Section A: Three questions @ 5 marks. Total 15 marks.

1. [5 marks] Consider the utility function U(X,Y)=12X+6Y, where X and Y are two goods. Assume the price of X is $6, the price of Y is $2 and the consumer has an income of $1200. Derive the optimal consumption bundle for the consumer.

Budget constraint: $6x + 2y = 1{,}200$

Slope of utility function: $-MRS_{x,y} = -\frac{MU_x}{MU_y} = \frac{-12}{6} = -2$

Slope of the budget line: $-\frac{P_x}{P_y} = \frac{-6}{2} = -3$

Comparing both slopes in absolute value:

$$MRS_{x,y} = 2 < 3 = \frac{P_x}{P_x}$$

The utility function is flatter than the budget constraint, thus the optimal bundle is a corner solution.

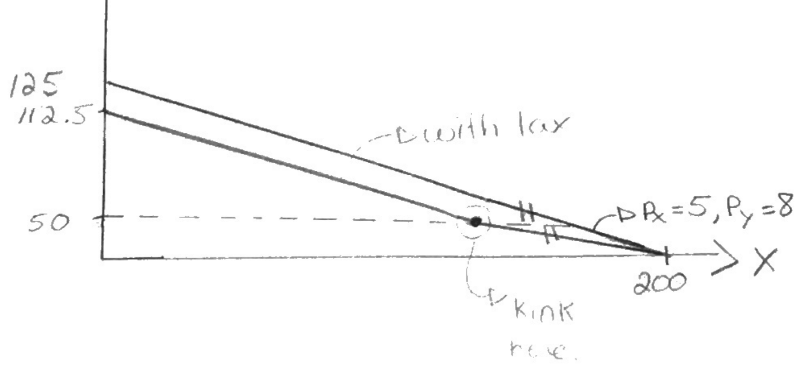
$$\boxed{x^* = 0 \quad y^* = 600}$$

2. [5marks] A consumer has $1000 in income and purchases two goods, X, which has a price of $5 and Y, which has a price of $8. Draw and appropriately label the consumer's budget constraint. Now suppose the government imposes a tax on good Y at the rate of $2 per unit, but the tax is levied only on the first fifty units purchased. Draw and appropriately label the new budget constraint.

Budget constraint: $5x + 8y = 1{,}000$

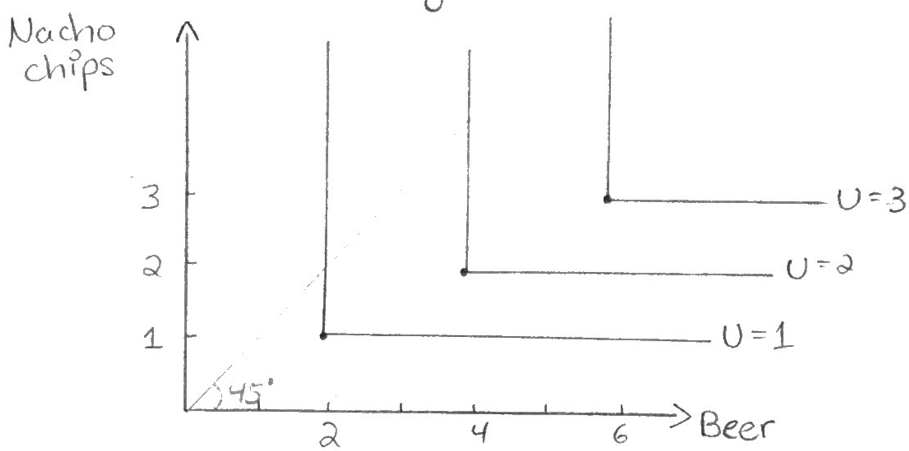
To find the intercept of the new (with tax) budget constraint, the maximum you can buy of good Y is $50 \times 10\$ = 500$, $\$1{,}000 - \$500 = \$500$, $\$500 \div 8 = 62.5$

Thus, the consumer can buy 50 units at $10 and 62.5 units at $8 maximum.



3. [5marks]Each Sunday Bill sits down to watch football on television. Bill drinks two bottles of beer and eats one bags of nacho chips during each football game he watches. Write an equation that describes Bill's preferences over beer, B, and nachos, N. Each Sunday Bill watches three football games. Draw and appropriately label Bill's indifference curve.

