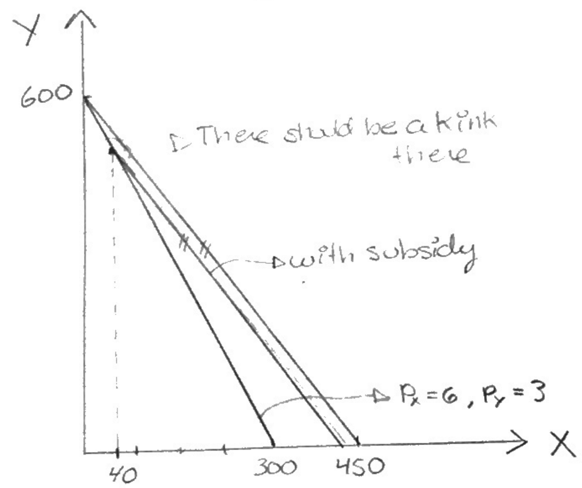
Section A: Three questions @ 5 marks. Total 15 marks.

1. [5 marks] Art has $1800 to spend on two goods, X, and Y. The price of X is $6 and the price of Y is $3. Draw and appropriately label Art's budget constraint. The government has decided to encourage the consumption of good X, but only after 40 units have been consumed. The government offers a subsidy of $2 per unit on the price of good X after 40 units have been purchased. Draw and appropriately label Art's new budget constraint. If Art's preferences are given by U=2X+3Y, how effective will the subsidy be? Explain.

Art's budget constraint: $6x + 3y = 1,800$



X and Y are perfect substitutes. Comparing the slope of the utility function to the slope of the budget line:

$$|-MRS_{x,y}| = \frac{2}{3} < \frac{6}{3} = \left|\frac{-P_x}{P_y}\right| \text{ and } \frac{2}{3} < \frac{4}{3} = \left|\frac{-P_x^S}{P_y}\right|$$

Since they are not equal, we will get a corner solution. The utility function is flatter than the budget constraint which means Art will only consumer good Y and no good X.
The subsidy will not encourage the consumption of good X.

2. [5marks]Given that Q^D=900-10P and Q^S=100+10P, solve for the equilibrium price and quantity in the market. Calculate the elasticity of demand at the equilibrium.

At the equilibrium, $Q^D = Q^S$

$$900 - 10P = 100 + 10P$$
$$800 = 20P$$
$$\boxed{40 = P^*} \text{ equilibrium price}$$

At $P^* = 40$, $Q^{d*} = 900 - 10 \cdot 40 = \boxed{500}$
At $P^* = 40$, $Q^{S*} = 100 + 10 \cdot 40 = \boxed{500}$

Elasticity of demand at equilibrium price is:

$$\frac{\partial Q^d}{\partial P} \cdot \frac{P}{Q^d} = -10 \cdot \frac{40}{500} = \boxed{-0.8}$$

3. **[5marks]Given the utility function U=5X+2Y and parameter values where P_X=4, P_Y=3 and income, I=$1200, determine the consumer's optimal bundle of goods X and Y.**

Budget constraint : $4x + 3y = 1{,}200$

Slope of budget constraint : $-\frac{P_x}{P_y} = \frac{-4}{3}$

Slope of utility function : $-MRS_{x,y} = -\frac{MU_x}{MU_y} = -\frac{5}{2}$

Comparing both slopes in absolute value $\Rightarrow$

$MRS_{x,y} = \frac{5}{2} > \frac{4}{3} = \frac{P_x}{P_y}$

Budget line is flatter than the utility function thus the optimal bundle is a corner solution

$\boxed{x^* = 300 \; ; \; y^* = 0}$

Section B: Three question @ 15 marks- 5 for each part of each question. Total 45 marks.

