ECON 351* -- NOTE 21: Summary

Using Dummy Variables to Test for Coefficient Differences

- **Two Separate Regression Equations for Group 1 and Group 2**

Group 1 PRE: $Y_i = \alpha_1 + \alpha_2 X_{2i} + \alpha_3 X_{3i} + u_{1i}, \qquad i = 1, ..., N_1$ **... (2.1)**

Group 2 PRE: $Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + u_{2i}, \qquad i = 1, ..., N_2$ **... (2.2)**

- **Two Approaches to Testing for Inter-Group Coefficient Differences**

Both approaches involve using an ***F-test*** to perform a test of

H_0: $\alpha_1 = \beta_1$ *and* $\alpha_2 = \beta_2$ *and* $\alpha_3 = \beta_3$ (3 coefficient restrictions)

against

H_1: $\alpha_1 \neq \beta_1$ *and/or* $\alpha_2 \neq \beta_2$ *and/or* $\alpha_3 \neq \beta_3$.

Approach 1: *Separate* Regressions Approach

- Does *not* use indicator (or dummy) variables as regressors.

Approach 2: *Pooled* Regression Approach

- Uses indicator (or dummy) variables as regressors.
- Let $\mathbf{D1_i}$ be the **Group 1 dummy variable**, defined as follows:

$D1_i = 1$ if observation i belongs to Group 1 ($\forall\ i \in$ Group 1)
$= 0$ if observation i does not belong to Group 1 ($\forall\ i \notin$ Group 1).

- Let $\mathbf{D2_i}$ be the **Group 2 dummy variable**, defined as follows:

$D2_i = 1$ if observation i belongs to Group 2 ($\forall\ i \in$ Group 2)
$= 0$ if observation i does not belong to Group 2 ($\forall\ i \notin$ Group 2).

- **Adding-Up Property of the Indicator Variables $D1_i$ and $D2_i$**

  For each and every i (population member or sample observation):

  if $D1_i = 1$ then $D2_i = 0$

  and

  if $D2_i = 1$ then $D1_i = 0$.

  Adding-Up Property: The definition of the indicator variables $D1_i$ and $D2_i$ thus implies that they satisfy the following **adding-up property**:

  $$\mathbf{D1_i + D2_i = 1} \qquad \forall\ \mathbf{i} \qquad \text{i.e., for all } i = 1, ..., N.$$

  Implications of Adding-Up Property:

  $$\mathbf{D1_i = 1 - D2_i} \qquad \forall\ \mathbf{i} \qquad \text{i.e., for all } i = 1, ..., N.$$
  $$\mathbf{D2_i = 1 - D1_i} \qquad \forall\ \mathbf{i} \qquad \text{i.e., for all } i = 1, ..., N.$$

❑ **<u>The Restricted and Unrestricted Models</u> Corresponding to H_0 and H_1**

- The **<u>unrestricted</u> model** corresponding to the **alternative hypothesis H_1** consists of the Group 1 PRE (2.1) and the Group 2 PRE (2.2):

Group 1 PRE: $Y_i = \alpha_1 + \alpha_2 X_{2i} + \alpha_3 X_{3i} + u_{1i}, \qquad i = 1, ..., N_1$ **... (2.1)**

Group 2 PRE: $Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + u_{2i}, \qquad i = 1, ..., N_2$ **... (2.2)**

- The **<u>restricted</u> model** corresponding to the **null hypothesis H_0** is obtained by imposing on the unrestricted model the coefficient restrictions specified by H_0.

  That is, set $\alpha_1 = \beta_1$ *and* $\alpha_2 = \beta_2$ *and* $\alpha_3 = \beta_3$ in the Group 1 PRE.

  The **restricted PRE** for the full sample of $N = N_1 + N_2$ observations is:

$$Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + u_i, \quad \forall \; i = 1, ..., N = N_1 + N_2 \qquad \textbf{... (1)}$$

4. Approach 2: Pooled (Full-Interaction) Regression Approach

- **Strategy:** Estimate a **single pooled regression equation** on the **full sample of $N = N_1 + N_2$ observations**. This equation incorporates the full set of coefficient differences between the PREs for Group 1 and Group 2.

