Economics 212

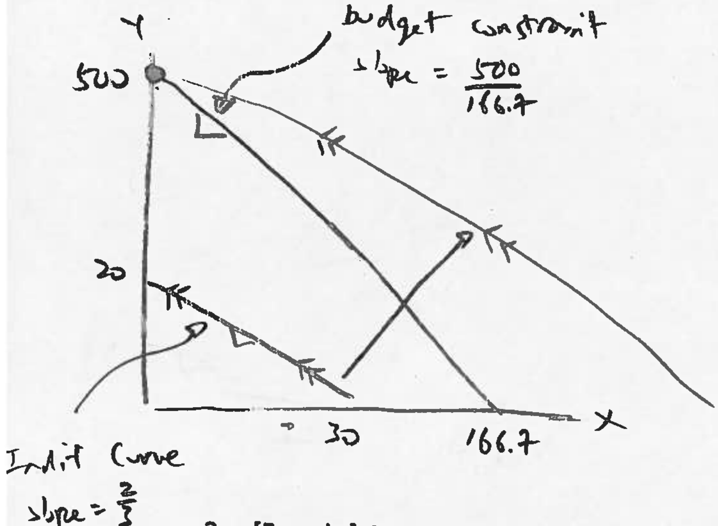
Midterm Exam

March 5, 2012

Student Number: Answer Key

Section A: Three questions @ 5 marks. Total 15 marks.

1. [5 marks] Consider the utility function U(X,Y)=2X+3Y, where X and Y are two goods. Draw and appropriately label two indifference curves for this consumer. Assume the price of X is \$6, the price of Y is \$2 and the consumer has an income of \$1000. Derive the optimal consumption bundle for the consumer.



$MU_x = 2$

$MU_y = 3$

Note: $\frac{MU_x}{P_x}$: Marginal util received per dollar spend on acquiring a marg. unit

$\frac{MU_x}{P_x} = \frac{2}{6} = \frac{1}{3}$

$\frac{MU_y}{P_y} = \frac{3}{2}$

$\frac{MU_x}{P_x} < \frac{MU_y}{P_y}$

$\Rightarrow$ $\boxed{\text{Optimal bundle } (X,Y) = (0,500)}$

2. [5marks] A consumer has \$125 in income and a utility of income function described by $U(I)=I^{1/3}$. The consumer is considering a wager that requires her to put up the entire \$125, with a probability of .3 that she will end up with \$512 and a probability of .7 that she will end up with \$27. Explain whether the consumer should accept or reject the wager.

$U(\text{Reject}) = u(125) = 125^{\frac{1}{3}} = 5$

$U(\text{Accept}) = 0.7\,u(27) + 0.3\,u(512)$

$= 0.7\,(3) + 0.3\,(8)$

$= 4.5$

$U(R) > U(A) \Rightarrow \boxed{\text{Reject}}$

since $|\varepsilon_D| = \varepsilon_S$

the tax burden is shared equally

3. [5marks] Assume that market demand is given by Q^D=1000-2P and market supply is given by Q^S=2P+200, where Q is quantity and P is price. Derive the equilibrium values of price and quantity. If a per unit tax were levied on this good, would producers or consumers bear the greater burden of the tax. Explain.

① Equilibrium $\Rightarrow Q^S = Q^D$

$$2P + 200 = 1000 - 2P$$

$$\boxed{P = 200, \quad Q = 600}$$

② Tax Burden

Demand Elasticity $= \varepsilon_D = \frac{dQ_D}{dP}\frac{P}{Q_D} = -2\left(\frac{P}{Q}\right) = -2\left(\frac{200}{600}\right) = \frac{-2}{3}$

Supply Elasticity $= \varepsilon_S = \frac{dQ_S}{dP}\frac{P}{Q_S} = \frac{2}{3}$

Section B: Three question @ 15 marks- 5 for each part of each question. Total 45 marks.