Bill's utility function $U(B,N) = \min\{\frac{B}{2}, N\}$



Section B: Three question @ 15 marks- 5 for each part of each question. Total 45 marks.

1. Ian consumes two goods, X and Y, according to the utility function $U(X,Y)=X^{1/2}Y^2$. Ian has an income, I, and faces prices for the two goods given by P_X and P_Y.
   a) [5 marks] Derive Ian's demand functions for the goods X and Y.

Optimization problem: $\max_{X,Y} U(X,Y)$ subject to $P_X X + P_Y Y = I$

$$\mathcal{L} = X^{1/2}Y^2 + \lambda\left[I - P_X X - P_Y Y\right]$$

$$\frac{\partial \mathcal{L}}{\partial X} \Rightarrow \frac{1}{2}X^{-1/2}Y^2 - P_X\lambda = 0$$

$$\frac{\partial \mathcal{L}}{\partial Y} \Rightarrow 2X^{1/2}Y - P_Y\lambda = 0$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} \Rightarrow I - P_X X - P_Y Y = 0$$

$$\frac{\frac{1}{2}X^{-1/2}Y^2}{2X^{1/2}Y} = \frac{P_X\cancel{\lambda}}{P_Y\cancel{\lambda}}$$

$$\frac{Y}{4X} = \frac{P_X}{P_Y}$$

$$Y = \frac{4XP_X}{P_Y}$$

Substituting in budget constraint:

$$P_X X + \cancel{P_Y}\frac{4XP_X}{\cancel{P_Y}} = I$$

$$\boxed{X^* = \frac{I}{5P_X}} \Rightarrow Y = \frac{4\cancel{P_X}}{P_Y}\cdot\frac{I}{5\cancel{P_X}} \Rightarrow \boxed{Y^* = \frac{4I}{5P_Y}}$$

b) [5 marks] Assume that Ian's income is $1500, the price of X is $5 and the price of Y is $1. Calculate his demand for each good. What is the elasticity of demand for X at this bundle?

$$X^* = \frac{1{,}500}{5 \cdot 5} = 60$$

$$Y^* = \frac{4 \cdot 1{,}500}{5 \cdot 1} = 1{,}200$$

$$5 \cdot 60 + 1{,}200 \cdot 1 = 1{,}500$$

$$U(x^*, y^*) = x^{*1/2} y^{*2} = 11{,}154{,}192.04$$

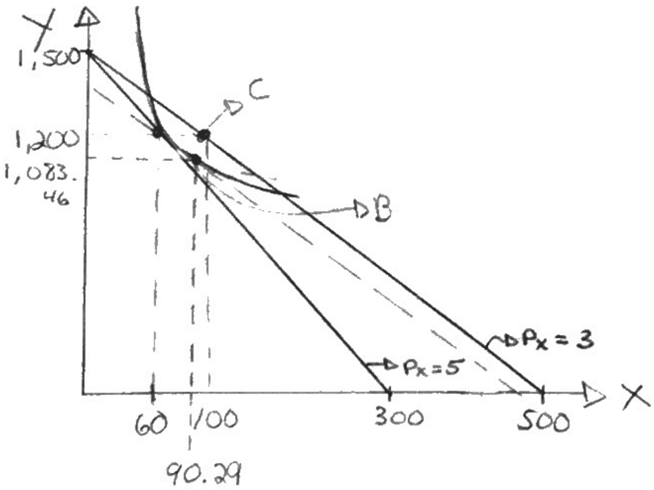
Elasticity of demand for X:

$$\frac{\partial Q_x^d}{\partial P_x} \frac{P_x}{Q_x^d} = -\frac{I}{5P_x^2} \cdot \frac{P_x}{Q_x^d} = \frac{-I}{5 P_x Q_x^d}$$

$$= -\frac{1{,}500}{5 \cdot 5 \cdot 60} = \boxed{-1}$$

c) [5 marks] Suppose the price of X decreases to $3. Determine the new demand for the goods and calculate the income and substitution effects of the price change.

