Solution

Economics 212

Section 001

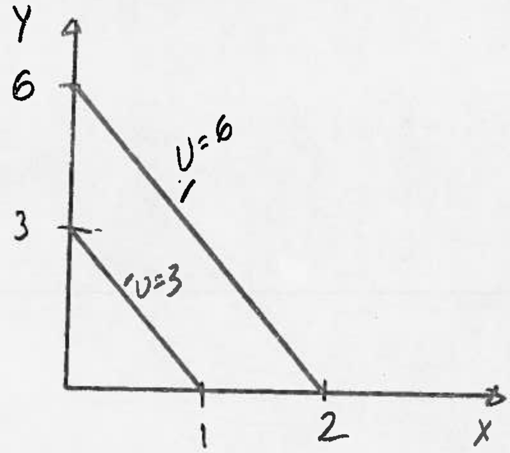
Midterm Exam

March 5, 2013

Student Number:

Section A: Three questions @ 5 marks. Total 15 marks.

1. [5 marks] Consider the utility function U(X,Y)=3X+Y, where X and Y are two goods. Draw and appropriately label two indifference curves for this consumer. Assume the price of X is \$12, the price of Y is \$4 and the consumer has an income of \$1200. Derive the optimal consumption bundle for the consumer.



$$MRS_{x,y} = \frac{MU_x}{MU_y} = \frac{3}{1} = 3$$

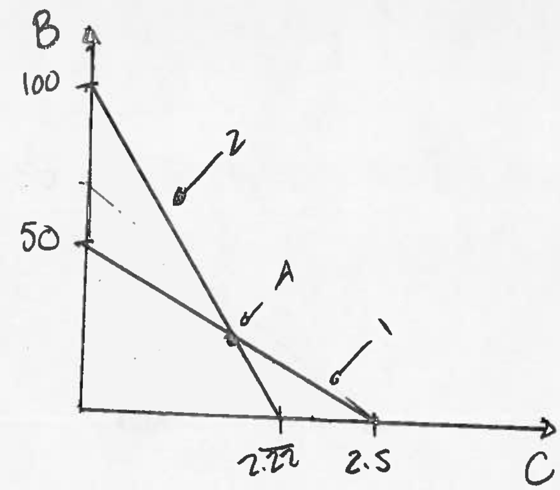
$$\frac{P_x}{P_y} = \frac{12}{4} = 3$$

$$\Rightarrow \frac{MU_x}{P_x} = \frac{MU_y}{P_y}$$

So he is indifferent between any bundle that satisfies the budget constraint with equality.

i.e. $12x + 4y = 1200$; any combination will give $U = 300$.

2. [5marks] Martin consumes bread, B, and cheese, C. The price of cheese is \$40 per unit and the price of bread is \$2 per unit. Martin has an income of \$100 to spend on the two goods. Draw and appropriately label his budget constraint. Now suppose the government imposes a tax equal to \$5 per unit on cheese and offers a subsidy of \$1 per unit on bread. Show how these actions affect Martin's consumption opportunities.



1. max $C = \frac{100}{40} = 2.5$

   max $B = \frac{100}{2} = 50$

2. max $C = \frac{100}{45} = 2.\overline{22}$

   max $B = \frac{100}{1} = 100$

Turn clockwise at point A.

Intercept of Bread ↑ from 50 to 100

(·) of Cheeze ↓ from 2.5 to 2.$\overline{22}$

3. [5marks] Assume that market demand is given by $Q^D=2000-2P+3I$ and market supply is given by $Q^S=4P+200-10W$, where Q is quantity, P is price, I is income and W is the wage paid to workers. Derive the equilibrium values of price and quantity (the expressions for P and Q will contain the terms I and W).

$$Q^D = 4P + 200 - 10W = 2000 - 2P + 3I = Q^S$$

$$\Leftrightarrow \quad 6P = 1800 + 3I + 10W$$

$$\Leftrightarrow \quad P = 300 + \frac{I}{2} + \frac{5}{3}W$$

$$\text{So} \quad Q^D = Q^S = Q^* = 4\left(300 + \frac{I}{2} + \frac{5}{3}W\right) + 200 - 10W$$

$$= 1400 + 2I - \frac{10}{3}W$$

Section B: Three question @ 15 marks- 5 for each part of each question. Total 45 marks.

1. For entertainment, Wei consumes movies, M, and dinners, D, according to the utility function $U(M,D)=M^{1/2}D$. The price of a movie is P_M, the price of a dinner is P_D, and Wei's income is I.
   a) [5 marks] Derive Wei's demand functions for the two goods.

