Economics 212

Section B

Midterm Exam

March 3, 2011

Student Number:

Section A: Three questions @ 5 marks. Total 15 marks.

1. [5 marks] Consider the utility function U(X,Y)=2X+Y, where X and Y are two goods. Assume the price of X is $10, the price of Y is $10 and the consumer has an income of $2000. Derive the optimal consumption bundle for the consumer.

2. [5marks] A consumer has $1000 in income and purchases two goods, X, which has a price of $4 and Y, which has a price of $5. Draw and appropriately label the consumer's budget constraint. Now suppose the government imposes a tax on good Y at the rate of $1 per unit, but the tax is levied only on units beyond the first thirty units purchased. Draw and appropriately label the new budget constraint.

3. [5marks]Each Sunday Tomas sits down to watch soccer on television. Tomas drinks three bottles of beer during each soccer game he watches. Write an equation that describes Tomas's preferences over beer, B, and soccer games, F. Each Sunday Tomas watches two soccer games. Draw and appropriately label his indifference curve.

Section B: Three question @ 15 marks- 5 for each part of each question. Total 45 marks.

1. Kate consumes two goods, X and Y, according to the utility function $U(X,Y)=X^2 Y^{1/2}$. Kate has an income, I, and faces prices for the two goods given by P_X and P_Y.
   a) [5 marks] Derive Kate's demand functions for the goods X and Y.

b) [5 marks] Assume that Kate's income is $1,000, the price of X is $5 and the price of Y is $1. Calculate her demand for each good. What is the elasticity of demand for Y at this bundle?

c) [5 marks]Suppose the price of X increases to $7. Determine the new demand for the goods and calculate the income and substitution effects of the price change.

2. Art has 112 hours per week to divide between leisure, R, and work. When he works, Art earns \$25 per hour. He values both leisure and consumption, C, according to the utility function U(R,C)=Min{25R ; C}. The price of the consumption good is unity.
   a) [5marks] Derive Art's optimal bundle. How much does he work?

   b) [5 marks] Explain Art's allocation of time between work and leisure in terms of the arguments in his utility function. [Hint: think about how R and C contribute to his well-being and how work and C are related].

c) Starting from the solution to part (a), assume Art's boss tells him that new regulations for the workplace mean Art can work no more than 45 hours per week. Calculate Art's utility level under the new rules and show that he is worse off because of the new rules.

3. [5 marks] Emily works in the present period and earns an income of $5,000,000. In the future period, Emily is retired and earns nothing. Her preferences over present consumption, C_P, and future consumption, C_F, are given by $U(C_P,C_F)=C_P^{1/2}C_F$. Emily's savings earn an interest rate of 80%.

a) Derive Emily's optimal consumption bundle and her level of savings.

b) Suppose that Emily had treated present and future consumption as perfect complements rather than according to her original utility function. Write an equation for Emily's preferences (perfect complements) that would lead Emily to consume equal amounts in each period of her life. Prove that this utility function does lead to equal consumption in each period.

c) Explain and illustrate how Emily's original budget line would change if the government taxed both her present earnings and the interest earned on her savings at the rate of 40%.

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SECTION B.

Elements of solution.

SECTION A.

QUESTION #1

$u(x,y) = 2x + y$

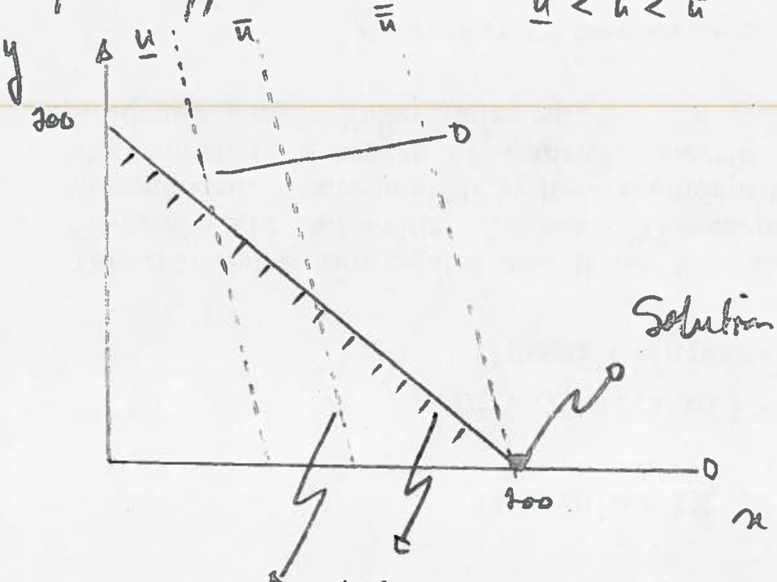
$p_x = \$10$; $p_y = \$10$; $I = \$2000$

