takes effect. Even people whose incomes are somewhat higher than $22,000 might take steps to drive taxable income below $20,000 to avoid the head tax. This contraction in taxable income is a kind of deadweight loss that must be taken into account when assessing the effects of lump sum taxation, and the marginal cost of public funds must rise accordingly. The flat line in figure 5 begins to curve upward soon after the transition from lump sum transfer to lump sum tax. This consideration would normally rule out the lump sum tax as a practical proposition. Societies typically employ lump sum subsidies rather than lump sum taxes, but such subsidies are targeted rather than universal: welfare for the poor, unemployment insurance and the old age pension. The simple story in figure 3 may be essentially right.

Nor is it really true that the head tax and demogrant are entirely distortion-free. They would be distortion-free in the ideal economy - postulated in economic models to procure clear and straightforward lessons for public finance - of ageless hermaphrodites who all work, who are never born, who never die, and whose presence can be detected flawlessly by the tax collector. A moment's thought about how a head tax or demogrant would be administered should be enough to convince one that these instruments are not completely distortion-free. Would the head tax or demogrant be imposed per person (every man, woman and child), per adult, per family or per worker? Each mode of administration invokes distortions, individually-advantageous but socially-disadvantageous behaviour with costs comparable to the cost of the tax on water. A strict head tax per person induces people to have smaller families, to conceal their children, or even to abandon them. A tax per worker induces people not to work; it would be the very opposite of the low-wage supplements in force in some countries today. A tax per family induces people to combine in fake families. The larger the lump sum tax or demogrant, the larger the waste in such dodges is likely to be. The point is not that a head tax or demogrant should not in any circumstances be imposed, but that no fiscal instruments are entirely distortion-free and that the distortion increases with the size of the tax or subsidy. This, and other considerations suggest that the ratio of mean to median income is a lower limit to the marginal cost of public funds rather than a rigid number to which the marginal cost of public funds is confined.

The Preferences of the Median Voter: Assumption 3 conceals a significant distinction between the political proposition that the will of the median voter prevails and the economic or psychological proposition that the median voter never looks beyond his immediate short-term interest in the narrowest possible sense. The one does not imply the other. There are several reasons why the median voter might support more redistribution than is in his own, immediate self-interest. He may be altruistic with particular concern for people worse off than himself. He may be risk averse. He may want generous assistance to be available in the event that he or his family become poor. He may fear the wrath of the impoverished masses in the event that their standard of living is allowed to fall too low.

The weightier these considerations in the assessment of the median voter, the more his preferences come to resemble a social welfare function, the higher the "benefit to the median voter per dollar of demogrant" as shown in figure 5, and the larger the marginal cost of public funds.[2] There is no telling, within the confines of his analysis how large the marginal cost of

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[2]The impact of lump sum taxes or transfers is the subject of Agnar Sandmo's "Redistribution and the Marginal Cost of Public Funds", *Journal of Public Economics*, 1998, 365-82. Sandmo derives the marginal cost of public funds by double maximization: First, consumers choose labour in response to given taxes, a given demogrant and a given supply of public goods. Then the government chooses these variables, taking account of the private sector response and the government's budget constraint. In deriving the marginal cost of public funds, Sandmo assumes that a) the government maximizes a utilitarian social welfare function (rather than the welfare of the median voter) and b) all public expenditure is either for the purchase of a Samuelson public good or for the provision of a lump sum tax or transfer. The crossing of the demand and supply curves in figure 1 is replaced by a rather general formula for the equilibrium marginal cost of public funds.

The equilibrium marginal cost of public funds, mcpf, becomes

$$\text{mcpf} = \Sigma m^i/q = \frac{\{1 - [(na + zq)/zq]\varepsilon_{hz}\}\{1 + \partial_{hw}\}}{\{[1 + \partial_m][1 + \varepsilon_{ht}]\}}$$

where m^i is the marginal valuation of public goods by person i, z is the supply of public goods, q public funds may tun out to be.

