QUEEN'S UNIVERSITY AT KINGSTON
Department of Economics

ECONOMICS 351* - Section A

Introductory Econometrics

Fall Term 1997 **MID-TERM EXAM** M.G. Abbott

DATE: **Thursday October 30, 1997.**

TIME: **80 minutes; 4:00 p.m. - 5:20 p.m.**

INSTRUCTIONS: The exam consists of **FIVE (5)** questions. Students are required to answer **ALL FIVE (5)** questions.

Answer all questions in the exam booklets provided. Be sure your ***name*** and ***student number*** are printed clearly on the front of all exam booklets used.

Do not write answers to questions on the front page of the first exam booklet.

Please label clearly each of your answers in the exam booklets with the appropriate number and letter.

Please write legibly.

MARKING: The marks for each question are indicated in parentheses immediately above each question. **Total marks** for the exam **equal 100**.

GOOD LUCK!

QUESTIONS: **Answer ALL FIVE questions.**

All questions pertain to the simple (two-variable) linear regression model for which the population regression equation can be written in conventional notation as:

$$Y_i = \beta_1 + \beta_2 X_i + u_i \qquad \textbf{(1)}$$

where Y_i and X_i are observable variables, β_1 and β_2 and unknown (constant) regression coefficients, and u_i is an unobservable random error term. The Ordinary Least Squares (OLS) sample regression equation corresponding to regression equation (1) is

$$Y_i = \hat{\beta}_1 + \hat{\beta}_2 X_i + \hat{u}_i \qquad (i = 1, ..., N) \qquad \textbf{(2)}$$

where $\hat{\beta}_1$ is the OLS estimator of the intercept coefficient β_1, $\hat{\beta}_2$ is the OLS estimator of the slope coefficient β_2, $\hat{u}_i$ is the OLS residual for the i-th sample observation, and N is sample size (the number of observations in the sample).

(15 marks)

1. State the Ordinary Least Squares (OLS) estimation criterion. State the OLS normal equations. Show how the OLS normal equations are derived from the OLS estimation criterion.

(15 marks)

2. Stating explicitly all required assumptions, prove that the OLS slope coefficient estimator $\hat{\beta}_2$ is an unbiased estimator of the slope coefficient β_2. Include in your answer a definition of unbiasedness.

(10 marks)

3. Explain how you would compute a two-sided $100(1-\alpha)$ percent confidence interval for the slope coefficient β_2. Define all terms required for the computation. Explain how the confidence interval for β_2 is interpreted.

(10 marks)

4. Derive the OLS decomposition equation for $TSS \equiv \sum_{i=1}^{N} y_i^2 = \sum_{i=1}^{N} (Y_i - \bar{Y})^2$, the total sum-of-squares of the observed Y_i values around their sample mean $\bar{Y}$. State all the computational properties of the OLS sample regression equation (2) on which the decomposition equation depends.

(50 marks)

5. A researcher is studying the relationship between percentage grade points on an exam (Y_i) and hours spent studying for the exam (X_i) for a sample of 30 students. Preliminary analysis of the sample data produces the following sample information:

$$\sum_{i=1}^{N} Y_i = 2400 \qquad \sum_{i=1}^{N} X_i = 450 \qquad N = 30$$

$$\sum_{i=1}^{N} x_i y_i = 3600 \qquad \sum_{i=1}^{N} x_i^2 = 900 \qquad \sum_{i=1}^{N} y_i^2 = 78{,}912 \qquad \sum_{i=1}^{N} \hat{u}_i^2 = 64{,}512$$

where $x_i \equiv X_i - \bar{X}$ and $y_i \equiv Y_i - \bar{Y}$ for i = 1, ..., N. Answer all the following questions. **Show explicitly all formulas and calculations**.

(10 marks)

(a) Use the above information to compute the OLS estimates of the intercept coefficient β_1 and the slope coefficient β_2. Interpret the slope coefficient estimate -- i.e., explain what the value of $\hat{\beta}_2$ means.

(5 marks)

(b) Use the above information to compute the value of R^2, the coefficient of determination for the estimated OLS sample regression equation. Briefly interpret what the calculated value of R^2 means.

(10 marks)

(c) Use the above information to calculate the estimated variance and estimated standard error of the slope coefficient estimate $\hat{\beta}_2$.

(10 marks)

(d) Perform a test of the null hypothesis H_0: $\beta_2 = 0$ against the alternative hypothesis H_1: $\beta_2 \neq 0$ at the 1% significance level (i.e., for significance level $\alpha = 0.01$). Show how you calculate the required test statistic, and state its null distribution. State the decision rule you use, and the inference you would draw from the test.

(5 marks)

(e) Compute the two-sided 99% confidence interval for the slope coefficient β_2.

(10 marks)

(f) Perform a test of the null hypothesis H_0: $\beta_2 \geq 2$ against the alternative hypothesis H_1: $\beta_2 < 2$ at the 5% significance level (i.e., for significance level $\alpha = 0.05$). Show how you calculate the required test statistic, and state its null distribution. State the decision rule you use, and the inference you would draw from the test.

Percentage Points of the t-Distribution