Addendum to NOTE 4

Sampling Distribution of OLS Estimator of β_2

A Monte Carlo Simulation

The True Model: is given by the **population regression equation (PRE)**

$$Y_i = \beta_1 + \beta_2 X_i + u_i = 90 + 0.90 X_i + u_i \qquad (1)$$

where

β_1 = 90.0 and **β_2 = 0.90**;

Y_i = weekly consumption expenditures of the i-th household;

X_i = weekly disposable income of the i-th household;

u_i = an iid random error term that is assumed to be $N(0, \sigma^2)$.

Two Alternative Models: specify different values for $\sigma^2 = Var(u_i \mid X_i)$.

Model 1: sets $\sigma^2 = Var(u_i \mid X_i) = 900$, $\sigma = \sqrt{Var(u_i \mid X_i)} = se(u_i \mid X_i) = 30$.

Model 2: sets $\sigma^2 = Var(u_i \mid X_i) = 32{,}400$, $\sigma = \sqrt{Var(u_i \mid X_i)} = se(u_i \mid X_i) = 180$.

The Monte Carlo Simulations

- Set N = 40 observations.

- Set populations values of X, β_1 and β_2, and $\sigma^2 = \text{Var}(u_i \mid X_i)$.

- Generate 1,000 independent random samples of Y_i, X_i and u_i values.

- For each of these 1,000 independent random samples, compute the values of the OLS coefficient estimators:

$$\hat{\beta}_2 = \frac{\sum_i x_i y_i}{\sum_i x_i^2} \tag{2}$$

$$\hat{\beta}_1 = \overline{Y} - \hat{\beta}_2 \overline{X} \tag{3}$$

where $x_i \equiv X_i - \overline{X}$, $y_i \equiv Y_i - \overline{Y}$, $\overline{X} = \sum_i X_i / N$, and $\overline{Y} = \sum_i Y_i / N$.

- Tabulate the 1,000 estimates of β_2, and the 1,000 estimates of β_1.

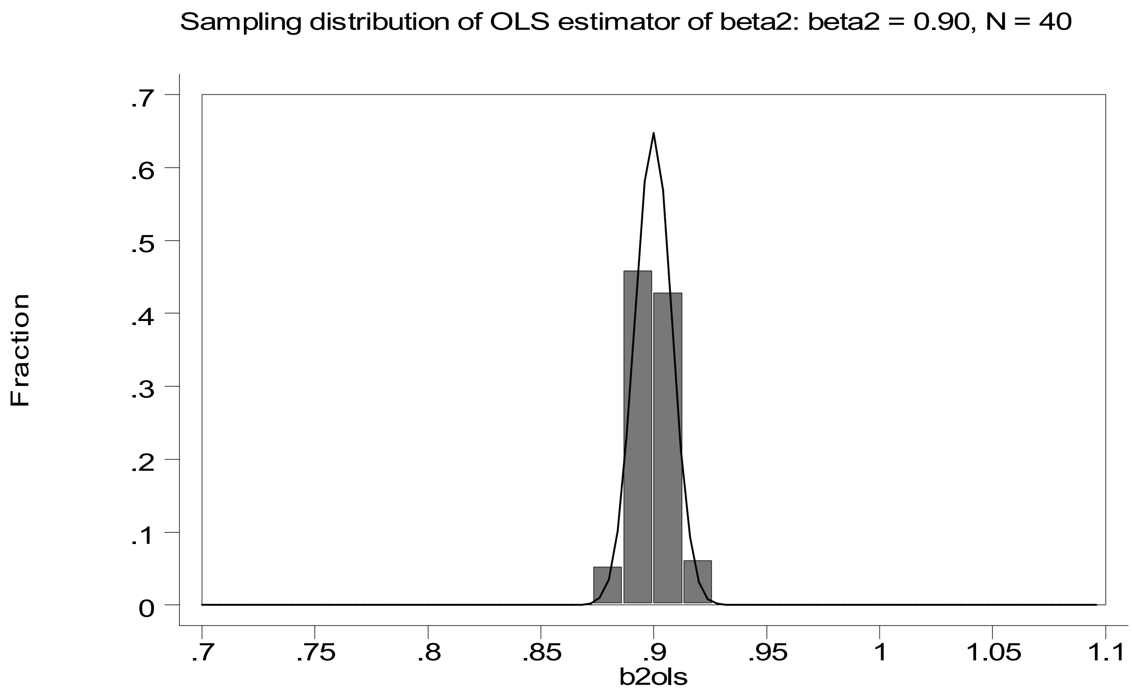
Summary statistics for simulation of Model 1 for which σ^2 = 900, σ = 30:

```
. summarize

Variable |     Obs        Mean   Std. Dev.       Min        Max
---------+-----------------------------------------------------
   b1ols |    1000    90.12619    10.07493   61.00864    129.009
   b2ols |    1000    .8998154     .008213   .8695259   .9269965
```