1. Ella and Sarah have been stranded on a desert island. Ella has found 20 coconuts, C, and 20 bananas, B. Her preferences over the two goods are described by $U_E=CB^2$. Sarah has found 10 coconuts and 40 bananas. Her preferences are given by $U_S=C^2B$. They have agreed to trade 2 bananas for 1 coconut.

   a) [5 marks] Derive Ella's optimal consumption bundle. Describe the trade she must make to move from her endowment (what she found) to her desired bundle.

$$\frac{MU_B^E}{P_B} = \frac{MU_C^E}{P_C} \Rightarrow \frac{2BC}{1} = \frac{B^2}{2} \Rightarrow \boxed{4C = B \quad (1)}$$

budget constraint (bc): $\underbrace{20P_C + 20P_B}_{\text{Income/Endowment}} = \underbrace{BP_B + CP_C}_{\text{Consumption}}$

$$\Rightarrow 20(2) + 20(1) = B(1) + C(2) \Rightarrow \boxed{60 = B + 2C \quad (2)}$$

(1) & (2) $\Rightarrow 60 = 6C$

$$\boxed{\begin{matrix} C^E = 10 \\ B^E = 40 \end{matrix}}$$

b) [5 marks] Derive Sarah's optimal consumption bundle. Describe the trade Sarah must make to move from her endowment to her optimal bundle

$$\frac{MU_B^S}{P_B} = \frac{MU_C^S}{P_C} \Rightarrow \boxed{C = B \quad (3)}$$

$$bc: \quad 10P_C + 40P_B = CP_C + BP_B$$

$$\boxed{60 = C + 2B \quad (4)}$$

$$(3) \ \& \ (4) \Rightarrow \quad 60 = 3B$$

$$\boxed{\begin{aligned} B^S &= 20 \\ C^S &= 20 \end{aligned}}$$

c) [5 marks] Will Sarah and Ella be able to arrange a trade the leaves each consuming her optimal bundle? Explain.

① Recall $B^E = 40 \qquad B^S = 20$

$C^E = 10 \qquad C^S = 20$

② Total desired Consumption: $\left.\begin{aligned} B &= B^E + B^S = 60 \\ C &= C^E + C^S = 30 \end{aligned}\right\}$

③ Total Endowment: $\left.\begin{aligned} \text{Bananas} &= 20 + 40 = 60 \geq B \\ \text{Coconuts} &= 20 + 10 = 30 \geq C \end{aligned}\right\}$

Since total endowment ≥ total desired consumption,
Feasible

2. Alex has 16 hours per day to divide between leisure, R, and work. When he works, Alex earns $30 per hour. He values both leisure and consumption, C, according to the utility function U(R,C)=RC. The price of the consumption good is unity.

a) [5marks] Derive Alex's optimal bundle. How much does he work?

bc: $\boxed{(16-R)W = C \quad (1)}$

$$\left.\begin{aligned} \frac{MU_R}{P_R} &= \frac{C}{W} \\ \frac{MU_C}{P_C} &= \frac{R}{1} = R \end{aligned}\right\} \quad \boxed{\frac{C}{W} = R \quad (2)}$$

(1) & (2) $\Rightarrow$ $\frac{(16-R)W}{W} = R \Rightarrow$ $\boxed{\begin{aligned} R &= 8 \\ C &= 240 \end{aligned}}$

b) [5 marks] The government imposes a tax at the rate of 10% on Alex. Explain how the tax affects Alex's optimal choice and his work effort.

assuming its an Income tax

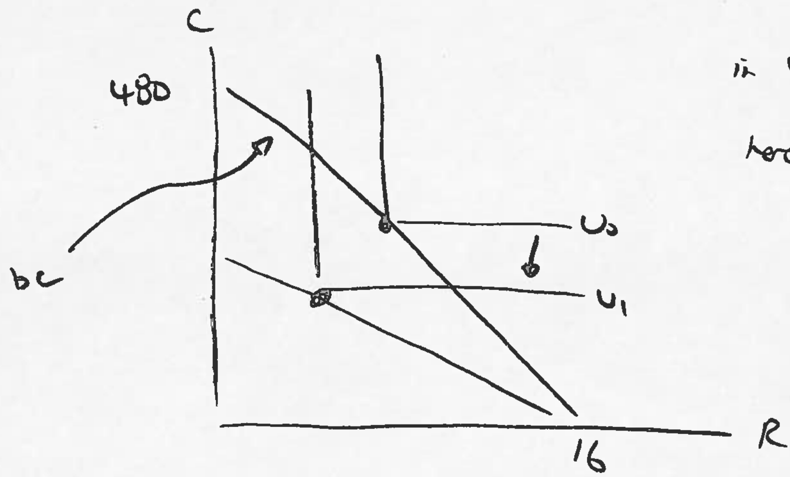
bc: $\boxed{(16-R)(1-t)W = C \quad (3)}$