① Set of preferences : $\bar{u} = 2x + y \iff y = \bar{u} - 2x$

② Budget constraint : $p_x x + p_y y \leq I \iff y \leq 200 - x$

③ Graphically,

$\underline{u} < \bar{u} < \bar{\bar{u}}$



Solution : $(x^*, y^*) = (200, 0)$

$y = \bar{u} - 2x$. $y \leq 200 - x$.

QUESTION #2.

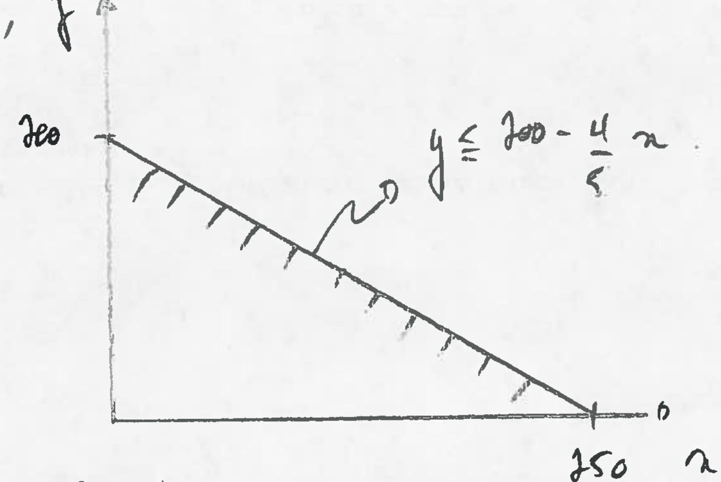
$I = \$1000$; $p_x = \$4$; $p_y = \$5$

Tax levied on units beyond the first 30 units of y ($t = \$1/\text{unit}$).

i) Before introducing the tax

$$p_x x + p_y y \leq I \iff y \leq \frac{I}{p_y} - \frac{p_x}{p_y} x = 200 - \frac{4}{5} x$$

Graphically,



2) New budget constraint.

Now, there is a tax of \$1/unit on y above 30 units. Said otherwise when $y \geq 30$ and $x \leq 250 - \frac{150}{4} = \frac{425}{2}$. The budget constraint in that region:

