ECON 351* -- NOTE 7

Interval Estimation in the Classical Normal Linear Regression Model

This note outlines the basic elements of interval estimation in the Classical Normal Linear Regression Model (the CNLRM). Interval estimation -- i.e., the construction of confidence intervals for unknown population parameters -- is one of the two alternative approaches to statistical inference; the other is hypothesis testing.

1. Introduction

- We have previously derived **point estimators** of all the unknown population parameters in the Classical Normal Linear Regression Model (CNLRM) for which the **population regression equation**, or **PRE**, is

$$Y_i = \beta_1 + \beta_2 X_i + u_i \qquad \text{where } u_i \text{ is iid as } N(0, \sigma^2) \qquad (i = 1, \ldots, N). \qquad \textbf{(1)}$$

  - The **unknown parameters** of the PRE are

    (1) the **regression coefficients β_1 and β_2**

    and

    (2) the **error variance σ^2**.

  - The **point estimators** of these unknown population parameters are

    (1) the ***unbiased*** **OLS regression coefficient estimators $\hat{\beta}_1$ and $\hat{\beta}_2$**

    and

    (2) the ***unbiased*** **error variance estimator $\hat{\sigma}^2$**.

- Assume that we have computed the point estimates $\hat{\beta}_1$, $\hat{\beta}_2$, and $\hat{\sigma}^2$ of the unknown parameters for a given set of sample data (Y_i, X_i), i = 1, ..., N.

- We therefore begin with the following **OLS sample regression equation** (or **OLS-SRE**):

$$Y_i = \hat{\beta}_1 + \hat{\beta}_2 X_i + \hat{u}_i \qquad (i = 1,\ldots,N). \qquad \textbf{(2)}$$

where

$$\hat{\beta}_2 = \frac{\sum_i x_i y_i}{\sum_i x_i^2} = \frac{\sum_i (X_i - \overline{X})(Y_i - \overline{Y})}{\sum_i (X_i - \overline{X})^2} = \text{ OLS estimate of } \beta_2;$$

$$\hat{\beta}_1 = \overline{Y} - \hat{\beta}_2 \overline{X} = \text{ OLS estimate of } \beta_1;$$

$$\hat{\sigma}^2 = \frac{\sum_i \hat{u}_i^2}{N-2} = \text{ unbiased OLS estimate of } \sigma^2;$$

$$\hat{\text{Var}}(\hat{\beta}_2) = \frac{\hat{\sigma}^2}{\sum_i x_i^2} = \frac{\hat{\sigma}^2}{\sum_i \left(X_i - \overline{X}\right)^2};$$

$$\hat{\text{se}}(\hat{\beta}_2) = \sqrt{\hat{\text{Var}}(\hat{\beta}_2)} = \left(\frac{\hat{\sigma}^2}{\sum_i x_i^2}\right)^{\frac{1}{2}} = \frac{\hat{\sigma}}{\sqrt{\sum_i x_i^2}};$$

$$\hat{\text{Var}}(\hat{\beta}_1) = \frac{\hat{\sigma}^2 \sum_i X_i^2}{N \sum_i x_i^2} = \frac{\hat{\sigma}^2 \sum_i X_i^2}{N \sum_i \left(X_i - \overline{X}\right)^2};$$

$$\hat{\text{se}}(\hat{\beta}_1) = \sqrt{\hat{\text{Var}}(\hat{\beta}_1)} = \left(\frac{\hat{\sigma}^2 \sum_i X_i^2}{N \sum_i x_i^2}\right)^{\frac{1}{2}}.$$

- Under the assumptions of the Classical Normal Linear Regression Model (CNLRM) -- including in particular the ***normality assumption A9*** -- the **sample t-statistics for $\hat{\beta}_2$ and $\hat{\beta}_1$** each have the **t-distribution with (N – 2) degrees of freedom**: i.e.,

$$t(\hat{\beta}_2) = \frac{\hat{\beta}_2 - \beta_2}{\sqrt{\hat{\text{Var}}(\hat{\beta}_2)}} = \frac{\hat{\beta}_2 - \beta_2}{\hat{\text{se}}(\hat{\beta}_2)} \sim t[N-2];$$

$$t(\hat{\beta}_1) = \frac{\hat{\beta}_1 - \beta_1}{\sqrt{\hat{\text{Var}}(\hat{\beta}_1)}} = \frac{\hat{\beta}_1 - \beta_1}{\hat{\text{se}}(\hat{\beta}_1)} \sim t[N-2].$$

