FIXED CAPITAL FORMATION IN THE BRITISH ECONOMY, 1956-1965

J.C.R. Rowley
Queen’s University

Department of Economics
Queen’s University
94 University Avenue
Kingston, Ontario, Canada
K7L 3N6

11-1969
FIXED CAPITAL FORMATION IN
THE BRITISH ECONOMY,
1956-1965

J. C. R. Rowley

Discussion Paper No. 10

November 1969.

This research was completed at the London School of Economics and Political Science during the first half of 1968. A first version of this paper was circulated in July 1968 and a modified version was distributed in the following February. J. D. Sargan suggested the topic.
In a recent series of papers, Dale W. Jorgenson and associated authors\(^1\) have combined rationally distributed lag functions with a neo-classical model for desired capital services so that they could investigate the determinants of real net investment in physical assets other than dwellings. This neo-classical model included features of tax practice and the cost of capital and, when combined with a temporally stable lag structure, it provided valuable insight into the two major economic problems of efficacy and timing of instrumental changes in tax practice. Although the model was overtly based on the work of Irving Fisher, the particular formulation was sufficiently distinctive to warrant a separate name and we shall refer to it as the Jorgenson Model.

Few econometric analyses dealing with aspects of the British post-war economy are available. In 1968, a study sponsored by the Brookings Institution\(^2\) was compelled to acknowledge the absence of any econometric investigation of the British system of investment incentives. In following sections, this omission is remedied and some empirical results are tabulated for a group of models applied to data derived for one decade of the post-war British economy.\(^3\) These models include explicitly variables representing three types of capital allowances and a variable representing a composite tax-rate. The latter takes account of Income Tax, Profits Tax and a number of other special levies.

---


3. 'Time-Series for the British Post-War Economy', Discussion Paper No. 9, Institute of Economic Research, Queen's University.
The Jorgenson Model provided a convenient starting point in the process of building these models although its translation to a different economy involved the rejection of his specification for the system of tax allowances chargeable against gross income as defined for tax purposes. An alternative specification was substituted and there are marked differences between the expression for the "user-cost" of capital services derived on the basis of the Jorgenson Model and the corresponding expressions derived for our Alternative Models. All of the models share deficiencies in the final expressions for desired capital services arising from the use of the Cobb Douglas characterization of the production function. These will be indicated in a later section.

Level of Enquiry

In most accounting and financial literature, the level of discussion is usually set at that of the firm. We shall use the ambiguous term 'enterprise', which will not be coincident with firm although the variables considered in the context of the decisions within the enterprise will always be aggregates of those associated with the formulation of investment programmes by individual firms. The enterprise is a convenient fiction and is essentially the whole economy or a substantial sector of it treated as an entity, or perhaps averaged in some sense. The individual firm can pursue many alternative policies in the hope of achieving an ample future stream of net income after payments of tax. It can choose to purchase new financial assets, second-hand assets, research and development, subsidiaries and consumers' loyalty by an extension of either trade credit or brand advertising.

4. These deficiencies can be eliminated by a number of additional "correction" terms. For example, see 'Investment Functions: Which Production Function?', Discussion Paper No. 6, Institute of Economic Research, Queen's University.
Alternatively, it can change its pricing policies or retain its existing stock of fixed assets in service for a larger period.\(^5\) When a firm is sold as a continuing concern, the price will reflect fixed assets, financial assets and 'goodwill' or invisible assets. Consequently a complete theory of the firm should take these policies into account explicitly. Douglas Vickers\(^6\) has attempted to develop a theory of the firm embodying both real capital and money capital but this represents only a partial adjustment although a significant one. Conventionally these other policies are neglected under a principle of abstraction and theory is developed in a framework of a 'typical firm'. We use the term 'enterprise' instead of 'typical firm' in order to avoid the undertones of micro-detail that the conventional terminology retains but which are wholly absent from the theoretical framework used here.

We are interested in explanations of the stream of investment expenditures based on changes in output, relative prices, the cost of capital, and tax practice. The final influence, if significant, will be too complicated for full coverage and the following analyses will only take account of some of the features of corporate taxation. This restriction affects the choice of data so the series derived for capital allowances and the tax rate are biased by their neglect of differences in tax treatment accorded to corporate and unincorporated sectors. The enterprise is treated as if it were a generalized corporate body.

\(^5\) Tibor Barna has suggested that actual expenditures corresponding to some of these policies by British firms in the post-war period were of the same order of magnitude as those for fixed capital formation. 


A major deficiency of this study is the failure to take account of the choice between purchases of new fixed assets, purchases of existing second-hand assets and leasing or renting assets. There was a rapid growth of equipment leasing and hire purchase agreements during the sample period but we shall not discuss the factors that encouraged this growth in any satisfactory detail even though ownership was an important qualification for claimants of some capital allowances. Organizational shifts associated with the desire to take advantage of prevailing tax practice may affect the level of aggregate investment expenditures and the rates of utilization of capital stocks but these shifts will be ignored here. The enterprise is restricted to the purchase of new assets outright. This restriction is less severe as the level of enquiry becomes more aggregative. Published series for gross domestic fixed capital formation attach assets to purchasers so that sectoral allocations of assets have been increasingly biased when interpreted as indicative of new assets in use within sectors. Similarly, sectoral series contain a component for the net change attendant to transactions in second-hand assets between sectors. The distinction between user and owner and the net balance of second-hand transactions become less important as sectors are combined.

'Net Worth'

Maximization of net worth is the criterion of optimality used in our models. The series for the tax-rate depends on a particular specification for the recipients of net worth. We did not distinguish between dividends and retentions and both are included in net worth. This choice implies a behavioural assumption that dividend policies and investment policies are

simultaneously determined even though the former are ignored below. There is little available evidence on interdependence of concomitant policies within British firms so the choice is arbitrary.

For convenience, we shall refer to the quantities discounted to form net worth as the 'elements' of net worth and the element corresponding to a chosen period of time will be labelled 'elemental net worth'. In developing his theoretical model, Jorgenson subtracted tax assessments from net income on current and capital account and considered that quantity as elemental net worth. Major objections to this choice stem from the existence of a lag between income receipts and payments of tax assessed on these earnings. The average length of this lag for British firms in the pre-Corporation Tax period was 18 months and depended on the relative rates of Income Tax and Profits Tax as well as the distribution of earnings over companies' financial years and the distribution of terminal dates of companies' accounts over calendar months. In cases of dispute between a firm and the Inland Revenue authorities a further delay could occur although in some such cases an interest charge might be made.

We might ask whether elemental gross income should be reduced by contemporaneous actual tax payments on previous earnings, by contemporaneous adjustments to tax reserves or by anticipated future assessments on current income in absolute or discounted units. In practice, the availability of data has tended to provide the response to this question. Jorgenson, in contrast to his theoretical discussion, took the first alternative - subtraction from current gross income of contemporaneous payments on previous and unrelated income - so that his tax rate series, in the absence of other influences, was biased downwards with the extent of the bias determined by the rate of growth of gross income. During the post-war period, there have
been marked changes in American tax practice. The Internal Revenue Code of 1954 authorized the use of the declining-balance method of depreciation (at twice the straight-line rate previously used) and the use of the sum-of-the-years-digits method of depreciation. In July 1962, some months before the corresponding change in Britain, the Revenue Procedure 62-21 shortened the periods used for lives of assets in calculations of depreciation allowances. Finally, the investment tax credit was included in the Revenue Act of 1962. Each of these changes would affect the bias attendant on the payment lag.