$$p_x x + (p_y + t)(y - 30) + 30 p_y \leq I$$

$$\iff p_x x + (p_y + t) y - (p_y + t)(30) + 30 p_y \leq I$$

$$\iff p_x x + (p_y + t) y \leq I + 30 t$$

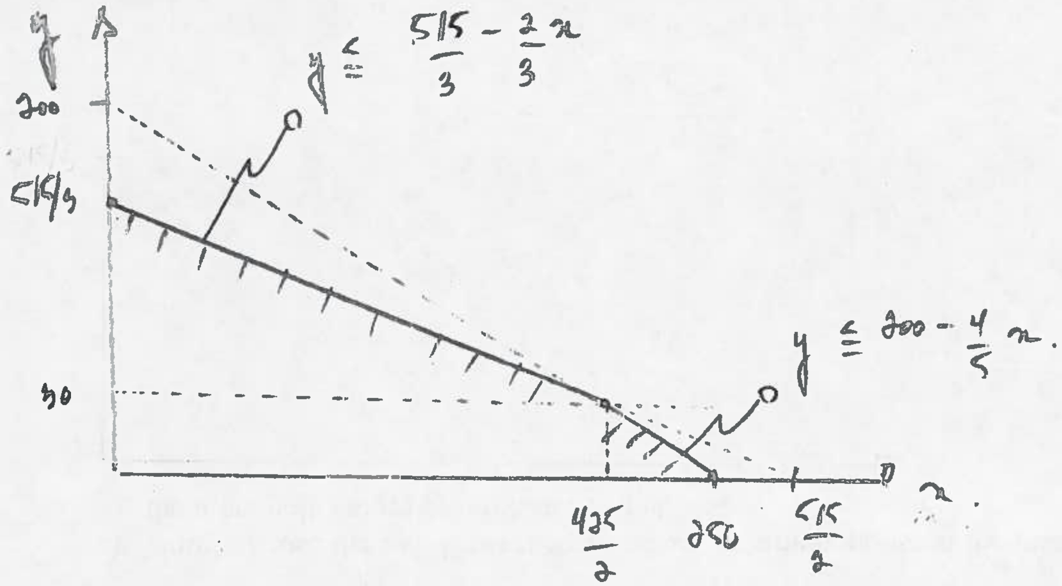
$$\iff y \leq \frac{I + 30t}{p_y + t} - \frac{p_x}{p_y + t} x = \frac{1030}{6} - \frac{4}{6} x$$

$$\iff y \leq \frac{515}{3} - \frac{2}{3} x$$

For the region $y \leq 30$ and $x \geq 435/2$, no tax levied, hence the budget constraint:

$$p_x x + p_y y \leq I \iff y \leq 200 - \frac{4}{5} x.$$

Graphically,



QUESTION #3.

3 bottles of beer each soccer game he watches.
each Sunday he watches 2 soccer games.

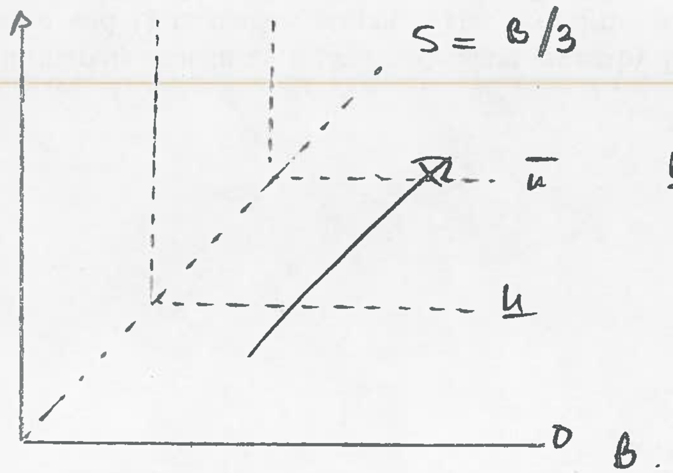
⇒ Every Sunday Tomas drinks

- 6 bottles of beer for (3 B for one game)
- 2 soccer games.

⇒ Perfect complement as beers and soccer games go hand to hand together:

$u(B, S) = 2\min\{B/3 ; S\}$.

Graphically,



SECTION B.

QUESTION #1

$u(x,y) = x^2 y^{1/2}$; $I; p_x; p_y$

(a) $\max u(x,y)$ s.t.: $p_x x + p_y y \leq I$.

$x, y \geq 0$

(i) Cobb-Douglas utility function: optimality condition $MRS_{x,y} = \frac{p_x}{p_y}$

(ii) $MRS_{x,y} = \dfrac{\partial u(\cdot)/\partial x}{\partial u(\cdot)/\partial y} = \dfrac{2 x y^{1/2}}{\frac{1}{2} x^2 \cdot y^{-1/2}} = \dfrac{4y}{x}$

(iii) at optimality:

$$MRS_{x,y} = p_x/p_y \iff y = \frac{p_x x}{4 p_y}$$

(iv) $y = p_x x / 4p_y$ in budget constraint (at equality)

$$p_x x + p_y \left[\frac{p_x x}{4 p_y} \right] = I$$

$$\iff x^* = \frac{4I}{5p_x} \quad \text{and} \quad y^* = \frac{I}{5p_y}.$$

5/.

