Economics 212

Section 001

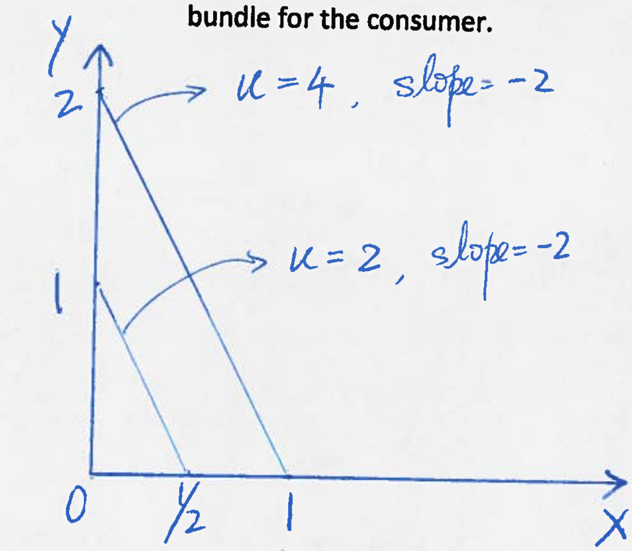
Midterm Exam

March 3, 2014

Student Number:

Section A: Three questions @ 5 marks. Total 15 marks.

1. [5 marks] Consider the utility function U(X,Y)=4X+2Y, where X and Y are two goods. Draw and appropriately label two indifference curves for this consumer. Assume the price of X is $4, the price of Y is $4 and the consumer has an income of $200. Derive the optimal consumption bundle for the consumer.



IC: $Y = -2X + \frac{\bar{u}}{2}$

B.C: $P_x X + P_y Y = I \iff 4X + 4Y = 200$

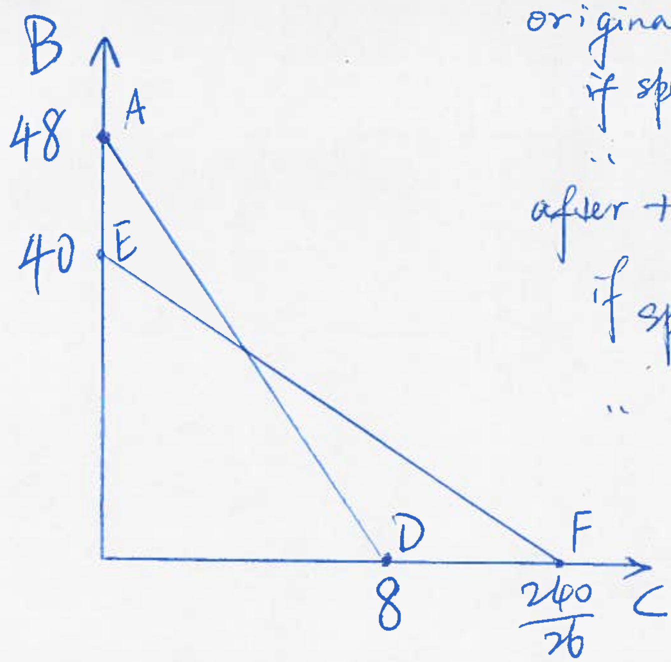
$$\left.\begin{aligned} MRS_{x,y} &= \frac{MU_x}{MU_y} = \frac{4}{2} = 2 \\ \frac{P_x}{P_y} &= \frac{4}{4} = 1 \end{aligned}\right\} \Rightarrow$$

$$\frac{MU_x}{MU_y} > \frac{P_x}{P_y} \Rightarrow \frac{MU_x}{P_x} > \frac{MU_y}{P_y} \Rightarrow$$

opt. to spend all on X:

$$\begin{cases} X^* = \frac{I}{P_x} = 50 \\ Y^* = 0 \end{cases}$$

2. [5marks] Martin consumes bread, B, and cheese, C. The price of cheese is $30 per unit and the price of bread is $5 per unit. Martin has an income of $240 to spend on the two goods. Draw and appropriately label his budget constraint. Now suppose the government imposes a tax equal to $1 per unit on bread and offers a subsidy of $4 per unit on cheese. Show how these actions affect Martin's consumption opportunities.



original B.C: $30C + 5B = 240$ (line: AD)

if spend all on B, most can get $B = 48$

" " " C, " " " $C = 8$

after tax & subsidy: $26C + 6B = 240$

(line: EF)

if spend all on B, most can get $B = 40$

" " " C, " " " $C = \frac{240}{26} > 8$

3. [5marks] Assume that market demand is given by $Q^D=2000-2P+3I$ and market supply is given by $Q^S=4P+200-10W$, where Q is quantity, P is price, I is income and W is the wage paid to workers. Derive the equilibrium values of price and quantity (the expressions for P and Q will contain the terms I and W).

at eq^m: $Q^D = Q^S \Rightarrow 2000-2P+3I = 4P+200-10W \Rightarrow$

$$P^* = 300 + \frac{I}{2} + \frac{5W}{3} \Rightarrow$$

$$Q^* = 4P^* + 200 - 10W = 1400 + 2I - \frac{10W}{3}$$

Section B: Three question @ 15 marks- 5 for each part of each question. Total 45 marks.