1. **Kate consumes two goods, X and Y, according to the utility function U(X,Y)=Min{3X; 2Y}. Kate has an income, I, and faces prices for the two goods given by P_X and P_Y.**
   a) **[5 marks] Derive Kate's demand functions for the goods X and Y.**

Budget constraint : $P_x \cdot X + P_y \cdot y = I$

With a Leontief utility function, set $3X = 2Y$

$\therefore \quad X = \frac{2Y}{3}$

Substituting in the budget constraint $\Rightarrow$

$P_x \cdot \frac{2Y}{3} + P_y \cdot Y = I$

$\boxed{Y = \frac{I}{\frac{2}{3}P_x + P_y} = \text{Demand for good } y}$

Similarly, $Y = \frac{3X}{2} \Rightarrow P_x \cdot X + P_y \cdot \frac{3X}{2} = I$

$\boxed{X = \frac{I}{P_x + \frac{3}{2}P_y} - \text{Demand for good } X}$

b) [5 marks] Let Kate's income be \$2000, the price of X be \$1 and the price of Y be \$3. How much of each good should Kate consume?

$$Y = \frac{2{,}000}{\frac{2}{3}\cdot 1 + 3} = \boxed{\frac{6{,}000}{11} \approx 545.45}$$

$$X = \frac{2{,}000}{1 + \frac{3}{2}\cdot 3} = \boxed{\frac{4{,}000}{11} \approx 363.64}$$

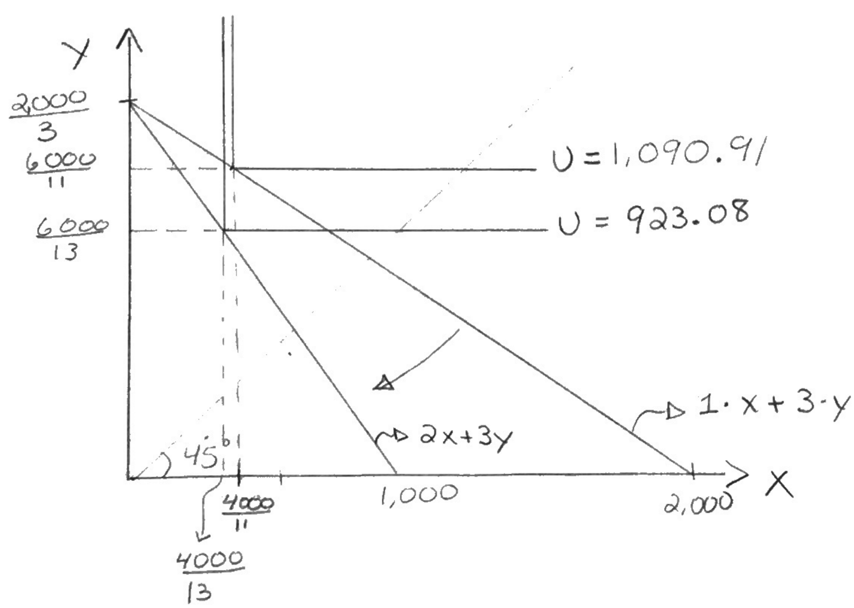
$$1 \cdot \frac{4{,}000}{11} + 3 \cdot \frac{6{,}000}{11} = 2{,}000$$

c) [5 marks]Suppose the price of X increases to \$2. Determine the new demand for the goods and calculate the income and substitution effects of the price change. (You may do this graphically for full credit, assuming you do so correctly).

$$Y = \frac{2{,}000}{\frac{2}{3}\cdot 2 + 3} = \frac{6{,}000}{13} \approx 461.54$$

$$X = \frac{2{,}000}{2 + \frac{3}{2}\cdot 3} = \frac{4{,}000}{13} \approx 307.69$$

With perfect complements, the substitution effect is 0 and the income effect accounts for the total change.



2. Ahmad has 112 hours per week to divide between leisure, R, and work, L. When he works, Ahmad earns \$25 per hour. He values both leisure and consumption, C, according to the utility function U(R,C)=RC. The price of the consumption good is unity.
   a) [5marks] Derive Ahmad's optimal bundle. How much does he work? Calculate Ahmad's level of utility.