- **Three Alternative Forms of the *Pooled* Full-Interaction Regression Equation -- The *Unrestricted* Model Corresponding to H_1**

  There are ***three* different but equivalent ways** of writing the pooled full-interaction regression equation:

1. **No base group**:

$$Y_i = \alpha_1 D1_i + \alpha_2 D1_i X_{2i} + \alpha_3 D1_i X_{3i} + \beta_1 D2_i + \beta_2 D2_i X_{2i} + \beta_3 D2_i X_{3i} + u_i \qquad \textbf{(12.0)}$$

2. **Group 1 selected as base group**: Set $D1_i = 1 - D2_i$ in equation (12.0).

$$Y_i = \alpha_1 + \alpha_2 X_{2i} + \alpha_3 X_{3i} + \gamma_1 D2_i + \gamma_2 D2_i X_{2i} + \gamma_3 D2_i X_{3i} + u_i \qquad \textbf{(12.1)}$$

where $\gamma_1 = \beta_1 - \alpha_1$; $\gamma_2 = \beta_2 - \alpha_2$; $\gamma_3 = \beta_3 - \alpha_3$;
$\beta_1 = \gamma_1 + \alpha_1$; $\beta_2 = \gamma_2 + \alpha_2$; $\beta_3 = \gamma_3 + \alpha_3$.

3. **Group 2 selected as base group**: Set $D2_i = 1 - D1_i$ in equation (12.0).

$$Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + \delta_1 D1_i + \delta_2 D1_i X_{2i} + \delta_3 D1_i X_{3i} + u_i \qquad \textbf{(12.2)}$$

where $\delta_1 = \alpha_1 - \beta_1$; $\delta_2 = \alpha_2 - \beta_2$; $\delta_3 = \alpha_3 - \beta_3$;
$\alpha_1 = \delta_1 + \beta_1$; $\alpha_2 = \delta_2 + \beta_2$; $\alpha_3 = \delta_3 + \beta_3$.

Note: The γ_j coefficients in pooled regression (12.1) are **equal in magnitude** but **opposite in sign** to the δ_j coefficients in pooled regression (12.2).

$$\gamma_j = \beta_j - \alpha_j = -\delta_j \qquad \text{or} \qquad \delta_j = \alpha_j - \beta_j = -\gamma_j \qquad j = 1, 2, 3.$$

- **Properties of Pooled Regression Equations (12.0), (12.1) and (12.2)**

Equations (12.0), (12.1), and (12.2) are *observationally equivalent*: OLS estimation of equations (12.0), (12.1), and (12.2) yield ***identical* values** of

RSS_1 ≡ ***unrestricted* residual sum-of-squares**

$\hat{\sigma}^2$ ≡ the ***unrestricted* estimator of the error variance σ^2**

R_U^2 ≡ the **R-squared value** for the ***unrestricted* OLS SRE**.

- **Derivation of Pooled Regression Equation 12.0**

1. **Multiply** equation (2.1) by $D1_i$ and equation (2.2) by $D2_i$:

$$D1_i Y_i = \alpha_1 D1_i + \alpha_2 D1_i X_{2i} + \alpha_3 D1_i X_{3i} + D1_i u_i \tag{10.1}$$

$$D2_i Y_i = \beta_1 D2_i + \beta_2 D2_i X_{2i} + \beta_3 D2_i X_{3i} + D2_i u_i \tag{10.2}$$

Note: We again assume that $u_{1i} = u_{2i} = u_i$ -- i.e., that u_{i1} and u_{i2} have identical distributions, so that they have equal variances $\sigma_1^2 = \sigma_2^2 = \sigma^2$.