2. Interval Estimation: Some Basic Ideas

2.1 General Form of a Confidence Interval

A **two-sided confidence interval for the slope coefficient β_2** takes the general form

$$\Pr(\hat{\beta}_{2L} \le \beta_2 \le \hat{\beta}_{2U}) = \Pr(\hat{\beta}_2 - \hat{\delta} \le \beta_2 \le \hat{\beta}_2 + \hat{\delta}) = 1 - \alpha \qquad \textbf{(3)}$$

where

α = the ***significance*** **level** ($0 < \alpha < 1$),

$1 - \alpha$ = the ***confidence*** **level** (or confidence coefficient),

$\hat{\delta}$ = a positively-valued sample statistic,

$\hat{\beta}_{2L} = \hat{\beta}_2 - \hat{\delta}$ = the ***lower*** **confidence limit**,

$\hat{\beta}_{2U} = \hat{\beta}_2 + \hat{\delta}$ = the ***upper*** **confidence limit**.

The interval $[\hat{\beta}_{2L}, \hat{\beta}_{2U}] = [\hat{\beta}_2 - \hat{\delta}, \hat{\beta}_2 + \hat{\delta}]$ is called the **two-sided $(1 - \alpha)$-level confidence interval**, or **two-sided $100(1 - \alpha)$ percent confidence interval**, for the slope coefficient β_2.

2.2 Interpretation of Confidence Intervals

$$\Pr(\hat{\beta}_{2L} \le \beta_2 \le \hat{\beta}_{2U}) = \Pr(\hat{\beta}_2 - \hat{\delta} \le \beta_2 \le \hat{\beta}_2 + \hat{\delta}) = 1 - \alpha \qquad \textbf{(3)}$$

1. The confidence interval $[\hat{\beta}_{2L}, \hat{\beta}_{2U}]$ is a <u>random</u> interval.

- The **confidence limits** $\hat{\beta}_{2L} = \hat{\beta}_2 - \hat{\delta}$ and $\hat{\beta}_{2U} = \hat{\beta}_2 + \hat{\delta}$ **are *random variables*** (or ***sample statistics***) that vary in value from one sample to another because the values of $\hat{\beta}_2$ and $\hat{\delta}$ vary from sample to sample.

- But for any one sample of data and the corresponding estimates of $\hat{\beta}_2$ and $\hat{\delta}$, the confidence limits $\hat{\beta}_{2L} = \hat{\beta}_2 - \hat{\delta}$ and $\hat{\beta}_{2U} = \hat{\beta}_2 + \hat{\delta}$ are simply fixed numbers, i.e., they take fixed values. Therefore, any one confidence interval calculated for a particular sample of data is a fixed -- meaning nonrandom -- interval.

2. The **correct interpretation of the confidence interval $[\hat{\beta}_{2L}, \hat{\beta}_{2U}]$** is based on the **concept of *repeated sampling***.

- In ***repeated*** **samples of the same *size* from the same *population*, 100(1 – α) percent of the confidence intervals** constructed using the formulas for $\hat{\beta}_{2L}$ and $\hat{\beta}_{2U}$ **will contain the true value of the population parameter β_2**.

  For example, if the confidence level $1 - \alpha = 0.95$, **95 percent of the confidence intervals** computed using repeated samples of the same size from the same population **will contain the true value of β_2**.

- But any one confidence interval for β_2, based on one sample of data, may or may not contain the true value of β_2. Since the true value of β_2 is unknown, we do not know whether that value does or does not lie inside any one confidence interval.

3. Confidence Intervals for the Regression Coefficients β_1 and β_2

3.1 Confidence Interval for β_2: Derivation

A **two-step** derivation:

Step 1: Start with a probability statement formulated in terms of $t(\hat{\beta}_2)$, the t-statistic for $\hat{\beta}_2$. This probability statement ***implicitly*** **defines** the two-sided $(1-\alpha)$-level confidence interval for β_2.

Step 2: Re-arrange this probability statement to obtain an equivalent probability statement formulated in terms of β_2 rather than $t(\hat{\beta}_2)$. The resultant probability statement ***explicitly*** **defines** the two-sided $(1-\alpha)$-level confidence interval for β_2.