In the empirical work presented here, an alternative approach was used. This was based on anticipated future assessments on current gross income in current undiscounted prices. Given the practice of discounting future elements of net worth, the use of discounting weights might have been preferable. This was precluded by the difficulties in taking account of the heterogeneity of lags.

The treatment of Tax Reserve Certificates and of Franked Investment Income for tax purposes during the period under survey implies that, in the absence of other influences, this alternative approach leads to an upward bias in the series for the tax rate. The extent of the bias would be determined by behaviour of firms with respect to their liquid assets and by the level of aggregation used for analyses. During the sample period, Tax Reserve Certificates were available and bore interest in the range 2½% to 3¾% p.a., free of both Income Tax and Profits Tax (provided they were used for payment of tax). The interest may be considered a minimum and sure post-tax return on liquid funds available to firms. Of the tax paid for 1960/61, 18% of Income Tax (other than PAYE and other deductions at source) and 18% of Profits Tax and the Excess Profits Levy was provided by
encashment of these certificates. A sample, collected by the Bank of England in January 1961, revealed a mode for the life of the certificates of 12 months and a mean of 14 months. These figures may provide an underestimate of the average levels for the whole of the period since changes in the accruing rate of interest during 1960 led to a high level of surrenders (paid without interest) and offsetting purchases of new certificates bearing a higher rate of interest.

The bias induced by the existence of Franked Investment Income falls as the level of aggregation increases and may be ignored at the level chosen for this study. The enterprise is treated as a group of companies taken as a whole or as a company without subsidiaries or investments in financial assets.

Although the payments lag will be reduced during the currency of the Corporation Tax, the complexity of transitional provisions may have induced a contrary shift for individual firms during the period of transition. There is insufficient evidence to provide an adequate basis for the specification of a particular pattern of total response during this period. When a reasonably stable pattern of payments is re-established, the problem of the payments lag will be reduced but not eliminated. R.S. Sayers has suggested that the potential destabilising effect of the lag during the currency of the Corporation Tax could be reduced by the introduction of a system of interim payments.

A major feature of the expression chosen to represent elemental net worth is the emphasis that the profitability aspects of the depreciation

allowances receive. Since four-fifths of tax payments did not involve the encashment of Tax Reserve Certificates, liquidity aspects of these allowances and other tax provisions may have been significant. After the publication of the Final Report of the Royal Commission on the Taxation of Profits and Income in 1955, considerable attention was devoted in the economic literature to the liquidity aspects of capital allowances. Alan Williams\textsuperscript{10} has provided a summary of the debate and we shall not provide any further discussion of it. The use of available aggregate time-series precludes an adequate treatment of liquidity and the only element in the adopted model that can be associated with this discussion is the adherence to original cost in the specification of annual, or wear and tear, allowances wherever possible. The relevance of this element is greatly reduced by the difficulties of introducing annual allowances into our mathematical framework, especially those due to the identification of these allowances with capital consumption.

Cost of Capital

The 'cost of capital' is the rate at which the elements of net worth are discounted by the enterprise. When this rate is specified a priori, several difficulties arise. For example, suppose the cost of capital is assumed to be the rate which the enterprise earns on the funds it lends, then two problems arise. First, we must acknowledge that the firm has financial assets as well as fixed assets and that internal funds are reinvested and earn an additional return so that the expression for elemental net worth has to be revised. If this revision is intended to take account of non-subsidiary investments, a severe valuation problem emerges. In the context of their investigation into the choice of accounting rules by

American companies with non-subsidiary investments, Dopuch and Drake\(^{11}\) concluded that "... it is difficult, if not virtually impossible, to obtain the market values of most assets held by a firm for any period of time worthy of analysis". If we modified elemental net income to take account of these investments, the tax expression would become very complicated as the tax treatment of Capital Gains and of Franked Investment Income would have to be taken into account. Deficiencies in the treatment of the substantial non-subsidiary financial investments undertaken by firms would tend to invalidate all results including those for real investments in fixed assets.

Second, there is the familiar problem arising from the multiplicity of rates varying with risk and withdrawal restrictions. To eliminate this problem we need to assess the risk associated by the purchaser to the income stream expected from the output produced with the services of the purchased asset. Jorgenson used the long-term bond rate as the cost of capital for his model in his original exposition\(^{12}\), an approach characterized as the "side-step approach" by Modigliani and Miller\(^{13}\). We considered a number of rates as approximations for the cost of capital. Some of these rates were published series and others were constants. A comparison of the rates will be provided elsewhere and only the results based on a specification that the dividend yield on industrial ordinary shares is an appropriate choice for the cost of capital will be tabulated here. Presence of measurement

\(^{11}\) N. Dopuch and D.F. Drake, 'Empirical Research in Accounting, Selected Studies, 1966'.


error in the estimate of the depreciation parameter preclude the direct estimation of an optimal fixed cost of capital by non-linear methods.

**Time Span**

A panel of data is available for the period extending from the first quarter of 1956 to the present\textsuperscript{14} and this may be extended by splicing to cover the earlier post-war period. Only a truncated sample was used for this study and this extended, for the dependent variable, from the third quarter of 1958 to the final quarter of 1965 inclusive. This truncation conflicts with conventional procedures but is in the spirit of the Principle of Selective Estimation proposed by Franklin M. Fisher\textsuperscript{15}: "Far from it being incumbent upon us to accept all observations sent us by Nature we must be highly selective about our observations if we want our results to have any economic meaning at all. In any given problem, we must use all the a priori information at our command to select from the experiments performed for us by Nature those which can reasonably be considered controlled". Factors that led to truncation are listed below. Terminal truncation is explained first.

During the currency of the system of Corporation Tax, all profits will be chargeable to Corporation Tax and, in addition, the distributed portion will be chargeable to Income Tax at the standard rate under Schedule F. Since the expression for elemental net worth does not distinguish between distributions and retentions, the estimated tax rate must take account of the additional tax on distributed profits. To extend the series for the

\textsuperscript{14} 'Economic Trends', October 1967 and subsequent issues.

tax rate into the recent period would require data on actual partitions of corporate net income. In the absence of such data due to delays in collection or other factors, two approaches can be taken. First, the time-span of enquiry can be curtailed until additional data becomes available. This approach was adopted in this study. Second, the expression for elemental net worth can be revised and net income partitioned into distributions and retentions. Then the calculation of a time series for the tax rate during the pre-Corporation Tax era requires data on actual partitions of corporate income during that period. The enterprise would be treated as an entity having existence independent of its shareholders. The problem of franked investment income would be introduced again since the distribution of dividends among recipients would need to be known so that franked investment income can be distinguished from funds accruing to recipients outside the corporate sector. Choice between the alternative approaches is arbitrary since we have insufficient evidence indicating how firms considered the partition of their anticipated income in decision-making.

One aspect of the transition to a new tax system has not received adequate discussion in economic literature - the possibility of a change in accounting practice and the reduction in departmental separation of responsibility due to a short-interval sequence of new and major tax innovations. For example, the introduction of the Corporation Tax, the Selective Employment Tax and the system of Investment Grants in a relatively short period of time may have resulted in a revision of the decision procedures adopted by British firms and in a closer integration of accounting and planning departments. If this revision is sufficiently marked to affect the stream of investment expenditures significantly, a major problem faces all econometric work based on a sample spanning both the pre-Corporation
Tax period and the subsequent period. This problem is independent of those arising from the complexity of transitional tax provisions. One justification of structural estimation is the possibility of reconciling changes with an earlier structure 16 but there would appear to be little hope of such a reconciliation in the assumed situation until a considerable period of time has elapsed and relevant information on business practice in both parts of the sample has been sought and analysed. Truncation of the sample provides a simple temporary expedient although it cannot be accepted as a solution to the problem of behavioural changes.