2. **Combine equations (10.1) and (10.2)** by ***adding them together*** for each observation $i = 1, ..., N$.

$$\begin{aligned} D1_i Y_i + D2_i Y_i &= \alpha_1 D1_i + \alpha_2 D1_i X_{2i} + \alpha_3 D1_i X_{3i} + D1_i u_i \\ &\quad + \beta_1 D2_i + \beta_2 D2_i X_{2i} + \beta_3 D2_i X_{3i} + D2_i u_i \\ &= \alpha_1 D1_i + \alpha_2 D1_i X_{2i} + \alpha_3 D1_i X_{3i} \\ &\quad + \beta_1 D2_i + \beta_2 D2_i X_{2i} + \beta_3 D2_i X_{3i} \\ &\quad + D1_i u_i + D2_i u_i \end{aligned}$$

or

$$\begin{aligned} (D1_i + D2_i) Y_i &= \alpha_1 D1_i + \alpha_2 D1_i X_{2i} + \alpha_3 D1_i X_{3i} \\ &\quad + \beta_1 D2_i + \beta_2 D2_i X_{2i} + \beta_3 D2_i X_{3i} \\ &\quad + (D1_i + D2_i) u_i \end{aligned} \tag{11}$$

3. Use the *adding-up property* to set $D1_i + D2_i = 1$ for all i in equation (11):

$$Y_i = \alpha_1 D1_i + \alpha_2 D1_i X_{2i} + \alpha_3 D1_i X_{3i} + \beta_1 D2_i + \beta_2 D2_i X_{2i} + \beta_3 D2_i X_{3i} + u_i \quad \forall\ i \tag{12.0}$$

- ***Result:*** Equation (12.0) is the ***pooled full-interaction PRE* corresponding to the alternative hypothesis H_1**:

$$Y_i = \alpha_1 D1_i + \alpha_2 D1_i X_{2i} + \alpha_3 D1_i X_{3i} + \beta_1 D2_i + \beta_2 D2_i X_{2i} + \beta_3 D2_i X_{3i} + u_i \quad \textbf{(12.0)}$$

- **Characteristics of equation (12.0):**

1) Equation (12.0) has ***no intercept* coefficient**.

2) Equation (12.0) **contains the *full set of regression coefficients*** for both the Group 1 PRF (α_1, α_2, and α_3) and the Group 2 PRF (β_1, β_2, and β_3).

Estimation of the pooled full-interaction regression equation (12.0) thus yields coefficient estimates of both the Group 1 PRF and the Group 2 PRF.

3) Both the **Group 1 and Group 2 PREs can be obtained from equation (12.0)**.

- **Group 1 PRE** is obtained by setting $D1_i = 1$ and $D2_i = 0$ in (12.0):

$$Y_i = \alpha_1 + \alpha_2 X_{2i} + \alpha_3 X_{3i} + u_i \qquad \forall\ i \text{ such that } D1_i = 1$$

- **Group 2 PRE** is obtained by setting $D2_i = 1$ and $D1_i = 0$ in (12.0):

$$Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + u_i \qquad \forall\ i \text{ such that } D2_i = 1$$

Approach 2: Pooled Regression Approach -- The *Unrestricted* Model

OLS-SREs for the three pooled full-interaction regression equations, estimated by OLS on the full sample of $N = N_1 + N_2$ observations.

$$Y_i = \hat{\alpha}_1 D1_i + \hat{\alpha}_2 D1_i X_{2i} + \hat{\alpha}_3 D1_i X_{3i} + \hat{\beta}_1 D2_i + \hat{\beta}_2 D2_i X_{2i} + \hat{\beta}_3 D2_i X_{3i} + \hat{u}_i \qquad \textbf{(12.0)}$$

$$Y_i = \hat{\alpha}_1 + \hat{\alpha}_2 X_{2i} + \hat{\alpha}_3 X_{3i} + \hat{\gamma}_1 D2_i + \hat{\gamma}_2 D2_i X_{2i} + \hat{\gamma}_3 D2_i X_{3i} + \hat{u}_i \qquad \textbf{(12.1)}$$

where $\hat{\gamma}_1 = \hat{\beta}_1 - \hat{\alpha}_1$; $\hat{\gamma}_2 = \hat{\beta}_2 - \hat{\alpha}_2$; $\hat{\gamma}_3 = \hat{\beta}_3 - \hat{\alpha}_3$.

$$Y_i = \hat{\beta}_1 + \hat{\beta}_2 X_{2i} + \hat{\beta}_3 X_{3i} + \hat{\delta}_1 D1_i + \hat{\delta}_2 D1_i X_{2i} + \hat{\delta}_3 D1_i X_{3i} + \hat{u}_i. \qquad \textbf{(12.2)}$$

where $\hat{\delta}_1 = \hat{\alpha}_1 - \hat{\beta}_1$; $\hat{\delta}_2 = \hat{\alpha}_2 - \hat{\beta}_2$; $\hat{\delta}_3 = \hat{\alpha}_3 - \hat{\beta}_3$.