Step 1: The **two-sided $(1 - \alpha)$-level confidence interval for β_2** is implicitly defined by the probability statement

$$\Pr\left(-t_{\alpha/2}[N-2] \le t(\hat{\beta}_2) \le t_{\alpha/2}[N-2]\right) = 1-\alpha \qquad \textbf{(4)}$$

where

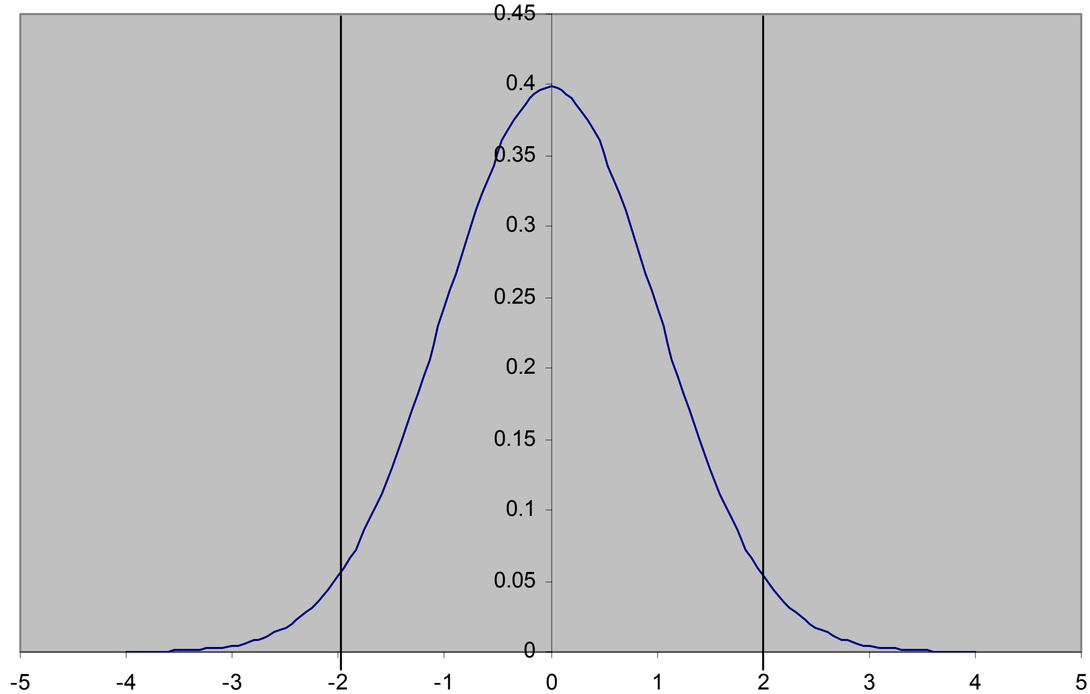
$1 - \alpha$ = the ***confidence*** **level** attached to the confidence interval;
α = the ***significance*** **level**, where $0 < \alpha < 1$;
$t_{\alpha/2}[N-2]$ = the ***critical value*** of the t-distribution with N−2 degrees of freedom at the $\alpha/2$ (or $100\alpha/2$ percent) significance level;

and $t(\hat{\beta}_2)$ is the t-statistic for $\hat{\beta}_2$ given by

$$t(\hat{\beta}_2) = \frac{\hat{\beta}_2 - \beta_2}{\sqrt{\widehat{\mathrm{Var}}(\hat{\beta}_2)}} = \frac{\hat{\beta}_2 - \beta_2}{\widehat{\mathrm{se}}(\hat{\beta}_2)}. \qquad \textbf{(5)}$$

The $t(\hat{\beta}_2)$ statistic has the t[N−2] distribution

area = $\alpha/2$ ↓ area = $1 - \alpha$ ↓ area = $\alpha/2$



$-\,t_{\alpha/2}$ $t_{\alpha/2}$

↑ ↑

$\Pr(t < -\,t_{\alpha/2}) = \alpha/2$ ↑ $\Pr(-\,t_{\alpha/2} \le t \le t_{\alpha/2}) = 1-\alpha$ ↑ $\Pr(t > t_{\alpha/2}) = \alpha/2$

<u>Step 2</u>: Express the double inequality inside the brackets in probability statement (4) in terms of β_2 rather than $t(\hat{\beta}_2)$.