The Corporation Tax can be introduced in the models developed below without any change in the mathematical expressions since the adjustment would only require an extension of the series for the tax rate. Similarly, the inclusion of Investment Grants would lead to no change in the expressions if the rate for Investment Allowances is reinterpreted as the rate of either these allowances or the grants. Unfortunately these adjustment would hide two major features of the new tax system and the transition to it. Transitional provisions have reduced the speed of adjustment but the change in the payments lag, corresponding to the introduction of Corporation Tax, must affect corporate liquidity. Any influence of this change on the stream of investment expenditures will not be represented in our models and, again, two approaches may be adopted in response to the omission. We can truncate the sample to exclude this unrepresented shift or we can derive a general model containing our present models as special cases and additional elements corresponding to liquidity and other latent factors. The second approach is difficult to implement and deficiencies in the original models may be augmented by a host of additional errors introduced by the enlargement.

The second hidden feature of the new system is the increased selectivity of the grants system. The British government uses this single instrument in an attempt to achieve two goals. It seeks to encourage fixed capital formation and to provide a more even spatial distribution of economic activity. Regional differences in the provision of grants invalidate certain methods that we have used for the earlier period. Derivation of an aggregate rate of grant from individual rates requires data representing a two-fold classification of investment expenditures - by type of asset and by region where used. Effectively, the number of asset-types has been doubled by the introduction of regional differentials into applicable rates without any increase in published information. A prerequisite of any investigation in this field after recent changes must be the collection of regional statistics of expenditures by asset-type. Until such data are available the validity of any aggregate series for Investment Grants is severely restricted.

There were two more influences on the decision to terminate the sample. Both involve considerations outside the period of transition and are less important than those discussed above. First, in the construction of the models, we assume that replacement investment and capital consumption are proportional to capital stock. This assumption is a reasonably accurate representation for the available series (based on a gross concept of capital stock) provided the period spanned by the study is short. Stability of the proportionality factor, or 'depreciation parameter', might be interpreted as a weak constraint on the length of the sample but the measurement errors associated with the parameter and the order of estimated changes in it imply that little attention need be given to the constraint. Second, the models emphasize profitability aspects of depreciation allowances and they do not represent many qualitative
influences. At certain times during the post-war period such influences might have had a significant role in the determination of the stream of investment expenditures. In particular, we can distinguish a 'novelty' effect occurring when policy changes take the form of an introduction of measures that have not previously been used in this economy. Obviously the effect is absent when policy changes take the form of adjustments to familiar instruments. Two examples of this effect might be provided by the introduction of Investment Allowances in 1954 and the announcement of the Corporation Tax in 1964. The former case had the additional influence associated with removal of direct controls and the general dominance of a theme of expansion in government announcements. J.C.R. Dow\textsuperscript{17} has suggested that "the effects of the 1954 investment allowances may have been considerable - not only because they give a subsidy on investment of about 10\% to any firm earning profits but because they symbolized the ethos of an expanding economy". Similarly, in his study of small - and medium-sized firms, A. Mackintosh\textsuperscript{18} points out the atmosphere-creating effects of the newly introduced allowances. The second example might be dated by the speech of the Chancellor of the Exchequer. This established the government's intention to reverse the past trend of official advice and to introduce a clear distinction between individual and corporate taxation. The immediate response to the speech may have been augmented in subsequent months by the uncertainty induced by a delay in the revision of depreciation treatment to accompany the shift in tax practice. It would be foolish to exaggerate the significance of novelty effects but they may have been important on

\textsuperscript{17} J.C.R. Dow, 'The Management of the British Economy 1945-60', Cambridge University Press, 1964. Chapters III, VI and VII.

the two cited occasions and are not represented in the expressions that we use. A minor check was introduced in this study. Separate regression equations were estimated for net investment over two samples, one containing data for 1965 and one excluding it.

An attempt to explain fixed capital formation in terms of expected contribution to net worth of the enterprise's purchases is primarily concerned with demand factors in the market for investment goods. One result of this approach is the interpretation of the estimated equations as those describing investment demand. Such an interpretation precludes the use of data that corresponds to a period when conditions of supply were major influences in this market. The initial truncation of the sample was determined by two considerations - the decline in reliability of published statistics and the intrusion of supply factors as the sample is extended into the early post-war period.

In his study of the effects of direct controls in post-war Britain, Dow distinguished five major instruments of control - restrictions on investment, consumer rationing, import controls, allocation of materials, and price controls - each of which may have a significant effect on the stream of investment expenditures. His survey contains a detailed chronological account of variations in controls and will not be reproduced here. Similarly, we shall discuss the controversy associated with the investment-export nexus only incidentally as Peter Kenen provides a description of this controversy.

We can distinguish two influences of controls on investment decisions. They may affect purchases directly by supply restrictions and their removal may induce a response, similar to a novelty effect, in excess of that

---

predictable on grounds of elimination of restrictions directly. Although we assume 'rational' actions undertaken by the decision-makers in economic models, it is important to remember that actual behaviour in extra-ordinary situations may be difficult to integrate into the framework of such schemes and that we are attempting to study actual behaviour rather than hypothetical behaviour albeit in the theoretical framework. Both influences are significant since neither is adequately represented in the models that we use in this study. The indirect effect has received unanimous support from economists who have described the first post-war decade in Britain and is characterized by the 'ethos of an expanding economy' cited by Dow above. We shall concentrate on direct influences of direct controls.

Apart from the survey of Dow, the Treasury 'Bulletin for Industry' provides clear indications of the evolution of government policy and of supply restrictions in the first post-war decade. For example, it reported that the government had direct control over 40% of the investment field in July 1950 and building controls provided the major instrument exercised over the remainder. In September 1952, it reported steel shortages. Finally, in May 1954 it acknowledged that investment in the previous five years had been severely curtailed by steel shortages, the defence programme and the need to increase engineering exports although these restrictions were not operative continuously. R. Nurkse\textsuperscript{20} suggests that the British government tried to curtail fixed capital formation in each post-war payments crisis prior to the appearance of his paper: "Whenever something went wrong with the balance of payments, domestic investment was hit on the head". Kenen demonstrates that this hypothesis cannot be accepted for the period

1948-1950 but may be a reasonably accurate description of behaviour in the subsequent period. If the sole restrictions were in the provisions of capital allowances or in tax rate changes, our models would be adequate. Similarly the inclusion of output as an argument for desired capital stock might take account of the consumers' goods boom attendant with the withdrawal of direct controls but the steel and sulphur shortages would not be represented in our models. Nor would these models represent the shifts in corporate liquidity experienced in the post-Korean War period. Meyer and Kuh have indicated concomitant influences in the American economy and suggest that a substantial period of adjustment followed. Although there were marked differences in characteristics between the two economies, no reasonable case can be presented for the proposition that dislocations in the British economy were less in extent or in duration. Truncation of the sample to avoid taking account of the deficiencies of the models in this situation appears a reasonable approach to take. Although the presence of controls does not necessarily indicate that they were effective, available evidence for the first post-war decade suggests that influences extraneous to our models were effective during certain periods of crises.

