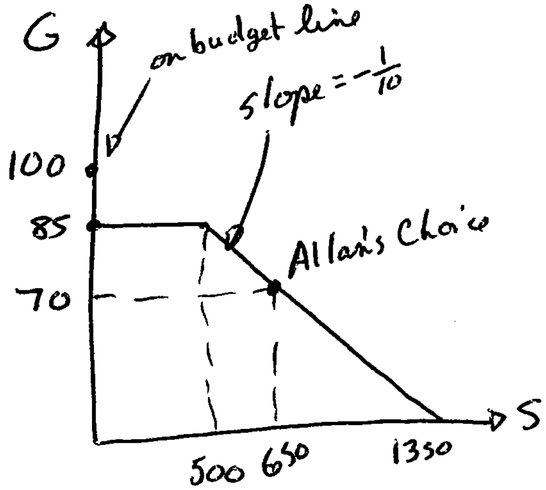
212 A MIDTERM

Section A [15 marks in total]

1. [5 marks] Allan has just signed up for internet service, S. His plan provides that he gets 500 minutes per month for a flat fee of $15, with all minutes beyond 500 charged at the rate of $.10 (i.e., 10 cents) per minute. Allan has $100 per month to spend on internet service and all other goods, G. Draw and appropriately label Allan's budget constraint. Allan has chosen to spend $30 on internet service and $70 on all other goods. If his utility function is given by U = S + 10G, is Allan maximizing his utility? Explain.



Indifference curves are linear with slope of $-1/10$. Utility maximizing choices lie along budget line with slope $= -1/10$

Combination where Allan spends $70 on other goods + buys 650 minutes of service is on this segment and is a utility maximizing choice

2. [5 marks] The market for premium cheese is characterized by a demand function of the form $Q^D = 4{,}000 - 300P$ and s supply function of the form $Q^S = 200P - 1{,}000$, where Q is quantity and P is price. Calculate the equilibrium values of price and quantity and determine the elasticity of demand at the equilibrium.

Equilibrium: $Q^D = Q^S$

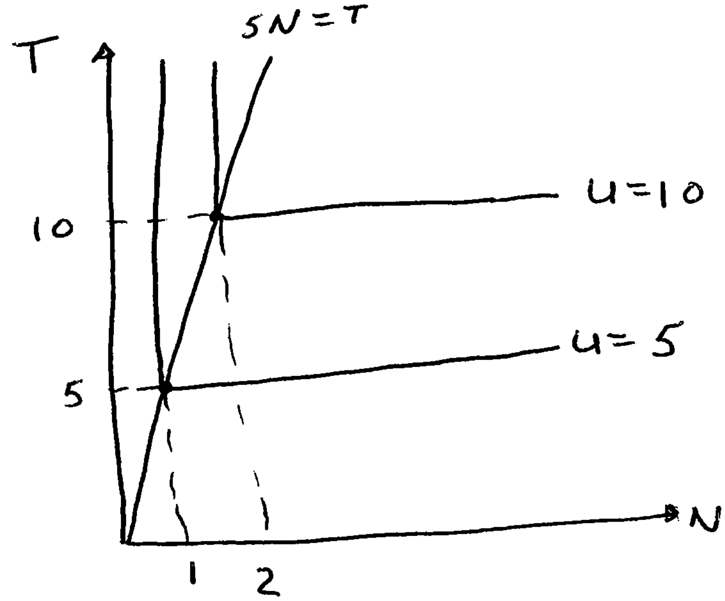
$$4000 - 300P = 200P - 1000$$

$$500P = 5000 \longrightarrow \boxed{P = 10}$$

Sub into Q^D: $Q = 4{,}000 - (300)(10) \longrightarrow \boxed{Q = 1000}$

$$\varepsilon_D = \frac{dQ^D}{dP} \cdot \frac{P}{Q^D} = (-300)\frac{(10)}{(1000)} \longrightarrow \boxed{\varepsilon_D = -3}$$

3. [5 marks] Margerite likes novels, N, and tea, T, with her preferences defined by the utility function U(N, T) = Min {5N : T}. Draw and appropriately label two of Margerite's indifference curves. Please put N on the horizontal axis.



