QUEEN'S UNIVERSITY AT KINGSTON
Department of Economics

ECONOMICS 351* - Section A

<u>Introductory Econometrics</u>

Fall Term 2000 **<u>MID-TERM EXAM ANSWERS</u>** M.G. Abbott

<u>DATE</u>: **Monday October 23, 2000.**

<u>TIME</u>: **80 minutes; 2:30 p.m. - 3:50 p.m.**

<u>INSTRUCTIONS</u>: The exam consists of **<u>FIVE</u> (5)** questions. Students are required to answer **ALL FIVE (5)** questions.

Answer all questions in the exam booklets provided. Be sure your ***name*** and ***student number*** are printed clearly on the front of all exam booklets used.

Do not write answers to questions on the front page of the first exam booklet.

Please label clearly each of your answers in the exam booklets with the appropriate number and letter.

Please write legibly.

<u>MARKING</u>: The marks for each question are indicated in parentheses immediately above each question. **Total marks** for the exam **equal 100**.

<u>QUESTIONS</u>: **Answer ALL <u>FIVE</u> questions.**

All questions pertain to the simple (two-variable) linear regression model for which the population regression equation can be written in conventional notation as:

$$Y_i = \beta_1 + \beta_2 X_i + u_i \qquad \textbf{(1)}$$

where Y_i and X_i are observable variables, β_1 and β_2 are unknown (constant) regression coefficients, and u_i is an unobservable random error term. The Ordinary Least Squares (OLS) sample regression equation corresponding to regression equation (1) is

$$Y_i = \hat{\beta}_1 + \hat{\beta}_2 X_i + \hat{u}_i \qquad (i = 1, ..., N) \qquad \textbf{(2)}$$

where $\hat{\beta}_1$ is the OLS estimator of the intercept coefficient β_1, $\hat{\beta}_2$ is the OLS estimator of the slope coefficient β_2, $\hat{u}_i$ is the OLS residual for the i-th sample observation, and N is sample size (the number of observations in the sample).

(15 marks)

1. State the Ordinary Least Squares (OLS) estimation criterion. State the OLS normal equations. Derive the OLS normal equations from the OLS estimation criterion.

<u>ANSWER</u>:

(3 marks)

- State the Ordinary Least Squares (OLS) estimation criterion. **(3 marks)**

The OLS coefficient estimators are **those formulas or expressions for $\hat{\beta}_1$ and $\hat{\beta}_2$ that <u>minimize</u> the sum of <u>squared</u> residuals RSS** for any given sample of size N.

The **OLS estimation criterion** is therefore:

$$\underset{\{\hat{\beta}_j\}}{\text{Minimize}}\ \text{RSS}\left(\hat{\beta}_1, \hat{\beta}_2\right) = \sum_{i=1}^{N} \hat{u}_i^2 = \sum_{i=1}^{N} \left(Y_i - \hat{\beta}_1 - \hat{\beta}_2 X_i\right)^2$$

(4 marks)

- State the OLS normal equations. **(4 marks)**

The ***first*** **OLS normal equation** can be written in *any one* of the following forms:

$$\begin{aligned} \Sigma_i\, Y_i - N\hat{\beta}_1 - \hat{\beta}_2\, \Sigma_i\, X_i &= 0 \\ -N\hat{\beta}_1 - \hat{\beta}_2\, \Sigma_i\, X_i &= -\Sigma_i\, Y_i \\ N\hat{\beta}_1 + \hat{\beta}_2\, \Sigma_i\, X_i &= \Sigma_i\, Y_i \end{aligned} \qquad \textbf{(N1)}$$

The ***second*** **OLS normal equation** can be written in *any one* of the following forms:

$$\begin{aligned} \Sigma_i\, X_i Y_i - \hat{\beta}_1\, \Sigma_i\, X_i - \hat{\beta}_2\, \Sigma_i\, X_i^2 &= 0 \\ -\hat{\beta}_1\, \Sigma_i\, X_i - \hat{\beta}_2\, \Sigma_i\, X_i^2 &= -\Sigma_i\, X_i Y_i \\ \hat{\beta}_1\, \Sigma_i\, X_i + \hat{\beta}_2\, \Sigma_i\, X_i^2 &= \Sigma_i\, X_i Y_i \end{aligned} \qquad \textbf{(N2)}$$

Question 1 (continued)

(8 marks)

- Show how the OLS normal equations are derived from the OLS estimation criterion.

