ANSWER KEY

ECONOMICS 212
SECTION B

MIDTERM EXAM
FEBRUARY 16, 2005

STUDENT NUMBER:

Section A: Three questions @ 5 marks. Total 15 marks.

1. [5 marks] Assume the market demand function for a good is given by $Q^D = 1000 - 2P + 4I$ and the market supply function is given by $Q^S = 2P + 200 - 20W$, where Q is quantity, P is price of the good, I is income and W is the wage paid to labour. Calculate the equilibrium values of price and quantity.

$$Q^D = Q^S \Rightarrow 1000 - 2P + 4I = 2P + 200 - 20W.$$

$$\Rightarrow 4P = 800 + 20W + 4I.$$

$$\Rightarrow P^* = 200 + 5W + I$$

$$\Rightarrow Q^* = 1000 - 2(200 + 5W + I) + 4I$$

$$= 1000 - 400 - 10W - 2I + 4I$$

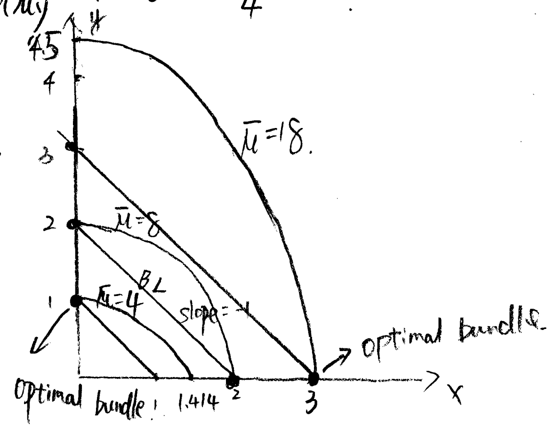
$$= 600 - 10W + 2I.$$

2. [5 marks] Consider the utility function $U(X, Y) = 2X^2 + 4Y$, where X and Y are two goods. Calculate the marginal utilities of the two goods and the marginal rate of substitution. If $P_x/P_y = 1$, derive the optimal amount of good X for the consumer?

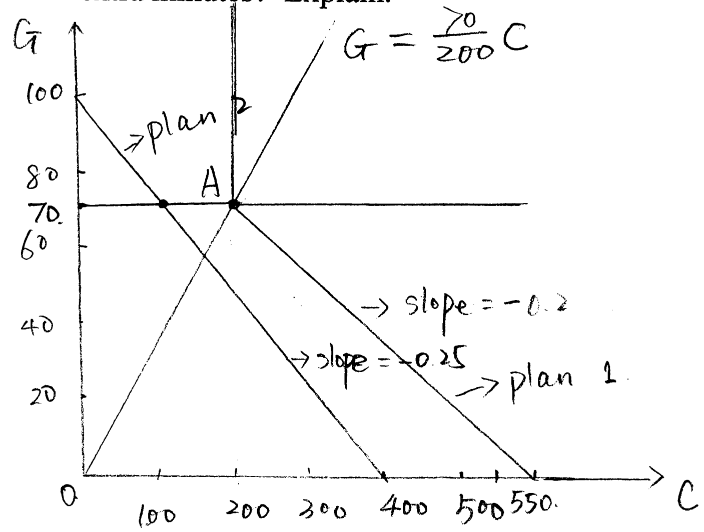
$MU_x = 4X \qquad MU_y = 4 \qquad \frac{MU_x}{MU_y} = MRS = \frac{4X}{4} = X.$

Note here $2x^2$ is a convex function, so optimality condition for interior solution does not hold here

- when $\frac{I}{P} < 2$, $\{x^* = 0,\ y^* = \frac{I}{P}\}$
- when $\frac{I}{P} = 2$, $\{x^* = \frac{I}{P} = 2,\ y^* = 0\}$ or $\{x^* = 0,\ y^* = \frac{I}{P} = 2\}$
- when $\frac{I}{P} > 2$, $\{x^* = \frac{I}{P},\ y^* = 0\}$



3. [5 marks] A consumer is considering two plans for cell phone service. The first plan has a fixed monthly fee of $30 and allows 200 minutes of calls per month. All minutes beyond 200 cost 20 cents each. The second plan has no monthly fee but each minute costs 25 cents. The consumer has $100 per month to spend on cell phone service, C, and a composite consumption good, G, whose price is $1 per unit. Draw and appropriately label the budget constraint associated with each plan. Is it possible that the consumer would purchase the first plan, but never buy extra minutes? Explain.