$$\boxed{X^* = \frac{1{,}500}{5 \cdot 3} = 100} \quad \boxed{Y^* = 1{,}200 \text{ is unchanged}}$$



To find the decomposition basket (B), 2 equations must hold:

$$1 \Rightarrow X_B^{1/2} Y_B^2 = 11{,}154{,}192.04$$

$$2 \Rightarrow \frac{MU_X}{MU_Y} = \frac{P_X}{P_Y} = \frac{3}{1} = 3$$

$$\therefore \frac{\frac{1}{2} X^{-1/2} Y^2}{2X^{1/2} Y} = 3 \quad \Big| \quad \frac{Y}{4X} = 3 \quad \Big| \quad Y = 12X$$

Substituting in condition 1, we get

$$X^{1/2}(12X)^2 = 11{,}154{,}192.04$$

$$X^{2.5} = 77{,}459.67 \quad \boxed{X_B^* = 90.29}$$

Thus, $\boxed{Y_B^* = 1{,}083.46}$

The substitution effect is the increase in the purchase of good X as Ian moves along the initial indifference curve. Thus, $\boxed{SE = 90.29 - 60 = 30.2}$

The income effect is the increase in the purchase of good X as Ian moves from the decomposition basket (B) to the final consumption basket (C). Thus, the $\boxed{IE = 100 - 90.29 = 9.71}$

$SE + IE$ = total change, $30.29 + 9.71 = 40 = 100 - 60$

2. Al has 126 hours per week to divide between leisure, R, and work. When he works, Al earns $25 per hour. He values both leisure and consumption, C, according to the utility function U(R,C)=Min{10R ; C}. The price of the consumption good is unity.

   a) [5marks] Derive Al's optimal bundle. How much does he work?

with a Leontief utility function $\Rightarrow 10R = C$

Substituting in the budget constraint $C = (126 - R)25$

$$\Rightarrow 10R = (126-R)25$$

$$35R = 3{,}150$$

$$R^* = 90 \quad \Rightarrow \quad C^* = (126-90)25 = 900$$

Al will work 36 hours.

Utility level is 900

   b) [5 marks] Explain how Al's allocation of time between work and leisure changes if Al is taxed on his earnings at the rate of $5 per hour (assume his wage falls to $20 per hour). Would Al be willing to take a second job to improve his level of well-being? Explain.

$$10R = (126-R)20$$

$$30R = 2{,}520$$

$$R^* = 84 \quad \Rightarrow \quad C^* = (126-84)20 = 840$$

Al will work 42 hours. Utility level is 840.
Clearly, Al is made worse off by this tax as his utility drops by 60. If Al cannot reduce the amount of hours worked at his first job, he would not want to take a second job because it would increase consumption without increasing leisure, which is not desirable with a Leontief preference as the ratio of leisure/consumption is important.

c) Starting from the solution to part (a), assume Al's boss tells him that new rules mean Al must work 44 hours per week. Calculate Al's utility level under the new rules and show that he is worse off because of the new rules.

$R^* = 126 - 44 = 82$

$C^* = (126-82)25 = 1{,}100$

Al's utility level is now 820 instead of 900 in part a). He is thus worse off because of the new rules.

3. [5 marks] Emily works in the present period and earns an income of \$8,000,000. In the future period, Emily is retired and earns nothing. Her preferences over present consumption, C_P, and future consumption, C_F, are given by $U(C_P,C_F)=C_P^{1/2}\,C_F^{1/2}$. Emily's savings earn an interest rate of 100%.

a) Derive Emily's optimal consumption bundle and her level of savings.

Budget constraint $\Rightarrow C_P + \frac{C_F}{1+r} = 8{,}000{,}000$

Optimization problem: $\max_{C_P, C_F} U(C_P, C_F)$ s.t. $C_P + \frac{C_F}{1+r} = 8{,}000{,}000$

$\mathcal{L} = C_P^{1/2} C_F^{1/2} + \lambda\left[8{,}000{,}000 - C_P - \frac{C_F}{1+1}\right]$

$\frac{\partial \mathcal{L}}{\partial C_P} \Rightarrow \frac{1}{2} C_P^{-1/2} C_F^{1/2} - \lambda = 0$

