ECONOMICS 212A
MIDTERM EXAM

OCTOBER 26, 2005

NAME: ANSWER KEY

STUDENT NUMBER:

INSTRUCTIONS: Please answer all questions in this exam booklet. If you run out of space on a question, please continue your answer on the back of the question paper. The grade assigned to each question is indicated beside the question.

Part A

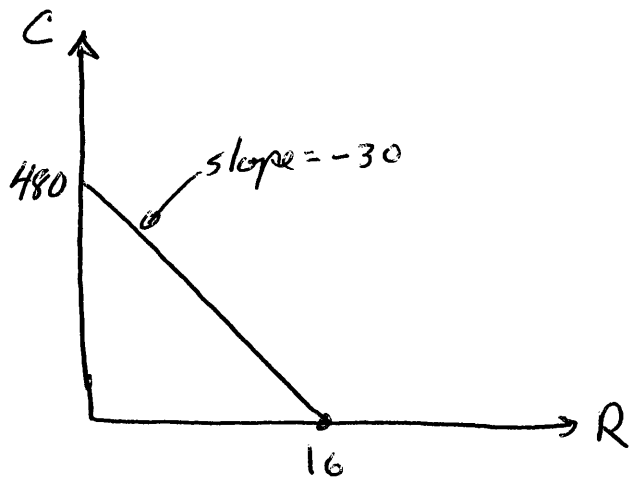
Question One [5 marks]
The demand for apples is given by $Q^D = 700 - 2P_A + P_O + .2I$, where I is income, P_A is the price of apples and P_O is the price of oranges. Are apples and oranges substitutes in consumption or complements in consumption? Explain. Are apples a normal good? Explain.

$$\varepsilon_O^{A,O} \frac{\partial Q^D}{\partial P_O} \frac{P_O}{Q^D} = (1)\frac{P_O}{Q_D} = \frac{P_O}{Q_D} > 0 \Rightarrow \text{substitutes}$$

$$\varepsilon_I = \frac{\partial Q^D}{\partial I} \frac{I}{Q^D} = (.2)\frac{I}{Q_D} > 0 \Rightarrow \text{normal}$$

— Don't have to use elasticities. Students can explain directly also.

Question Two [5 marks]
James has 16 hours per day to allocate between leisure, R, and work. When he works, James earns $30 per hour which he uses to buy a composite consumption good, C, with a price of one. Draw and appropriately label James' budget constraint. Write a utility function between consumption and leisure such that James optimal consumption bundle involves all lesisure.



Many would work. The simplest is U only depends on R.

Question Three [5 marks]
Grace has a utility function defined by $U(X, Y) = X^2 + Y^2$, where X and Y are two goods. If the price of X is two, the price of Y is four and Grace's income is \$200, find Grace's optimal consumption bundle.

Interior $|MRS| = \frac{2}{4} \rightarrow \frac{X}{Y} = \frac{2}{4}$ $\quad X = \frac{2Y}{4}$

Sub into b.c. $2\left[\frac{2Y}{4}\right] + 4Y = 200 \rightarrow 5Y = 200 \quad Y = 40$

$\Rightarrow X = 20$

$\rightarrow$ not U MAX b/c U fn convex & Indiff. Curves concave. $U = 2000$

Corners

All X $\Rightarrow X = 100 \quad U = 10000$

All Y $\Rightarrow Y = 50 \quad U = 2500$

All X is U MAX

Solution

Part B

Question One
Mikhail has \$300 in income and is considering spending it on a wager where there is a 20% probability that he finishes with \$1,000 and an 80% probability that he finishes with \$0.

a) [5 marks] Mikhail is a risk-lover with a utility of income function given by $U(I) = I^2$. Will Mikhail accept the wager? Explain.

Sure thing $U(300) = 300^2 = 90000$

wage $EU = (.2)(1000)^2 + (.8)(0)^2$

$= 200,000$

Accept wager b/c $EU > U(\text{sure thing})$

b) [5 marks] What is the smallest winning outcome (given a winning probability of 20%) that would induce Mikhail to accept the wager?

MUST have $EU = U(\text{sure thing}) = 90000$

$EU = (.2)(X)^2 = 90000$

$X^2 = 450000$

$X = 670.82$

c) [5 marks] Beginning from part a), what is the smallest probability of winning $1000 that would just induce Mikhail to accept this wager?

As above except

$EU = p(1000)^2 = 90000$

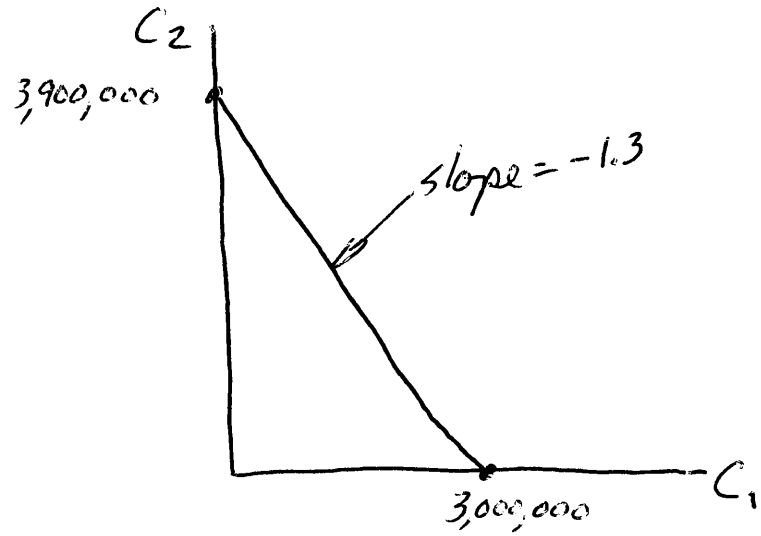
$p = \frac{90000}{1000000}$

$p = .09$

Question Two

Yu earns $3,000,000 when working and nothing when retired. The interest rate between her working life and retired life is 30%.

a) [5 marks] Draw and appropriately label Yu's budget constraint. Write the equation of this budget constraint in its present value form.



