ECONOMICS 212
SECTION A

MIDTERM EXAM
FEBRUARY 15, 2005

STUDENT NUMBER: ANSWER KEY

Section A: Three questions @ 5 marks. Total 15 marks.

1. [5 marks] Assume the market demand function for a good is given by $Q^D = 2000 - 4P + 8I$ and the market supply function is given by $Q^S = 4P + 400 - 40W$, where Q is quantity, P is price of the good, I is income and W is the wage paid to labour. Calculate the equilibrium values of price and quantity.

$Q^S = Q^D$ so $400 + 4P - 40W = 2000 - 4P + 8I$

$8P = 1600 + 40W + 8I$

$\boxed{P^* = 200 + 5W + I}$

sub in $Q = 400 + 4(200 + 5W + I) - 40W$

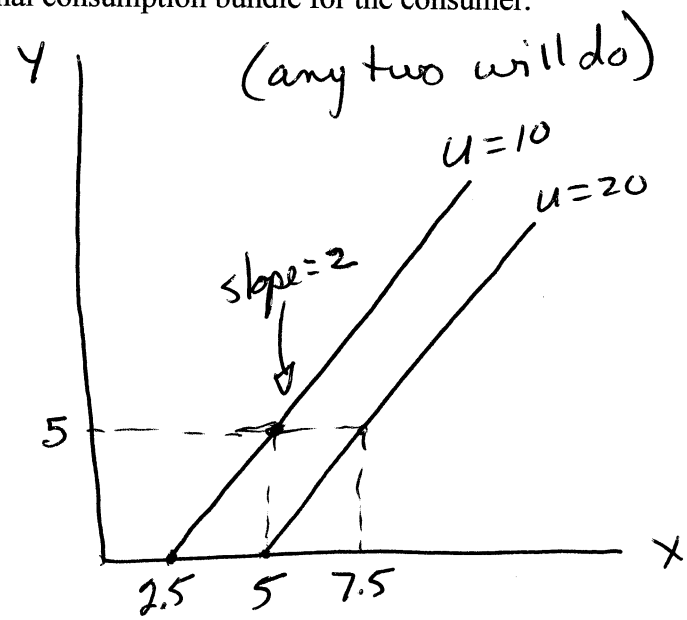
$Q = 400 + 800 + 20W + 4I - 40W$

$\boxed{Q^* = 1200 - 20W + 4I}$

2. [5 marks] Consider the utility function U(X, Y) = 4X - 2Y, where X and Y are two goods. Draw and appropriately label two indifference curves for this consumer. Assume the price of X is \$10, the good Y is free and the consumer's income is \$200. Derive the optimal consumption bundle for the consumer.

Because Y is a bad consume only X

$\boxed{X^* = \frac{200}{10} = 20}$



3. [5 marks] A consumer buys two goods, X and Y. The price of X is \$8, the price of Y is \$2 and the consumer's income is \$240. Draw and appropriately label the consumer's budget constraint. Now suppose that the government wishes to encourage consumption of Y and discourage consumption of X. They announce that the first 10 units of Y will be subsidized at the rate of \$4 per unit and that all units of good X will be subject to a tax of \$2 per unit. Draw and appropriately label the new budget constraint.