Acceptance of the principle of selective estimation, as we interpret it, indicates a need for all evidence to be spatially and temporally fixed. In particular, evidence from other economies and from the British economy in earlier periods, e.g. the evidence presented by the Oxford Economists' Research Group in 1938-1939, is not necessarily relevant to our investigation even when free from collection and interpretive dispute. Behaviour is adaptive and may be variable over time and culture so that specification of an adequate representation of behaviour is extremely difficult. While recognizing the optimality of a stable relation taking account explicitly
of changes in behaviour, the suggestion by Zvi Griliches\textsuperscript{21} that only stable relations should be considered must be rejected. In a review of the Brookings' Model for the U.S. economy, he claims that "... there seems to be an implicit assumption at work that one should not expect coefficients to remain 'stable' for any length of time. Thus, if a re-estimation of the model for 1953-1960 period turns up with significantly different coefficients, this is only natural in a changing world. One drops off the 1947-1953 data and proceeds further as if nothing had happened instead of treating it as an indication of serious model mis-specification which calls for a thoroughgoing re-examination of its original design." With our approach the earlier data might be discardable on the grounds that variables which are latent during the second period may have been major factors in the early period. This approach acknowledges the imprecision of our present economic knowledge. An interval is chosen so that, during this period, a given set of variables may provide an adequate basis for the explanation of investment expenditures. Any period during which extraneous variables are suspected as significant influences on these expenditures is excluded from a particular analysis. The demand that the model be re-specified exaggerates our ability to derive general models and the discrimination that is possible with time series data. Truncation of sample is consistent with a re-examination of original design.

This truncation appears to discard 'information' since some of the available time series are not used. For example, Griliches adds: "In fact, there may be more 'information' in the fluctuating 1947-1952 period than in the relatively smooth 1953-1960 years". Similarly, Klein\textsuperscript{22} suggests


that "there would not be four times as much information, but there would be much more information" in a discussion of the use of quarterly data as compared to the use of annual data. Both views are superficial to the extent that they fail to define the concept of 'information'. Appeal to intuition is an inadequate basis for their conclusions. Annual and quarterly data may illustrate different but concomitant aspects of behaviour. Similarly, the cause of fluctuations during 1947-1952 may not be operative in the succeeding period and may distort an analysis of variables that are considered operative throughout the whole period. The characteristics of supply functions tell us little about those of demand functions so that data pertaining to a period in which supply factors predominate may be of little significance in an analysis of investment demand. Meyer and Glauber,23 without adopting the principle of selective estimation, have reached a very similar view-point: "The intention here .. is not to argue that the economic world is totally devoid of all behavioural regularities. Identification of these regularities may not be an easy task and may require new tools and concepts, but it would still appear possible .. It is .. perhaps better to know limitations than to exceed them".

**Notation**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>Real Output</td>
</tr>
<tr>
<td>L</td>
<td>Real Labour Input</td>
</tr>
<tr>
<td>K</td>
<td>Real Stock of Capital</td>
</tr>
<tr>
<td>I</td>
<td>Real Gross Investment or Capital Consumption</td>
</tr>
<tr>
<td>( \dot{K} )</td>
<td>Expansion of Capital Stock</td>
</tr>
</tbody>
</table>

---

k  Supply of Capital Services
m  Efficiency in Use of Capital Stock
N  Supply of Labour Services
n  Efficiency in Use of Labour Input
p  Output price
w  Wage Rate
q  Investment Good's Price
r_1  Cost of Capital
δ  Replacement Rate of Capital Stock, the Depreciation Parameter
r_2  Interest Rate payable on Debt
c  User-Cost of Capital Stock

T  Amount of Direct Tax assessed on Current Income
Y  Gross Income defined for Tax Purposes
A  Total Charges against gross income defined for tax purposes
Z  Net Income on Current and Capital Account
W  Net Worth of the Enterprise
u  Rate of Taxation for net income defined for tax purposes
B  Total Debt, other than trade credit and overdraft facilities
A_1  Charge for Depreciation of Capital Stock against Y
A_2  Charge for Debt Burden against Y
v_1  Rate of Annual, or Wear and Tear, Allowances
v_2  Rate of Allowances for Fixed Interest Payments

Mathematical Representation of the Jorgenson Model

Definitional Identities:

(1) \( \dot{K} = I - R \)
(2) \( m = k/K \)
(3) \( n = N/L \)
(4) \( \delta = R/K \)
(5) \( W = \int_{0}^{\infty} (Z-T) \exp(-r_1 t) \, dt \)
(6) \( T = u(Y-A) \)

Supplementary Specifications:

(7) \( Y = pX-wL \)
(8) \( Z = Y-qI \)
(9) \( A = A_1 + A_2 \)
(10) \( A_2 = v_2 r_2 B \)
(11) \( A_1 = v_1 qR \)
(12) \( B = qK \)

Alternative and mathematically equivalent specifications of (10), (12):

(i) \( (10') A_2 = r_2 B \)
(ii) \( (10'') A_2 = v_{21} r_2 B \)

(12') \( B = v_2 qK \)
(12'') \( B = v_{22} qK; v_{21} v_{22} = v_2 \)

The enterprise is assumed to maximise net worth, as defined by (5), subject to the particular specifications provided in the list above.

To remove indeterminacy Jorgenson adds three more specifications:

(13) \( m, n, \delta, r_1, r_2, p, q, v_1, v_2 \) and \( w \) are constant.
(14) \( X, k \) and \( N \) are related by a 'production function' so that we can find a relation associating \( X, k \) and \( L \), say \( F(X, K, L) = 0 \). This second relation will also be called a production function where context precludes confusion with the first relation. The latter function will be taken as strictly convex and twice-differentiable with positive marginal rates of substitution between inputs and positive marginal productivities of both inputs \( K \) and \( L \).
(15) \( r = r_1 = r_2 \)

For the moment, specification (15) is neglected and the following Lagrangean expression \( \Omega \) is defined.

\[
Z - T = Y - qI - uY + uA \\
= (1-u)pX - (1-u)wL - qI + (uv_2 r_2 q + v_1 q u_2)K \\
= \mu_1 X + \mu_2 L + \mu_3 I + \mu_4 K, \text{ say.}
\]

\[
f(t) = (Z - T)\exp(-r_1 t) + \lambda_0 F(X, K, L) + \lambda_1 (I - K - \delta K)
\]

where \( \lambda_0 \) and \( \lambda_1 \) are multipliers.

\[
\Omega = \int_0^\infty f(t) \, dt
\]

We see that:

\[
\frac{\partial f}{\partial X} = \mu_1 \exp(-r_1 t) + \lambda_0 \frac{\partial F}{\partial X} \\
\frac{\partial f}{\partial L} = \mu_2 \exp(-r_1 t) + \lambda_0 \frac{\partial F}{\partial L} \\
\frac{\partial f}{\partial I} = \mu_3 \exp(-r_1 t) + \lambda_1 \\
\frac{\partial f}{\partial K} = \mu_4 \exp(-r_1 t) + \lambda_0 \frac{\partial F}{\partial K} - \lambda_1 \delta
\]

\[
\frac{d(\frac{\partial f}{\partial K})}{dt} = - \frac{d\lambda_1}{dt} = - \lambda_1
\]