(4 marks)

Step 1: **Partially differentiate** the RSS $\left(\hat{\beta}_1, \hat{\beta}_2\right)$ function with respect to $\hat{\beta}_1$ and $\hat{\beta}_2$, using

$$\hat{u}_i = Y_i - \hat{\beta}_1 - \hat{\beta}_2 X_i \qquad \Rightarrow \qquad \frac{\partial \hat{u}_i}{\partial \hat{\beta}_1} = -1 \qquad \text{and} \qquad \frac{\partial \hat{u}_i}{\partial \hat{\beta}_2} = -X_i \,.$$

$$\frac{\partial RSS}{\partial \hat{\beta}_1} = \sum_{i=1}^{N} 2\hat{u}_i \left(\frac{\partial \hat{u}_i}{\partial \hat{\beta}_1} \right) = \sum_{i=1}^{N} 2\,\hat{u}_i (-1) = -2 \sum_{i=1}^{N} \hat{u}_i = -2 \sum_{i=1}^{N} \left(Y_i - \hat{\beta}_1 - \hat{\beta}_2 X_i \right) \tag{1}$$

$$\frac{\partial RSS}{\partial \hat{\beta}_2} = \sum_{i=1}^{N} 2\hat{u}_i \left(\frac{\partial \hat{u}_i}{\partial \hat{\beta}_2} \right) = \sum_{i=1}^{N} 2\hat{u}_i (-X_i) = -2 \sum_{i=1}^{N} X_i \hat{u}_i$$

$$= -2 \sum_{i=1}^{N} X_i \left(Y_i - \hat{\beta}_1 - \hat{\beta}_2 X_i \right) \qquad \text{since } \hat{u}_i = Y_i - \hat{\beta}_1 - \hat{\beta}_2 X_i \tag{2}$$

$$= -2 \sum_{i=1}^{N} \left(X_i Y_i - \hat{\beta}_1 X_i - \hat{\beta}_2 X_i^2 \right)$$

(4 marks)

Step 2: **Obtain the first-order conditions (FOCs)** for a minimum of the RSS function by **setting the partial derivatives (1) and (2) *equal to zero*** and then dividing each equation by –2 and re-arranging:

$$\frac{\partial RSS}{\partial \hat{\beta}_1} = 0 \quad \Rightarrow \quad -2 \sum_i \hat{u}_i = 0 \quad \Rightarrow \quad -2 \sum_i \left(Y_i - \hat{\beta}_1 - \hat{\beta}_2 X_i \right) = 0$$

$$\Rightarrow \quad \sum_i \left(Y_i - \hat{\beta}_1 - \hat{\beta}_2 X_i \right) = 0$$

$$\Rightarrow \quad \sum_i Y_i - N\hat{\beta}_1 - \hat{\beta}_2 \sum_i X_i = 0$$

$$\Rightarrow \quad \sum_i Y_i = N\hat{\beta}_1 + \hat{\beta}_2 \sum_i X_i \tag{N1}$$

$$\frac{\partial RSS}{\partial \hat{\beta}_2} = 0 \quad \Rightarrow \quad -2 \sum_i X_i \hat{u}_i = 0 \quad \Rightarrow \quad -2 \sum_i X_i \left(Y_i - \hat{\beta}_1 - \hat{\beta}_2 X_i \right) = 0$$

