Economics 212B, Fall 2003

Suggested Answers for Midterm [1]

Section A:

Question 1

a) The horizontal intercept is the maximum consumption if Wei spends all his life-time income in period 1 and consumes nothing in period 2(That is, Wei not only spends all his period 1 income, but also borrow all his period 2 income and spends it in period 1). The value is: $800000 + \frac{100000}{1.04} = 896153.85$

The vertical intercept is the maximum consumption in period two if Wei consumes nothing in period 1 and saves all his period 1 income. The value is: $800000(1 + 0.04) + 100000 = 932000$.

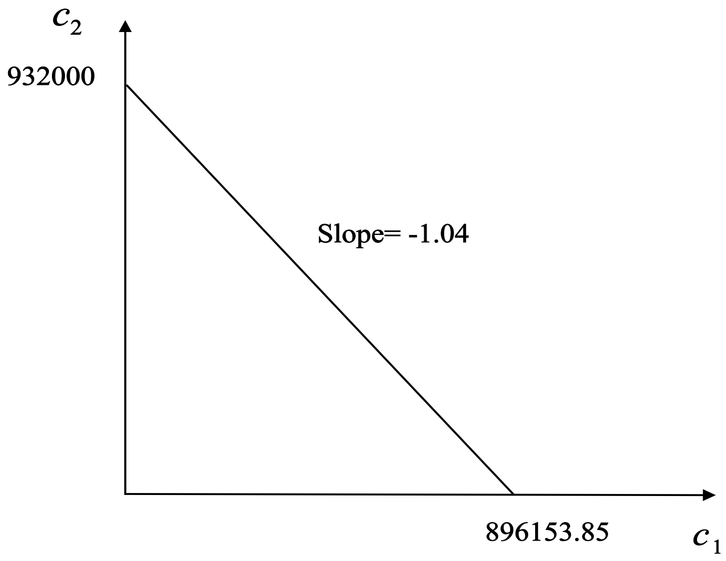

Figure 1: A1-a

b) For utility function $U(C_1, C_2) = Min(C_1; 3C_2)$, we have $C_1 = 3C_2$, substitute this into the budget constraint:

$$C_1(1 + r) + C_2 = I_1(1 + r) + I_2$$

[1] The actual answer may not need to be so detailed as shown here

we have

$$\begin{aligned}
3C_2(1+r)+C_2 &= I_1(1+r)+I_2 \\
3C_2(1+0.04)+C_2 &= 800000(1+0.04)+100000 \\
4.12C_2 &= 932000 \\
C_2 &= 226213.59
\end{aligned}$$

And $C_1 = 3C_2 = 678640.78$, because C_1 is smaller than Wei's period one income, so for the first period, Wei's level of savings is $I_1 - C_1 = 800000 - 678640.78 = 122359.22$

c) Similar to part b), we have

$$\begin{aligned}
3C_2(1+r)+C_2 &= I_1(1+r)+I_2 \\
3C_2(1+0.06)+C_2 &= 800000(1+0.06)+100000 \\
4.18C_2 &= 948000 \\
C_2 &= 226794.26
\end{aligned}$$

And $C_1 = 3C_2 = 680382.78$, since $I_1 > C_1$, Wei has a saving of $I_1 - C_1 = 800000 - 680382.78 = 119617.22$

Question 2

a) For $U = N^{\frac{1}{3}}S^{\frac{2}{3}}$ we have

$$\begin{aligned}
MU_N &= \frac{\partial U}{\partial N} = \frac{1}{3}N^{-\frac{2}{3}}S^{\frac{2}{3}} \\
MU_S &= \frac{\partial U}{\partial S} = \frac{2}{3}N^{\frac{1}{3}}S^{-\frac{1}{3}} \\
MRS_{N,S} &= \frac{MU_N}{MU_S} = \frac{\frac{1}{3}N^{-\frac{2}{3}}S^{\frac{2}{3}}}{\frac{2}{3}N^{\frac{1}{3}}S^{-\frac{1}{3}}} = \frac{S}{2N}
\end{aligned}$$