Then the Euler Extremal Conditions are given by:

\[
- \lambda_0 (\frac{\partial F}{\partial K}) \exp(r_1 t) = \mu_1 \\
- \lambda_0 (\frac{\partial F}{\partial L}) \exp(r_1 t) = \mu_2 \\
- \lambda_1 = \mu_3 \exp(-r_1 t) \\
- \lambda_0 (\frac{\partial F}{\partial L}) \exp(r_1 t) = \mu_4 - (\lambda_1 \delta - \lambda_1) \exp(r_1 t)
\]

Substitute and we have:

\[
- \frac{\partial F}{\partial K} = \frac{\partial X}{\partial K} = - \frac{\mu_3 (\delta + r_1)}{\mu_1} \\
= \frac{q(\delta + r_1 - uv_2 r_2 uv_1 \delta)}{p(1-u)} \\
= \frac{c}{p}, \text{ say.}
\]
In particular, under (15), user cost is defined as
\[
    c = q \frac{[(1-u)\delta + (1-u\omega)\sigma]}{(1-u)} = q/\phi, \text{ say.}
\]

Given a particular form for the production function, we shall replace \( \mathbb{K} \) by \( \mathbb{K}^* \), to be interpreted as 'desired capital stock', and treat the expression \( (\partial \mathbb{X}/\partial \mathbb{K}^*) = c/p \) as a definition of \( \mathbb{K}^* \). For example, the choice of the Cobb-Douglas characterization of the production function yields the following expression for \( \mathbb{K}^* \).

\[(14.1) \quad \mathbb{X} = \mathbb{M}^\alpha \mathbb{L}^\beta \text{ leads to } \mathbb{K}^* = \frac{\alpha}{q} \mathbb{X}^\phi. \text{ The same expression for } \mathbb{K}^* \text{ is obtained with any production function of the class defined by:}
\]

\[(14.2) \quad \mathbb{X} = \mathbb{K}^\alpha \cdot g(\mathbb{L}) \text{ for some function } g(\mathbb{L}).\]

The choice of the Constant Elasticity of Substitution form results in a different expression for \( \mathbb{K}^* \).

\[(14.3) \quad \mathbb{X}^{\alpha/\gamma} = \alpha_0 \mathbb{K}^\rho + \beta_0 \mathbb{L}^\rho \quad \text{leads to } \mathbb{K}^* = (\alpha_0 \gamma)^a \left(\frac{\beta \gamma}{\phi q}\right)^b \mathbb{X}^\phi, \text{ where}
\]

\[a = \frac{1}{1-\rho} \quad \text{and} \quad b = \frac{a(1-\rho)}{\gamma}.
\]

Note that, given convexity, the choice of the Cobb-Douglas form results in the introduction of a single additional parameter (\( \alpha \)) whereas the Constant Elasticity of Substitution form involves three parameters (\( \alpha_0, \gamma, \rho \)). In both cases desired capital stock is expressed in terms of real output, relative prices and an adjustment factor for tax provisions. The parallel representation in terms of real labour input instead of real output leads to intractability for (14.3) and the intrusion of the scale parameter \( \mathbb{M} \) for (14.1). Bert Hickman\(^{24}\) has presented an alternative expression containing the wage rate. This is consistent with the framework that we have outlined.

---

when proportionality of the wage rate and product price is assumed and
the constant reinterpreted.

One important feature of this derivation of desired capital stock
is the redundancy of the second extremal condition. Due to this redun-
dancy the expression for desired capital stock is unaffected by certain
changes in elemental net worth (Z-T). For example, we can introduce an
extra term into either the specified relation for tax allowances, (9),
or a redefined tax identity, (6), provided this term has the single
argument, real labour input. In particular, the expression for user-cost
is unaffected by the introduction of an index of the Selective Employment
Tax or of an index of National Insurance Contributions into the tax identity.
This feature persists when a multidimensional concept of real labour input,
based on the distinction between men and man-hours, is introduced and net
worth is maximized with respect to each dimension.

The Alternative Models

The six definitional identities and the specifications (7), (8),
(13) and (14) of the Jorgenson Model will be called the 'Basis' since they
are common to the two alternative models presented below.

Additional notation:

\( A_3 \) Charge for Initial Allowances against Income
\( A_4 \) Charge for Investment Allowances against Income
\( v_3 \) Rate of Initial Allowances
\( v_4 \) Rate of Investment Allowances.
ALTERNATIVE A.

This is formed by the basis, (11) and the following three additional specifications.

\[(16) \quad A = A_1 + A_3 + A_4\]
\[(17) \quad A_3 = v_3qI\]
\[(18) \quad A_4 = v_4qI\]

The corresponding elemental net worth is

\[Z-T = (1-u)pX - (1-u)wL - (1-uv_3 - uv_4)qI + uv_1q\delta K,\]

and the expression for user-cost under this alternative is

\[c = \frac{q[(1-uv_3 - uv_4)(\delta+r_1) - uv_1\delta]}{(1-u)} = \frac{q}{\phi_A}, \text{ say.}\]

ALTERNATIVE B.

This is formed by the basis, (17), (18) and the single additional specification.

\[(19) \quad A = A_3 + A_4\]

The corresponding elemental net worth is

\[Z-T = (1-u)pX - (1-u)wL - (1-uv_3 - uv_4)qI,\]

and the expression for user-cost under this alternative is

\[c = \frac{q(1-uv_3 - uv_4)(\delta+r_1)}{(1-u)} = \frac{q}{\phi_B}, \text{ say.}\]

By deletion of parameters in the final forms of the two alternative models, a third group of ad hoc expressions for user-cost were obtained. This resulted in twenty-one different expressions for user-cost, where a variable rate was used for the cost of capital. Many other expressions can be obtained by taking different constant values of the cost of capital. Regression equations were fitted for these alternatives but the results will not be discussed here.
Prices.

Specification (13) was maintained in the basis. This assumed constancy of prices and wages in the decision model and may be contrasted with the inflation experienced during the period under review. Let \( p \) and \( q \) be independent of \( X, K, \) and \( L \) but variable over time. Then the expressions for \( \partial f / \partial X, \partial f / \partial L, \partial f / \partial I \) and \( \partial f / \partial K \) are unchanged and the only change in the expression for user-cost resulting from this modified specification must stem from the change in \( \lambda_1 \) due to the temporal dependence of \( \mu_3. \)

Before modification:

\[
\begin{align*}
1/\phi &= (1-uv_1)\delta + (1-uv_2)r \\
1/\phi_A &= (1-uv_3-uv_4)(\delta+r) - uv_1\delta \\
1/\phi_B &= (1-uv_3-uv_4)(\delta+r) \\
\end{align*}
\]

After modification:

\[
\begin{align*}
1/\phi &= (1-uv_1)\delta + (1-uv_2)r - \dot{q}/q \\
1/\phi_A &= (1-uv_3-uv_4)(\delta+r) - uv_1\delta + \dot{\mu}_3/q \\
1/\phi_B &= (1-uv_3-uv_4)(\delta+r) + \dot{\mu}_3/q \\
\end{align*}
\]

where \( \mu_3 = -(1-uv_3-uv_4)q \)

If only \( q \) and \( p, \) or \( q, \) vary over time, these expressions become:

\[
\begin{align*}
1/\phi_A &= (1-uv_3-uv_4)(\delta+r-\dot{q}/q) - uv_1\delta \\
1/\phi_B &= (1-uv_3-uv_4)(\delta+r-\dot{q}/q) \\
\end{align*}
\]

In the empirical work reported here we ignored the problem of
changing prices on the grounds that the series for the change in the price of investment goods suffers from two major deficiencies: the low number of significant figures of the two value series used to obtain the implicit deflator used as the price of investment goods, and the possibility of magnification of recognized errors in these series by repeated division and subtraction. The official commentary on British statistics contains a 'subjective' grouping of investment expenditures into measurement error categories. The value of such a grouping is difficult to assess but levels of error indicated by statisticians collecting the data are uncomfortably high.

