QUEEN'S UNIVERSITY AT KINGSTON
Department of Economics

ECONOMICS 351* - Section A

Introductory Econometrics

Fall Term 1998 **MID-TERM EXAM** M.G. Abbott

DATE: **Monday October 19, 1998.**

TIME: **80 minutes; 2:30 p.m. - 3:50 p.m.**

INSTRUCTIONS: The exam consists of **SIX (6)** questions. Students are required to answer **ALL SIX (6)** questions.

Answer all questions in the exam booklets provided. Be sure your ***name*** and ***student number*** are printed clearly on the front of all exam booklets used.

Do not write answers to questions on the front page of the first exam booklet.

Please label clearly each of your answers in the exam booklets with the appropriate number and letter.

Please write legibly.

MARKING: The marks for each question are indicated in parentheses immediately above each question. **Total marks** for the exam **equal 100**.

GOOD LUCK!

QUESTIONS: **Answer ALL FIVE questions.**

All questions pertain to the simple (two-variable) linear regression model for which the population regression equation can be written in conventional notation as:

$$Y_i = \beta_1 + \beta_2 X_i + u_i \qquad \textbf{(1)}$$

where Y_i and X_i are observable variables, β_1 and β_2 and unknown (constant) regression coefficients, and u_i is an unobservable random error term. The Ordinary Least Squares (OLS) sample regression equation corresponding to regression equation (1) is

$$Y_i = \hat{\beta}_1 + \hat{\beta}_2 X_i + \hat{u}_i \qquad (i = 1, ..., N) \qquad \textbf{(2)}$$

where $\hat{\beta}_1$ is the OLS estimator of the intercept coefficient β_1, $\hat{\beta}_2$ is the OLS estimator of the slope coefficient β_2, $\hat{u}_i$ is the OLS residual for the i-th sample observation, and N is sample size (the number of observations in the sample).

(15 marks)

1. State and briefly explain the following assumptions of the Classical Linear Regression Model:

(a) the assumption of zero mean error;

(b) the assumption of homoskedastic errors.

State the implications of each assumption for the distribution of the dependent variable Y_i in population regression equation (1).

(15 marks)

2. Answer parts (a) to (c) below.

(a) State the Ordinary Least Squares (OLS) estimation criterion.

(b) State the OLS normal equations.

(c) Show how the OLS normal equations are derived from the OLS estimation criterion.

(15 marks)

3. Answer parts (a) and (b) below.

(a) Explain the meaning of the following statement: The estimator $\hat{\beta}_2$ is an unbiased estimator of the slope coefficient β_2.

(b) Stating explicitly all required assumptions, prove that the OLS slope coefficient estimator $\hat{\beta}_2$ is an unbiased estimator of the slope coefficient β_2.

(10 marks)

4. Answer parts (a) and (b) below.

(a) Write the expressions (or formulas) for $\text{Var}(\hat{\beta}_2)$, the variance of $\hat{\beta}_2$, and $\text{Var}(\hat{\beta}_1)$, the variance of $\hat{\beta}_1$.

(b) Which of the following factors makes $\text{Var}(\hat{\beta}_2)$ and $\text{Var}(\hat{\beta}_1)$ ***smaller***?

(1) a smaller value of N, sample size

(2) smaller values of $x_i^2 = (X_i - \bar{X})^2$, i = 1, …, N

(3) a larger value of σ^2, the error variance

(4) a smaller value of σ^2, the error variance

(5) a larger value of N, sample size

(6) larger values of $x_i^2 = (X_i - \bar{X})^2$, i = 1, …, N.

(10 marks)

5. State the Gauss-Markov theorem. Explain fully what it means.

[Question 6 on next page …]

(35 marks)

6. A researcher is studying the relationship between house prices Y_i (measured in *thousands* of dollars) and X_i house size (measured in square feet) for a sample of 14 homes in a particular neighborhood. Preliminary analysis of the sample data produces the following sample information:

$N = 14$ $\quad \sum_{i=1}^{N} Y_i = 2{,}834.9$ $\quad \sum_{i=1}^{N} X_i = 26{,}832.0$

$\sum_{i=1}^{N} Y_i^2 = 612{,}628.0$ $\quad \sum_{i=1}^{N} X_i^2 = 53{,}890{,}858.0$ $\quad \sum_{i=1}^{N} X_i Y_i = 5{,}664{,}696.0$

$\sum_{i=1}^{N} x_i y_i = 231{,}407.5$ $\quad \sum_{i=1}^{N} x_i^2 = 2{,}465{,}417.0$ $\quad \sum_{i=1}^{N} y_i^2 = 38{,}581.01$ $\quad \sum_{i=1}^{N} \hat{u}_i^2 = 16{,}860.79$

where $x_i \equiv X_i - \overline{X}$ and $y_i \equiv Y_i - \overline{Y}$ for i = 1, ..., N. Answer all the following questions. **Show explicitly all formulas and calculations**.

(10 marks)

(a) Use the above information to compute OLS estimates of the intercept coefficient β_1 and the slope coefficient β_2.

(5 marks)

(b) Interpret the slope coefficient estimate you calculated in part (a) -- i.e., explain what the numeric value you calculated for $\hat{\beta}_2$ means.

(5 marks)

(c) Use the above information to calculate an estimate of σ^2, the error variance. What is the estimate of σ^2 used for?

(5 marks)

(d) Let $\hat{Y}_i = \hat{\beta}_1 + \hat{\beta}_2 X_i$ (i = 1, …, N). What is the value of $\sum_{i=1}^{N} \hat{Y}_i$ for the sample regression equation you have estimated? Explain briefly how you obtained your answer.

(5 marks)

(e) What is the value of $\sum_{i=1}^{N} X_i \hat{u}_i$ for the sample regression equation you have estimated? Explain briefly how you obtained your answer.

(5 marks)

(f) Use the above information to compute the value of R^2, the coefficient of determination for the estimated OLS sample regression equation. Briefly explain what the calculated value of R^2 means.