The Content of Public Expenditure: Assumption 2 is completely false in one sense, but valid as an approximation and quite useful in another. It is false in the sense that public projects, programs and activities do not always allocate benefits in proportion to people's pre-tax incomes. Most projects are directed to sub-groups of the population. One project is beneficial to my city. Another project is beneficial to yours. One project is for people in the North. Another is for people in the South. Higher education is beneficial to the rich. Public housing is beneficial to the poor. The closer you look at public expenditure, the more various does it become. An unfortunate implication of the diversity of the beneficiaries of public projects, programs and activities is the disintegration of the implied ordering of projects in the demand curve for public expenditure in figure 1, for different people's orderings of projects, programs and activities are no longer the same. I rank projects beneficial to me ahead of projects beneficial to you. You do the opposite. There is, in practice, no commonly-accepted ordering of projects as would arise if Assumptions1 and 2 were entirely valid.

On the other hand, benefit-cost analysis is intended to supply rules for all possible projects, programs and activities. These rules are partly political and partly economic. Their political role is to forestall the great scramble for advantage and privilege that would inevitable occur if projects were assessed by the legislature one by one. An omniscient, omnipotent and benevolent planner would maximize the value of a social welfare function, weighing the benefits of all projects according to the incomes of the people to whom those benefits accrue. A

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is the cost of public goods in terms of private goods, n is population, and a is the demogrant per person. The original Samuelson formula was $\Sigma m^i/q = 1$. The formula is less complicated and unfamiliar than it may at first appear. The marginal cost of public funds places a wedge in equilibrium between the sum of the benefits and the cost of an additional dollar's worth of the public good. The epsilons are weighted elasticities: ε_{ht} is the average elasticity over the entire population of hours of labour, h, with respect to the tax rate, t, and ε_{hz} is the average elasticity over the entire population of hours of labour with respect to the supply of the public good. The deltas are covariances over the entire population with respect to the marginal utility of income as derived from the social welfare function: δ_{hw} is the covariance with respect to income, wh, and δ_m is covariance with respect to the marginal valuation of public goods with respect to private goods. Drop the deltas, make everybody alike, and the formula reverts to the original Atkinson and Stern (1974) formula for the marginal cost of public funds.

In a similar vein, Wilson (1991) shows that only distributional considerations can drive the marginal cost of public funds above 1 in the event that partial lump sum taxation is feasible. He goes on to argue that distortionary taxation may actually lead to an increase in the optimal provision of the public good by increasing people's marginal valuations of the public good. I am not persuaded that there may not be a compensating increase in the marginal cost of the public good. Regardless, what Wilson calls distributionary considerations are implicit in the assumptions of this paper.

democratic society must eschew such fine computations. A great virtue of benefit-cost analysis is that, like justice, it is blind. Costs are weighed against benefits, to whomsoever the benefits accrue. A simple rule suppresses conflict among potential recipients, avoiding the scramble for public largess that would otherwise occur. A single rule for all projects is a prescription for harmony in circumstances where the alternative would be divisive and chaotic. The economic role of benefit-cost analysis is to make the typical citizen as well off as possible in the long run, and the "right" measure of the marginal cost of public funds is determined accordingly. Never entirely true, Assumption 2 is appropriate if it points to the right measure of the marginal cost of public funds.

Assumption 2 acquires plausibility from the tendency of the rich to place high values on public goods or public services, even when amounts available to everybody are the same. Benefits of public services can only be measured by what people are prepared to pay for them. Consider two people in a bread and cheese economy, where both people have identical preferences and both consume exactly the same amount of cheese. If one of these two people consumes more bread, then that person must place a higher value on cheese and would be prepared to pay more - to give up more bread per pound of cheese acquired - for an additional amount of cheese. Similarly, if a new project or program supplies everybody with the same additional medical service or with the same access to a new road, then the benefit of the project - in the sense of what one would be willing to pay for it - is greater for the rich than for the poor.

It is customary in the literature of public finance to abstract from the diversity of government expenditure by assuming - an assumption based more on the connotations of words than on the content of public expenditure - that the public sector produces a uniform stuff called "public goods" which appears, along with private consumption, in every person's utility function. The assumption is only an inch away our Assumption 2, for, the greater one's consumption of private goods, the higher one's marginal valuation of access to a given supply of public goods. This extreme assumption can be unfortunate in diverting attention from the variety of public sector activities and from the potential conflict of interest among recipients and would-be recipients of public sector favour. It is less unfortunate in the context of identifying the appropriate marginal cost of public funds to be applied over a wide range of circumstances.