$$\Rightarrow \quad \sum_i X_i \left(Y_i - \hat{\beta}_1 - \hat{\beta}_2 X_i \right) = 0$$

$$\Rightarrow \quad \sum_i \left(X_i Y_i - \hat{\beta}_1 X_i - \hat{\beta}_2 X_i^2 \right) = 0$$

$$\Rightarrow \quad \sum_i X_i Y_i - \hat{\beta}_1 \sum_i X_i - \hat{\beta}_2 \sum_i X_i^2 = 0$$

$$\Rightarrow \quad \sum_i X_i Y_i = \hat{\beta}_1 \sum_i X_i + \hat{\beta}_2 \sum_i X_i^2 \tag{N2}$$

(15 marks)

2. Answer parts (a), (b) and (c) below.

(3 marks)

(a) Explain the meaning of the following statement: The estimator $\hat{\beta}_2$ is an unbiased estimator of the slope coefficient β_2. **(3 marks)**

The mean of the sampling distribution of $\hat{\beta}_2$ is equal to β_2: $E(\hat{\beta}_2) = \beta_2$.
(An appropriate diagram would also be sufficient.)

(4 marks)

(b) Show that the OLS slope coefficient estimator $\hat{\beta}_2$ is a *linear* function of the Y_i sample values.

$$\hat{\beta}_2 = \frac{\sum_i x_i y_i}{\sum_i x_i^2} = \frac{\sum_i x_i (Y_i - \bar{Y})}{\sum_i x_i^2} = \frac{\sum_i x_i Y_i}{\sum_i x_i^2} - \frac{\bar{Y} \sum_i x_i}{\sum_i x_i^2}$$

$$= \frac{\sum_i x_i Y_i}{\sum_i x_i^2} \qquad \text{because } \sum_i x_i = 0 \qquad \textbf{(4 marks)}$$

$$= \sum_i k_i Y_i \qquad \text{where } k_i \equiv \frac{x_i}{\sum_i x_i^2}.$$

(8 marks)

(c) Stating explicitly all required assumptions, prove that the OLS slope coefficient estimator $\hat{\beta}_2$ is an unbiased estimator of the slope coefficient β_2.

(1) **Substitute for Y_i** the expression $Y_i = \beta_1 + \beta_2 X_i + u_i$ from the population regression equation (or PRE). **(4 marks)**

$$\begin{aligned}
\hat{\beta}_2 &= \sum_i k_i Y_i \\
&= \sum_i k_i (\beta_1 + \beta_2 X_i + u_i) && \text{since } Y_i = \beta_1 + \beta_2 X_i + u_i \text{ by assumption (A1)} \\
&= \sum_i (\beta_1 k_i + \beta_2 k_i X_i + k_i u_i) \\
&= \beta_1 \sum_i k_i + \beta_2 \sum_i k_i X_i + \sum_i k_i u_i \\
&= \beta_2 + \sum_i k_i u_i, && \text{since } \sum_i k_i = 0 \text{ and } \sum_i k_i X_i = 1
\end{aligned}$$

(2) Now **take expectations** of the above expression for $\hat{\beta}_2$: **(4 marks)**

$$\begin{aligned}
E(\hat{\beta}_2) &= E(\beta_2) + E[\textstyle\sum_i k_i u_i] \\
&= \beta_2 + \textstyle\sum_i k_i E(u_i) && \text{since } \beta_2 \text{ is a constant and the } k_i \text{ are nonstochastic} \\
&= \beta_2 + \textstyle\sum_i k_i 0 && \text{since } E(u_i) = 0 \text{ by assumption (A2)} \\
&= \beta_2.
\end{aligned}$$

(10 marks)

3. Explain what is meant by each of the following statements about the estimator $\hat{\theta}$ of the population parameter θ, and explain the difference between the two statements.

(a) $\hat{\theta}$ is a minimum variance estimator of θ.
(b) $\hat{\theta}$ is an efficient estimator of θ.

