Economics 212

Section A

Midterm Exam

March 3, 2011

Student Number:

Section A: Three questions @ 5 marks. Total 15 marks.

1. [5 marks] Consider the utility function U(X,Y)=3X+2Y, where X and Y are two goods. Assume the price of X is $30, the price of Y is $20 and the consumer has an income of $4000. Derive the optimal consumption bundle for the consumer.

2. [5marks] A consumer has $2000 in income and purchases two goods, X, which has a price of $5 and Y, which has a price of $4. Draw and appropriately label the consumer's budget constraint. Now suppose the government imposes a tax on good Y at the rate of $2 per unit, but the tax is levied only on units beyond the first twenty units purchased. Draw and appropriately label the new budget constraint.

3. [5marks]Each Saturday Claude sits down to watch hockey on television. Claude drinks two bottles of beer during each hockey game he watches. Write an equation that describes Claudes's preferences over beer, B, and hockey games, H. Each Saturday, Claude watches four hockey games. Draw and appropriately label his indifference curve.

Section B: Three question @ 15 marks- 5 for each part of each question. Total 45 marks.

1. Kate consumes two goods, X and Y, according to the utility function $U(X,Y)=X^{1/2}\ Y$. Kate has an income, I, and faces prices for the two goods given by P_X and P_Y.
   a) [5 marks] Derive Kate's demand functions for the goods X and Y.

b) [5 marks] Assume that Kate's income is \$2,000, the price of X is \$2 and the price of Y is \$4. Calculate her demand for each good. What is the elasticity of demand for Y at this bundle?

c) [5 marks]Suppose the price of X increases to \$4. Determine the new demand for the goods and calculate the income and substitution effects of the price change.

2. Al has 126 hours per week to divide between leisure, R, and work. When he works, Al earns $40 per hour. He values both leisure and consumption, C, according to the utility function U(R,C)=Min{20R ; C}. The price of the consumption good is unity.

   a) [5marks] Derive Al's optimal bundle. How much does he work?

   b) [5 marks] Explain Al's allocation of time between work and leisure in terms of the arguments in his utility function. [Hint: think about how R and C contribute to his well-being and how work and C are related].

c) Starting from the solution to part (a), assume Al's income is taxed at the rate of 20%. Calculate Al's new optimal bundle and amount of work and show that he is worse off because of the tax.

3. [5 marks] Emily works in the present period and earns an income of $10,000,000. In the future period, Emily is retired and earns nothing. Her preferences over present consumption, C_P, and future consumption, C_F, are given by $U(C_P,C_F)=C_PC_F$. Emily's savings earn an interest rate of 100%.
   a) Derive Emily's optimal consumption bundle and her level of savings.

b) Suppose Emily learns that she will inherit $2,000,000 at the start of the future period and that she can borrow against it if she wishes. Draw Emily's original budget line and show how it is affected by the inheritance. Is it possible that for some utility function Emily might choose to borrow against this future income? Briefly explain.

c) Explain and illustrate how Emily's original budget line would change if the government taxed both her present earnings and the interest earned on her savings at the rate of 25%.

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SECTION A.

Elements of solution.

SECTION A

QUESTION #1.