Section B [45 marks in total]

1. Donna is a jazz singer who dines out frequently when she is on the road. Consumes dinners, D, and wine, W, according to the utility function U(D, W) = $D^{1/2}W$. The price of a dinner is P_D, the price of wine is P_W, and her income is I.
   a) [5 marks] Derive Donna's demand functions for dinners and wine.

① $\left.\begin{array}{l} MU_D = \frac{1}{2} D^{-1/2} W \\ MU_W = D^{1/2} \end{array}\right\} \Rightarrow MRS = -\frac{MU_D}{MU_W} = \frac{\frac{1}{2} D^{-1/2} W}{D^{1/2}} = -\frac{1}{2} D^{-1} W = -\frac{W}{2D}$

② Equate to price ratio + solve $\rightarrow -\frac{W}{2D} = -\frac{P_D}{P_W} \rightarrow W = \frac{2DP_D}{P_W}$ + $D = \frac{WP_W}{2P_D}$

③ Sub into constraint $\rightarrow P_D D + P_W \left[\frac{2DP_D}{P_W}\right] = I \rightarrow 3P_D D = I \rightarrow \boxed{D^* = \frac{I}{3P_D}}$

④ Sub into constraint $\rightarrow P_D \left[\frac{WP_W}{2P_D}\right] + P_W W = I \rightarrow 3P_W W = 2I \rightarrow \boxed{W^* = \frac{2I}{3P_W}}$

   b) [5 marks] If Donna's income is \$200, the price of a dinner is \$20 and the price of a glass of wine is \$10, calculate Donna's optimal consumption bundle.

$D = \frac{200}{(3)(20} = \frac{200}{60} \rightarrow \boxed{D = 3.33}$

$W = \frac{(2)(200)}{(3)(10)} = \frac{400}{30} \rightarrow \boxed{W = 13.33}$

c) [5 marks] If the price of a dinner increases to $25, calculate Donna's new optimal bundle and the income and substitution effects of the price change.

New optimal bundle is $D' = \frac{200}{(3)(25)} \Rightarrow \boxed{D' = 2.67}$

$W' = \frac{(200)(2)}{(3)(10)} \Rightarrow \boxed{W' = 13.33}$

Need original utility level to find decomposition bundle

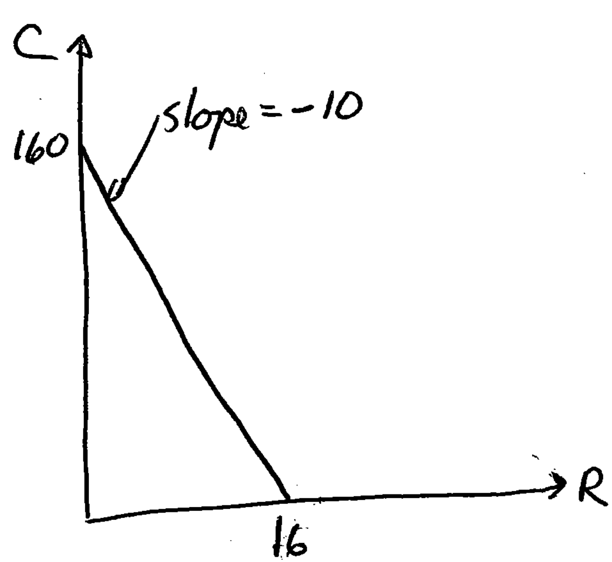
$U = D^{1/2}W = (3.33)^{1/2}(13.33) = (1.82)(13.33) = 24.26$

At decomp bundle must have $\frac{W}{2D} = \frac{25}{10}$ so $W = \frac{50D}{10} \Rightarrow 5D = W$

Also on original ~~bun~~ utility so $24.26 = (D^{1/2})(5D) \rightarrow D^{1.5} = 4.85 \rightarrow D = (4.85$

At decomp bundle $D = 2.88$ (see back)

2. Mats has 16 hours per day to allocate between work and leisure, R. Mats has a utility function defined over two goods, leisure and a composite consumption good, C, with a price of one. The utility function is of the form U(R, C) = Min{6R ; C}. Mats receives a wage of $10 per hour when he works.
   a) [5 marks] Draw and appropriately label Mats budget constraint. ~~Write an equation that describes the future value form of this budget constraint.~~



Brain dead! Cannot do this here.

Entire 5 marks for drawing correctly.

