Student Number : **Fall 2009**

Name : **Good Luck !**

ECON 212

Quiz 2 for Section B (two pages)

Question 1 [9 marks] You now live in a two periods world. Your income this year is I_1 and I_2 next year. You can borrow or lend at an interest rate r (borrowing/lending an amount denoted by b_1 today means paying/collecting $(1+r)b_1$ tomorrow). Your preferences are given by a Cobb-Douglas Utility function (i.e. an interior solution exists) over consumption in period 1, C_1, and period 2, C_2. Answer the following questions and leave trace of your thinking/calculation. **[Hint: Chap.4 - To lend of not to lend?]**

(A)[5 marks] Write down the budget constraint for t=1, t=2 and derive the present value version.

(B) [4 marks] On two separate graphs show the optimal bundle (solution) for a borrower, show the solution with no borrowing or lending and identify the preferences directions.

Question 2 [11 marks] Tony Stark[i], the billionaire playboy, has a Cobb-Douglas Utility Function given by $U(x,y) = x^{1/3}y^{2/3}$. His disclosed income per year is $ 25b (billions). Initial price for good x is p_{x1} = 5b and subsequently increase to p_{x2} = 10 while the price for good y stays constant at p_y = 1b. Answer the following questions and leave trace of your thinking/calculation.

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(A)[7 marks] Find the numerical values of the income and substitution effects on good x consumption.

(B)[4 marks] Graph Part (A) result. [Hint: Make sure to identify (1) initial optimal bundle, (2) wealth effect and (3) substitution effect]

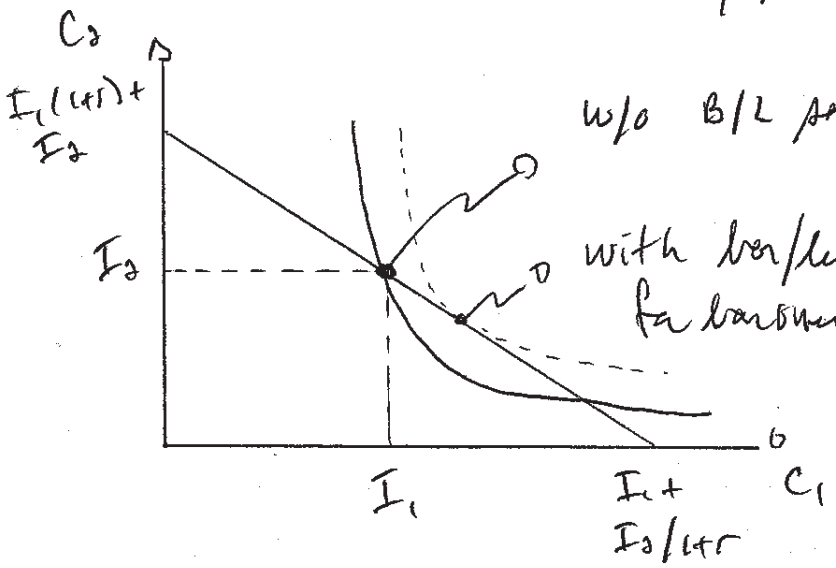
[i] Iron Man 2 : coming soon to a Theater near you in 2010.

QUESTION 1. SECTION B.

(A) See SECTION A.

(B) for borrower see SECTION A.

No borrowing and no lending
⇒ Endowment every periods.



no B/L for a "borrower"

QUESTION 2. SECTION B.

(A) As SECTION A for $x^* = \frac{\alpha I}{p_x}$ + $y^* = (1-\alpha) I$.

Using book 3 steps

(1) Initial basket with $p_x = 5$
(2) Final basket with $p_x' = 10$
(3) Intermediate decomposition basket (same utility but new price).

(1) Initial basket with $p_x = 5$

$$x_I^* = \frac{1}{3} \cdot \frac{25}{5} = \frac{5}{3} \quad + \quad y_I^* = \frac{2}{3} \cdot 25 = \frac{50}{3}.$$

(2) Final basket with $p_x' = 10$

$$x_F^* = \frac{1}{3} \cdot \frac{25}{10} = \frac{5}{6} \quad + \quad y_F^* = y_I^* = \frac{50}{3}.$$

(3) Intermediate decomposition basket
* same utility as initial basket

$$\bar{u}^I = \left(\frac{5}{3}\right)^{1/3} \left(\frac{50}{3}\right)^{2/3}$$

* tangency with new price: $MRS = p_x'$

$$y = \frac{2/3}{1/3} \cdot x \cdot 10 = 20x$$

$$\Rightarrow x^{1/3} \left[20x\right]^{2/3} = \bar{u}^I$$

$$\Leftrightarrow x_D^* = \frac{\bar{u}^I}{20^{2/3}} \Rightarrow y_D^* = 20 \left[\frac{\bar{u}^I}{20^{2/3}}\right] = 20^{1/3} \bar{u} I$$

Subs. effect: $x_D^* - x_I^*$
Income effect: $x_F^* - x_D^*$

(B) As section A.

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QUESTION 1 — SECTION A.