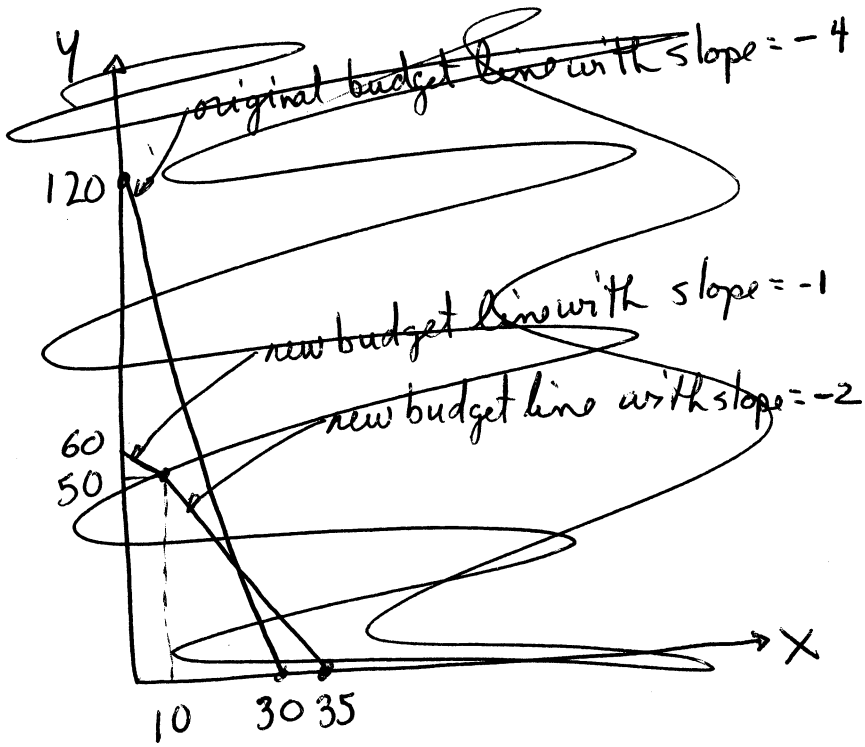
Explanation

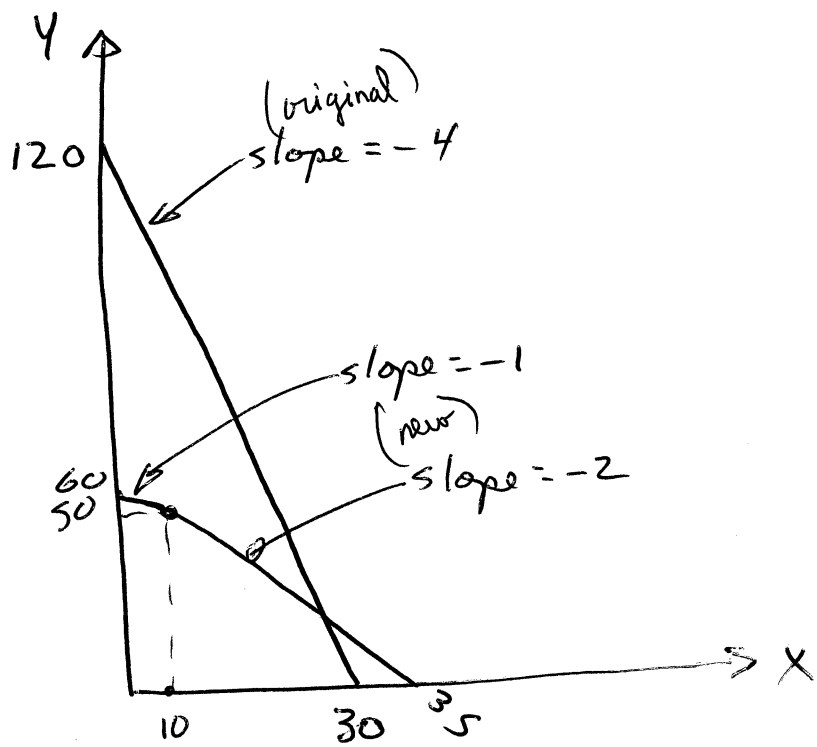
1st 10X at price of 4
⇒ spend \$40 so \$200
left → if all of X at
$P_X = 8$ buy 25 more
∴ horiz intercept is 35

P_Y now priced at \$4 ∴
max is 60

1st 10 X $\frac{P_X}{P_Y} = 1$

Rest of X $\frac{P_X}{P_Y} = 2$





Section B: Three questions @15 marks – 5 for each part of each question. Total 45 marks.

1. For entertainment, Lee consumes movies, M, and dinners, D, according to the utility function U (M, D) = $M^{1/2}D^{1/2}$. The price of a movie is P_M, the price of a dinner is P_D and Lee's income is I.

   (a) [5 marks] Derive Lee's demand functions for movies and dinners.

$MU_M = \frac{1}{2} M^{-1/2} D^{1/2}$

$MU_D = \frac{1}{2} M^{1/2} D^{-1/2}$

$|MRS| = \frac{D}{M}$ so $\frac{D}{M} = \frac{P_M}{P_D}$

$D = \frac{M P_M}{P_D}$

$M = \frac{D P_D}{P_M}$

sub in $P_M \frac{D P_D}{P_M} + P_D D = I$ $\longrightarrow$ $\boxed{D^* = \frac{I}{2P_D}}$

sub in $P_M M + P_D \frac{M P_M}{P_D} = I$ $\longrightarrow$ $\boxed{M^* = \frac{I}{2P_M}}$

   (b) [5 marks] Suppose that the price of a movie is \$10, the price of a dinner is \$10, and Lee's entertainment budget is \$200. Determine Lee's optimal bundle.