IF do write budget equation it is

$C = 160 - 10R$

(c) continued

$$\text{Subs} = \underset{\text{original bundle}}{\text{D at}} - \underset{\text{decomp bundle}}{\text{D at}} = [3.33 - 2.88] = .45$$

$\boxed{\text{Subs effect is decrease of .45 in D}}$

$$\text{Inc} = \underset{\text{decomp bundle}}{\text{D at}} - \underset{\text{final bundle}}{\text{D at}} = [2.88 - 2.67] = .21$$

$\boxed{\text{Inc effect is decrease of .21 in D}}$

b) [5 marks] Derive Mats' optimal bundle of leisure and consumption.

→ budget constraint: $C = 160 - 10R$

→ opt cond'n for perf comp where two arguments in U fn are equal so where $C = 6R$

→ sub into constraint → $6R = 160 - 10R$ → $16R = 160$

↳ $\boxed{R^* = 10}$ → implies work 6 hours so $\boxed{C^* = (6)(10) = 60}$

c) Mats has been offered a choice by his boss. Mats can continue to freely choose his hours of work and be paid \$10 per hour or he can move to a new job that pays him \$15 per hour, but requires that he must work 8 hours per day. Which would Mats prefer? Explain.

Prefer one yielding higher utility

Existing job → $U = \text{Min}\{6R; C\} = \text{Min}\{60; 60\}$ → $U = 60$

New job must work 8 hours so $R = 8$ ⇒ $C = (8)(15) = 120$

New job → $U = \text{Min}\{6R; C\} = \text{Min}\{48; 120\}$ → $U = 48$

$\boxed{\text{Mats prefer existing job b/c higher utility}}$

3. Ricardo has \$200 in income and is considering spending the income on one of two lotteries, A and B. For lottery A, there is a 70% probability that Ricardo will end with a final income of \$0 and a 30% probability that he will end with a final income of \$700. For lottery B, there is a 50% probability that he will end with a final income of \$60 and a 50% probability that he will end with a final income of \$360.

   a) [5 marks] Show that the two lotteries have the same expected value, but different degrees of riskiness.

$$\left.\begin{array}{l} EV_A = (.7)(0) + (.3)(700) = 0 + 210 = 210 \\ EV_B = (.5)(60) + (.5)(360) = 30 + 180 = 210 \end{array}\right\} \text{same EV}$$

→ Calculate variance of outcomes for each.

Riskiness of A

$(210-0)^2 = 0 \times .7 = 0$

$(210-700)^2 = 240{,}100 \times .3 = 72{,}030$

$\text{Variance}_A = 0 + 72{,}030$

$= \boxed{72{,}030}$

Riskiness of B

$(210-60)^2 = 22{,}500 \times .5 = 11{,}225$

$(210-360)^2 = 22{,}500 \times .5 = 11{,}225$

$\text{Variance}_B = 11{,}225 + 11{,}225$

$= \boxed{22{,}500}$

Different Riskiness

   b) [5 marks] Suppose that Ricardo is a risk-averse person with a utility of income function given by $U(I) = I^{1/2}$, where I is his income. Will Ricardo prefer to keep his initial \$200 or would he prefer one of the two lotteries? Explain.

Keep \$200 with certainty $U = I^{1/2} = \sqrt{200} = \underline{\underline{14.14}}$

Expected utility for A + B

$EU_A = (.7)\sqrt{0} + (.3)\sqrt{700} = 0 + 7.94 = \underline{\underline{7.94}}$

$EU_B = (.5)\sqrt{60} + (.5)\sqrt{360} = 3.87 + 9.49 = \underline{\underline{13.36}}$

Ricardo would prefer to keep his \$200 b/c this choice yields highest utility

/

should be 700 to something bigger!!! OR about lottery B

error here

c) [5 marks] If the winning outcome for lottery A were to increase from \$360 to \$525, would it be the preferred choice for Ricardo? Explain.

Answer A

as written

Calculate expected utility of new version of $A \rightarrow A'$

$$EU_{A'} = (.7)\sqrt{0} + (.3)\sqrt{525} = 0 + 6.87 = \underline{6.87}$$

→ would not accept this lottery

Answer B

uses lottery B

$$EU_{B'} = (.5)(60) + (.5)(525) = 3.87 + 11.46 = 15.33$$

→ would now prefer B to sure thing b/c highest EU

Accept either answer