$\frac{\partial \mathcal{L}}{\partial C_F} \Rightarrow \frac{1}{2} C_P^{1/2} C_F^{-1/2} - \frac{\lambda}{2} = 0$

$\frac{1}{2} C_P^{-1/2} C_F^{1/2} = \lambda$

$C_P^{1/2} C_F^{-1/2} = \lambda$

$\frac{1}{2} C_P^{-1/2} C_F^{1/2} = C_P^{1/2} C_F^{-1/2}$

$\boxed{C_F = 2C_P}$

Substituting in budget constraint $\Rightarrow C_P + \frac{2C_P}{2} = 8{,}000{,}000$

$\boxed{C_P^* = 4{,}000{,}000}$

$\boxed{C_F^* = 8{,}000{,}000}$

Emily will save \$4,000,000 in the present period.

b) Assume the government implements a law that requires every person who earns $8,000,000 or more in the present period to save at least $5,000,000. Show that this law would make Emily worse off.

If Emily saves $5,000,000 in the present, this means that $C_P^* = \$3,000,000$ and $C_F^* = 10,000,000$

This will give her a utility level of

$u(C_P^*, C_F^*) = 3,000,000^{1/2}\ 10,000,000^{1/2} = 5,477,225.575$

In part a) the $U(C_P^*, C_F^*) = 4,000,000^{1/2}\ 8,000,000^{1/2} = 5,656,854.249$

Emily is therefore worse off with this law.

c) Explain and illustrate how Emily's original budget line would change if the government taxed both her present earnings at the rate of 25% and the interest earned on her savings at the rate of 10%.

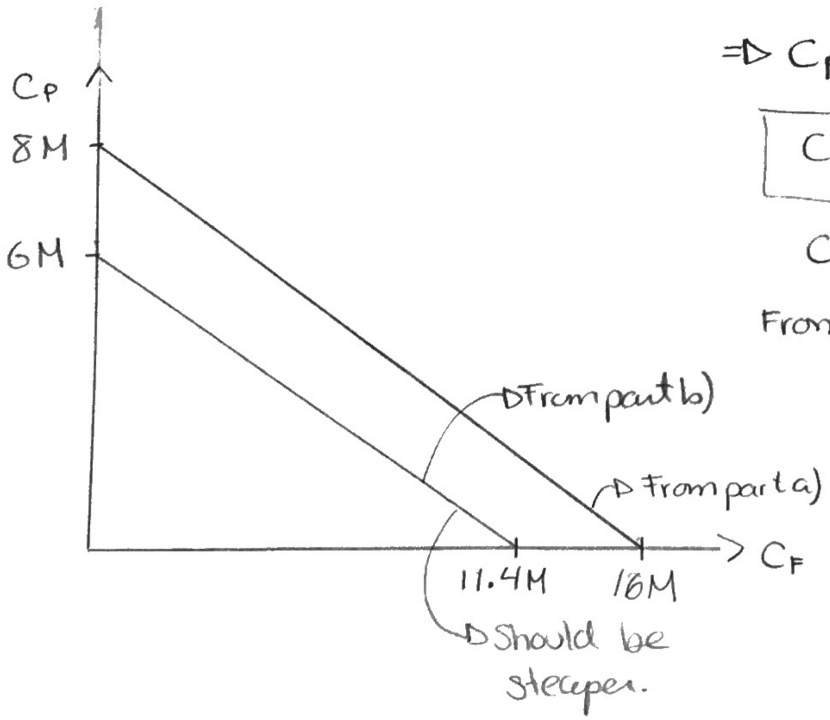
Budget constraint: $C_P + \dfrac{C_F}{1+1(1-0.1)} = 8,000,000(1-0.25)$

$\Rightarrow C_P + \dfrac{C_F}{1.9} = 8,000,000 \cdot 0.75$

$$\boxed{C_P + \dfrac{C_F}{1.9} = 6,000,000}$$

$C_P = 6000,000 - \dfrac{C_F}{1.9}$

From part a) $C_P = 8,000,000 - \dfrac{C_F}{2}$



In addition to having less money due to the tax on earnings, Emily earns less interest on her savings which means that the part b)'s budget line is below part a)'s budget line and steeper.