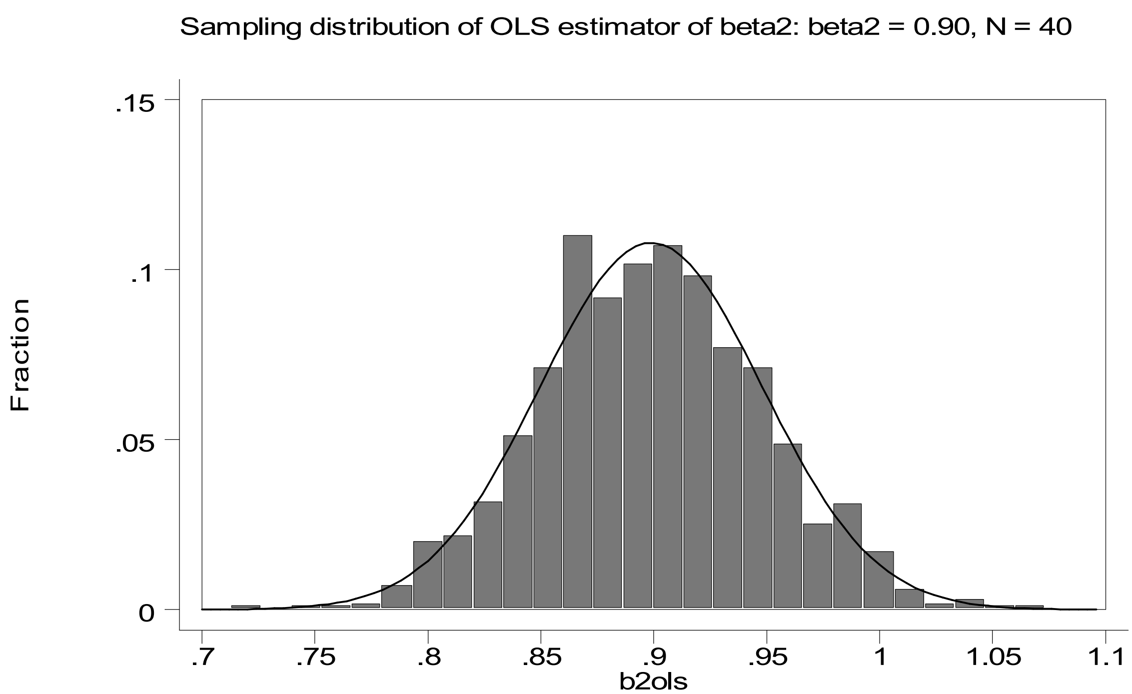
Summary statistics for simulation of Model 2 for which σ^2 = 32,400, σ = 180:

```
. summarize

Variable |     Obs        Mean   Std. Dev.       Min        Max
---------+-----------------------------------------------------
   b1ols |    1000    90.75711    60.44958  -83.94814   324.0542
   b2ols |    1000    .8988922    .0492779   .7171556   1.061979
```

- Comparison of Sampling Distributions of $\hat{\beta}_2 = \dfrac{\sum_i x_i y_i}{\sum_i x_i^2}$:



Model 1: E(u) = 0, Var(u) = 900, SE(u) = 30, N = 40



Model 2: E(u) = 0, Var(u) = 32,400, SE(u) = 180, N = 40