$3{,}000{,}000 = C_1 + \frac{C_2}{1.3}$

b) [5 marks] Derive Yu's optimal consumption bundle given that her utility function is given by $U(C_1, C_2) = C_1C_2$, where C_1 is consumption when working and C_2 is consumption when retired.

$MU_1 = C_2$
$MU_2 = C_1$
$\Big\}$ $|MRS| = \frac{C_2}{C_1}$ At opt $\frac{C_2}{C_1} = 1.3 \Rightarrow C_2 = 1.3C_1$

sub into bc. $3{,}000{,}000 = C_1 + \frac{1.3C_1}{1.3}$

$2C_1 = 3{,}000{,}000$

$C_1 = 1{,}500{,}000$

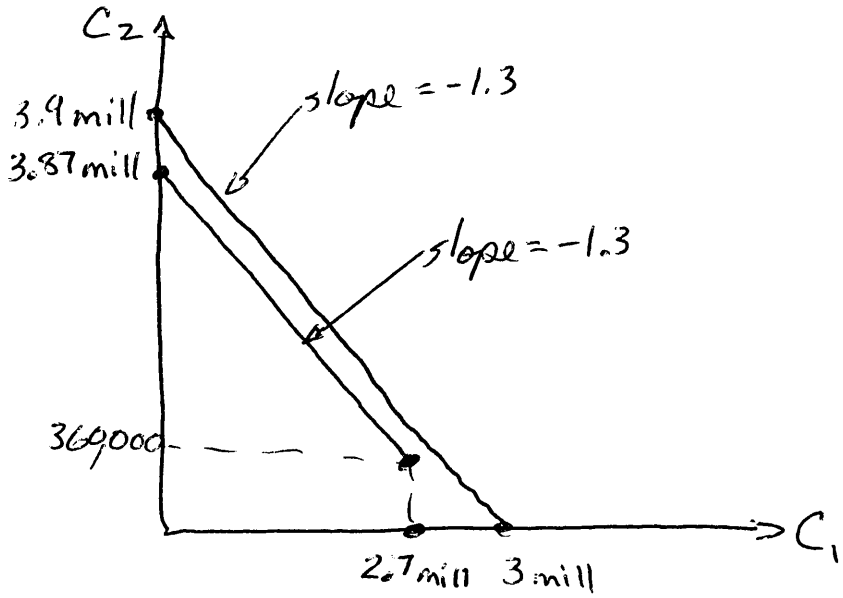
$C_2 = 1{,}950{,}000$

c) [5 marks] Suppose the government introduces a mandatory pension plan during Yu's working life. The plan is financed by a 10% tax on Yu's earnings in the first period and promises to pay her this amount plus interest at 20% during her retirement. Draw and appropriately label her new budget constraint.

10% tax leaves
2.7 mill in period 1
& collects #300,000

Returns (300,000)(1.2)
= 360,000

Further choices to save earn 30%



Question Three

Marc consumes apples, A, and cheese, C, according to he utility function $U(A, C) = A^{2/3}C^{1/3}$. The price of apples is P_A and the price of cheese is P_C. Marc's income is I.

a) [5 marks] Derive Marc's demand functions for apples and cheese.

$MU_A = \frac{2}{3}A^{-1/3}C^{1/3}$

$MU_C = \frac{1}{3}A^{2/3}C^{-2/3}$

$|MRS| = \frac{2C}{A}$ at opt $\frac{2C}{A} = \frac{P_A}{P_C}$ → $C = \frac{AP_A}{2P_C}$ → $A = \frac{2CP_C}{P_A}$

sub into b.c. $P_A\left[\frac{2CP_C}{P_A}\right] + P_C C = I \rightarrow 3P_C C = I$ $\boxed{C^* = \frac{I}{3P_C}}$

sub into b.c $\frac{2P_A A}{2} + P_C\left[\frac{AP_A}{2P_C}\right] = I$ $3P_A A = 2I$

$\boxed{A^* = \frac{2I}{3P_A}}$

b) [5 marks] Marc's income is \$240, the price of apples is \$4, and the price of cheese is \$8. Determine Marc's optimal consumption bundle. What is his elasticity of demand for cheese?

$$C = \frac{240}{(3)(8)} = 10$$

$$A = \frac{(2)(240)}{(3)(4)} = 40$$

$\varepsilon_D = -1$ b/c demand is constant elasticity form with exponent on $P_C = -1$

c) [5 marks] Suppose the price of cheese increases to \$12. Calculate Marc's new consumption bundle and the income and substitution effects of the price increase.

New bundle $A = \frac{(2)(240)}{(3)(4)} = 40$ $C = \frac{240}{(3)(12)} = 6.67$

Original U level $U = (40)^{2/3}(10)^{1/3} = (11.7)(2.15) = 25.16$

New opt condn $\frac{2C}{A} = \frac{4}{12} \Rightarrow C = \frac{4A}{24}$ so $C = \frac{A}{6}$

Find decomposition bundle $A^{2/3}C^{1/3} = 25.16$

$A^{2/3}\left(\frac{A}{6}\right)^{1/3} = 25.16$

$A\left(\frac{1}{6}\right)^{1/3} = 25.16$

$A = \frac{25.16}{.55}$

$A = 45.75$

$C = \frac{45.75}{6} = 7.62$

$\therefore$ subs effect $\downarrow C$ from 10 to 7.62

$\downarrow$ income effect $\downarrow C$ from 7.62 to 6.67