$$\Pr\left(-t_{\alpha/2}[N-2] \le t(\hat{\beta}_2) \le t_{\alpha/2}[N-2]\right) = 1-\alpha \qquad \textbf{(4)}$$

(1) Substitute in the double inequality

$$-t_{\alpha/2}[N-2] \le t(\hat{\beta}_2) \le t_{\alpha/2}[N-2]$$

the expression for $t(\hat{\beta}_2)$ given in (5) above:

$$-t_{\alpha/2}[N-2] \le \frac{\hat{\beta}_2 - \beta_2}{\hat{se}(\hat{\beta}_2)} \le t_{\alpha/2}[N-2]. \qquad (6.1)$$

(2) Multiply the double inequality (6.1) by the positive number $\hat{se}(\hat{\beta}_2) > 0$:

$$-t_{\alpha/2}\hat{se}(\hat{\beta}_2) \le \hat{\beta}_2 - \beta_2 \le t_{\alpha/2}\hat{se}(\hat{\beta}_2). \qquad (6.2)$$

(3) Subtract $\hat{\beta}_2$ from both sides of inequality (6.2):

$$-\hat{\beta}_2 - t_{\alpha/2}\hat{se}(\hat{\beta}_2) \le -\beta_2 \le -\hat{\beta}_2 + t_{\alpha/2}\hat{se}(\hat{\beta}_2). \qquad (6.3)$$

(4) Multiply all terms in inequality (6.3) by –1, remembering to reverse the direction of the inequalities:

$$\hat{\beta}_2 - t_{\alpha/2}\hat{se}(\hat{\beta}_2) \le \beta_2 \le \hat{\beta}_2 + t_{\alpha/2}\hat{se}(\hat{\beta}_2). \qquad (6.4)$$

RESULT: The probability statement (4) can be written as

$$\Pr\left(\hat{\beta}_2 - t_{\alpha/2}[N-2]\hat{se}(\hat{\beta}_2) \le \beta_2 \le \hat{\beta}_2 + t_{\alpha/2}[N-2]\hat{se}(\hat{\beta}_2)\right) = 1-\alpha. \qquad \textbf{(7)}$$

The **two-sided (1 − α)-level confidence interval for β_2** can therefore be written as

$$\hat{\beta}_2 - t_{\alpha/2}[N-2]\hat{se}(\hat{\beta}_2) \le \beta_2 \le \hat{\beta}_2 + t_{\alpha/2}[N-2]\hat{se}(\hat{\beta}_2)$$

or more compactly as

$$\hat{\beta}_2 \pm t_{\alpha/2}[N-2]\hat{se}(\hat{\beta}_2) \quad \text{or} \quad \left[\hat{\beta}_2 - t_{\alpha/2}[N-2]\hat{se}(\hat{\beta}_2),\ \hat{\beta}_2 + t_{\alpha/2}[N-2]\hat{se}(\hat{\beta}_2)\right]$$