<u>ANSWER</u>:

(4 marks)

- (a) $\hat{\theta}$ is a minimum variance estimator of θ. **(4 marks)**

  The variance of the estimator $\hat{\theta}$ is ***smaller than*** the variance of *any other* estimator of the parameter θ.

  If $\tilde{\theta}$ is any other estimator of θ, then $\hat{\theta}$ is a ***minimum variance*** **estimator** of θ if

  $$\mathrm{Var}(\hat{\theta}) \le \mathrm{Var}(\tilde{\theta}).$$

(4 marks)

- (b) $\hat{\theta}$ is an efficient estimator of θ. **(4 marks)**

  The estimator $\hat{\theta}$ is an efficient estimator if it is ***unbiased*** and has ***smaller variance*** than *any other* ***unbiased*** estimator of the parameter θ.

  If $\tilde{\theta}$ is *any other* ***unbiased*** *estimator* of θ, then $\hat{\theta}$ is an ***efficient*** **estimator** of θ if

  $$\mathrm{Var}(\hat{\theta}) \le \mathrm{Var}(\tilde{\theta}) \qquad \text{where } E(\hat{\theta}) = \theta \text{ and } E(\tilde{\theta}) = \theta.$$

(2 marks)

- The important difference between statements (a) and (b) is that **an *efficient* estimator must be *unbiased*** whereas a minimum variance estimator may be biased or unbiased.

  An ***efficient*** **estimator** is the ***minimum variance*** **estimator in the class of all *unbiased* estimators** of the parameter θ.

(10 marks)

4. State the error normality assumption. State and explain the implications of the error normality assumption for (1) the distribution of the Y_i sample values and (2) the sampling distribution of the OLS slope coefficient estimator $\hat{\beta}_2$.

ANSWER:

(3 marks)

- **Statement of Error Normality Assumption (A9):** The random error terms u_i are **independently and identically distributed (iid)** as the **normal distribution** with **zero mean** and **constant variance σ^2**. *OR* The random error terms u_i are **normally and independently distributed (NID)** with **zero mean** and **constant variance σ^2**.

  the u_i are iid as $N(0, \sigma^2)$ for all i. *OR* the u_i are $NID(0, \sigma^2)$.

(7 marks)

- **Two Implications of (A9):** Follow from the ***linearity property* of the normal distribution**.

- ***Linearity property* of the normal distribution**: any *linear* function of a normally distributed random variable is itself normally distributed. **(1 mark)**

- **Two** implications of error normality assumption (A9): **(3 marks each)**

  (3 marks)

  (1) The **Y_i values are normally distributed**: Y_i are $NID(\beta_1 + \beta_2 X_i, \sigma^2)$ **(2 marks)**

  Why? Because the PRE states that **the Y_i values are *linear* functions of the u_i**: $Y_i = \beta_1 + \beta_2 X_i + u_i$. **(1 mark)**

  (3 marks)

  (2) The OLS slope coefficient estimator **$\hat{\beta}_2$ is normally distributed: $\hat{\beta}_2 \sim N(\beta_2, Var(\hat{\beta}_2))$**

  where $Var(\hat{\beta}_2) = \dfrac{\sigma^2}{\sum_i x_i^2}$. **(2 marks)**

  Why? Because **$\hat{\beta}_2$ can be written as a *linear* function of the Y_i values**: $\hat{\beta}_2 = \sum_i k_i Y_i$.
  (1 mark)

(50 marks)

