Economics 212

Section 002

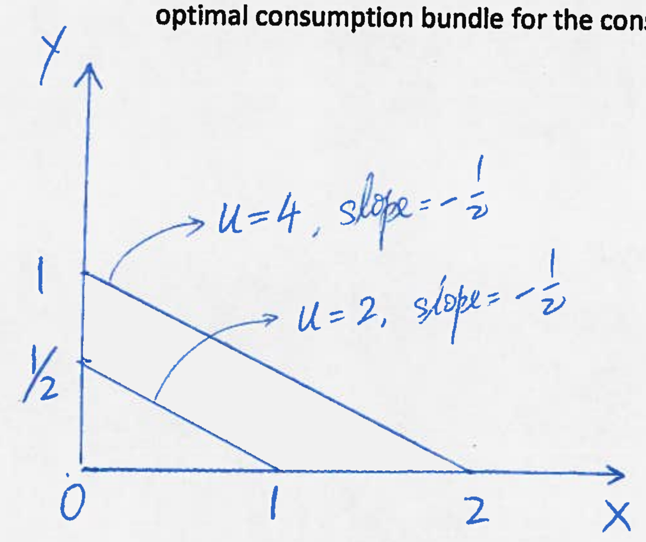
Midterm Exam

March 4, 2014

Student Number:

Section A: Three questions @ 5 marks. Total 15 marks.

1. [5 marks] Consider the utility function U(X,Y)=2X+4Y, where X and Y are two goods. Assume the price of X is $6, the price of Y is $5 and the consumer has an income of $1200. Derive the optimal consumption bundle for the consumer.



IC: $Y = -\frac{X}{2} + \frac{\bar{u}}{4}$

B.C: $P_x X + P_y Y = I \iff 6X + 5Y = 1200$

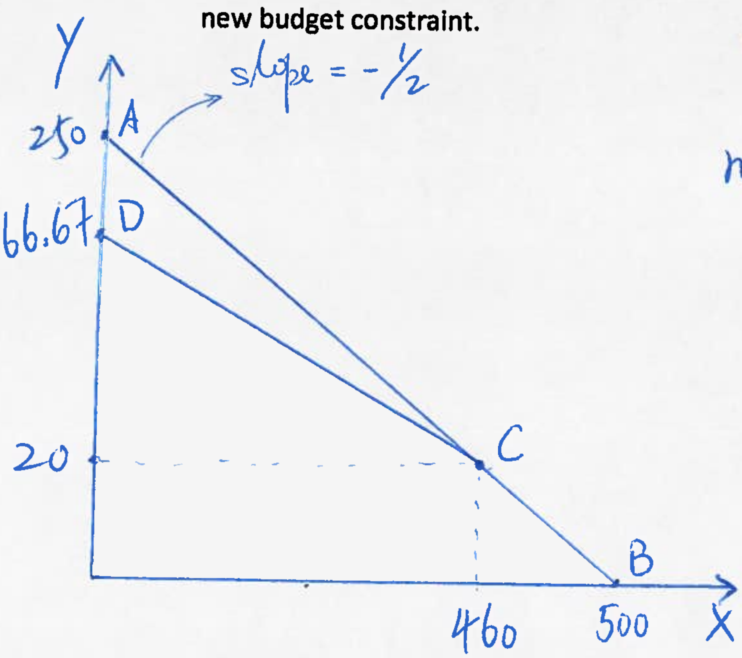
$$\left.\begin{array}{l} MRS_{x,y} = \frac{MU_x}{MU_y} = \frac{2}{4} = \frac{1}{2} \\ \frac{P_x}{P_y} = \frac{6}{5} \end{array}\right\} \Rightarrow$$

$\frac{MU_x}{MU_y} < \frac{P_x}{P_y} \Rightarrow \frac{MU_x}{P_x} < \frac{MU_y}{P_y} \Rightarrow$

opt. to spend all on Y:

$$\begin{cases} X^* = 0 \\ Y^* = \frac{I}{P_y} = 240 \end{cases}$$

2. [5marks] A consumer has $1000 in income and purchases two goods, X, which has a price of $2 and Y, which has a price of $4. Draw and appropriately label the consumer's budget constraint. Now suppose the government imposes a tax on good Y at the rate of $2 per unit, but the tax is levied only on units beyond the first twenty units purchased. Draw and appropriately label the new budget constraint.