Budget constraint: $C = (112 - R)25$

Optimization problem is $\Rightarrow$

$$\max_{R,C} U(R,C) \quad \text{subject to} \quad C = (112-R)25$$

$$\mathcal{L} = RC + \lambda[(112-R)25 - C]$$

$$\frac{\partial \mathcal{L}}{\partial C} \Rightarrow R - \lambda = 0 \quad | \quad R = \lambda \qquad \frac{\partial \mathcal{L}}{\partial \lambda} \Rightarrow (112-R)25 - C = 0$$

$$\frac{\partial \mathcal{L}}{\partial R} \Rightarrow C - \lambda 25 = 0 \quad | \quad C = 25R$$

Substituting in $\frac{\partial \mathcal{L}}{\partial \lambda}$ equation $\Rightarrow$ $25R = (112-R)25 \quad | \quad 50R = 2{,}800 \quad \boxed{R^* = 56}$

and $C^* = (112-56)25 = \boxed{1{,}400}$ units of consumption.

Ahmad work 56 hours and has a utility level of 78,400

   b) [5 marks] The government decides to levy a tax at the rate of \$5 per hour on Ahmad's earnings (assume his wage falls to \$20/hour). Find Ahmad's new optimal bundle, including the amount of work and leisure chosen. In words, explain this outcome in terms of the income and substitution effects of the tax.

New budget constraint: $C = (112-R)20$

$$\mathcal{L} = RC + \lambda[(112-R)20 - C]$$

$$\frac{\partial \mathcal{L}}{\partial C} \Rightarrow R - \lambda = 0$$

$$\frac{\partial \mathcal{L}}{\partial R} \Rightarrow C - 20\lambda = 0$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} \Rightarrow (112-R)20 - C = 0$$

$\Rightarrow C = 20R$

$20R = (112-R)20$

$40R = 2{,}240$

$\boxed{R^* = 56}$

$(112-56)20 = \boxed{1{,}120 = C^*}$

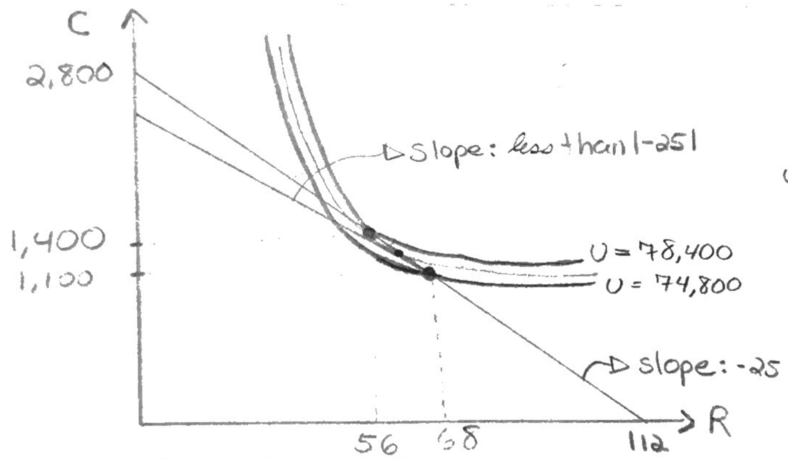
The tax makes consumption more expensive relative to leisure. Thus, Ahmad will move away from consumption and have more leisure, however, since he is made poorer by this tax, he will reduce both leisure and consumption. As a result, consumption falls but leisure happens to be unaffected as both effects cancel each other out.

c) Starting from the solution to part (a), assume Ahmad's boss tells him that he must work 44 hours per week. Show that Ahmad is worse off. Starting from the situation where Ahmad works only 44 hours per week, show graphically that Ahmad would be willing to accept a second job at a wage lower than his current wage.

If Ahmad has to work 44 hours $\Rightarrow R^* = 68$ and $C^* = (112-68)25 = 1,100$

$U(R^*, C^*) = 68 \cdot 1,100 = 74,800$ which is lower than $U(R^*, C^*)$ of part a).

Ahmad is therefore worse off.



We can clearly see from the graph that $\exists$ a wage smaller than 25 for which Ahmad would be willing to work in order to increase his utility.