1. For entertainment, Wei consumes movies, M, and dinners, D, according to the utility function $U(M,D)=MD^{1/2}$. The price of a movie is P_M, the price of a dinner is P_D, and Wei's income is I.

   a) [5 marks] Derive Wei's demand functions for the two goods.

at opt. : $MRS_{M,D} = \frac{MU_M}{MU_D} = \frac{D^{1/2}}{\frac{1}{2}MD^{-1/2}} = \frac{2D}{M} = \frac{P_M}{P_D} \Rightarrow$

$M = 2D\frac{P_D}{P_M}$ sub. into B.C.: $P_M \cdot M + P_D \cdot D = I \Rightarrow$

$$D^* = \frac{I}{3P_D}, \quad M^* = \frac{2I}{3P_M}$$

b) [5 marks] Assume the price of a movie is \$10, the price of a dinner is \$20 and Wei has an entertainment budget of \$600. Determine Wei's optimal bundle.

$$D^* = \frac{I}{3P_D} = \frac{600}{3(20)} = 10$$

$$M^* = \frac{2I}{3P_M} = \frac{2(600)}{3(10)} = 40$$

c) [5 marks] Assume that the price of a movie increases to \$20. Determine the new optimal bundle and the income and substitution effects of the price increase.

new opt. bundle: $M^N = \frac{2I}{3P_M^N} = \frac{2(600)}{3(20)} = 20$

$D^N = D^* = 10$ (no change in P_D)

original utility: $U_0 = M^*(D^*)^{1/2} = 40\sqrt{10}$

new price: $MRS_{M,D} = \frac{P_M^N}{P_D} \Rightarrow \frac{2D}{M} = \frac{20}{20} \Rightarrow D = \frac{M}{2}$

with decomposition bundle to get U_0 $\Rightarrow$

$$40\sqrt{10} = M^D \cdot (D^D)^{1/2} = M^D \cdot \left(\frac{M^D}{2}\right)^{1/2} \Rightarrow$$

$M^D \approx 31.75$

~~sub. effect~~ : $\cancel{M^N - M^* = \overset{20}{31.75} - 40 = -8.25}$ $\cancel{-20}$

~~income effect~~ : $\cancel{M^* - M^D = 40 - 31.75} =$

sub. effect: $M^D - M^* = 31.75 - 40 = -8.25$

income effect: $M^N - M^D = 20 - 31.75 = -11.75$

2. Alexa has 126 hours per week to divide between leisure, R, and work. When she works, Alex earns \$15 per hour. She values both leisure and consumption, C, according to the utility function U(R,C)=Min{20R;C}. The price of the consumption good is unity.

   a) [5marks] Derive Alexa's optimal bundle. How much does she work?

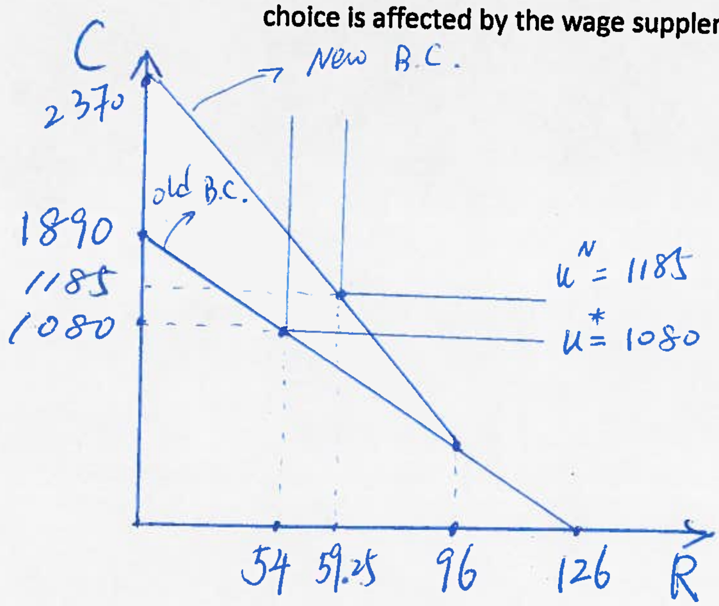
opt. at $20R = C$ with B.C: $C = 15(126-R)$

$\Rightarrow 20R = 15(126-R) \Rightarrow R^* = 54$

$C^* = 20R^* = 1080$

she works $126 - R^* = 72$ hours

   b) [5 marks] The government tells Alexa that it will supplement her wage by \$5 per hour, but only if she works more than 30 hours per week and only on hours worked above 30 hours. Draw and appropriately label the new budget constraint. Explain how Alexa's choice is affected by the wage supplement.



new problem with B.C.:

$C^N = 20[(126-30) - R^N] + 15(30)$

opt. at $20R^N = C^N \Rightarrow$

$R^N = 59.25 \quad C^N = 1185$

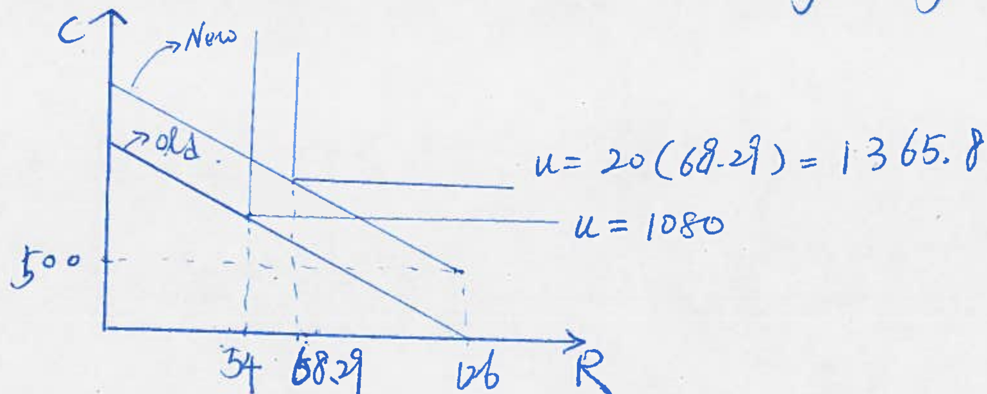
Alexa is better off since $R^N > R^*$, $C^N > C^*$ and get higher utility level (can see from graph).

c) **If Alexa wins $500 in the lottery, will she work more or less? Explain your answer.**

compare result with part a) :

now : $20R = 15(126 - R) + 500 \Rightarrow R^* = 68.29 > 54$

so Alexa will work less, and get higher utility.



3. **[5 marks]Jonas is a music star who earns $100,000,000 during his working life and nothing when he retires. The interest rate between his working life and retirement is 100%. His preferences over present consumption, C_P, and future consumption, C_F, are given by $U(C_P,C_F)=C_P\,C_F$.**

   a) **Derive Jonas optimal consumption bundle and his level of savings.**

B.C. : $C_p + \frac{C_F}{1+r} = I_p + 0 = I_p \Rightarrow C_p + \frac{C_F}{2} = 100M$.

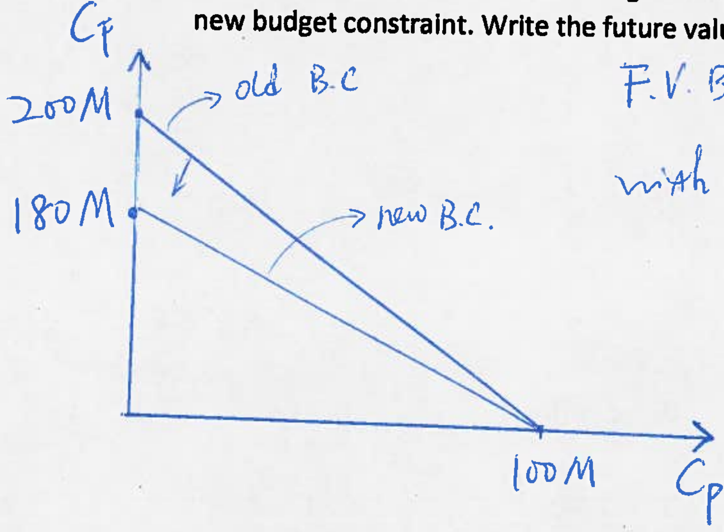
opt. must be: $MRS_{C_p,C_F} = \frac{MU_{C_p}}{MU_{C_F}} = \frac{C_F}{C_p} = 1+r = 2 \Rightarrow$

$C_F = 2C_p$ sub. into B.C. $\Rightarrow$

$C_p^* = 50M$, $C_F^* = 2C_p^* = 100M$

saving $= I_p - C_p^* = 100M - 50M = 50M$.

b) Draw and appropriately label Jonas budget constraint. Suppose the government decides to tax the interest earned on his savings at the rate of 20%. Draw and appropriately label the new budget constraint. Write the future value form of this new budget constraint.



F.V. B.C. : $(1+r)C_P + C_F = (1+r)I_P$

with tax : $C_P\left[1+r(1-T)\right] + C_F = \left[1+r(1-T)\right]I_P$

with $r=1$, $T=0.2$, $I_P = 100M$

$\Rightarrow \; 1.8C_P + C_F = 1.8\,I_P$

c) [5 marks] How would your answer to part (a) change if Jonas had preferences given by $U(C_P, C_F) = 3C_P + C_F$? Explain your answer.

$$MRS_{C_P, C_F} = \frac{MU_{C_P}}{MU_{C_F}} = 3$$

$$\frac{P_{C_P}}{P_{C_F}} = 1 + r = 2$$

$$\Rightarrow MRS_{C_P, C_F} = \frac{MU_{C_P}}{MU_{C_F}} > \frac{P_{C_P}}{P_{C_F}}$$

$$\Rightarrow \frac{MU_{C_P}}{P_{C_P}} > \frac{MU_{C_F}}{P_{C_F}}$$

$\Rightarrow$ spend all on present $\quad C_P = \dfrac{I_P}{P_{C_P}} = 100M$

$C_F = 0$.