5. A researcher is using data for a sample of 526 paid workers to investigate the relationship between hourly wage rates Y_i (measured in dollars per hour) and years of formal education X_i (measured in years). Preliminary analysis of the sample data produces the following sample information:

$$N = 526 \qquad \sum_{i=1}^{N} Y_i = 3101.35 \qquad \sum_{i=1}^{N} X_i = 6608.0 \qquad \sum_{i=1}^{N} Y_i^2 = 25446.29$$

$$\sum_{i=1}^{N} X_i^2 = 87040.0 \qquad \sum_{i=1}^{N} X_i Y_i = 41140.65 \qquad \sum_{i=1}^{N} x_i y_i = 2179.204$$

$$\sum_{i=1}^{N} y_i^2 = 7160.414 \qquad \sum_{i=1}^{N} x_i^2 = 4025.43 \qquad \sum_{i=1}^{N} \hat{u}_i^2 = 5980.682$$

where $x_i \equiv X_i - \overline{X}$ and $y_i \equiv Y_i - \overline{Y}$ for i = 1, ..., N. Use the above sample information to answer all the following questions. **Show explicitly all formulas and calculations**.

(10 marks)

(a) Use the above information to compute OLS estimates of the intercept coefficient β_1 and the slope coefficient β_2.

- $\hat{\beta}_2 = \dfrac{\sum_i x_i y_i}{\sum_i x_i^2} = \dfrac{2179.204}{4025.43} = 0.5413593 = \underline{\mathbf{0.54136}}$ **(5 marks)**

- $\hat{\beta}_1 = \overline{Y} - \hat{\beta}_2 \overline{X}$

  $\overline{Y} = \dfrac{\sum_{i=1}^{N} Y_i}{N} = \dfrac{3101.35}{526} = 5.896103$ and $\overline{X} = \dfrac{\sum_{i=1}^{N} X_i}{N} = \dfrac{6608.0}{526} = 12.56274$

  Therefore

  $\hat{\beta}_1 = \overline{Y} - \beta_2 \overline{X} = 5.896103 - (0.54136)(12.56274) = 5.896103 - 6.800965 = \underline{\mathbf{-0.90486}}$

  (5 marks)

(5 marks)

(b) Interpret the slope coefficient estimate you calculated in part (a) -- i.e., explain what the numeric value you calculated for $\hat{\beta}_2$ means.

<u>Note</u>: $\hat{\beta}_2$ = **0.54136**. Y_i is measured in ***dollars per hour***, and X_i is measured in ***years***.

The estimate **0.54136** of β_2 means that a ***one-year increase*** **(decrease) in years of formal education** X_i is associated on average with **an increase (decrease) in hourly wage rate of *0.54136 dollars* per hour**, or ***54.136 cents*** **per hour**.

(5 marks)

(c) Calculate an estimate of σ^2, the error variance.

$$\hat{\sigma}^2 = \frac{RSS}{N-2} = \frac{\sum_{i=1}^{N}\hat{u}_i^2}{N-2} = \frac{5980.682}{526-2} = \frac{5980.682}{524} = 11.413515 = \underline{\mathbf{11.4135}}$$

(6 marks)

(d) Compute the value of R^2, the coefficient of determination for the estimated OLS sample regression equation. Briefly explain what the calculated value of R^2 means.

(1) $ESS = TSS - RSS = \sum_{i=1}^{N} y_i^2 - \sum_{i=1}^{N}\hat{u}_i^2 = 7160.414 - 5980.682 = 1179.732$

$$R^2 = \frac{ESS}{TSS} = \frac{\sum_{i=1}^{N}\hat{y}_i^2}{\sum_{i=1}^{N} y_i^2} = \frac{1179.732}{7160.414} = 0.1647575 = \underline{\mathbf{0.1648}}$$

or **(4 marks)**

(2) $$R^2 = 1 - \frac{RSS}{TSS} = 1 - \frac{\sum_{i=1}^{N}\hat{u}_i^2}{\sum_{i=1}^{N} y_i^2} = 1 - \frac{5980.682}{7160.414} = 1 - 0.8352 = \underline{\mathbf{0.1648}}$$

Interpretation of R^2 = 0.1648: The value of 0.1648 indicates that **16.48 percent of the total sample (or observed) variation in Y_i** (hourly wage rates) **is *attributable to*, or *explained by*, the regressor X_i** (years of formal education). **(2 marks)**

(12 marks)

(e) Perform a test of the null hypothesis H_0: $\beta_2 = 0$ against the alternative hypothesis H_1: $\beta_2 \neq 0$ at the 5% significance level (i.e., for significance level $\alpha = 0.05$). Show how you calculated the test statistic. State the decision rule you use, and the inference you would draw from the test. Briefly explain what the test outcome means.