original B.C.: $2X + 4Y = 1000$

line AB: slope $= -\frac{1}{2}$

new B.C. $\begin{cases} 2X + 4Y = 1000, & Y \le 20 \quad ① \\ 2X + 6Y = 1000, & Y > 20 \quad ② \end{cases}$

①: line BC: slope $= -\frac{1}{2}$

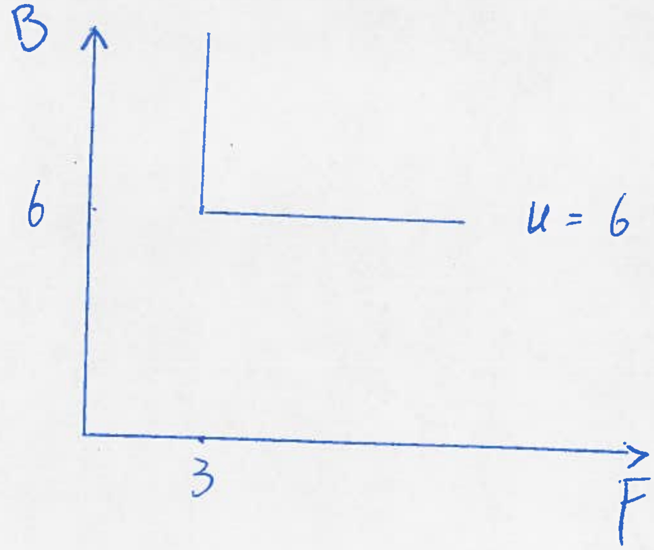
if $Y = 20 \Rightarrow X = 460$

②: line CD: slope $= -\frac{1}{3}$

if $X = 0 \Rightarrow Y = \frac{1000}{6} \approx 166.67$

3. [5marks]Each Sunday Tomas sits down to watch soccer on television. Tomas drinks two bottles of beer during each soccer game he watches. Write an equation that describes Tomas's preferences over beer, B, and soccer games, F. Each Sunday Tomas watches three soccer games. Draw and appropriately label his indifference curve.

$U(B,F) = \min\{B, 2F\}$



Section B: Three question @ 15 marks- 5 for each part of each question. Total 45 marks.

1. Kate consumes two goods, X and Y, according to the utility function $U(X,Y)=X\,Y^{1/2}$. Kate has an income, I, and faces prices for the two goods given by P_X and P_Y.
   a) [5 marks] Derive Kate's demand functions for the goods X and Y.

at opt.: $MRS_{x,y} = \frac{MU_x}{MU_y} = \frac{Y^{1/2}}{\frac{1}{2}XY^{-1/2}} = \frac{2Y}{X} = \frac{P_x}{P_y} \Rightarrow$

$X = 2Y\frac{P_y}{P_x}$ sub. into B.C.: $P_x \cdot X + P_y \cdot Y = I \Rightarrow$

$Y^* = \frac{I}{3P_y}$, $X^* = \frac{2I}{3P_x}$

b) [5 marks] Assume that Kate's income is \$1,800, the price of X is \$4 and the price of Y is \$2. Calculate her demand for each good. What is the elasticity of demand for Y at this bundle?

$$Y^* = \frac{I}{3P_Y} = \frac{1800}{3(2)} = 300$$

$$X^* = \frac{2I}{3P_X} = \frac{2(1800)}{3(4)} = 300.$$

$$\frac{dY}{dP_Y} = \frac{I}{3}\cdot(-1)\cdot\frac{1}{P_Y^2} = -\frac{Y}{P_Y} \Rightarrow$$

$$\varepsilon_Y = \frac{dY}{dP_Y}\cdot\frac{P_Y}{Y} = -1$$

c) [5 marks]Suppose the price of X decreases to \$3. Determine the new demand for the goods and calculate the income and substitution effects of the price change.

new opt. bundle: $X^N = \frac{2I}{3P_X^N} = \frac{2(1800)}{3(3)} = 400$

$Y^N = Y^* = 300$ (no change in P_Y)

original utility: $U_0 = X^*(Y^*)^{1/2} = 3000\sqrt{3}$

new price: $MRS_{X,Y} = \frac{2Y}{X} = \frac{P_X^N}{P_Y} = \frac{3}{2} \Rightarrow Y = \frac{3X}{4}$

with decomposition bundle to get $U_0 \Rightarrow$

$$3000\sqrt{3} = X^D\cdot\left(\frac{3X^D}{4}\right)^{1/2} \Rightarrow X^D \approx 330.19$$

sub. effect: $X^D - X^* = 330.19 - 300 = 30.19$

income effect: $X^N - X^D = 400 - 330.19 = 69.81$

2. Art has 126 hours per week to divide between leisure, R, and work. When he works, Art earns $30 per hour. He values both leisure and consumption, C, according to the utility function U(R,C)=Min{30R ; C}. The price of the consumption good is unity.