$D = \frac{200}{(2)(10)} = 10$

$M = \frac{200}{(2)(10)} = 10$

New demand is $M = \frac{200}{(2)(5)} = 20$

(c) [5 marks] Suppose that the price of a movie decreases to \$5. Determine the new optimal bundle and the income and substitution effects of the price change for movies.

original utility level is $U = (10)^{1/2}(10)^{1/2} = 10$

Find decomposition bundle must have

$M^{1/2} D^{1/2} = 10$ (on original indiff curve)

and $|MRS| = \frac{P_M}{P_D} \Rightarrow \frac{D}{M} = \frac{1}{2}$ (opt condn)

sub 2nd into 1st

$M^{1/2}\left(\frac{M}{2}\right)^{1/2} = 10 \rightarrow M = 14.14$ → subs effect P X from 10 to 14.14

Income effect P X from 14.14 to 20

2. Suppose that Sandra earns \$3,000,000 while working and nothing when retired. The interest rate is 10% and Sandra's utility function is given by $U = C_1^2 + 2C_2^2$, where C_1 is consumption during working life and C_2 is consumption during retirement.

(a) [5 marks] Derive Sandra's optimal consumption bundle and her level of savings.

Budget constraint is $1.1C_1 + C_2 = 3{,}300{,}000$

$MU_1 = 2C_1$, $MU_2 = 4C_2$ $\Big\}$ $|MRS| = \frac{C_1}{2C_2}$ at opt $\frac{C_1}{2C_2} = 1.1 \rightarrow C_1 = 2.2C_2$

sub in $(1.1)(2.2C_2) + C_2 = 3{,}300{,}000 \rightarrow C_2^* = 964{,}912.28$

$C_1 = 2.2C_2 \rightarrow C_1^* = 2{,}122{,}807$

Savings $S = I_1 - C_1 \rightarrow S^* = 877{,}193$

(b) [5 marks] Assume that Sandra knows she will inherit $500,000 at the start of her retirement. Derive her new optimal consumption bundle and level of savings. Is consumption during retirement a normal good? Explain.

Inheritance adds I_2 to b.c. so $3{,}800{,}000 = 1.1C_1 + C_2$

same process as part (a)

$C_2^* = 2{,}444{,}444.40$

$C_1^* = 1{,}111{,}111.10$

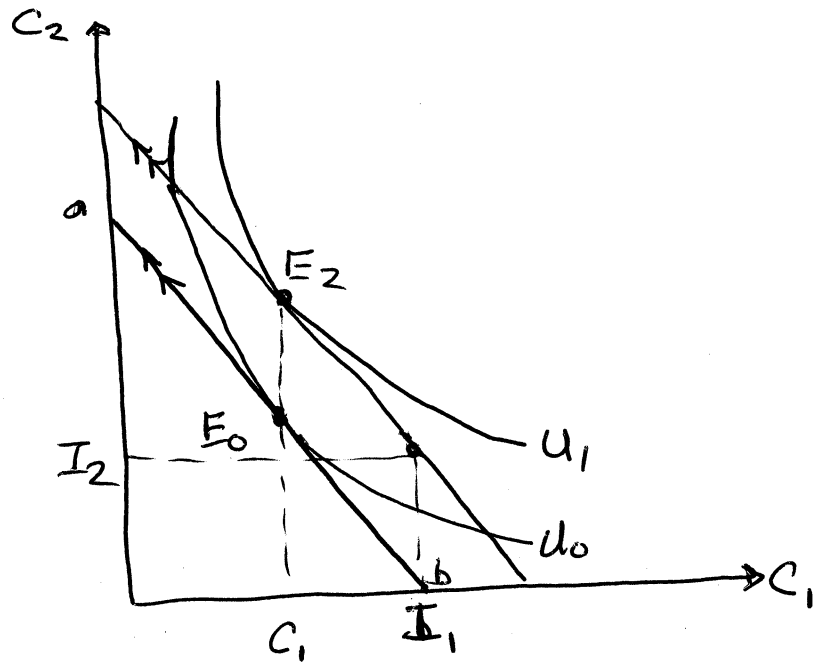
$S^* = 555{,}555.60$

C_2 is normal good b/c ↑ as Income ↑

(c) [5 marks] Suppose Sandra's tastes change and her utility function becomes $U(C_1, C_2) = C_1^2 + C_2$. Show graphically how the inheritance affects her consumption and her level of savings.