Approach 1: Separate Regressions Approach -- The *Unrestricted* Model

1. The **Group 1 OLS-SRE** for the subsample of N_1 observations for Group 1 is

$$Y_i = \hat{\alpha}_1 + \hat{\alpha}_2 X_{2i} + \hat{\alpha}_3 X_{3i} + \hat{u}_{1i}, \quad i = 1, ..., N_1, \qquad \textbf{(5.1)}$$

$$RSS_{(1)} = \sum_{i=1}^{N_1} \hat{u}_{1i}^2 \quad \text{with} \quad df_{(1)} = N_1 - k_0 = N_1 - 3$$

2. The **Group 2 OLS-SRE** for the subsample of N_2 observations for Group 2 is

$$Y_i = \hat{\beta}_1 + \hat{\beta}_2 X_{2i} + \hat{\beta}_3 X_{3i} + \hat{u}_{2i}, \quad i = 1, ..., N_2, \qquad \textbf{(6.1)}$$

$$RSS_{(2)} = \sum_{i=1}^{N_2} \hat{u}_{2i}^2 \quad \text{with} \quad df_{(2)} = N_2 - k_0 = N_2 - 3$$

The ***unrestricted*** **residual sum-of-squares** is: $RSS_1 = RSS_U = \sum_{i=1}^{N} \hat{u}_i^2$

$$\mathbf{RSS_1 = RSS_U = RSS(12.0) = RSS(12.1) = RSS(12.2) = RSS_{(1)} + RSS_{(2)}}$$

with $df_1 = df_U = df_{(1)} + df_{(2)} = N - 2k_0 = N - 6$.

Note: The RSS from OLS estimation of pooled equations (12.0), (12.1), and (12.2) equals the sum of the RSS values from separate OLS estimation of the Group 1 and Group 2 regression equations.

The *Restricted* Model

Corresponds to the **null hypothesis H_0**, which is

$$H_0: \ \alpha_1 = \beta_1 \ \textit{and} \ \alpha_2 = \beta_2 \ \textit{and} \ \alpha_3 = \beta_3 \qquad \text{(3 coefficient restrictions)}$$

The **restricted regression equation** for the full sample of $N = N_1 + N_2$ observations is obtained by imposing the three coefficient restrictions specified by H_0 on any of the unrestricted regression equations (12.0), (12.1) and (12.2).

1. Set $\alpha_1 = \beta_1$ *and* $\alpha_2 = \beta_2$ *and* $\alpha_3 = \beta_3$ in the unrestricted pooled regression equation (12.0)

$$Y_i = \alpha_1 D1_i + \alpha_2 D1_i X_{2i} + \alpha_3 D1_i X_{3i} + \beta_1 D2_i + \beta_2 D2_i X_{2i} + \beta_3 D2_i X_{3i} + u_i . \quad \textbf{(12.0)}$$

$$\begin{aligned} Y_i &= \beta_1 D1_i + \beta_2 D1_i X_{2i} + \beta_3 D1_i X_{3i} + \beta_1 D2_i + \beta_2 D2_i X_{2i} + \beta_3 D2_i X_{3i} + u_i \\ &= \beta_1 (D1_i + D2_i) + \beta_2 (D1_i + D2_i) X_{2i} + \beta_3 (D1_i + D2_i) X_{3i} + u_i \\ &= \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + u_i \qquad \text{because } D1_i + D2_i = 1 \text{ for all i.} \end{aligned}$$

2. Set $\gamma_1 = \beta_1 - \alpha_1 = 0$, $\gamma_2 = \beta_2 - \alpha_2 = 0$, *and* $\gamma_3 = \beta_3 - \alpha_3 = 0$ in the unrestricted pooled regression equation (12.1)

$$Y_i = \alpha_1 + \alpha_2 X_{2i} + \alpha_3 X_{3i} + \gamma_1 D2_i + \gamma_2 D2_i X_{2i} + \gamma_3 D2_i X_{3i} + u_i . \quad \textbf{(12.1)}$$

$$Y_i = \alpha_1 + \alpha_2 X_{2i} + \alpha_3 X_{3i} + u_i \qquad \forall \ \text{i.}$$