Cobb-Douglas. ⇒ interior solution.

$$MRS_{M,D} = \frac{\frac{1}{2}\frac{D}{M^{1/2}}}{M^{1/2}} = \frac{1}{2}\frac{D}{M} = \frac{P_M}{P_D} \Leftrightarrow D = 2M\frac{P_M}{P_D} \quad (*)$$

Plug $(*)$ in Budget constraint ⇒ $P_M M + P_D\left(2M\frac{P_M}{P_D}\right) = I$

$$\Leftrightarrow P_M M + 2M P_M = I$$

$$3M P_M = I$$

$$M = \frac{I}{3P_M} \quad (**)$$

Plug $(**)$ in equation for D ⇒

$$D = 2\left(\frac{I}{3P_M}\right)\frac{P_M}{P_D}$$

$$= \frac{2}{3}\frac{I}{P_D}$$

125

b) [5 marks] Assume the price of a movie is \$12, the price of a dinner is \$24 and Wei has an entertainment budget of \$500. Determine Wei's optimal bundle.

$$D = \frac{2}{3}\frac{I}{P_D} = \frac{2}{3}\cdot\frac{500}{24} = \frac{1000}{72} = \frac{125}{9} \approx 13.89$$

$$M = \frac{1}{3}\frac{I}{P_M} = \frac{1}{3}\frac{500}{12} = \frac{500}{36} = \frac{125}{9} \approx 13.89$$

c) [5 marks] Assume that the price of a movie increases to \$15. Determine the new optimal bundle and the income and substitution effects of the price increase.

D stays the same. $M_0 = \frac{125}{9}$

$$M_1 = \frac{500}{45} = \frac{100}{9} \approx 11.\overline{11}$$

At initial bundle $U_0 = \left(\frac{125}{9}\right)^{1/2}\cdot\frac{125}{9} = \left(\frac{125}{9}\right)^{3/2}$

$$MRS_{M,D} = \frac{1}{2}\frac{D}{M} = \frac{15}{24} \Rightarrow D = \frac{30}{24}M = \frac{5}{4}M$$

Plug in $\left(\frac{125}{9}\right)^{3/2} = M^{1/2}\cdot\frac{5}{4}M$

$$\left(\frac{125}{9}\right)^{3/2} = M^{3/2}\cdot\frac{5}{4}$$

$$\left(\frac{4}{5}\right)^{2/3}\cdot\frac{125}{9} = M \Rightarrow M = 11.97$$

Income effect

$11.\overline{11} - 11.97 = -0.86$

Substitution effect

$11.97 - \frac{125}{9} = -1.92$

2. Alexa has 168 hours per week to divide between leisure, R, and work. When she works, Alex earns $20 per hour. She values both leisure and consumption, C, according to the utility function U(R,C)=Min{60R;C}. The price of the consumption good is unity.

   a) [5marks] Derive Alexa's optimal bundle. How much does she work?

$\min\{60R, C\} \Rightarrow 60R = C$

Budget constraint: $C = (168-R)\cdot 20$

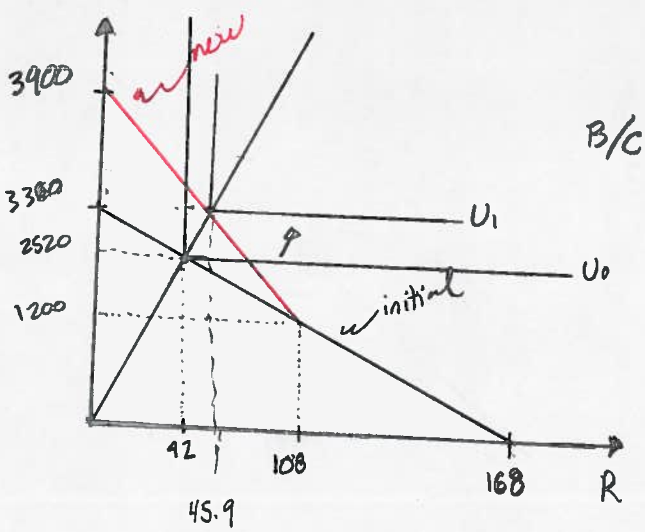
$\Rightarrow 60R = 20\cdot 168 - 20R$

$80R = 20\cdot 168$

$R = \frac{20\cdot 168}{80} = 42$, So she works $168-42 = 126$ hour

And consumes $126\cdot 20 = 2520$

   b) [5 marks] The government tells Alexa that it will supplement her wage by $5 per hour, but only if she works more than 60 hours per week and only on hours worked above 60 hours. Draw and appropriately label the new budget constraint. Explain how Alexa's choice is affected by the wage supplement.