Yes, it is possible. For example, suppose the consumer's utility fn is given by $U = \min\{70C, 200G\}$, then the consumer will purchase the first plan & her optimal bundle will be $A = \{C^* = 200, G^* = 70\}$, that is, she will only use 200 minutes of call and never buy extra minutes.

Section B: Three questions @15 marks – 5 for each part of each question. Total 45 marks.

1. Richard consumes steak, S, and ale, A, according to the utility function U (S, A) = $S^{1/3}A^{2/3}$. The price of steak is P_S, the price of ale is P_A and Richard's income is I.
   (a) [5 marks] Derive Richard's demand functions for steak and ale.

① $\max_{(S,A)} u(S,A) = S^{1/3}A^{2/3}$

s.t. $P_S \cdot S + P_A \cdot A = I$

$$MRS = \frac{MU_S}{MU_A} = \frac{\frac{1}{3}S^{-\frac{2}{3}}A^{\frac{2}{3}}}{\frac{2}{3}S^{\frac{1}{3}}\cdot A^{-\frac{1}{3}}} = \frac{A}{2S} = \frac{P_S}{P_A} \Rightarrow A = 2S\cdot\frac{P_S}{P_A}$$

$$P_S\cdot S + \cancel{P_A}\cdot 2S\cdot\frac{P_S}{\cancel{P_A}} = I \Rightarrow S^* = \frac{I}{3P_S}, \quad A^* = 2S\cdot\frac{P_S}{P_A} = \frac{2I}{3P_A}$$

or ② $$S^* = \frac{a}{a+b}\cdot\frac{I}{P_S} = \frac{\frac{1}{3}}{\frac{1}{3}+\frac{2}{3}}\frac{I}{P_S} = \frac{I}{3P_S}$$

$$A^* = \frac{b}{a+b}\cdot\frac{I}{P_A} = \frac{\frac{2}{3}}{\frac{1}{3}+\frac{2}{3}}\frac{I}{P_A} = \frac{2I}{3P_A}$$

   (b) [5 marks] Suppose that the price of steaks is \$5, the price of ale is \$4, and Richard's income is \$300. Determine Richard's optimal bundle.

$$S^* = \frac{I}{3P_S} = \frac{300}{3\times5} = 20$$

$$A^* = \frac{2I}{3P_A} = \frac{2\times300}{3\times4} = 50.$$

(c) [5 marks] Suppose that the price of ale increases to \$5. Determine the new optimal bundle and the income and substitution effects of the price change for ale.

① new optimal bundle $\{S_1, A_1\}$

$$S_1 = \frac{I}{3P_S} = \frac{300}{3\times 5} = 20 \quad A_1 = \frac{2I}{3P_A} = \frac{2\times 300}{3\times 5} = 40$$

② Decomposition bundle $\{S_2, A_2\}$

utility level given by initial optimal bundle $\{S_0 = 20, A_0 = 50\}$

$$u_0 = S_0^{\frac{1}{3}} A_0^{\frac{2}{3}} = 20^{\frac{1}{3}} \times 50^{\frac{2}{3}} = 36.84$$

$$\min_{(S,A)} \quad 5\cdot S + 5A$$

$$\text{s.t} \quad u = S^{\frac{1}{3}}\cdot A^{\frac{2}{3}} = 36.84$$

$$MRS = \frac{MU_S}{MU_A} = \frac{\frac{1}{3}S^{-\frac{2}{3}}A^{\frac{2}{3}}}{\frac{2}{3}S^{\frac{1}{3}}A^{-\frac{1}{3}}} = \frac{A}{2S} = \frac{P_S}{P_A} = \frac{5}{5} = 1 \Rightarrow A = 2S$$

$$\Rightarrow S^{\frac{1}{3}}(2S)^{\frac{2}{3}} = 36.84 \Rightarrow S_2 = 23.21,\ A_2 = 2S_2 = 46.42$$

Substitution Effect: $A_2 - A_0 = 46.42 - 50 = -3.58$ ↓ by 3.58

Income Effect: $A_1 - A_2 = 40 - 46.42 = -6.42$ ↓ by 6.42

2. Suppose that Marie earns \$2,000,000 while working and nothing when retired. The interest rate is 5% and Marie's utility function is given by U = Min $\{C_1; 4C_2\}$, where C_1 is consumption during working life and C_2 is consumption during retirement.