3. [5 marks] Emily works in the present period and earns an income of \$5,000,000. In the future period, Emily is partly retired and earns \$2,000,000. Her preferences over present consumption, C_P, and future consumption, C_F, are given by $U(C_P,C_F)=C_P C_F^{1/2}$. Emily's can save or borrow at an interest rate of 100%.

a) Derive Emily's optimal consumption bundle and her level of savings or borrowing.

Emily's budget constraint: $C_P + \frac{C_F}{1+r} = 5,000,000 + \frac{2,000,000}{1+r}$

Optimization problem: $\max_{C_P, C_F} C_P C_F^{1/2}$ s.t. budget constraint

$$\mathcal{L} = C_P C_F^{1/2} + \lambda \left[5,000,000 + \frac{2,000,000}{1+r} - C_P - \frac{C_F}{1+r} \right]$$

$\frac{\partial \mathcal{L}}{\partial C_P} \Rightarrow C_F^{1/2} - \lambda = 0$ | $C_F^{1/2} = \lambda$ | $C_F^{1/2} = C_P C_F^{-1/2}$

$\frac{\partial \mathcal{L}}{\partial C_F} \Rightarrow \frac{1}{2} C_P C_F^{-1/2} - \frac{\lambda}{1+1} = 0$ | $C_P C_F^{-1/2} = \lambda$ | $\boxed{C_P = C_F}$

Substituting in the budget constraint $\Rightarrow C_P + \frac{C_P}{2} = 5,000,000 + \frac{2,000,000}{2}$

$\Rightarrow C_P^* = 4,000,000$

$\Rightarrow C_F^* - 4,000,000$

Emily will save 1,000,000 in the first period.

b) Now assume that Emily's future income is zero. How does this change your answer to part a)? If the reduction of future income is a pure income effect, what does this tell you about the nature of the goods C_P and C_F?

Budget constraint becomes $C_P + \frac{C_F}{2} = 5{,}000{,}000$

$$\mathcal{L} = C_P C_F^{1/2} + \lambda\left[5{,}000{,}000 - C_P - \frac{C_F}{2}\right]$$

$\frac{\partial \mathcal{L}}{\partial C_P} \Rightarrow C_F^{1/2} - \lambda = 0$

$\frac{\partial \mathcal{L}}{\partial C_F} \Rightarrow \frac{1}{2} C_P C_F^{-1/2} - \frac{\lambda}{2} = 0$

$C_F^{1/2} = C_P C_F^{-1/2}$

$\boxed{C_F = C_P}$

In budget constraint $\Rightarrow$

$C_P + \frac{C_P}{2} = 5{,}000{,}000$

$\boxed{C_P^* = C_F^* = 3{,}333{,}333.333}$

Since the reduction in income causes a reduction in the optimal quantity of both C_P and C_F, we can then conclude that C_P and C_F are Normal goods.

c) Return to the situation described in part a) and assume that the interest rate between the two periods is now 0%. How does this change your answer to part a)?

Budget constraint: $C_P + C_F = 5{,}000{,}000 + 2{,}000{,}000$

$$\mathcal{L} = C_P C_F^{1/2} + \lambda\left[5{,}000{,}000 + 2{,}000{,}000 - C_P - C_F\right]$$

$\frac{\partial \mathcal{L}}{\partial C_P} \Rightarrow C_F^{1/2} - \lambda = 0$

$\frac{\partial \mathcal{L}}{\partial C_F} \Rightarrow \frac{1}{2} C_P C_F^{-1/2} - \lambda = 0$

$C_F^{1/2} = \lambda$

$\frac{1}{2} C_P C_F^{-1/2} = \lambda$

$C_F^{1/2} = \frac{1}{2} C_P C_F^{-1/2}$

$\boxed{C_F = \frac{1}{2} C_P}$

Substituting in budget constraint:

$C_P + \frac{C_P}{2} = 7{,}000{,}000$

$\boxed{C_P^* = 4{,}666{,}666.67}$ $\boxed{C_F^* = 2{,}333{,}333.33}$

Emily will now save \$333,333.33 instead of \$1,000,000 in part a).