Solution

Economics 212

Section 002

Midterm Exam

March 5, 2013

Student Number:

Section A: Three questions @ 5 marks. Total 15 marks.

1. [5 marks]The market for bushels of squash is characterized by a demand function of the form $Q^D = 1800 - 10P$ and a supply function of the form $Q^S = 290P - 300$, where Q is quantity and P is price. Calculate the equilibrium price and quantity in the market and the elasticity of demand at the equilibrium.

$$Q^D = Q^S \Rightarrow 1800 - 10P = 290P - 300$$

$$2100 = 300P$$

$$7 = \frac{2100}{300} = P^*$$

$$Q^* = 1800 - 10(7) = 1730$$

$$\varepsilon_d = \frac{dQ^D}{dP}\frac{P}{Q} = -10 \cdot \frac{7}{1730} = \frac{-70}{1730} = \frac{-7}{173} \quad \text{inelastic}$$

2. [5marks] A consumer is always willing to give up four units of good X in exchange for one unit of good Y. The price of X is four and the price of Y is ten. The consumer has an income of 1000 dollars. Determine the consumer's optimal bundle.

Perfect substitutes : $U(x,y) = x + 4y$

$$\frac{MU_x}{MU_y} = MRS = \frac{1}{4} < \frac{4}{10} = \frac{P_x}{P_y}$$

$$\Leftrightarrow \frac{MU_x}{P_x} < \frac{MU_y}{P_y}$$

Corner solution. Will consume only Y.

$$Y^* = \frac{1000}{10} = 100 \qquad U(x,y) = 400$$

$$X^* = 0$$

3. [5marks] Anthony has \$2,000 in income and is considering spending it on one of two lotteries. Lottery A gives Anthony a 40% probability of \$1,400 in final income and a 60% probability of \$2500 in final income. Lottery B gives Anthony a 40% probability of \$1700 in final income and a 60% probability of \$2,200 in final income. Anthony is risk averse with a utility of income function given by $U(I) = i^{1/2}$, where i is his income. Will Anthony prefer to keep his ~~\$1,000~~ 2000\$ or will he choose one of the lotteries? Explain.

Lottery A: $E(u) = \frac{4}{10}\sqrt{1400} + \frac{6}{10}\sqrt{2500} = 44.96 > 44.72$

B: $E(u)\ \frac{4}{10}\sqrt{1700} + \frac{6}{10}\sqrt{2200} = 44.63 < 44.72$

No lottery : $\sqrt{2000} = 44.72$

He will choose lottery A.

Section B: Three question @ 15 marks- 5 for each part of each question. Total 45 marks.

1. Katie consumes two goods, X and Y, according to the utility function $U(X,Y)=X^{1/2}\,Y$. Katie has an income, I, and faces prices for the two goods given by P_X and P_Y.
   a) [5 marks] Derive Katie's demand functions for the goods X and Y.

$MRS_{x,y} = \dfrac{\frac{1}{2}\frac{Y}{X^{1/2}}}{X^{1/2}} = \frac{1}{2}\frac{Y}{X} = \frac{P_x}{P_y} \Rightarrow Y = \frac{2P_x}{P_y}X$ Plug in B/c

B/c : $P_x X + \cancel{P_y}\left(2\frac{P_x}{\cancel{P_y}}X\right) = I$

$\Rightarrow \quad 3P_x X = I \Rightarrow X = \frac{I}{3P_x}\ ;\ Y = \frac{2}{3}\frac{I}{P_y}$

b) [5 marks] Assume that Katie's income is \$2,000, the price of X is \$5 and the price of Y is \$3. Calculate her demand for each good. What is the elasticity of demand for Y at this bundle?

$$X = \frac{2000}{3 \cdot 5} = \frac{2000}{15} \quad ; \quad Y = \frac{2 \cdot 2000}{3 \cdot 3} = \frac{4000}{9}$$

$$\varepsilon_Y = \frac{dY}{dP_Y}\frac{P_Y}{Y} = -\frac{2}{3}\cdot\frac{2000}{P_Y^2}\cdot\frac{P_Y}{4000/9} = -\frac{2}{\cancel{3}}\cdot\frac{2000}{\cancel{3}}\cdot\frac{\cancel{9}}{4000}$$

$$= -\frac{4000}{4000} = -1$$, unitary elastic

c) [5 marks]Suppose the price of X increases to \$8. Determine the new demand for the goods and calculate the income and substitution effects of the price change.

Y stays the same; $X = \frac{2000}{3 \cdot 8} = \frac{250}{3}$

At initial bundle $U_0 = \left(\frac{2000}{15}\right)^{1/2} \cdot \frac{4000}{9} \approx 5132.$

$$MRS = \frac{1}{2}\frac{Y}{X} = \frac{8}{3} \Rightarrow Y = \frac{16}{3}X$$

$$5132 = X^{1/2} \cdot \frac{16}{3}X$$

$$= X^{3/2} \cdot \frac{16}{3}$$

$$\left(\frac{3}{16}\right)^{2/3}(5132)^{2/3} = X$$

$$97.47 = X$$

Income effect

$$\frac{250}{3} - 97.47 = -14.14$$

Substitution effect

$$97.47 - \frac{2000}{15} = -35.86$$

2. Al has 24 hours per day to divide between leisure, R, and work. When he works, Al earns $30 per hour. He values both leisure and consumption, C, according to the utility function U(R,C)= $R^{1/2}C$. The price of the consumption good is unity.
   a) [5marks] Derive Al's optimal bundle. How much does he work?

$$MRS_{R,C} = \frac{1}{2}\frac{C}{R} = \frac{P_R}{P_C} = \frac{1}{2}\frac{C}{R} = P_R = w \iff C = 2Rw$$

$$B/C \quad C = (24-R)\cdot w$$

$$\iff 2R\cancel{w} = (24-R)\cancel{w}$$

$$2R = 24 - R$$

$$3R = 24$$

$$R = 8 \;;\quad C = 16\cdot 30 = 480 \;;\ \text{work } 16\ H$$

   b) [5 marks] Al's boss tells him that a reorganization of the workplace has occurred and that Al must work either longer or shorter hours. The longer shift is 12 hours and the shorter shift is 4 hours. Which shift would Al prefer? Explain your reasoning.