CAN reach higher indifference curve. More C and more R.

B/C She was already working more than 60 h

New problem:

$60R = (108-R)25 + 20\cdot 60$

$85R = 108\cdot 25 + 1200$

$R = 45.9$

$C = 2752.5$

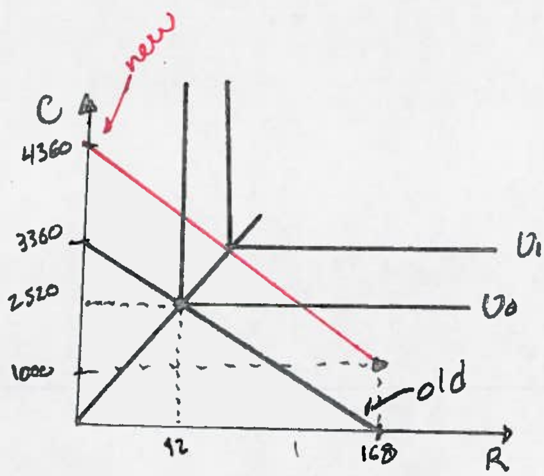
(compare to (a))

c) If Alexa wins $1000 in the lottery, will she work more or less? Explain your answer.

Problem become $60R = (168 - R)20 + 1000$

$80R = 20 \cdot 168 + 1000$

$R = \frac{20 \cdot 168 + 1000}{80} > \frac{20 \cdot 168}{80}$ So she work less.



3. [5 marks]Jonas is a music star who earns $100,000,000 during his working life and nothing when he retires. The interest rate between his working life and retirement is 100%. His preferences over present consumption, C_P, and future consumption, C_F, are given by $U(C_P,C_F)=C_P^{1/2}\,C_F$.

a) Derive Jonas optimal consumption bundle and his level of savings.

$MRS_{P,F}$ = SAME THAN Section B.1

$$\frac{1}{2}\frac{CF}{CP} = \frac{P_P}{P_F}(1+r) \Rightarrow C_F = 2C_P(1+r)$$

Budget constraints = $100\,000\,000 = C_P + s$, $C_F = (1+r)s$ $\Big\}$ $100\,000\,000 = C_P + \frac{C_F}{(1+r)}$

Substituting $\Rightarrow$ $100\,000\,000 = C_P + \frac{2C_P(1+r)}{(1+r)} = 3C_P$

$$C_P = \frac{100\,000\,000}{3} \Rightarrow s = \frac{2}{3}(100\,000\,000)$$

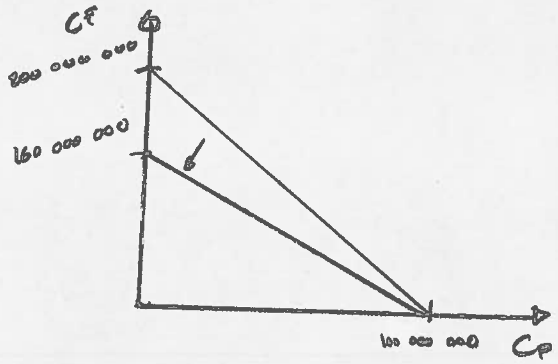
$$C_F = (1+r)\frac{2}{3}(100\,000\,000) = \frac{4}{3}(100\,000\,000)$$

b) Draw and appropriately label Jonas budget constraint. Suppose the government decides to tax the interest earned on his savings at the rate of 40%. Draw and appropriately label the new budget constraint. Write the future value form of this new budget constraint.

40% on interest earned $\Rightarrow$ $C_F = (1+r(1-0.4))s$

$= (1+0.6)s$

$= 1.6s$



c) [5 marks] How would your answer to part (a) change if Jonas had preferences given by $U(C_P, C_F) = 3C_P + C_F$? Explain your answer.

$MRS = 3$, $(1+r) = 2$ $\Rightarrow$ $MRS > \frac{P_P}{P_F}$

$\Leftrightarrow \frac{MU_P}{MU_F} > \frac{P_P}{P_F} \Leftrightarrow \frac{MU_P}{P_P} > \frac{MU_F}{P_F}$

Corner solution. He will consume everything in the present and consume nothing in the future.