3. Set $\delta_1 = \alpha_1 - \beta_1 = 0$, $\delta_2 = \alpha_2 - \beta_2 = 0$, *and* $\delta_3 = \alpha_3 - \beta_3 = 0$ in the unrestricted pooled regression equation (12.2)

$$Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + \delta_1 D1_i + \delta_2 D1_i X_{2i} + \delta_3 D1_i X_{3i} + u_i . \quad \textbf{(12.2)}$$

$$Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + u_i \qquad \forall \ \text{i.}$$

Result: The ***restricted*** **model** is given by the population regression equation

$$Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + u_i \qquad \forall\ i = 1, \ldots, N = N_1 + N_2.$$

The ***restricted*** **OLS-SRE** for the full sample of Group 1 and Group 2 observations is

$$Y_i = \tilde{\beta}_1 + \tilde{\beta}_2 X_{2i} + \tilde{\beta}_3 X_{3i} + \tilde{u}_i, \quad i = 1, ..., N = N_1 + N_2 \qquad \textbf{... (4.1)}$$

The ***restricted*** **residual sum-of-squares** is

$$RSS_0 = RSS_R = \sum_{i=1}^{N} \tilde{u}_i^2 \qquad \text{with } df_0 = df_R = N - k_0 \qquad \textbf{... (4.2)}$$

The F-Statistic for Testing for Complete Coefficient Equality

H_0: $\alpha_1 = \beta_1$ *and* $\alpha_2 = \beta_2$ *and* $\alpha_3 = \beta_3$ (3 coefficient restrictions)

against

H_1: $\alpha_1 \neq \beta_1$ *and/or* $\alpha_2 \neq \beta_2$ *and/or* $\alpha_3 \neq \beta_3$.

The ***sample value*** **of the F-statistic** for testing the null hypothesis of complete coefficient equality is

$$F_0 = \frac{(RSS_0 - RSS_1)/(df_0 - df_1)}{RSS_1/df_1} = \frac{(RSS_0 - RSS_1)/k_0}{RSS_1/(N - 2k_0)}.$$

- ***Null distribution*** **of F_0:** $F_0 \sim F(k_0, N - 2k_0)$ under H_0.
- ***Decision Rule:*** At the 100α percent significance level

1. ***reject*** **H_0** if $F_0 \geq F_\alpha(k_0, N - 2k_0)$ *or* p-value for $F_0 \leq \alpha$;

2. ***retain*** **H_0** if $F_0 < F_\alpha(k_0, N - 2k_0)$ *or* p-value for $F_0 > \alpha$.

❑ Equivalence of Three F-Tests

The **three F-tests** based on pooled full-interaction regression equations (12.0), (12.1), and (12.2) ***are equivalent to each other*** **and** ***to the F-test computed using the separate regressions approach***.

$$F(12.0) = F(12.1) = F(12.2) = F_{SR} \sim F\left(k_0, N - 2k_0\right) \text{ under } H_0.$$

where all four F-statistics F(12.0) = F(12.1) = F(12.2) = F_{SR} = F_0 and

$$F_0 = \frac{\left(RSS_0 - RSS_1\right)/\left(df_0 - df_1\right)}{RSS_1/df_1} = \frac{\left(RSS_0 - RSS_1\right)/k_0}{RSS_1/\left(N - 2k_0\right)}.$$

❑ Advantages of Pooled Regression Approach (Approach 2)

1. Approach 2 is more *informative*.

- It permits **t-tests of individual coefficient differences** between the Group 1 and Group 2 regression functions.

  This advantage is particularly evident when a base group is selected for the pooled full-interaction regression equation.

2. Approach 2 is more *flexible*.

- **Approach 2** can be used **to test for coefficient equality between any subset of regression coefficients** in the Group 1 and Group 2 PRFs.

  Approach 1 can only test the hypothesis of **complete coefficient equality** between the Group 1 and Group 2 regression functions.

- ***Illustrations of Approach 2:*** Tests for equality of subsets of coefficients.