(a) [5 marks] Derive Marie's optimal consumption bundle and her level of savings.

$$BL: C_1 + \frac{C_2}{1+r} = I_1 + \frac{I_2}{1+r} \Rightarrow C_1 + \frac{C_2}{1.05} = 2\times 10^6 + 0$$

$$\max_{(C_1, C_2)} \quad U = \min\{C_1; 4C_2\}$$

$$\text{s.t} \quad C_1 + \frac{C_2}{1.05} = 2\times 10^6$$

$$C_1 = 4C_2 \Rightarrow 4C_2 + \frac{C_2}{1.05} = 2\times 10^6 \Rightarrow 4.952C_2 = 2\times 10^6 \Rightarrow$$

$$C_2^* = 4.04\times 10^5; \quad C_1^* = 4C_2^* = 1.616\times 10^6$$

$$S^* = I_1 - C_1 = 2\times 10^6 - 1.616\times 10^6 = 3.84\times 10^5$$

(b) [5 marks] The government has decided to impose a tax at the rate of 25% on Marie's earnings from work. The interest on her savings is not taxed. Derive Marie's new optimal bundle. How does the tax affect her savings?

New budget line: $C_1 + \frac{C_2}{1.05} = 2\times10^6 \times (1-0.25) = 1.5\times10^6$

$$\max_{(C_1,C_2)} U = \min\{C_1, 4C_2\}$$

$$\text{s.t} \quad C_1 + \frac{C_2}{1.05} = 1.5\times10^6$$

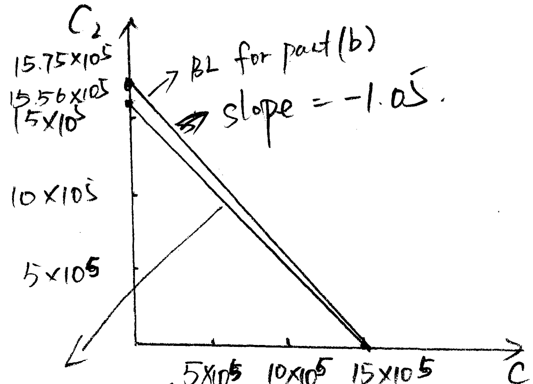
$$C_1 = 4C_2 \Rightarrow 4C_2 + \frac{C_2}{1.05} = 1.5\times10^6 \Rightarrow 4.952\,C_2 = 1.5\times10^6$$

$$\Rightarrow C_2^* = 3.03\times10^5 \;;\quad C_1^* = 4C_2^* = 1.212\times10^6$$

$$S^* = I_1 - C_1^* = (1.5 - 1.212)\times10^6 = 2.88\times10^5 < 3.84\times10^5$$

Her savings decrease after the tax is levied.

(c) [5 marks] Draw and appropriately label the budget constraint from part (b) of this question. Show how the budget constraint would change if the interest on Marie's savings were also subject to the 25% tax.



BL if interest on savings are also subject to 25% tax

slope $= -1 - 0.05\times0.75 = -1.0375$

part (b)

horizontal intercept: 1.5×10^6

Vertical intercept: $1.5\times1.05\times10^6 = 1.575\times10^6$

Slope $= -1.05$

If the interest on savings are subject to the tax.

horizontal intercept: 1.5×10^6

Vertical intercept: $0.05\times1.5\times10^6\times(1-0.25) + 1.5\times10^6 = 1.55625\times10^6$

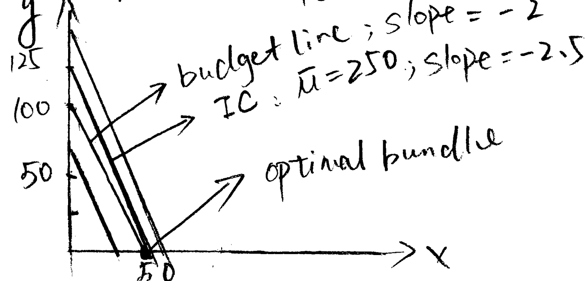
slope $= -\frac{1.55625\times10^6}{1.5\times10^6} = -1.0375$.