H_0: $\beta_2 = 0$
H_1: $\beta_2 \neq 0$ a ***two-sided* alternative hypothesis** $\Rightarrow$ a ***two-tailed* test**

- Test statistic is $t(\hat{\beta}_2) = \dfrac{\hat{\beta}_2 - \beta_2}{\hat{se}(\hat{\beta}_2)} \sim t[N-2]$. (1)
- Calculate the estimated standard error of $\hat{\beta}_2$:

$$V\hat{a}r(\hat{\beta}_2) = \frac{\hat{\sigma}^2}{\sum_i x_i^2} = \frac{11.413515}{4025.43} = 0.002835352$$

$$\hat{se}(\hat{\beta}_2) = \sqrt{V\hat{a}r(\hat{\beta}_2)} = \sqrt{0.002835352} = \mathbf{0.053248}.$$ **(1 mark)**

- Calculate the ***sample value* of the t-statistic** (1) under H_0: set $\beta_2 = 0$ in (1).

$$t_0(\hat{\beta}_2) = \frac{\hat{\beta}_2 - \beta_2}{\hat{se}(\hat{\beta}_2)} = \frac{0.54136 - 0.0}{0.053248} = \frac{0.54136}{0.053248} = 10.16676 = \underline{\mathbf{10.167}}$$ **(3 marks)**

- **Null distribution** of $t_0(\hat{\beta}_2)$ is **t[N – 2] = t[524]**. **(1 mark)**

<u>**Decision Rule**</u> **-- Formulation 1:** At significance level α, **(2 marks)**

- ***reject* H₀** if $\left|t_0(\hat{\beta}_2)\right| > t_{\alpha/2}[524]$,
  i.e., if either (1) $t_0(\hat{\beta}_2) > t_{\alpha/2}[524]$ or (2) $t_0(\hat{\beta}_2) < -t_{\alpha/2}[524]$;

- ***retain* H₀** if $\left|t_0(\hat{\beta}_2)\right| \le t_{\alpha/2}[524]$, i.e., if $-t_{\alpha/2}[524] \le t_0(\hat{\beta}_2) \le t_{\alpha/2}[524]$.

Critical value of t[524]-distribution: from t-table, use **df = ∞**

- ***two-tailed* 5 percent critical value** = $t_{\alpha/2}[524] = t_{0.025}[\infty] = \mathbf{1.960}$. **(1 mark)**

Question 5(e) -- continued

<u>Inference</u>:

- At **5 percent significance level**, i.e., for $\alpha = 0.05$, **(3 marks)**

  $\left|t_0(\hat{\beta}_2)\right| = 10.167 > 1.960 = t_{0.025}[\infty] \quad \Rightarrow$ ***reject*** $\mathbf{H_0}$ vs. H_1 **at 5 percent level**.

- **<u>Inference</u>: At the 5% significance level**, the null hypothesis $\beta_2 = 0$ is rejected in favour of the alternative hypothesis $\beta_2 \neq 0$.

Meaning of test outcome: Rejection of the null hypothesis $\beta_2 = 0$ in favour of the alternative hypothesis $\beta_2 \neq 0$ means that **the sample evidence favours the existence of a relationship between *wage rates* and *years of education***. **(1 mark)**

(12 marks)

(f) Compute the two-sided 95% confidence interval for the slope coefficient β_2. Would the two-sided 99% confidence interval be wider or narrower than the two-sided 95% confidence interval for β_2? Why?