There is a parallel problem with Assumption 1. Some taxes bite hardest on the rich. Other taxes bite hardest on the poor. Design of the tax structure typically involves a trade-off between excess burden and redistributive impact. See Dahlby, (1998). Suppose extra revenue is to be acquired by altering the schedule of a progressive income tax. Raising marginal rates on high incomes places most of the extra burden of taxation upon the rich but at a high marginal cost of public funds, while raising marginal rates on low incomes is in some respects like introducing a small lump sum tax, placing more of the burden upon the poor but at a lower marginal cost of public funds.

The Shadow Price of Public Expenditure: Recognition of the diversity of public expenditure suggests two distinct meanings for the marginal cost of public funds, both representable as the equilibrium ratio $(\Delta R + \Delta L)\Delta R$ in equation (5) above, but different in their interpretations of the marginal excess burden, ΔL. In both interpretations, the marginal excess burden is the harm to consumers in the project-induced shift in purchasing power from more-taxed to less taxed goods. One interpretation focuses exclusively upon the effects of the tax increase to finance the project. The other interpretation focuses as well upon the effects of the appearance of the project itself. If, for example, the excess burden of taxation consists of a tax-induced shift from paid work to leisure, then a project, such as the establishment of a trade schools (Universities are the other way round.) that encourages people to work may be said to have a higher marginal cost of public funds on the first interpretation than on the second. On the first interpretation, the reduction in the excess burden of taxation from the project-induced incentive to work would be counted as an externality of the project, but would not be seen as affecting the supply curve of public funds.[3]

Finally, a word might be said about the argument, to be found in Hyland and Zeckhauser (1979) and in Kaplow (1996) among other places, that the marginal cost of public funds is confined to 1 - that any project ought to be undertaken when benefits exceed cost regardless of the deadweight loss in taxation - because the beneficiaries of any project can afford to bribe the rest of the community to undertake that project in the event that the sum of the benefits is greater than the cost. There are circumstances where that would undoubtedly be so. It would be so for an omniscient communist government that could set personalized lump sum taxes or demogrants. The beneficiaries of each and every project would be taxed in accordance with their benefits and, over and above that, additional grants or taxes would be imposed to attain the "appropriate" distribution of income. Hyland and Zeckhauser imagine a world where projects are horizontally

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[3]Equation (7) is based on the first interpretation. Atkinson and Stern (1974) incorporate the effect of a project on consumers' allocation of purchasing power between more taxed and less taxed goods by reinterpreting the term P_G in equation (7), converting the term from the actual price of the public good to a shadow price that accounts for the project-induced change in the composition of consumption. They are agnostic as to whether the marginal cost of public funds is greater or less than 1. "The correct benefit measure with distortionary taxation may exceed the marginal rates of substitution a) where the public good is complementary with the taxed private good, and b) where a rise in exogenous income would lead to a fall in the net tax paid." (From the conclusion on page 126). On the first interpretation of the marginal cost of public funds, consideration (a) would be switched to the demand side of figure 3. The other consideration, b, would imply that the curve connecting tax rate to tax base in figure 2 might be upward-sloping. While not impossible, this is very unlikely when account is taken of the many sources of deadweight loss in taxation and of the fact that the implicit loss of income in taxation is in part balanced by the additional public expenditure. There is an interesting proof in Wildasin (1984) that the income effect of the tax-induced change in the net wage can be ignored if the elasticity of labour with respect to the supply of the public good is appropriately defined.

equitable in the sense that each project conveys the same benefits not to everybody, but to everybody whose income is the same, so that the cost of any project can be imposed upon the beneficiaries by altering their tax assessments. So far as I can tell, Kaplow gets his result by supposing that a) the only tax-induced distortion is in the labour-leisure choice and b) that each person's supply of labour is dependent not just on his personal income, but on his full income inclusive of his benefit from public goods or from projects, programs or activities of the public sector. I do not think either of these hypotheses is sufficiently realistic to drag the marginal cost of public funds any significant distance toward 1.

Simple rules are required for the administration of an economy based on private ownership of the means of production in a society where no social welfare function is generally recognized as binding on the government and where too much discretion on the part of the government would be tantamount abandonment of people's property rights. The marginal cost of public funds is a parameter in the rules of benefit-cost analysis. The right value of the parameter is whatever serves to make the typical person as well off as possible in the long run. This paper is an assessment of a candidate for that parameter: the ratio of mean to median income. It is argued that this measure is what the median voter would prefer in a simple setting, but that it is a lower limit in more complex societies.

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