a) [5marks] Derive Art's optimal bundle. How much does he work?

opt. at $30R = C$ with B.C. $C = 30(126-R)$

$\Rightarrow 30R = 30(126-R) \Rightarrow R^* = 63$

$C^* = 30R^* = 1890$

Art works $126 - R^* = 63$ hours

b) [5 marks] Art is now subject to a tax of 20% on his wages (he fully bears the tax). Does Art work more or less because of the tax? Explain his choice in terms of the general direction of income and substitution effects.

now wage becomes to $30(1-20\%) = 24$

still opt. at $30R = C$ with $C = 24(126-R)$

$\Rightarrow R^* = 56$. $C^* = 30R^* = 1680$

Art works $126 - 56 = 70$ hours, more work now.

- income effect: Art got paid less wage, so he needs to work more, $\downarrow R$ and $\uparrow$ working hours
- sub. effect: wage decreases $\Rightarrow$ leisure is cheaper $\Rightarrow$ $\uparrow R$ and $\downarrow$ working hours
- our result shows that income effect dominates sub. effect.

c) Starting from the solution to part (a), assume Art's boss tells him that new regulations for the workplace mean Art can work no more than 40 hours per week. Calculate Art's utility level under the new rules and show that he is worse off because of the new rules.

part a): utility $U_a = \min\{30R^*; C^*\} = 1890$

now: if works 40 hrs: ~~u = min{30(126-40)}~~

then $R = 126 - 40 = 86$. $\Rightarrow$ $30R = 2580$

$C = 30(126-86)$

$= 1200$

$U_c = \min\{2580; 1200\} = 1200 < 1890$

Art is worse off under new rule.

3. [5 marks] Emily works in the present period and earns an income of \$8,000,000. In the future period, Emily is retired and earns nothing. Her preferences over present consumption, C_P, and future consumption, C_F, are given by $U(C_P,C_F)=C_P^{1/2}C_F$. Emily's savings earn an interest rate of 100%.

a) Derive Emily's optimal consumption bundle and her level of savings.

B.C.: $C_P + \frac{C_F}{1+r} = I_P \Rightarrow C_P + \frac{C_F}{2} = 8M$

opt. must be: $MRS_{C_P,C_F} = \frac{MU_{C_P}}{MU_{C_F}} = \frac{\frac{1}{2}C_P^{-1/2}C_F}{C_P^{1/2}} = \frac{C_F}{2C_P} = 1+r = 2 \Rightarrow$

$C_F = 4C_P$ sub. into B.C. $\Rightarrow$

$C_P^* = \frac{8}{3}M$, $C_F^* = 4C_P^* = \frac{32}{3}M$

saving $= I_P - C_P^* = \frac{16}{3}M$

b) Suppose that Emily had treated present and future consumption as perfect complements rather than according to her original utility function. Write an equation for Emily's preferences (perfect complements) that would lead Emily to consume equal amounts in each period of her life. Prove that this utility function does lead to equal consumption in each period.

$U(C_P, C_F) = \min\{C_P, C_F\}$ can lead Emily consume equal amounts.

for $\min\{C_P, C_F\}$ is opt. at $C_P = C_F$ sub. into B.C.

$\Rightarrow C_P + \frac{C_P}{2} = 8M \Rightarrow$

$C_P = C_F = \frac{16}{3}M$ is opt. bundle.

c) Explain and illustrate how Emily's original budget line would change if the government taxed both her present earnings and the interest earned on her savings at the rate of 40%.

original B.C. : $C_P + \frac{C_F}{1+r} = I_P$.

tax on $I_P \Rightarrow$ new : $I_P(1-T)$

tax on interest earned $\Rightarrow$ new price at: $1 + r(1-T)$ $\Rightarrow$

new B.C. $C_P + \frac{C_F}{1+r(1-T)} = I_P(1-T)$

compare to original : both max level of C_P & C_F decrease.

can see from graph that both intercept decrease.