(b) $I = \$1000$; $p_x = \$5$; $p_y = \$1$

(i) For demand ... plug in the values

$$x^I = \frac{4I}{5p_x} = \frac{4(1000)}{5(5)} = 160.$$

$$y^I = \frac{I}{5p_y} = \frac{1000}{5(1)} = 200.$$

(ii) Elasticity of demand at (x^I, y^I).

$$\eta = \frac{dy}{dp_y} \cdot \frac{p_y}{y^I} = \frac{-I}{(5)\,p_y^2} \cdot \frac{p_y}{y^I}$$

$$= \frac{-I}{5\,p_y \cdot y^I}$$

$$= \frac{-1000}{5.(1)(200)} = -1$$

(c) $p_x' = \$7$ from $p_x = \$5$.

Final basket : $x^F = \dfrac{4I}{5p_x'} = \dfrac{4(1000)}{5(7)} = \dfrac{800}{7}$.

$$y^F = \frac{I}{5p_Y} = 200.$$

(i) Initial basket : $(x^I, y^I) = (160, 200)$

(ii) Final basket : $(x^F, y^F) = (800/7, 200)$.

(iii) Decomposition basket : (x^D, y^D)

– Same utility as initial basket : $u(x^I, y^I) = \bar{u}^I = 160^2 (200)^{1/2}$

– Tangency new price and initial utility level :

$MRS_{x,Y} = \dfrac{p_x'}{p_Y} \Leftrightarrow y = \dfrac{p_x' x}{4p_Y}$

$\Rightarrow u(x, y(x)) = \bar{u}^I$

$\Leftrightarrow x^2 \left[\dfrac{p_x' x}{4p_Y}\right]^{1/2} = \bar{u}^I$

$\Leftrightarrow x^D = \left[\dfrac{4p_Y \bar{u}^I}{p_x'}\right]^{2/5}$

(iv) Income effect $= x^F - x^D$

Substitution effect $= x^D - x^I$

QUESTION # 2.

112 hours b/w R and L (labour).

wage $\equiv w = \$25$ /h.

$u(R,C) = \min\{25R; C\}$.

a) ① $C \leq 25(112-R) = 2800 - 25R$

$\Leftrightarrow C + 25R \leq 2800$. (Budget constraint)

② With Perfect Complement: solution is on the ray $25R = C$

③ Plugging $25R = C$ in budget constraint: (at equality)

$C + 25R = 2800$

$\Leftrightarrow R^* = \frac{2800}{50} = 56 \Rightarrow L^* = 112 - 56 = 56$ hours.

&

$C^* = 25(56) = 1400$.

& $u(R^*, C^*) = 1400$.

b). $u(R,C) = \min\{25R; C\}$

Perfect complements goods are goods the consumer always wants in fixed proportion to each other, here $\Delta R = 1 \Rightarrow \Delta C = 25$.

Using B.C.: $C = 25(112-R)$, this implies that at optimality we have $25R = 25(112-R) \Leftrightarrow R = \underbrace{112-R}_{L} \Rightarrow \Delta R = 1 \Rightarrow \Delta L = +1$

8/.