♦ **<u>Test #1</u>: Equality of *all* (both) slope coefficients**.

H_0:	$\alpha_2 = \beta_2$	and	$\alpha_3 = \beta_3$	in pooled PRE (12.0)
	$\gamma_2 = 0$	and	$\gamma_3 = 0$	in pooled PRE (12.1)
	$\delta_2 = 0$	and	$\delta_3 = 0$	in pooled PRE (12.2)

H_1:	$\alpha_2 \neq \beta_2$	and/or	$\alpha_3 \neq \beta_3$	in pooled PRE (12.0)
	$\gamma_2 \neq 0$	and/or	$\gamma_3 \neq 0$	in pooled PRE (12.1)
	$\delta_2 \neq 0$	and/or	$\delta_3 \neq 0$	in pooled PRE (12.2)

- ***Restricted* OLS-SRE** corresponding to H_0 is any one of the following three OLS sample regression equations, for which $\mathbf{k_0 = 4}$:

$$Y_i = \tilde{\alpha}_1 D1_i + \tilde{\beta}_1 D2_i + \tilde{\beta}_2 X_{2i} + \tilde{\beta}_3 X_{3i} + \tilde{u}_i \qquad \text{from (12.0)}$$

$$Y_i = \tilde{\alpha}_1 + \tilde{\alpha}_2 X_{2i} + \tilde{\alpha}_3 X_{3i} + \tilde{\gamma}_1 D2_i + \tilde{u}_i \qquad \text{from (12.1)}$$

$$Y_i = \tilde{\beta}_1 + \tilde{\beta}_2 X_{2i} + \tilde{\beta}_3 X_{3i} + \tilde{\delta}_1 D1_i + \tilde{u}_i \qquad \text{from (12.2)}$$

- The ***restricted* RSS under $\mathbf{H_0}$** is

$$RSS_0 = RSS_R = \sum_{i=1}^{N} \tilde{u}_i^2 \qquad \text{with } df_0 = df_R = N - k_0 = N - 4.$$

- The ***unrestricted* RSS under $\mathbf{H_1}$** is

$$RSS_1 = RSS_U = \sum_{i=1}^{N} \hat{u}_i^2 \qquad \text{with } df_1 = df_U = N - k = N - 6.$$

- The ***numerator* degrees of freedom** equal

$$df_{num} = df_0 - df_1 = (N - k_0) - (N - k) = k - k_0 = 6 - 4 = 2.$$

- The ***denominator* degrees of freedom** equal

$$df_{den} = df_1 = N - k = N - 6.$$

♦ **<u>Test #2</u>: Equality of *a subset of* slope coefficients, e.g. the coefficient of X_{3i}.**

H$_0$: $\alpha_3 = \beta_3$ in pooled PRE (12.0)
$\gamma_3 = 0$ in pooled PRE (12.1)
$\delta_3 = 0$ in pooled PRE (12.2)

H$_1$: $\alpha_3 \neq \beta_3$ in pooled PRE (12.0)
$\gamma_3 \neq 0$ in pooled PRE (12.1)
$\delta_3 \neq 0$ in pooled PRE (12.2)

- ***Restricted* OLS-SRE** corresponding to H_0 is any one of the following three OLS sample regression equations, for which $\mathbf{k_0 = 5}$:

$$Y_i = \tilde{\alpha}_1 D1_i + \tilde{\beta}_1 D2_i + \tilde{\alpha}_2 D1_i X_{2i} + \tilde{\beta}_2 D2_i X_{2i} + \tilde{\beta}_3 X_{3i} + \tilde{u}_i \qquad \text{from (12.0)}$$

$$Y_i = \tilde{\alpha}_1 + \tilde{\alpha}_2 X_{2i} + \tilde{\alpha}_3 X_{3i} + \tilde{\gamma}_1 D2_i + \tilde{\gamma}_2 D2_i X_{2i} + \tilde{u}_i \qquad \text{from (12.1)}$$

$$Y_i = \tilde{\beta}_1 + \tilde{\beta}_2 X_{2i} + \tilde{\beta}_3 X_{3i} + \tilde{\delta}_1 D1_i + \tilde{\delta}_2 D1_i X_{2i} + \tilde{u}_i \qquad \text{from (12.2)}$$