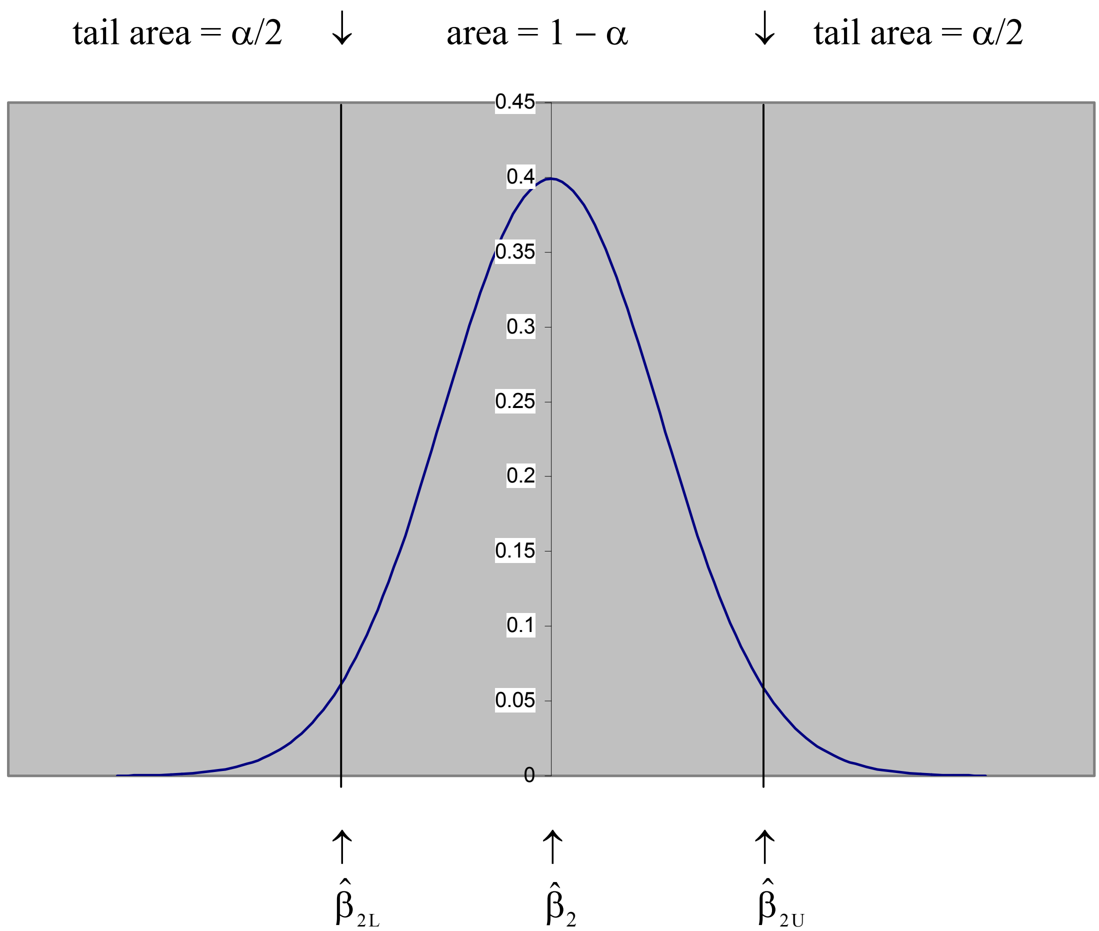
where at the (1 − α) confidence level, or 100(1 − α) percent confidence level,

$\hat{\beta}_{2L} = \hat{\beta}_2 - t_{\alpha/2}[N-2]\hat{se}(\hat{\beta}_2)$ = the ***lower*** **100(1 − α) percent confidence limit for β_2**

and

$\hat{\beta}_{2U} = \hat{\beta}_2 + t_{\alpha/2}[N-2]\hat{se}(\hat{\beta}_2)$ = the ***upper*** **100(1 − α) percent confidence limit for β_2**

Two-sided (1 − α)-level confidence interval for β_2 is centered around $\hat{\beta}_2$



$\hat{\beta}_{2L} = \hat{\beta}_2 - t_{\alpha/2}[N-2]\,\hat{se}(\hat{\beta}_2)$ = the ***lower*** **(1 − α)-level confidence limit for β_2**

$\hat{\beta}_{2U} = \hat{\beta}_2 + t_{\alpha/2}[N-2]\,\hat{se}(\hat{\beta}_2)$ = the ***upper*** **(1 − α)-level confidence limit for β_2**

3.2 Confidence Interval for β_1: Derivation

The confidence interval (or interval estimator) for the intercept coefficient β_1 is derived, interpreted, and constructed in exactly the same way as the confidence interval for the slope coefficient β_2.

1. The **two-sided (1 − α)-level confidence interval for β_1** is implicitly defined by the probability statement

$$\Pr\left(-t_{\alpha/2}[N-2] \le t(\hat{\beta}_1) \le t_{\alpha/2}[N-2]\right) = 1-\alpha \qquad \textbf{(8)}$$

where

$1-\alpha$ = the ***confidence*** **level** attached to the confidence interval;
α = the ***significance*** **level**, where $0 < \alpha < 1$;
$t_{\alpha/2}[N-2]$ = the ***critical value*** of the t-distribution with (N−2) degrees of freedom at the $\alpha/2$ (or $100(\alpha/2)$ percent) significance level;

and $t(\hat{\beta}_1)$ is the t-statistic for $\hat{\beta}_1$ given by

$$t(\hat{\beta}_1) = \frac{\hat{\beta}_1 - \beta_1}{\sqrt{\hat{Var}(\hat{\beta}_1)}} = \frac{\hat{\beta}_1 - \beta_1}{\hat{se}(\hat{\beta}_1)}. \qquad (9)$$

2. The double inequality inside the brackets in probability statement (8) can be expressed in terms of β_1 rather than $t(\hat{\beta}_1)$, using a derivation analogous to that used in deriving the confidence interval for β_2.