$\frac{MU_C}{P_C} = \frac{MU_R}{P_R} \Rightarrow \boxed{\frac{C}{C(1-t)} = R \quad (4)}$

(3) & (4) $\Rightarrow$ $\boxed{\begin{aligned} R &= 8 \\ C &= 216 \end{aligned}}$

c) Explain and illustrate how your answer to part b) would change, if at all, if Alex had preferences over leisure and consumption described by a perfect complements utility function.

Perfect Complement util: $U(R, C) = \min\{R, C\}$



in (b), R is constant

here (c) $\underline{\underline{R \downarrow}}$

3. [5 marks] Mario is a video game entrepreneur who is very successful. He earns $40,000,000 during his working life and nothing when he retires. The interest rate between his working life and retirement is 80%. His preferences over present consumption, C_P, and future consumption, C_F, are given by $U(C_P,C_F)=\text{Min}\{C_P;4C_F\}$.

a) Derive Mario's optimal consumption bundle and his level of savings.

$$\min\{C_P, 4C_F\} \Rightarrow \boxed{C_P = 4C_F \quad (1)}$$

bc Present: $I = S + C_P$

Future: $S(1+r) = C_F \Rightarrow S = \frac{C_F}{(1+r)}$

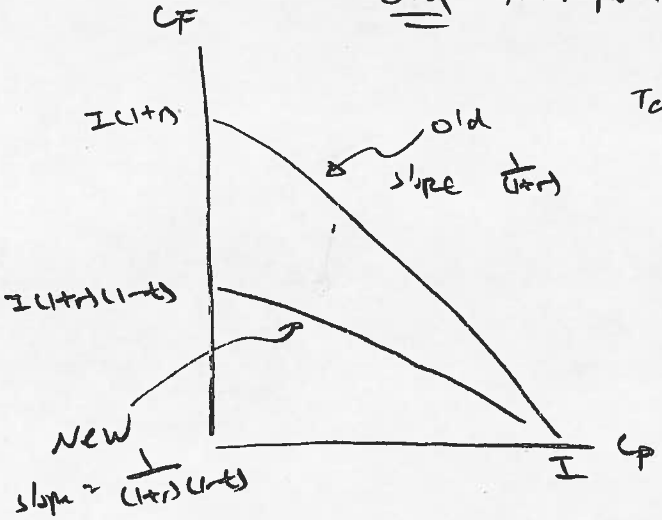
Intemporal bc: $\boxed{I = \frac{C_F}{(1+r)} + C_P \quad (2)}$

(1) & (2) $\Rightarrow I = \frac{C_F}{(1+r)} + 4C_F$

$$\boxed{\begin{array}{ll} C_F = 8{,}784{,}488 & S = 4{,}878{,}048 \\ C_P = 35{,}121{,}951 & \end{array}}$$

b) Draw and appropriately label Mario's budget constraint. Suppose the government decides to tax the interest earned on Mario's savings at the rate of 20%. Draw and appropriately label Mario's new budget constraint. Write the present value form of this new budget constraint.

old intemporal bc : $I = \frac{C_F}{(1+r)} + C_P$



Tax on Earnings ⇒ Future budget

$s(1+r)(1-t) = C_F \Rightarrow \frac{S_F}{(1+r)(1-t)}$

New intemporal bc :

$$\boxed{I = \frac{C_F}{(1+r)(1-t)} + C_P}$$

c) Derive Mario's new optimal bundle in the presence of the tax. How has the tax affected Mario's level of savings?

Given $I = \frac{C_F}{(1+r)(1-t)} + C_P$ & $C_P = 4C_F$

$\Rightarrow I(1+r)(1-t) = C_F + 4C_F(1+r)(1-t)$

$I(1.8)(0.8) = C_F + C_F(7.2)(0.8)$

$1.44\,I = 6.76\,C_F$

$C_F' = \frac{1.44\,I}{6.76} = 8{,}520{,}710$

$C_P' = 34{,}082{,}840$

$s' = 5{,}917{,}159$

$s' > s$