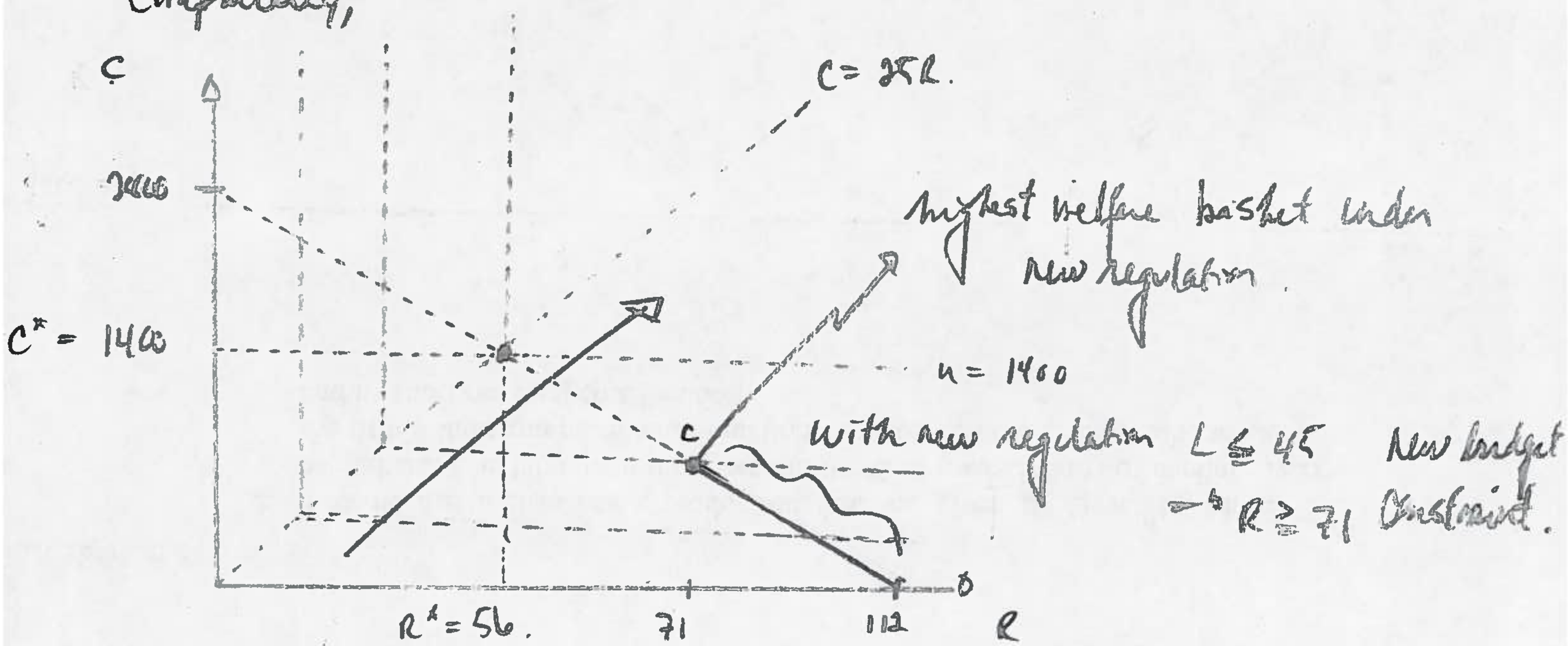
(c) In (a) $R^* = L^* = 56$ hours.

$C^* = u^* = 1400$.

New regulation: Cannot work more than 45 hours

$\Rightarrow$ leisure more than $116 - 45 = 71$ hours.

Graphically,



At point c : $R^{NR} = 71 \Rightarrow L^{NR} = 45$

$$\Rightarrow C^{NR} = 2800 - 25(71) = 1025$$

$$\Rightarrow u(R^{NR}, C^{NR}) = u^{NR} = 1025.$$

$\Rightarrow u^{NR} < u^*$

QUESTION #3.

$I_P = \$5{,}000{,}000$; $I_F = \$0$

$u(C_P, C_F) = C_P^{1/2} C_F$

$r = 80\%$.

(a) ① Present Period B.C. : $C_P + S \leq I_P$.

② Future Period B.C. : $C_F \leq S(1+r)$

③ ?. Intertemporal budget constraint ?

From future B.C. : $S = \frac{C_F}{1+r}$.

④ $S = \frac{C_F}{1+r}$ in present period B.C. (present value version).

$C_P + \frac{C_F}{1+r} \leq I_P \iff C_F \leq I_P(1+r) - (1+r)C_P$.

⑤ Solving : Cobb-Douglas at optimality $MRS_{P,F} = \frac{P}{P_0} = \frac{1}{\frac{1}{1+r}} = 1+r$.