- ab is original budget line & E_0 is optimal choice
- Savings are $(I_1 - C_1)$
- inheritance shifts budget line out parallel through (I_1, I_2)
- Key is utility fn is quasi-linear & vertically parallel ⇒ new opt at where C_1 doesn't Δ

⇒ No Δ in savings $(I_1 - C_1)$



3. Dolittle has 112 hours per week to divide between leisure, R, and work. When he works, Dolittle earns $10 per hour. He values both leisure and consumption, C, according to the utility function U (R, C) = Min {R ; C}. The price of the consumption good is one.

   (a) [5 marks] Derive Dolittle's optimal consumption bundle. How much does Dolittle work?

- budget constraint is $112W = C + RW \rightarrow 1120 = C + 10R$
- with perf complements opt is where $R = C$ → sub in

$1120 = C + 10C \rightarrow$ **$C^* = 101.82$ and $R^* = 101.82$**

Dolittle works 10.18 hours

   (b) [5 marks] The government tells Dolittle that they will supplement his wage by $10 per hour, but only if he works 30 hours or more per week. Draw and appropriately label Dolittle's original budget constraint and his budget line with the wage supplement. Explain whether Dolittle will accept or reject the wage supplement.

Dolittle currently has

$U = \text{Min}\{101.82; 101.82\}$

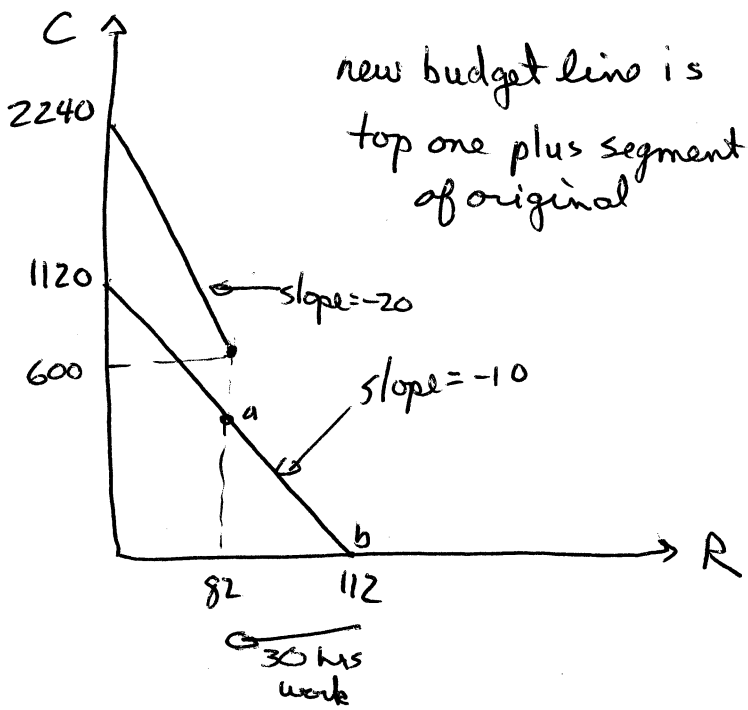
$= 101.82$

Under program + working minimum 30 hours

$U = \text{Min}\{82; 600\}$

$= 82 \rightarrow$ ↑ work & ↓ leisure & ↓ utility

⇒ Not accept offer



new budget line is top one plus segment ab of original

(c) [5 marks] Dolittle's friend, Idle, has received the same offer from the government. Idle is currently not working and spends his 112 hours per week in leisure. His utility function is given by U (R, C) = 15R + C. Idle has been offered a job at $10 per hour. Will Idle accept the job offer and the wage supplement? If so, how much will he work? Explain your answer.

Initially Idle does not work b/c corner solution of all leisure

compare $\frac{MU_R}{w} = \frac{15}{10}$ vs $\frac{MU_C}{P_C} = \frac{1}{1}$

→ leisure generates larger utility per $ spent

With the supplement

compare $\frac{MU_R}{w} = \frac{15}{20}$ vs $\frac{MU_C}{P_C} = \frac{1}{1}$

→ Now consumption yields more utility per $ spent so all consumption corner solution ⇒ works 112 hours & accepts offer