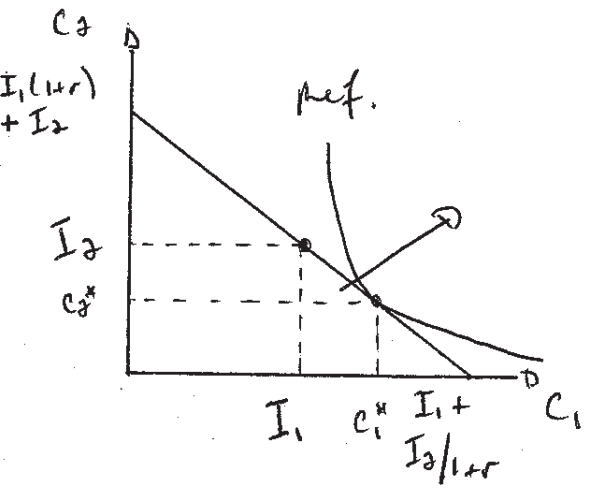
(A) $t=1$: $C_1 + b_1 = I_1$ budget const. $t=1$.

$t=2$: $C_2 = I_2 + b_1(1+r)$. budget const. $t=2$.

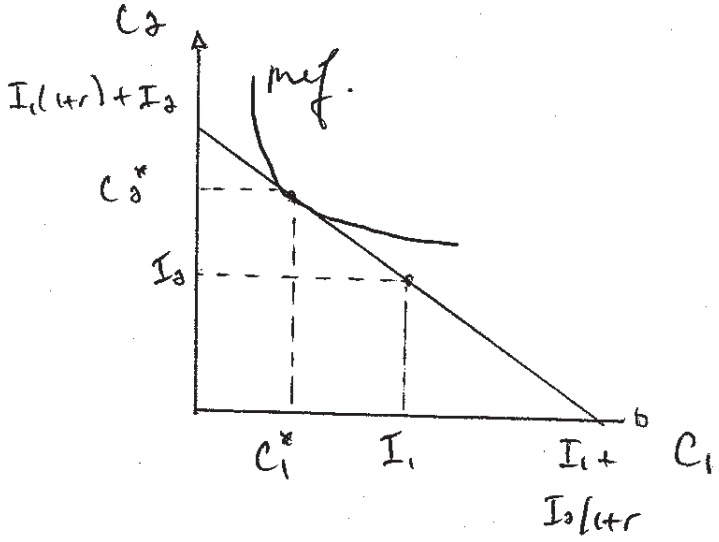
PV version: from $t=2 \Rightarrow 0$ $b_1 = \dfrac{C_2 - I_2}{1+r}$ (*)

$\Rightarrow$ plug (*)' in $t=1 \Rightarrow 0$ $C_1 + \dfrac{C_2}{1+r} = I_1 + \dfrac{I_2}{1+r}$.

(B) Borrower.



Lender



QUESTION 2. SECTION A.

(A) $\max_{x,y} x^{\alpha} y^{1-\alpha}$ s.t. $p_x x_m + y = I$

Since Cobb-Douglas $\Rightarrow$ interior solution. Optimality: $MRS = \frac{p_x}{p_y} = \frac{p_x}{1}$

$\Rightarrow$ Optimal solution given by:

$$MRS = \frac{MU_x}{MU_y} = \frac{\alpha \left(\frac{y}{x}\right)^{1-\alpha}}{(1-\alpha)\left(\frac{x}{y}\right)^{\alpha}} = \frac{\alpha y}{(1-\alpha)x}.$$

$$\Rightarrow MRS = p_x \Leftrightarrow \frac{\alpha y}{(1-\alpha)x} = p_x \Leftrightarrow y = \frac{(1-\alpha)x p_x}{\alpha}$$

$\Rightarrow$ plugging in budget constraint

$$p_x x + y = p_x x + \frac{(1-\alpha)x p_x}{\alpha} = I$$

$$\Leftrightarrow x^* = \frac{\alpha I}{p_x} \Rightarrow y^* = (1-\alpha) I$$

Using book 3 steps

1. Initial basket with $p_x = 10$
2. Final basket with $p_x' = 15$
3. Intermediate decomposition basket (same utility but new price).

SECTION A

① Initial basket with $p_x = 10$ and

$x_I^* = \frac{1}{5} \cdot \frac{30}{10} = 3/5$ and $y_I^* = \frac{4}{5} \cdot 30 = 24$

② Final basket with $p_x' = 15$

$x_F^* = \frac{1}{5} \cdot \frac{30}{15} = 2/5$ and $y_F^* = y_I^* = \frac{4}{5} \cdot 30 = 24$

③ Intermediate decomposition basket

* Same utility as initial basket

$$\bar{u}^I = \left(\frac{3}{5}\right)^{1/5} (24)^{4/5}$$

* Tangency with new price $MRS = p_x'$

$$y = \frac{4/5}{4/5} \cdot x \cdot 15 = 60x.$$

$$\Rightarrow x^{1/5} [60x]^{4/5} = \bar{u}^I \quad \text{(plug in } \bar{u}^I\text{)}$$

$$\Leftrightarrow x_D^* = \frac{\bar{u}^I}{60^{4/5}} \Rightarrow y_D^* = 60 \left[\frac{\bar{u}^I}{60^{4/5}}\right] = 60^{1/5} \bar{u}^I$$

Substitution effect: $x_D^* - x_I^*$

Income effect: $x_F^* - x_D^*$