RESULT: The probability statement (8) can be written as

$$\Pr\left(\hat{\beta}_1 - t_{\alpha/2}[N-2]\,\hat{se}(\hat{\beta}_1) \le \beta_1 \le \hat{\beta}_1 + t_{\alpha/2}[N-2]\,\hat{se}(\hat{\beta}_1)\right) = 1-\alpha\,. \qquad \textbf{(10)}$$

The **two-sided (1 − α)-level confidence interval for β_1** can therefore be written as

$$\hat{\beta}_1 - t_{\alpha/2}[N-2]\,\hat{se}(\hat{\beta}_1) \le \beta_1 \le \hat{\beta}_1 + t_{\alpha/2}[N-2]\,\hat{se}(\hat{\beta}_1)$$

or more compactly as

$$\hat{\beta}_1 \pm t_{\alpha/2}[N-2]\,\hat{se}(\hat{\beta}_1) \quad \text{or} \quad \left[\hat{\beta}_1 - t_{\alpha/2}[N-2]\,\hat{se}(\hat{\beta}_1),\ \hat{\beta}_1 + t_{\alpha/2}[N-2]\,\hat{se}(\hat{\beta}_1)\right]$$

where at the (1 − α) confidence level, or 100(1 − α) percent confidence level,

$\hat{\beta}_{1L} = \hat{\beta}_1 - t_{\alpha/2}[N-2]\,\hat{se}(\hat{\beta}_1) =$ the ***lower*** **100(1 − α) percent confidence limit for β_1**

and

$\hat{\beta}_{1U} = \hat{\beta}_1 + t_{\alpha/2}[N-2]\,\hat{se}(\hat{\beta}_1) =$ the ***upper*** **100(1 − α) percent confidence limit for β_1**

3.3 Procedure for Computing Confidence Intervals

Consider the problem of computing a confidence interval for the slope coefficient β_2. Recall that the **two-sided (1 − α)-level confidence interval for β_2** is given by the double inequality

$$\hat{\beta}_2 - t_{\alpha/2}[N-2]\hat{se}(\hat{\beta}_2) \le \beta_2 \le \hat{\beta}_2 + t_{\alpha/2}[N-2]\hat{se}(\hat{\beta}_2).$$

<u>Step 1</u>: After estimating the PRE (1) by OLS, retrieve from the estimation results the OLS estimate $\hat{\beta}_2$ of β_2 and the estimated standard error $\hat{se}(\hat{\beta}_2)$.

<u>Step 2</u>: Select the value of the confidence level (1 − α), which amounts to selecting the value of α. Although the choice of confidence level is essentially arbitrary, the values most commonly used in practice are:

$\alpha = 0.01 \Rightarrow (1 - \alpha) = 0.99$, i.e., the $100(1 - \alpha) = 100(0.99) = 99$ percent confidence level;

$\alpha = 0.05 \Rightarrow (1 - \alpha) = 0.95$, i.e., the $100(1 - \alpha) = 100(0.95) = 95$ percent confidence level;

$\alpha = 0.10 \Rightarrow (1 - \alpha) = 0.90$, i.e., the $100(1 - \alpha) = 100(0.90) = 90$ percent confidence level.

<u>Step 3</u>: Obtain the value of $t_{\alpha/2}[N-2]$, the α/2 critical value of the t-distribution with N−2 degrees of freedom, either from statistical tables of the t-distribution or from a computer software program.

<u>Step 4</u>: Use the values of $\hat{\beta}_2$, $\hat{se}(\hat{\beta}_2)$, and $t_{\alpha/2}[N-2]$ to compute the upper and lower 100(1 − α) percent confidence limits for β_2:

$\hat{\beta}_{2U} = \hat{\beta}_2 + t_{\alpha/2}[N-2]\hat{se}(\hat{\beta}_2)$ = ***upper*** **100(1 − α)% confidence limit for β_2**;

$\hat{\beta}_{2L} = \hat{\beta}_2 - t_{\alpha/2}[N-2]\hat{se}(\hat{\beta}_2)$ = ***lower*** **100(1 − α)% confidence limit for β_2**.