⑥ $MRS_{P,F} = \frac{\partial u(\cdot)/\partial C_P}{\partial u(\cdot)/\partial C_F} = \frac{\frac{1}{2}\frac{C_F}{C_P^{1/2}}}{C_P^{1/2}} = \frac{C_F}{2C_P}$

⑦ At optimality : $MRS_{P,F} = 1+r$

$\iff C_F = 2C_P(1+r)$

⑧ $C_F = 2C_P(1+r)$ in Intertemporal budget constraint :

$C_P + \frac{C_F}{1+r} = I_P$. and $S^* = \frac{C_F^*}{1+r}$

$\iff C_P^* = \frac{I_P}{3} = \frac{\$5M}{3}$. $\iff S^* = 2 \cdot C_P^* = \frac{\$10M}{3}$.

$C_F^* = 2(1+r)C_P^* = 2(1.8)\frac{\$5M}{3} = \$6M$.

10/.

ⓑ. Perfect complement instead of Cobb-Douglas utility fct.
$\Rightarrow$ to consume equal amounts in each period of her life.

① Perfect complement implying identical consumption in both periods: $u(C_P, C_F) = \min\{C_P, C_F\}$ at optimality $C_P^* = C_F^*$

② Remember intertemporal budget constraint from ⓐ

$$C_P + \frac{C_F}{1+r} = I_P$$

Using the optimality on the ray $C_P = C_F$

$$\Rightarrow C_P + \frac{C_F}{1+r} = I_P \Leftrightarrow C_P + \frac{C_P}{1+r} = I_P \Leftrightarrow C_P^* = \frac{(1+r)\,I_P}{2+r}.$$

and $C_F^* = C_P^* = \dfrac{(1+r)\,I_P}{2+r}.$

Solution: $C_F^* = C_P^* = \dfrac{(1+r)\,I_P}{2+r} = \dfrac{1.8 \cdot 85\,M}{2,8} = \dfrac{9}{14} \cdot 85M.$

(c) Govt taxes present saving and interest earned on saving at rate $t = 40\%$.

(1) Present period budget constraint:

$$C_P + S \leq I_P(1-t).$$

(2) Future period budget constraint:

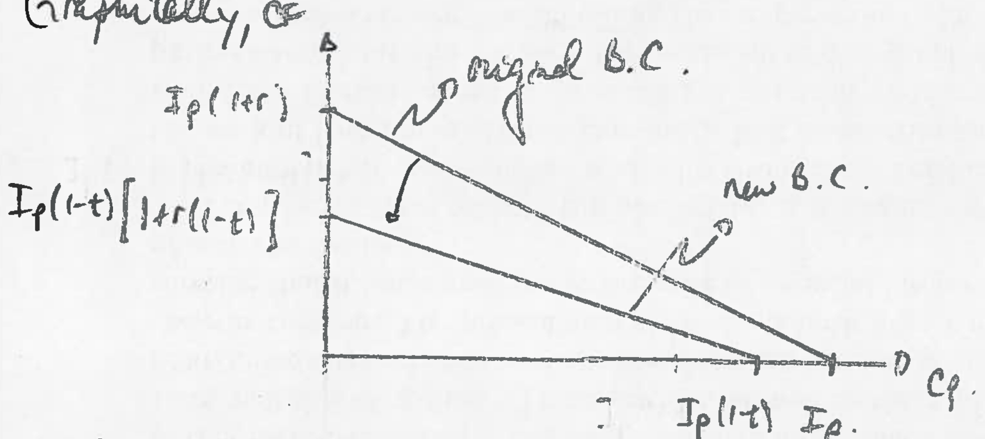
$$C_F \leq S(1 + r(1-t))$$

(3) Intertemporal budget constraint

from (2): $S = \dfrac{C_F}{1+r(1-t)}$.

in (1): $C_P + \dfrac{C_F}{1+r(1-t)} \leq I_P \cdot (1-t) \Leftrightarrow C_F \leq I_P(1-t)[1+r(1-t)] - [1+r(1-t)]C_P.$

Graphically,



Two effects: - Wealth effect (less revenue because of earning tax $I_P(1-t) < I_P$)

- Substitution effect (future consumption is now more costly because $\dfrac{1}{1+r(1-t)} > \dfrac{1}{1+r}$).

(c)