4 hours : $C = 4\cdot 20 = 80$; $R = 20$ $\quad U(20, 80) = 357.77$

12 hours : $C = 12\cdot 20 = 240$; $R = 12$ $\quad U(12, 240) = 831.38$.

Prefer the longer shift.

New wage $= \$20$

c) Starting from the solution to part (a), assume Al's income is taxed at the rate of $10per hour and that Al's wage decreases by the full $10. Calculate Al's new optimal bundle and amount of work and show that he is worse off because of the tax.

with Cobb-Douglas R and L are independent of w so

$R = 8$; $L = 16$ But $C = 16 \cdot 20$ instead of $16 \cdot 30$

So R stays the same but C ↓

3. [5 marks] Emily works in the present period and earns an income of $6,000,000. In the future period, Emily is retired and earns nothing. Her preferences over present consumption, C_P, and future consumption, C_F, are given by $U(C_P,C_F)= \text{Min}\ \{C_P\ ;\ 3C_F\}$. Emily's savings earn an interest rate of 100%.

a) Derive Emily's optimal consumption bundle and her level of savings.

$C_P = 3C_F$

B/c : $\left.\begin{aligned} 6\ 000\ 000 &= C_P + s \\ C_F &= (1+r)s \end{aligned}\right\}$ $6000\ 000 = C_P + \dfrac{C_F}{1+r}$

$[1+r = 1+1 = 2]$

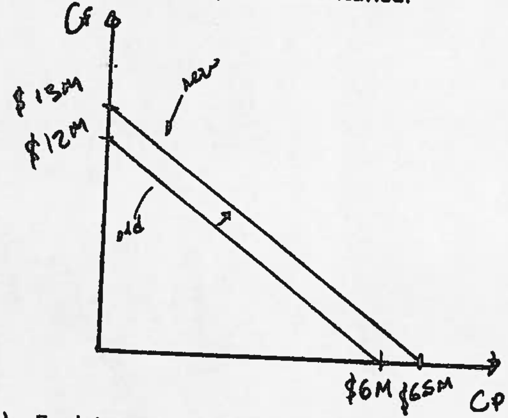
$\Rightarrow 6\ 000\ 000 = 3C_F + \dfrac{C_F}{2}$

$12\ 000\ 000 = 7C_F$

$\dfrac{12}{7}\ 000\ 000 = C_F$; $C_P = \dfrac{36\ 000\ 000}{7}$

$s \approx 857\ 000\ 000$

b) Suppose Emily learns that she will inherit $1,000,000 at the start of the future period and that she can borrow against it if she wishes. Draw Emily's original budget line and show how it is affected by the inheritance.



c) Explain and illustrate how Emily's budget line from part (b) would change if the government taxed both her earnings and the interest earned on her savings at the rate of 40%.

income + inheritance

$$(1-t)\,6\,000\,000 = C_p + s$$

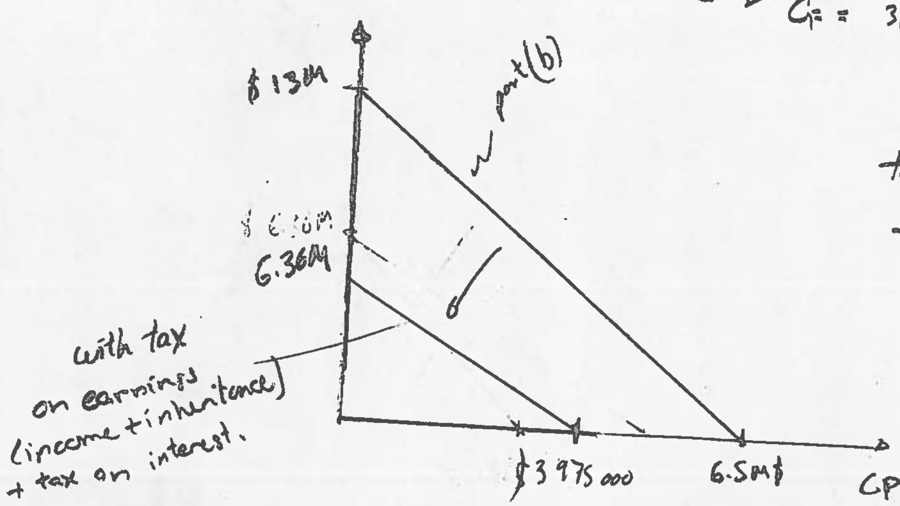
$$C_F = (1+r(1-t))s + 1\,000\,000\,(1-t)$$

$$\Rightarrow \frac{C_F - 600\,000}{1.6} = s$$

$$3\,600\,000 = C_p + \frac{C_F}{1.6} - \frac{600\,000}{1.6}$$

If consume all in present $\Rightarrow$ $C_p = 3\,975\,000$

future $\Rightarrow$ $C_F = 3{,}6M(1.6) + 600\,000 = \$6{,}36M$



Budget constraint moves toward the origin.

+ Slope changes

in (b) $\rightarrow$ -2

now $\rightarrow$ -1.6