QUEEN'S UNIVERSITY AT KINGSTON
Department of Economics

ECONOMICS 351* - Section A

Introductory Econometrics

Fall Term 1999 **MID-TERM EXAM** M.G. Abbott

DATE: **Thursday November 4, 1999.**

TIME: **80 minutes; 4:00 p.m. - 5:20 p.m.**

INSTRUCTIONS: The exam consists of **FOUR (4)** questions. Students are required to answer **ALL FOUR (4)** questions.

Answer all questions in the exam booklets provided. Be sure your ***name*** and ***student number*** are printed clearly on the front of all exam booklets used.

Do not write answers to questions on the front page of the first exam booklet.

Please label clearly each of your answers in the exam booklets with the appropriate number and letter.

Please write legibly.

MARKING: The marks for each question are indicated in parentheses immediately above each question. **Total marks** for the exam **equal 100**.

GOOD LUCK!

QUESTIONS: **Answer ALL FOUR questions.**

All questions pertain to the simple (two-variable) linear regression model for which the population regression equation can be written in conventional notation as:

$$Y_i = \beta_1 + \beta_2 X_i + u_i \qquad \textbf{(1)}$$

where Y_i and X_i are observable variables, β_1 and β_2 are unknown (constant) regression coefficients, and u_i is an unobservable random error term. The Ordinary Least Squares (OLS) sample regression equation corresponding to regression equation (1) is

$$Y_i = \hat{\beta}_1 + \hat{\beta}_2 X_i + \hat{u}_i \qquad (i = 1, ..., N) \qquad \textbf{(2)}$$

where $\hat{\beta}_1$ is the OLS estimator of the intercept coefficient β_1, $\hat{\beta}_2$ is the OLS estimator of the slope coefficient β_2, $\hat{u}_i$ is the OLS residual for the i-th sample observation, and N is sample size (the number of observations in the sample).

(15 marks)

1. State the Ordinary Least Squares (OLS) estimation criterion. State the OLS normal equations. Derive the OLS normal equations from the OLS estimation criterion.

(15 marks)

2. Give a general definition of the t-distribution. Starting from this definition, derive the t-statistic for the OLS slope coefficient estimator $\hat{\beta}_2$. State all assumptions required for the derivation.

(10 marks)

3. State the Gauss-Markov theorem. Explain fully what it means.

(60 marks)

4. A researcher is using data for a sample of 30 companies to investigate the relationship between annual business taxes paid Y_i (measured in *millions* of dollars per year) and gross business income X_i (measured in *millions* of dollars per year). Preliminary analysis of the sample data produces the following sample information:

$$N = 30 \qquad \sum_{i=1}^{N} Y_i = 28.730 \qquad \sum_{i=1}^{N} X_i = 166.045 \qquad \sum_{i=1}^{N} Y_i^2 = 33.17041$$

$$\sum_{i=1}^{N} X_i^2 = 1095.526 \qquad \sum_{i=1}^{N} X_i Y_i = 190.1279 \qquad \sum_{i=1}^{N} x_i y_i = 31.11217$$

$$\sum_{i=1}^{N} y_i^2 = 5.65665 \qquad \sum_{i=1}^{N} x_i^2 = 176.495 \qquad \sum_{i=1}^{N} \hat{u}_i^2 = 0.1722599$$

where $x_i \equiv X_i - \overline{X}$ and $y_i \equiv Y_i - \overline{Y}$ for i = 1, ..., N. Use the above sample information to answer all the following questions. **Show explicitly all formulas and calculations**.

(10 marks)
(a) Use the above information to compute OLS estimates of the intercept coefficient β_1 and the slope coefficient β_2.

(5 marks)
(b) Interpret the slope coefficient estimate you calculated in part (a) -- i.e., explain what the numeric value you calculated for $\hat{\beta}_2$ means.

(5 marks)
(c) Compute the value of R^2, the coefficient of determination for the estimated OLS sample regression equation. Briefly explain what the calculated value of R^2 means.

(5 marks)
(d) Calculate an estimate of σ^2, the error variance.

(5 marks)
(e) Calculate the estimated variance of $\hat{\beta}_2$.

(10 marks)
(f) Perform a test of the null hypothesis H_0: $\beta_2 = 0$ against the alternative hypothesis H_1: $\beta_2 \neq 0$ at the 5% significance level (i.e., for significance level $\alpha = 0.05$). State the decision rule you use, and the inference you would draw from the test. Would you draw the same inference if you performed the test at the 1% significance level (i.e., for significance level $\alpha = 0.01$)?

(10 marks)
(g) Compute the two-sided 95% confidence interval for the slope coefficient β_2.

(10 marks)
(h) Perform a test of the proposition that $\beta_2 < 0.20$ at the 5% significance level (i.e., for significance level $\alpha = 0.05$). State the null and alternative hypotheses, and show how you calculated the test statistic. State the decision rule you use, and the inference you would draw from the test. Would you draw the same inference if you performed the test at the 1% significance level (i.e., for significance level $\alpha = 0.01$)?

Percentage Points of the t-Distribution