4. Determinants of the Confidence Intervals for β_1 and β_2

Consider for example the **two-sided 100(1 − α)% confidence interval for β_2**:

$$\hat{\beta}_2 - t_{\alpha/2}[N-2]\hat{se}(\hat{\beta}_2) \le \beta_2 \le \hat{\beta}_2 + t_{\alpha/2}[N-2]\hat{se}(\hat{\beta}_2)$$

or

$$\left[\hat{\beta}_2 - t_{\alpha/2}[N-2]\hat{se}(\hat{\beta}_2),\ \hat{\beta}_2 + t_{\alpha/2}[N-2]\hat{se}(\hat{\beta}_2)\right]$$

The **two-sided confidence interval for β_2 is *wider***

(1) the ***greater* the value of $\hat{se}(\hat{\beta}_2)$, the estimated standard error of $\hat{\beta}_2$**, i.e., the ***less* precise is the estimate of β_2**;

(2) the ***greater* the critical value $t_{\alpha/2}[N-2]$**, i.e., the ***greater* the chosen value of the confidence level (1 − α)** for the given sample size N.

Explanation: Given sample size N, **the value of $t_{\alpha/2}[N-2]$ is *negatively* related to the value of** α, and so is ***positively* related to the value of (1 − α)**.

Example: Suppose sample size **N = 30**, so that the degrees-of-freedom **N-2 = 28**. Then from a table of percentage points for the t-distribution, we obtain the following values of $t_{\alpha/2}[N-2] = t_{\alpha/2}[28]$ for different values of α:

$\alpha = 0.01 \Rightarrow (1-\alpha) = 0.99$: $\alpha/2 = 0.005$ and $t_{0.005}[28] = 2.763$;

$\alpha = 0.02 \Rightarrow (1-\alpha) = 0.98$: $\alpha/2 = 0.01$ and $t_{0.01}[28] = 2.467$;

$\alpha = 0.05 \Rightarrow (1-\alpha) = 0.95$: $\alpha/2 = 0.025$ and $t_{0.025}[28] = 2.048$;

$\alpha = 0.10 \Rightarrow (1-\alpha) = 0.90$: $\alpha/2 = 0.05$ and $t_{0.05}[28] = 1.701$.

Note that higher values of (1 − α) -- i.e., higher confidence levels -- correspond to higher critical values of $t_{\alpha/2}[28]$.

5. Two-Sided 100(1 − α)% Confidence Intervals for β_2: Examples

Two-Sided 100(1 − α) Percent Confidence Interval for β_j: Formulas

In general, the **two-sided 100(1-α) percent confidence interval for regression coefficient β_j** is:

$$\left[\hat{\beta}_j - t_{\alpha/2}[N-k]\hat{se}(\hat{\beta}_j),\ \hat{\beta}_j + t_{\alpha/2}[N-k]\hat{se}(\hat{\beta}_j)\right]$$

where

$\hat{\beta}_{jU} = \hat{\beta}_j + t_{\alpha/2}[N-k]\hat{se}(\hat{\beta}_j)$ = ***upper*** **100(1−α) % confidence limit for β_2**

$\hat{\beta}_{jL} = \hat{\beta}_j - t_{\alpha/2}[N-k]\hat{se}(\hat{\beta}_j)$ = ***lower*** **100(1−α) % confidence limit for β_2**

DATA: **auto1.dta** A sample of 74 cars sold in North America in 1978.

MODEL: $\textbf{price}_i = \beta_1 + \beta_2\,\textbf{weight}_i + \textbf{u}_i$ **(i = 1, ..., N) N = 74**

Compute the two-sided 95% confidence interval for β_2

```
. regress price weight

  Source |       SS       df       MS              Number of obs =      74
---------+------------------------------           F(  1,    72) =   29.42
   Model |   184233937     1   184233937           Prob > F      =  0.0000
Residual |   450831459    72  6261548.04           R-squared     =  0.2901
---------+------------------------------           Adj R-squared =  0.2802
   Total |   635065396    73  8699525.97           Root MSE      =  2502.3

------------------------------------------------------------------------------
   price |      Coef.   Std. Err.       t     P>|t|       [95% Conf. Interval]
---------+--------------------------------------------------------------------
  weight |   2.044063   .3768341      5.424   0.000       1.292858    2.795268
   _cons |  -6.707353    1174.43     -0.006   0.995       -2347.89    2334.475
------------------------------------------------------------------------------

. display invttail(72, 0.025)
1.9934635
```