3. Mark consumes the goods X and Y according to the utility function U = 5X + 2Y. Assume the price of X is \$10, the price of Y is \$5 and Mark's income is \$500.
   (a) [5 marks] Derive Mark's optimal bundle. Draw and appropriately label a consumer choice diagram that shows this optimum.

$$\max_{(x,y)} \; u = 5x + 2y$$
$$\text{s.t} \quad 10x + 5y = 500$$

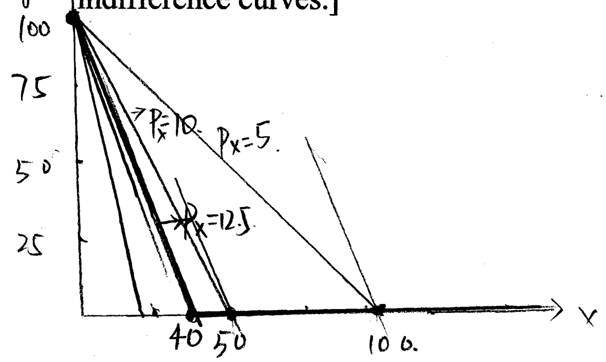
$$\frac{MU_x}{P_x} = \frac{5}{10} > \frac{MU_y}{P_y} = \frac{2}{5} \Rightarrow \text{spend all on good } x$$

$$\Rightarrow \{x^* = \frac{500}{10} = 50 \; ; \; y^* = 0\}$$

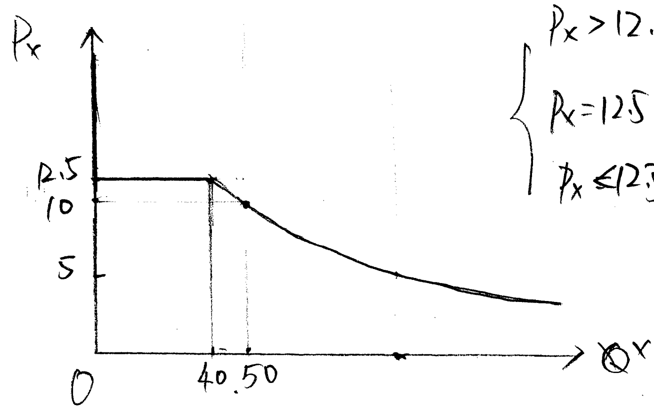


   (b) [5 marks] Given that Mark's income is \$500 and that the price of Y is \$5, draw and appropriately label Mark's demand curve for the good X. [Hint: go through the same graphical exercise as you would with well-behaved indifference curves.]

$\frac{500}{P_x = 12.5}$



$$X^* = \begin{cases} \frac{I}{P_x} = \frac{500}{P_x} & \text{if } P_x < 12.5 \\ \text{any point} \in [0, 40] & \text{if } P_x = 12.5 \\ 0 & \text{if } P_x > 12.5 \end{cases}$$



$$\begin{cases} P_x > 12.5 & \frac{MU_x}{P_x} = \frac{5}{P_x} < \frac{MU_y}{P_y} = \frac{2}{5}, \text{ spend all on } Y, \; X^* = 0 \\ P_x = 12.5 & \frac{MU_x}{P_x} = \frac{MU_y}{P_y} \text{ any point on } 12.5x + 5y = 500, \; X^* \in [0, 40] \\ P_x \leq 12.5 & \frac{MU_x}{P_x} > \frac{MU_y}{P_y} \text{ spend all on } X, \; X^* = \frac{500}{P_x} \end{cases}$$

(c) [5 marks] Suppose that Mark develops a strong distaste for the good Y and his utility function becomes U = 5X – 2Y. Given the income and price of Y as in part (b), draw Mark's new demand curve for the good X. Explain why it is shaped this way.

Y is the bad good. A consumer only wants to consume X.

$$X^* = \frac{I}{P_x} = \frac{500}{P_x}$$