A partial justification of our neglect of price changes can be derived from recent studies published by the Centre for Business Research at Manchester. Views presented by the economists at the Centre may be summarized by the following quotation due to Williams and Scott: "Forecasts of selling prices were also given relatively little attention - partly because of the same difficulty in forecasting the effects of competition, partly because of the conceptual difficulty of forecasting in an inflationary situation. Forecasts of prices and costs were generally based on current money values, and it was assumed that any changes would be balanced out".

Finally, there is a danger of introducing a 'halo' effect into any econometric analysis. Alan Prest has pointed out the self-generating character of some economic phenomena and asks ".. when the situation is


such that today no one under 40 can remember a period when prices were not rising, is it surprising that economic actions become more and more based on the assumption that this is a permanent state of affairs?". The current interest in inflation might cause the investigator to assume that the same weight was associated with it in the past by decision-takers, when this was not so.

**Debt.**

Fixed interest payments are chargeable against corporate income. In the derivation of the Jorgenson Model, this feature of tax practice is represented by (10) and (12) and by the alternative specifications indicated there. In the Alternative Models fitted to the post-war British economy, the feature is not represented. This alternative treatment exaggerates the effective rate of tax, avoids the need to equate \( r_1 \) and \( r_2 \), and does not distinguish between recipients of distributed corporate income as assessors of net worth. In order to include fixed interest payments in a suitable mathematical expression, Jorgenson assumed the proportionality of total debt and capital stock valued in terms of the price of investment goods. This assumption is not made in this study.

**Lag Structure**

Consider

\[
(20) \quad (I-R)_t = \psi(\theta) \cdot v \cdot K^*_t, \quad v = 1 - \theta,
\]

where \( \theta \) is the lag operator defined for any variable \( X_t \) by the equation:

\[
\theta X_t = X_{t-1}
\]

Assume that \( \psi(\theta) \) is a rational function of the following form:

\[
\psi(\theta) = \frac{a_0 \theta^2 + a_1 \theta^3 + a_2 \theta^4 + a_3 \theta^5}{1 - b_1 \theta - b_2 \theta^2},
\]

where \( \psi(1) = 1 \).
Then, (20) can be transformed to obtain:

$$
(21) \quad (I-R)_{t} = b(\theta)(I-R)_{t} + a(\theta) \nabla K^{*}_{t}.
$$

At this stage, the coefficients \(a_{j}, b_{j}\) are assumed constant over the sample period, (21) is assumed to be inexact and a disturbance is added. Finally, the disturbance is assumed to satisfy classical properties. The results listed in Tables Two and Three were derived by the Ordinary Least Squares method of estimation. Table One contains a classification of the lag structures used. These were obtained by imposing zero restrictions on the parameters of (21). Each regression is indicated by a paired index. (8, j) represents a regression model with the jth lag structure and based on Alternative A with the dividend yield on industrial ordinary shares as the cost of capital. (16, j) represents a regression model with the jth lag structure and based on Alternative B with the same cost of capital.

$$
K^{*}_{A} = \alpha_{P} X \phi_{A} \quad \text{for (8, j)}
$$

$$
K^{*}_{B} = \alpha_{P} X \phi_{B} \quad \text{for (8, j)}
$$

The regression results cover two spans of data. Taking the dependent variables as the standard, most of the regressions have a coverage from the third quarter of 1957 to the last quarter of 1965. However, the four terminal observations are omitted for the regression with second indices 9, 10, 15 and 16. The dependent variable is (non-dwelling) net investment, desesasonalized and valued in 1958 prices but not detrended.

All the equations involve the specification of unit elasticities. Available evidence\(^{28}\) indicates that this specification may be excessive with respect to the role of relative prices but that it leads to only a

---

28. 'Investment Functions: Which Production Function?', Discussion Paper No. 6, Institute of Economic Research, Queen's University.
slight exaggeration of the role for the composite tax factor $\phi$.

\textbf{Table One. Lag Structures}

<table>
<thead>
<tr>
<th>Second Index</th>
<th>Constraint</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$a_2 = a_3 = 0$</td>
</tr>
<tr>
<td>2</td>
<td>$a_3 = 0$</td>
</tr>
<tr>
<td>3, 10</td>
<td>NONE</td>
</tr>
<tr>
<td>4</td>
<td>$a_0 = 0$</td>
</tr>
<tr>
<td>5</td>
<td>$a_0 = a_3 = 0$</td>
</tr>
<tr>
<td>6</td>
<td>$b_2 = 0$; $a_3 = 0$</td>
</tr>
<tr>
<td>7</td>
<td>$b_2 = 0$</td>
</tr>
<tr>
<td>8, 9</td>
<td>$b_2 = 0$; $a_0 = a_3 = 0$</td>
</tr>
<tr>
<td>11, 15</td>
<td>$b_1 = b_2 = 0$; $a_0 = a_3 = 0$</td>
</tr>
<tr>
<td>12</td>
<td>$b_1 = b_2 = 0$; $a_3 = 0$</td>
</tr>
<tr>
<td>13</td>
<td>$b_1 = b_2 = 0$</td>
</tr>
<tr>
<td>14, 16</td>
<td>$b_1 = b_2 = 0$; $a_0 = 0$</td>
</tr>
<tr>
<td>17</td>
<td>$b_2 = 0$; $a_0 = 0$</td>
</tr>
</tbody>
</table>

The results for regression equations are arranged in cells for Table Two and Table Three. Each cell contains either two or three components. Whenever three numbers are given, they are the estimated coefficient, its estimated standard error and the corresponding student's $t$-statistic. The last number is omitted for cells with only two components. A horizontal index is introduced for these tables. This is the set of integers from one to six inclusive. One and Two indicate the presence of lagged dependent variables in the fitted equation. The former represents net investment
lagged one quarter and the latter represents net investment lagged two quarters. Three to Six represent lagged changes in desired capital variable with the lags extending from two quarters to five quarters.