The **two-sided (1 – α)-level,** or **100(1 – α) percent, confidence interval for β₂** is computed as

$$\hat{\beta}_2 - t_{\alpha/2}[N-2]\,\hat{se}(\hat{\beta}_2) \le \beta_2 \le \hat{\beta}_2 + t_{\alpha/2}[N-2]\,\hat{se}(\hat{\beta}_2)$$ **(2 marks)**

where

- $\hat{\beta}_{2L} = \hat{\beta}_2 - t_{\alpha/2}[N-2]\hat{se}(\hat{\beta}_2)$ = the ***lower*** **100(1 – α)% confidence limit for β_2**
- $\hat{\beta}_{2U} = \hat{\beta}_2 + t_{\alpha/2}[N-2]\hat{se}(\hat{\beta}_2)$ = the ***upper*** **100(1 – α)% confidence limit for β_2**
- $t_{\alpha/2}[N-2]$ = the **α/2 critical value of the t-distribution with N–2 degrees of freedom**.

$\hat{\beta}_2$ = **0.54136** $\qquad \hat{se}(\hat{\beta}_2) = \sqrt{\hat{Var}(\hat{\beta}_2)} = \sqrt{0.002835352}$ = **0.053248**

$1 - \alpha = 0.95 \Rightarrow \alpha = 0.05 \Rightarrow$ **α/2 = 0.025**: $\qquad t_{\alpha/2}[N-2] = t_{0.025}[524]$ = **1.960**

$$t_{\alpha/2}[N-2]\,\hat{se}(\hat{\beta}_2) = t_{0.025}[524]\,\hat{se}(\hat{\beta}_2) = t_{0.025}[\infty]\,\hat{se}(\hat{\beta}_2) = 1.960(0.053248) = \mathbf{0.104366}$$

- **Lower 95% confidence limit for β₂** is: **(4 marks)**

$$\hat{\beta}_{2L} = \hat{\beta}_2 - t_{\alpha/2}[N-2]\hat{se}(\hat{\beta}_2) = \hat{\beta}_2 - t_{0.025}[\infty]\hat{se}(\hat{\beta}_2)$$
$$= 0.54136 - 1.960(0.053248) = 0.54136 - 0.104366 = 0.436994 = \underline{\mathbf{0.4370}}$$

- **Upper 95% confidence limit for β₂** is: **(4 marks)**

$$\hat{\beta}_{2U} = \hat{\beta}_2 + t_{\alpha/2}[N-2]\hat{se}(\hat{\beta}_2) = \hat{\beta}_2 + t_{0.025}[25]\hat{se}(\hat{\beta}_2)$$
$$= 0.54136 + 1.960(0.053248) = 0.54136 + 0.104366 = 0.645726 = \underline{\mathbf{0.6457}}$$

- **<u>Result</u>:** The **two-sided 95% confidence interval for β₂** is:

  [0.4370, 0.6457].

Question 5(f) -- continued

(2 marks)

- The **two-sided *99%* confidence interval** for β_2 would be ***wider than*** the **two-sided *95%* confidence interval**.

  Reason: The critical value of the t[524] = t[∞] distribution is greater for the confidence level $1 - \alpha = 0.99$ than for the confidence level $1 - \alpha = 0.95$.

  For the **two-sided 95% confidence interval**:

  $\mathbf{1 - \alpha = 0.95} \;\Rightarrow\; \alpha = 0.05 \;\Rightarrow\; \alpha/2 = 0.025$: $t_{\alpha/2}[N-2] = t_{0.025}[524]$ $\mathbf{= t_{0.025}[\infty] \; = 1.960}$

  For the **two-sided 99% confidence interval**:

  $\mathbf{1 - \alpha = 0.99} \;\Rightarrow\; \alpha = 0.01 \;\Rightarrow\; \alpha/2 = 0.005$: $t_{\alpha/2}[N-2] = t_{0.005}[524]$ $\mathbf{= t_{0.005}[\infty] \; = \; 2.576}$