- The ***restricted* RSS under H_0** is

$$RSS_0 = RSS_R = \sum_{i=1}^{N} \tilde{u}_i^2 \qquad \text{with } df_0 = df_R = N - k_0 = N - 5.$$

- The ***unrestricted* RSS under H_1** is

$$RSS_1 = RSS_U = \sum_{i=1}^{N} \hat{u}_i^2 \qquad \text{with } df_1 = df_U = N - k = N - 6.$$

- The ***numerator* degrees of freedom** equal

$$df_{num} = df_0 - df_1 = (N - k_0) - (N - k) = k - k_0 = 6 - 5 = 1.$$

- The ***denominator* degrees of freedom** equal

$$df_{den} = df_1 = N - k = N - 6.$$

♦ **<u>Test #3</u>: Equality of *intercept* coefficients only**.

$$\begin{array}{lll} H_0: & \alpha_1 = \beta_1 & \text{in pooled PRE (12.0)} \\ & \gamma_1 = 0 & \text{in pooled PRE (12.1)} \\ & \delta_1 = 0 & \text{in pooled PRE (12.2)} \end{array}$$

$$\begin{array}{lll} H_1: & \alpha_1 \neq \beta_1 & \text{in pooled PRE (12.0)} \\ & \gamma_1 \neq 0 & \text{in pooled PRE (12.1)} \\ & \delta_1 \neq 0 & \text{in pooled PRE (12.2)} \end{array}$$

- ***Restricted* OLS-SRE** corresponding to H_0 is any one of the following three OLS sample regression equations, for which $\mathbf{k_0 = 5}$:

$$Y_i = \tilde{\beta}_1 + \tilde{\alpha}_2 D1_i X_{2i} + \tilde{\beta}_2 D2_i X_{2i} + \tilde{\alpha}_3 D1_i X_{3i} + \tilde{\beta}_3 D2_i X_{3i} + \tilde{u}_i \quad \text{from (12.0)}$$

$$Y_i = \tilde{\alpha}_1 + \tilde{\alpha}_2 X_{2i} + \tilde{\alpha}_3 X_{3i} + \tilde{\gamma}_2 D2_i X_{2i} + \tilde{\gamma}_3 D2_i X_{3i} + \tilde{u}_i \quad \text{from (12.1)}$$

$$Y_i = \tilde{\beta}_1 + \tilde{\beta}_2 X_{2i} + \tilde{\beta}_3 X_{3i} + \tilde{\delta}_2 D1_i X_{2i} + \tilde{\delta}_3 D1_i X_{3i} + \tilde{u}_i \quad \text{from (12.2)}$$

- The ***restricted* RSS under H_0** is

$$RSS_0 = RSS_R = \sum_{i=1}^{N} \tilde{u}_i^2 \qquad \text{with} \quad df_0 = df_R = N - k_0 = N - 5.$$

- The ***unrestricted* RSS under H_1** is

$$RSS_1 = RSS_U = \sum_{i=1}^{N} \hat{u}_i^2 \qquad \text{with} \quad df_1 = df_U = N - k = N - 6.$$

- The ***numerator* degrees of freedom** equal

$$df_{num} = df_0 - df_1 = (N - k_0) - (N - k) = k - k_0 = 6 - 5 = 1.$$

- The ***denominator* degrees of freedom** equal

$$df_{den} = df_1 = N - k = N - 6.$$

- **Computing Test #1, Test #2 and Test #3**

For each test, compute the ***sample value*** **of the general F-statistic**, and **then apply the conventional** ***decision rule***:

(1) If $\mathbf{F \geq F_{\alpha}(k-k_0, N-k)}$, or if the ***p-value*** **for** $\mathbf{F} \leq \alpha$, ***reject*** the coefficient restrictions specified by the ***null*** **hypothesis** $\mathbf{H_0}$ at the ***100α%*** significance level;

(2) If $\mathbf{F < F_{\alpha}(k-k_0, N-k)}$, or if the ***p-value*** **for** $\mathbf{F} > \alpha$, ***retain (do not reject)*** the coefficient restrictions specified by the ***null*** **hypothesis** $\mathbf{H_0}$ at the ***100α%*** significance level.