The vertical index of the Tables has been described above. Asterisks are added to the indices when the sample is shortened.
Table Two: Alternative A

<table>
<thead>
<tr>
<th>Index</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>100R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.1</td>
<td>.5710</td>
<td>.1545</td>
<td>.1978</td>
<td>.2293</td>
<td>.</td>
<td>.</td>
<td>91.03</td>
</tr>
<tr>
<td></td>
<td>.1914</td>
<td>.1675</td>
<td>.0717</td>
<td>.0661</td>
<td>.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.2</td>
<td>.4571</td>
<td>.1434</td>
<td>.0895</td>
<td>.3148</td>
<td>.1997</td>
<td>.</td>
<td>92.27</td>
</tr>
<tr>
<td></td>
<td>.1885</td>
<td>.1582</td>
<td>.0847</td>
<td>.0742</td>
<td>.0939</td>
<td>.</td>
<td></td>
</tr>
<tr>
<td>8.3</td>
<td>.4032</td>
<td>.1382</td>
<td>.0884</td>
<td>.2881</td>
<td>.2427</td>
<td>.0678</td>
<td>92.39</td>
</tr>
<tr>
<td></td>
<td>.2079</td>
<td>.1601</td>
<td>.0856</td>
<td>.0856</td>
<td>.1158</td>
<td>.1046</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.9396</td>
<td>.8630</td>
<td>1.0321</td>
<td>3.3648</td>
<td>2.0961</td>
<td>6.481</td>
<td></td>
</tr>
<tr>
<td>8.4</td>
<td>.4527</td>
<td>.1055</td>
<td>.2660</td>
<td>.3030</td>
<td>.0700</td>
<td>.</td>
<td>92.09</td>
</tr>
<tr>
<td></td>
<td>.2025</td>
<td>.1571</td>
<td>.0860</td>
<td>.1001</td>
<td>.1047</td>
<td>.</td>
<td></td>
</tr>
<tr>
<td>8.5</td>
<td>.5091</td>
<td>.1105</td>
<td>.2934</td>
<td>.2594</td>
<td>.</td>
<td></td>
<td>91.97</td>
</tr>
<tr>
<td></td>
<td>.1824</td>
<td>.1554</td>
<td>.0716</td>
<td>.0752</td>
<td>.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2.7918</td>
<td>.7110</td>
<td>4.0998</td>
<td>3.4502</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.6</td>
<td>.5987</td>
<td>.0744</td>
<td>.3118</td>
<td>.2025</td>
<td>.</td>
<td></td>
<td>92.05</td>
</tr>
<tr>
<td></td>
<td>.1054</td>
<td>.0828</td>
<td>.0739</td>
<td>.0936</td>
<td>.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.7</td>
<td>.5355</td>
<td>.0738</td>
<td>.2834</td>
<td>.2483</td>
<td>.0724</td>
<td>.</td>
<td>92.18</td>
</tr>
<tr>
<td></td>
<td>.1397</td>
<td>.0835</td>
<td>.0851</td>
<td>.1151</td>
<td>.1040</td>
<td>.</td>
<td></td>
</tr>
<tr>
<td>8.8</td>
<td>.6156</td>
<td>.2938</td>
<td>.2535</td>
<td>.</td>
<td></td>
<td></td>
<td>91.82</td>
</tr>
<tr>
<td></td>
<td>.1033</td>
<td>.0710</td>
<td>.0741</td>
<td>.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>5.9564</td>
<td>4.1412</td>
<td>3.4211</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.9*</td>
<td>.6445</td>
<td>.2727</td>
<td>.2397</td>
<td>.</td>
<td></td>
<td></td>
<td>91.80</td>
</tr>
<tr>
<td></td>
<td>.1020</td>
<td>.0636</td>
<td>.0651</td>
<td>.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.10*</td>
<td>.4399</td>
<td>.1094</td>
<td>.0466</td>
<td>.2430</td>
<td>.2738</td>
<td>.1000</td>
<td>92.34</td>
</tr>
<tr>
<td></td>
<td>.2126</td>
<td>.1699</td>
<td>.0774</td>
<td>.0784</td>
<td>.1026</td>
<td>.0967</td>
<td></td>
</tr>
<tr>
<td>8.11</td>
<td>.6459</td>
<td>.6281</td>
<td>.</td>
<td>.0571</td>
<td>.0570</td>
<td>.</td>
<td>82.16</td>
</tr>
<tr>
<td></td>
<td>.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>11.3162</td>
<td>11.0164</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table Two (cont.)

<table>
<thead>
<tr>
<th>Index</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>100R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.12</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>.1582</td>
<td>.6636</td>
<td>.4977</td>
<td></td>
<td></td>
<td></td>
<td>83.19</td>
</tr>
<tr>
<td></td>
<td>.1164</td>
<td>.0578</td>
<td>.1112</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.3592</td>
<td>11.4807</td>
<td>4.4741</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.13</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>.1149</td>
<td>.3639</td>
<td>.5649</td>
<td>.3310</td>
<td></td>
<td></td>
<td>88.08</td>
</tr>
<tr>
<td></td>
<td>.1005</td>
<td>.1000</td>
<td>.0972</td>
<td>.0960</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.14</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>.3388</td>
<td>.6609</td>
<td>.3447</td>
<td></td>
<td></td>
<td></td>
<td>87.56</td>
</tr>
<tr>
<td></td>
<td>.0981</td>
<td>.0493</td>
<td>.0957</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3.4555</td>
<td>13.4130</td>
<td>3.6018</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.15*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>.5879</td>
<td>.5703</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>79.21</td>
</tr>
<tr>
<td></td>
<td>.0615</td>
<td>.0606</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.16*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>.2922</td>
<td>.6229</td>
<td>.3481</td>
<td></td>
<td></td>
<td></td>
<td>86.63</td>
</tr>
<tr>
<td></td>
<td>.0926</td>
<td>.0514</td>
<td>.0916</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table Three: Alternative B