Percentage Points of the t-Distribution

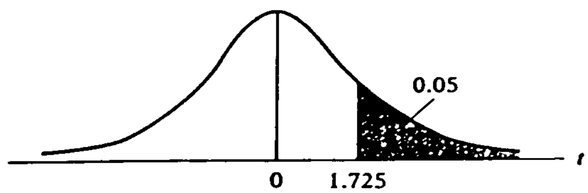
TABLE D.2
Percentage points of the t distribution

Example

$\Pr(t > 2.086) = 0.025$

$\Pr(t > 1.725) = 0.05$ for df = 20

$\Pr(|t| > 1.725) = 0.10$



df \ Pr	0.25 0.50	0.10 0.20	0.05 0.10	0.025 0.05	0.01 0.02	0.005 0.010	0.001 0.002
1	1.000	3.078	6.314	12.706	31.821	63.657	318.31
2	0.816	1.886	2.920	4.303	6.965	9.925	22.327
3	0.765	1.638	2.353	3.182	4.541	5.841	10.214
4	0.741	1.533	2.132	2.776	3.747	4.604	7.173
5	0.727	1.476	2.015	2.571	3.365	4.032	5.893
6	0.718	1.440	1.943	2.447	3.143	3.707	5.208
7	0.711	1.415	1.895	2.365	2.998	3.499	4.785
8	0.706	1.397	1.860	2.306	2.896	3.355	4.501
9	0.703	1.383	1.833	2.262	2.821	3.250	4.297
10	0.700	1.372	1.812	2.228	2.764	3.169	4.144
11	0.697	1.363	1.796	2.201	2.718	3.106	4.025
12	0.695	1.356	1.782	2.179	2.681	3.055	3.930
13	0.694	1.350	1.771	2.160	2.650	3.012	3.852
14	0.692	1.345	1.761	2.145	2.624	2.977	3.787
15	0.691	1.341	1.753	2.131	2.602	2.947	3.733
16	0.690	1.337	1.746	2.120	2.583	2.921	3.686
17	0.689	1.333	1.740	2.110	2.567	2.898	3.646
18	0.688	1.330	1.734	2.101	2.552	2.878	3.610
19	0.688	1.328	1.729	2.093	2.539	2.861	3.579
20	0.687	1.325	1.725	2.086	2.528	2.845	3.552
21	0.686	1.323	1.721	2.080	2.518	2.831	3.527
22	0.686	1.321	1.717	2.074	2.508	2.819	3.505
23	0.685	1.319	1.714	2.069	2.500	2.807	3.485
24	0.685	1.318	1.711	2.064	2.492	2.797	3.467
25	0.684	1.316	1.708	2.060	2.485	2.787	3.450
26	0.684	1.315	1.706	2.056	2.479	2.779	3.435
27	0.684	1.314	1.703	2.052	2.473	2.771	3.421
28	0.683	1.313	1.701	2.048	2.467	2.763	3.408
29	0.683	1.311	1.699	2.045	2.462	2.756	3.396
30	0.683	1.310	1.697	2.042	2.457	2.750	3.385
40	0.681	1.303	1.684	2.021	2.423	2.704	3.307
60	0.679	1.296	1.671	2.000	2.390	2.660	3.232
120	0.677	1.289	1.658	1.980	2.358	2.617	3.160
∞	0.674	1.282	1.645	1.960	2.326	2.576	3.090

Note: The smaller probability shown at the head of each column is the area in one tail; the larger probability is the area in both tails.

Source: From E. S. Pearson and H. O. Hartley, eds., *Biometrika Tables for Statisticians*, vol. 1, 3d ed., table 12, Cambridge University Press, New York, 1966. Reproduced by permission of the editors and trustees of *Biometrika*.