So we solve the following two equations:

$$\begin{aligned}
\frac{P_N}{P_S} &= \frac{S}{2N} \\
P_N N + P_S S &= I
\end{aligned}$$

that is

$$\begin{aligned}
P_N N + P_S \frac{2P_N N}{P_S} &= I \\
3NP_N &= I \\
N^* = \frac{I}{3P_N}
\end{aligned}$$

similarly

$$S^* = \frac{2I}{3P_S}$$

b) From part a), $N^* = \frac{I}{3P_N}$, $S^* = \frac{2I}{3P_S}$, when $I = 600$, $P_N = 10$, $P_S = 5$, we have

$$N^* = \frac{600}{3*10} = 20 \qquad S^* = \frac{1200}{3*5} = 80$$

$$U = N^{\frac{1}{3}}S^{\frac{2}{3}} = 20^{\frac{1}{3}}80^{\frac{2}{3}} = 50.3969$$

similarly, final optimal bundle is

$$N^* = \frac{600}{3*12} = 16.67 \qquad S^* = \frac{1200}{3*5} = 80$$

The MRS at the final bundle is

$$MRS_{N,S} = \frac{S}{2N} = \frac{P_N}{P_S} = \frac{12}{5} \Longrightarrow S = \frac{24N}{5}$$

substitute this into the original utility level

$$50.3969 = N^{\frac{1}{3}}S^{\frac{2}{3}} = N^{\frac{1}{3}}\left(\frac{24N}{5}\right)^{\frac{2}{3}} = 2.8455N \rightarrow N = 17.7$$

Substitution effect $= 17.7 - 20 = -2.3$, income effect $= 16.67 - 17.7 = -1.03$.

c) With the card, $P_N = 10$, and the income left after purchasing the card is $600 - 30 = 570$. Using the result of part a) we have

$$N^* = \frac{570}{3*10} = 19 \qquad S^* = \frac{570*2}{3*5} = 76$$

$$U = N^{\frac{1}{3}}S^{\frac{2}{3}} = 19^{\frac{1}{3}}76^{\frac{2}{3}} = 47.88$$

Without the card, the optimal bundle we computed in part b) is $N^* = 16.67$ and $S^* = 80$

$$U = N^{\frac{1}{3}}S^{\frac{2}{3}} = 16.67^{\frac{1}{3}}80^{\frac{2}{3}} = 47.429 < 47.88$$

So she will prefer to buy the card

Question 3

a) The expected value of the lottery is

$$EV = 0.5*2000 + 0.4*11000 + 0.1*75000 = 12900$$

In order to decide whether JP accepts the lottery of not, we need to compute his expected utility. Denote the expected utility for this lottery as U_1 and the utility for keeping his initial wealth as U_0.

$$\begin{aligned} EU_1 &= 0.5(10\sqrt{2000}) + 0.4(10\sqrt{11000}) + 0.1(10\sqrt{75000}) = 916.99 \\ U_0 &= 10\sqrt{10000} = 1000 > 916.99 \end{aligned}$$

So JP would like to keep the initial 10000, because the utility is larger than the expected utility of the lottery.

b) The definition for Risk Premium is

$$EU = U(EV - RP)$$

In part a), We already computed the expected value of the lottery and its expected utility , $EV = 12900$ and $EU_1 = 916.99$, so

$$\begin{aligned} 916.99 &= U(12900 - RP) \\ 916.99 &= 10\sqrt{12900 - RP} \\ &\Rightarrow RP = 4491.29 \end{aligned}$$

c) Expected utility for lottery 2 is

$$EU_2 = 0.5(10\sqrt{1000}) + 0.4(10\sqrt{10000}) + 0.1(10\sqrt{84000}) = 847.94$$

Since $U_0 = 1000 > EU_2$, so JP would prefer to keep his initial wealth. And since $EU_1 > EU_2$, if he were forced to participate in one of the two lotteries, he will choose the first one.

Section B:

Question one

Because Joshua always consume wine and dinner according to a proportion, it is a "Perfect Complements" utility function.

$$U = min\{W, 2D\}$$

(Note, for this question, any function in the form of $U = c * [Min(aW, bD)]$ with $a/b = 1/2$ is correct.)

The graph is shown below:

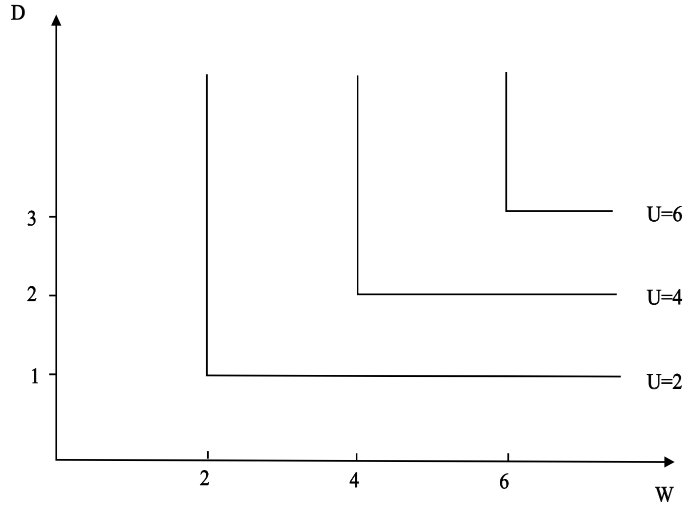

Figure 2: Section B: Question 1

Question Two

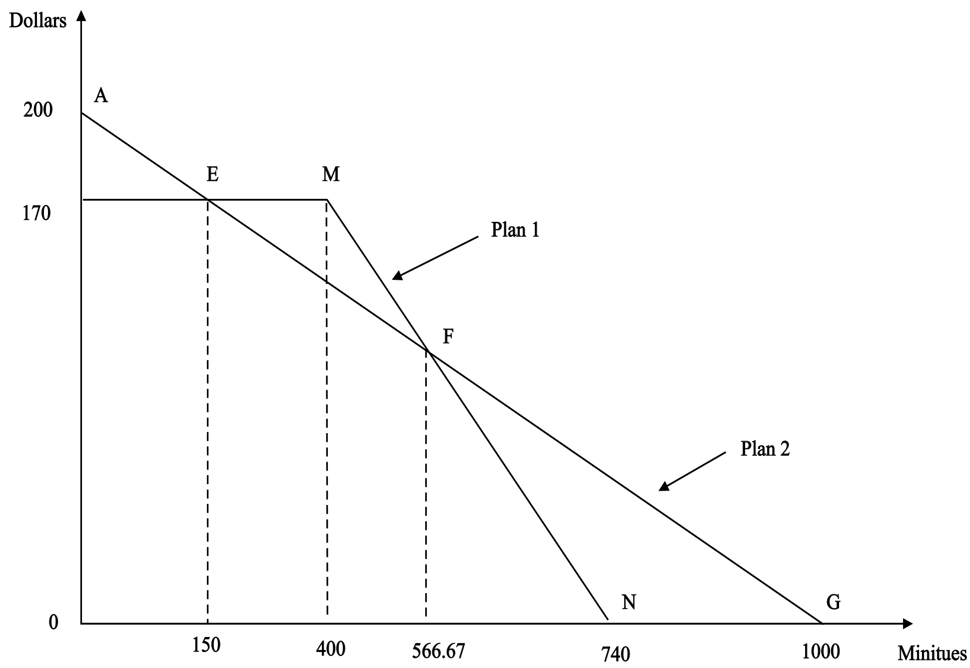

Figure 3: B-2

The explanation of the graph is as follows:

For plan 1, the total minutes with \$200 is $200/0.2 = 1000$.

For plan 2, the money left after the first 400 minutes is $200 - 30 = \$170$, which can buy another $170/.05 = 340$ minutes, so the sum is $400 + 340 = 740$.

For point E, since the amount spent on telephone service is \$30, so for plan 2, $30/0.2 =$ 150 minutes.

Point E is the cross of line AB and line MN. On line AB, the price for phone call is \$0.2, so the function for AB is $0.2x + y = 200$, where 200 is the budget. and because point M is $(400, 170)$ and the price for phone call on line MN is \$0.5, so the function for line MN is $(x - 400)0.5 + y = 170$, where 170 is the budget. [2] So solve:

$$\begin{aligned} 0.2x + y &= 200 \\ 0.5x + y &= 340 \\ \Rightarrow x &= 566.67 \end{aligned}$$

Lee will prefer the second plan when the budget curve of the second plan is higher than the budget curve of the first plan, that is when the service he needs is smaller than 150 minutes or larger than 566.67 minutes.

Question Three

Solve the two equations for the equilibrium:

$$\begin{aligned} Q^s &= 1995P - 6000 \\ Q^D &= 10000 - 5P \\ \Rightarrow P^* &= 8 \qquad Q^* = 9960 \end{aligned}$$

So the elasticity is:

$$\begin{aligned} \varepsilon &= \frac{dQ^D}{dP}\frac{P}{Q^D} \\ &= -5 * \frac{8}{9960} \\ &= -0.004016 \end{aligned}$$

[2] There is another method to get the function for line MN. Note that at point N, 740 minutes * 0.5=\$370, so it's just like Lee has \$340 dollars, and the budget line is $0.5x + y = 340$, which is the same as $(x - 400)0.5 + y = 170$.