<table>
<thead>
<tr>
<th>Index</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>100R^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>16.1</td>
<td>.6850</td>
<td>.0409</td>
<td>.1652</td>
<td>.1903</td>
<td></td>
<td></td>
<td>89.88</td>
</tr>
<tr>
<td></td>
<td>.1925</td>
<td>.1759</td>
<td>.0746</td>
<td>.0735</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>16.2</td>
<td>.5290</td>
<td>-.0090</td>
<td>.1108</td>
<td>.3333</td>
<td>.2052</td>
<td></td>
<td>91.98</td>
</tr>
<tr>
<td></td>
<td>.1836</td>
<td>.1604</td>
<td>.0705</td>
<td>.0850</td>
<td>.0757</td>
<td></td>
<td></td>
</tr>
<tr>
<td>16.3</td>
<td>.5153</td>
<td>-.0112</td>
<td>.1085</td>
<td>.3297</td>
<td>.2181</td>
<td>.0142</td>
<td>91.99</td>
</tr>
<tr>
<td></td>
<td>.2052</td>
<td>.1638</td>
<td>.0732</td>
<td>.0892</td>
<td>.1109</td>
<td>.0874</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2.5114</td>
<td>-.0681</td>
<td>1.4825</td>
<td>3.6971</td>
<td>1.9671</td>
<td>.1620</td>
<td></td>
</tr>
<tr>
<td>16.4</td>
<td>.5802</td>
<td>-.0135</td>
<td></td>
<td>.2544</td>
<td>.2732</td>
<td>.0397</td>
<td>91.34</td>
</tr>
<tr>
<td></td>
<td>.2047</td>
<td>.1672</td>
<td></td>
<td>.0748</td>
<td>.1067</td>
<td>.0875</td>
<td></td>
</tr>
<tr>
<td>16.5</td>
<td>.6242</td>
<td>-.0074</td>
<td></td>
<td>.2599</td>
<td>.2391</td>
<td></td>
<td>91.27</td>
</tr>
<tr>
<td></td>
<td>.1777</td>
<td>.1644</td>
<td></td>
<td>.0728</td>
<td>.0744</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3.5127</td>
<td>-.0448</td>
<td></td>
<td>3.5698</td>
<td>3.2123</td>
<td></td>
<td></td>
</tr>
<tr>
<td>16.6</td>
<td>.5215</td>
<td>.1108</td>
<td>.3325</td>
<td>.2047</td>
<td></td>
<td></td>
<td>91.98</td>
</tr>
<tr>
<td></td>
<td>.1237</td>
<td>.0693</td>
<td>.0823</td>
<td>.0739</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>16.7</td>
<td>.5065</td>
<td>.1086</td>
<td>.3289</td>
<td>.2171</td>
<td>.0137</td>
<td></td>
<td>91.99</td>
</tr>
<tr>
<td></td>
<td>.1569</td>
<td>.0719</td>
<td>.0867</td>
<td>.1078</td>
<td>.0856</td>
<td></td>
<td></td>
</tr>
<tr>
<td>16.8</td>
<td>.6181</td>
<td>.2593</td>
<td>.2387</td>
<td></td>
<td></td>
<td></td>
<td>91.27</td>
</tr>
<tr>
<td></td>
<td>.1107</td>
<td>.0702</td>
<td>.0726</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>5.5837</td>
<td></td>
<td>3.3933</td>
<td>3.2860</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>16.9*</td>
<td>.6723</td>
<td></td>
<td>.2374</td>
<td>.2179</td>
<td></td>
<td></td>
<td>90.99</td>
</tr>
<tr>
<td></td>
<td>.1062</td>
<td></td>
<td>.0634</td>
<td>.0643</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>16.10*</td>
<td>.5629</td>
<td>-.0163</td>
<td>.0691</td>
<td>.2756</td>
<td>.2349</td>
<td>.0431</td>
<td>91.55</td>
</tr>
<tr>
<td></td>
<td>.2167</td>
<td>.1775</td>
<td>.0679</td>
<td>.0827</td>
<td>.1039</td>
<td>.0864</td>
<td></td>
</tr>
<tr>
<td>16.11</td>
<td></td>
<td></td>
<td>.5953</td>
<td>.5897</td>
<td></td>
<td></td>
<td>82.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>.0508</td>
<td>.0511</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>11.7202</td>
<td>11.5420</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Index</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>$100R^2$</td>
</tr>
<tr>
<td>--------</td>
<td>----</td>
<td>-----</td>
<td>-----</td>
<td>-----</td>
<td>-----</td>
<td>-----</td>
<td>---------</td>
</tr>
<tr>
<td>16.12</td>
<td></td>
<td>.2535</td>
<td>.6426</td>
<td>.3866</td>
<td></td>
<td></td>
<td>87.06</td>
</tr>
<tr>
<td></td>
<td></td>
<td>.0426</td>
<td>.0866</td>
<td>.0750</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>3.3568</td>
<td>13.9028</td>
<td>5.1557</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>16.13</td>
<td>.1703</td>
<td>.4792</td>
<td>.4797</td>
<td>.1788</td>
<td></td>
<td></td>
<td>89.01</td>
</tr>
<tr>
<td></td>
<td>.0798</td>
<td>.0842</td>
<td>.0815</td>
<td>.0790</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>16.14</td>
<td>.3832</td>
<td>.6278</td>
<td>.2564</td>
<td></td>
<td></td>
<td></td>
<td>87.28</td>
</tr>
<tr>
<td></td>
<td>.0753</td>
<td>.0453</td>
<td>.0741</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>5.0895</td>
<td>13.8661</td>
<td>3.4583</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>16.15*</td>
<td></td>
<td>.5572</td>
<td>.5470</td>
<td></td>
<td></td>
<td></td>
<td>77.11</td>
</tr>
<tr>
<td></td>
<td></td>
<td>.0598</td>
<td>.0592</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>16.16*</td>
<td>.3255</td>
<td>.6086</td>
<td>.2898</td>
<td></td>
<td></td>
<td></td>
<td>85.85</td>
</tr>
<tr>
<td></td>
<td>.0751</td>
<td>.0498</td>
<td>.0723</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
SOME CONCLUSIONS

Some tentative conclusions may be offered on the basis of these empirical results. They are listed below.

(1) Despite the clear significance of the variable representing investment lagged one quarter, there is no indication from any of the tabulated results that a second lagged dependent variable has any significant role in the determination of investment expenditures. This feature of the results must introduce doubt as to the generality of the practice advocated for equations involving distributed lags by Dale Jorgenson. He suggests that we should constrain the choice between different lag structures by a requirement that the coefficients of the first two lagged investment variables are non-zero. If we accept, as he did, the criterion requiring minimization of the estimated standard error of regression but leave this choice unconstrained, then all the evidence generated by this study indicates rejection of the constraint. Similarly the constraint is rejected by all conventional statistical criteria.

(2) Since inclusion of a second lagged dependent variable as a regressor markedly affects estimates of the implied temporal pattern for the response of investment to a change in the level of desired capital, we are faced with the choice between omission of the variable and dismissal of the estimated pattern of response on the grounds that these estimates are unreliable due to the imprecision of estimated coefficients.


(3) Serial correlation of net investment provides an absolute minimum for the coefficient of multiple determination derived from regression equations involving lagged dependent variables. In a purely autoregressive specification, the adjusted coefficient $\bar{R}^2$ rises from 0.86 to 0.88 as the number of regressors increases from one to six. The inclusion of additional variables, which are explicitly derived from the theory of optimal capital accumulation, can only increase the coefficient by ten per cent so that any equation containing such variables and a lagged dependent variable must fit well whether the former are relevant or not. This line of argument is invalid if it is used to assert that the change in $\bar{R}^2$ induced by the inclusion of 'additional theoretical' variables is the sole indication of the relevance of the embodied theory. Suppose that we adopt a symmetric approach to both sets of variables. Then we can compare equation (8.13) with (8.7), or equation (16.13) with (16.7). Taking the first equation of each pair, the capital variables provide an absolute minimum for $\bar{R}^2$ of 0.88 or 0.89 even when no lagged variable is included as a regressor. If the change in $\bar{R}^2$ within either pair is the sole indicator of the relevance of the lagged dependent variable, the latter might be excluded. We conclude that the $\bar{R}^2$ for the autoregressive fit is an inappropriate basis upon which to judge the relevance of other variables.

(4) The estimated standard errors of coefficients and t-statistics reported in the two tables indicate that those variables indexed by one, four and five should be treated as significant for the explanation of the level of investment expenditures. This conclusion is supported by other evidence that has not been tabulated here. For example, the inclusion of the lagged dependent variable increases the Durbin-Watson d-statistic to
acceptable levels. Equations (8.8) and (16.8) satisfy all of the conventional statistical tests whereas equations (8.8) and (16.6) are indicated by the unconstrained standard-error criterion. The difference between the standard errors of (16.6) and (16.8) was of order 2½ percent, which is a slender basis for preference of (16.6).

(5) Whether a conventional statistical test or the alternative standard-error criterion is accepted, the empirical results indicate that the theory of optimal capital accumulation does provide an explanation of the pattern of British investment expenditures during the particular post-war decade under review. In particular, these expenditures may be explained by the composite influence of changes in three factors determining the level of desired capital. For stabilization purposes, the important implications of the results are three-fold. Capital allowances and the corporate tax-rate are effective instruments for the control of investment expenditures provided sufficient attention is given to the contemporaneous influence of relative prices and the cost of capital. The gestation period between a change in the level of an existing instrument and the inception of changes in expenditures is three quarters so that efficient management of the British Economy requires considerable foresight. Finally, the mean lag of response is between five and six quarters so that the response is concentrated within a few quarters. Government must have such foresight otherwise its policies must lead to instability.

(6) Although three of the larger residuals were present in the final year of the period, the changes in estimated coefficients were insufficient to justify the inference that a structural break was induced by the Labour Government when it came to office but the composite nature of the regressors might obscure such a break.