$\hat{\beta}_2 = \mathbf{2.0441}$ $\qquad$ $\hat{se}(\hat{\beta}_2) = \mathbf{0.37683}$

$N = 74 \qquad k = 2 \qquad \mathbf{N - k = 74 - 2 = 72}$

$1 - \alpha = 0.95 \quad \Rightarrow \quad \alpha = 1 - 0.95 = 0.05 \quad \Rightarrow \quad \boldsymbol{\alpha/2 = 0.05/2 = 0.025}$

$t_{\alpha/2}[N-2] = t_{0.025}[72] = \mathbf{1.9935}$

$t_{\alpha/2}[N-2]\hat{se}(\hat{\beta}_2) = t_{0.025}[N-2]\hat{se}(\hat{\beta}_2) = 1.9935(0.37683) = \mathbf{0.75121}$

$\hat{\beta}_{2U} = \hat{\beta}_2 + t_{\alpha/2}[N-2]\hat{se}(\hat{\beta}_2) = \mathbf{2.0441 + 0.75121} = 2.79531 = \underline{\mathbf{2.795}}$

$\hat{\beta}_{2L} = \hat{\beta}_2 - t_{\alpha/2}[N-2]\hat{se}(\hat{\beta}_2) = \mathbf{2.0441 - 0.75121} = 1.29289 = \underline{\mathbf{1.293}}$

***<u>Result</u>*: The two-sided *95%* confidence interval for β_2 is [1.293, 2.795].**

Compute the two-sided 99% confidence interval for β_2

```
. regress price weight, level(99)

      Source |       SS       df       MS              Number of obs =      74
-------------+------------------------------           F(  1,    72) =   29.42
       Model |   184233937     1   184233937           Prob > F      =  0.0000
    Residual |   450831459    72  6261548.04           R-squared     =  0.2901
-------------+------------------------------           Adj R-squared =  0.2802
       Total |   635065396    73  8699525.97           Root MSE      =  2502.3

------------------------------------------------------------------------------
       price |      Coef.   Std. Err.       t     P>|t|     [99% Conf. Interval]
-------------+----------------------------------------------------------------
      weight |   2.044063   .3768341      5.42   0.000     1.047015     3.04111
       _cons |  -6.707353    1174.43     -0.01   0.995    -3114.074    3100.659
------------------------------------------------------------------------------

. display invttail(72, 0.005)
2.6458519

. scalar b2u99 = _b[weight] + 2.6459*_se[weight]

. scalar b2l99 = _b[weight] - 2.6459*_se[weight]

. scalar list b2u99 b2l99
     b2u99 =   3.041128
     b2l99 =  1.0469972
```

$\hat{\beta}_2 = \mathbf{2.0441}$ $\quad\quad$ $\hat{se}(\hat{\beta}_2) = \mathbf{0.37683}$

$N = 74 \quad\quad k = 2 \quad\quad \mathbf{N - k = 74 - 2 = 72}$

$1 - \alpha = 0.99 \quad \Rightarrow \quad \alpha = 1 - 0.99 = 0.01 \quad \Rightarrow \quad \mathbf{\alpha/2 = 0.01/2 = 0.005}$

$t_{\alpha/2}[N-2] = t_{0.005}[72] = \mathbf{2.6459}$

$t_{\alpha/2}[N-2]\,\hat{se}(\hat{\beta}_2) = t_{0.005}[N-2]\,\hat{se}(\hat{\beta}_2) = 2.6459(0.37683) = \mathbf{0.99705}$

$\hat{\beta}_{2U} = \hat{\beta}_2 + t_{0.005}[N-2]\,\hat{se}(\hat{\beta}_2) = \mathbf{2.0441 + 0.99705} = 3.04115 = \underline{\mathbf{3.041}}$

$\hat{\beta}_{2L} = \hat{\beta}_2 - t_{0.005}[N-2]\,\hat{se}(\hat{\beta}_2) = \mathbf{2.0441 - 0.99705} = 1.04705 = \underline{\mathbf{1.047}}$

***<u>Result</u>*:** **The two-sided *99%* confidence interval for β_2 is [1.047, 3.041].**