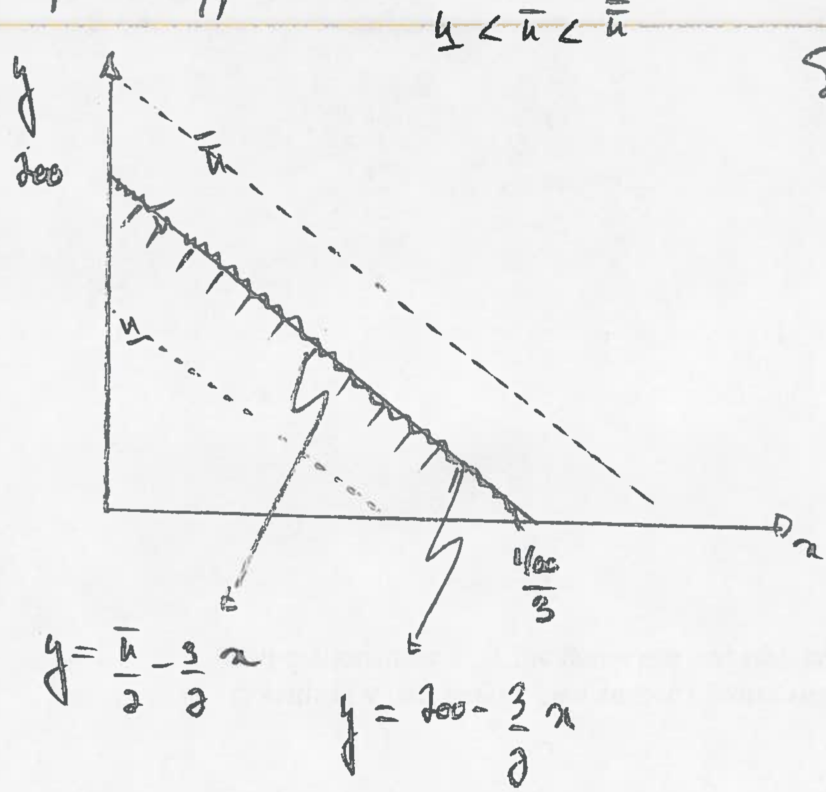
$u(x,y) = 3x + 2y$

$p_x = \$30;\ p_y = \$20;\ I = \$4000$

① Set of preferences : $\bar{u} = 3x + 2y \iff y = \frac{\bar{u}}{2} - \frac{3}{2}x$

② Budget Constraint : $p_x x + p_y y \leq I \iff y \leq 200 - \frac{3}{2}x$

③ Graphically,



Solution : All possible combination of x and y such that $y = 200 - \frac{3}{2}x$, $x \geq 0$, $y \geq 0$

Since slope of indiff. curves is the same as slope of budget constraint.

QUESTION #2.

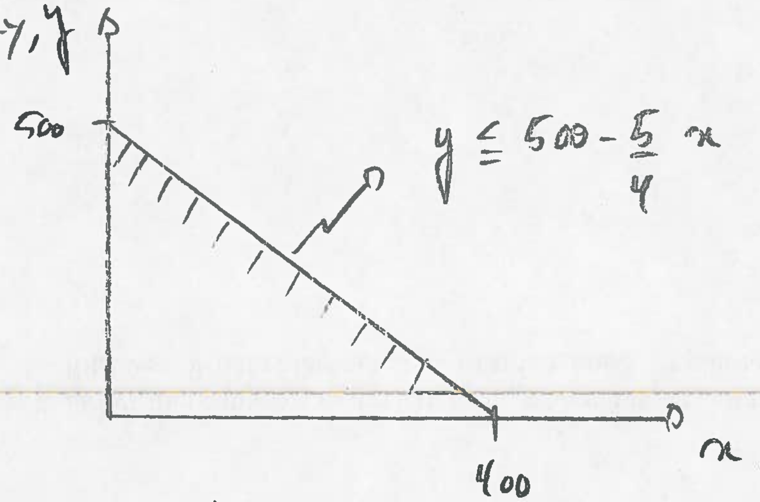
$I = \$2000$; $p_x = \$5$; $p_y = \$4$

Tax levied on units beyond the first 30 units of y ($t = \$2/\text{unit}$).

① Before introducing the tax

$$p_x x + p_y y \leq I \Rightarrow y \leq \frac{I}{p_y} - \frac{p_x}{p_y} x = 500 - \frac{5}{4} x$$

Graphically,



② New budget constraint

There is a tax of \$2/unit on y above 30 units. Said otherwise when $y \geq 30$ and $x \leq 400 - \frac{4}{5}(30) = 376$. The budget constraint in that region:

$$p_x x + (p_y + t)(y - 30) + 30 p_y \leq I$$

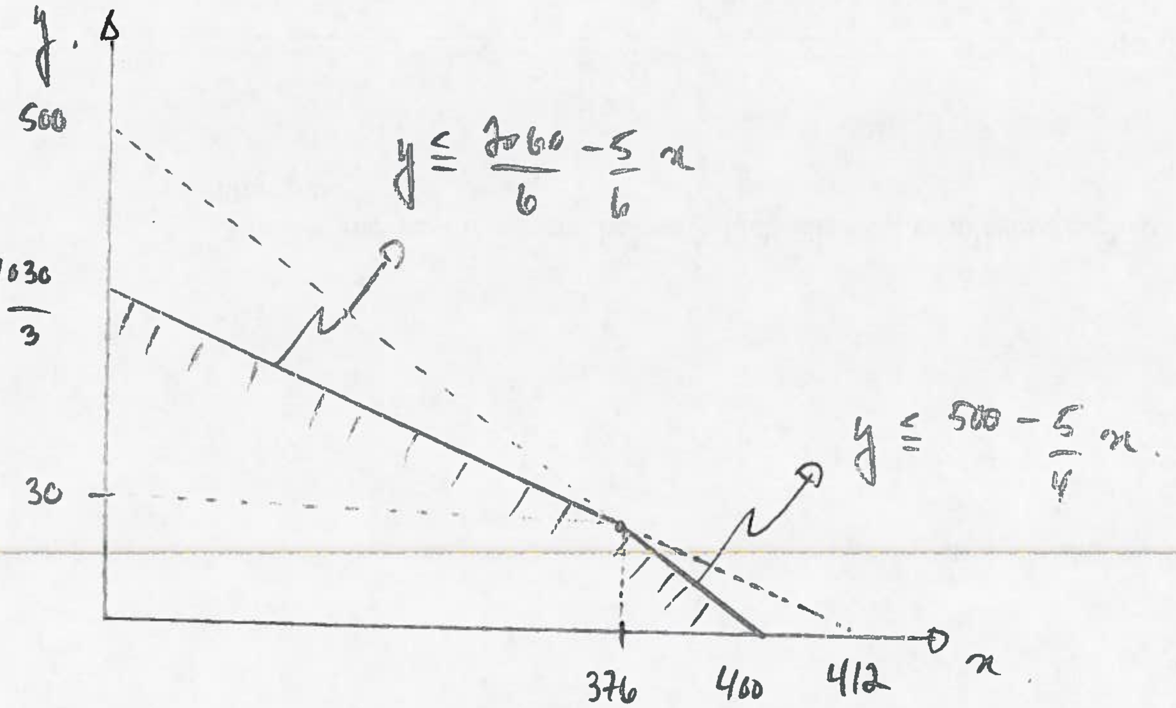
$$\Leftrightarrow p_x x + (p_y + t) y \leq I + 30t.$$

$$\Leftrightarrow y \leq \frac{I + 30t}{p_y + t} - \frac{p_x}{p_y + t} x = \frac{2060}{6} - \frac{5}{6} x = \text{[illegible]} - x.$$

Now, for the region $y \leq 30$ and $x \geq 376$, no tax levied, hence the budget constraint is

$$p_x x + p_y y \leq I \iff y \leq 500 - \frac{5}{4}x$$

Graphically,



QUESTION #3.

2 bottles of beer during each hockey game

each Sunday he watches 4 hockey game

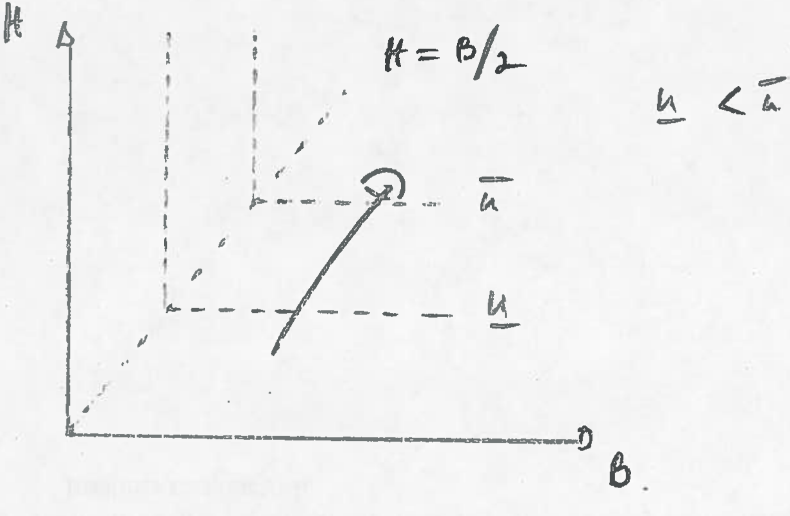
$\Rightarrow$ Each sunday

- 8 bottles of beer (2B for one game)
- 4 hockey games.

$\Rightarrow$ Perfect complement as beers and hockey games go hand to hand together

$u(B, H) = 4 \min\{B/2 \,;\, H\}$.

Graphically,



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SECTION B.

QUESTION #1

$$u(x,y) = x^{1/2} y \quad ; I ; p_x ; p_y$$

(a) $\max\limits_{x,y \geq 0} u(x,y)$ s.t. $p_x x + p_y y \leq I$

(i) Cobb-Douglas utility function: optimality condition $MRS_{x,y} = \frac{p_x}{p_y}$

(ii) $MRS_{x,y} = \frac{\partial u(\cdot)/\partial x}{\partial u(\cdot)/\partial y} = \frac{\frac{y}{2x^{1/2}}}{x^{1/2}} = \frac{y}{2x}$

(iii) At optimality: $MRS_{x,y} = p_x/p_y \iff y = 2x\frac{p_x}{p_y}$

(iv) $y = 2x\, p_x/p_y$ in budget constraint (at equality)

$$p_x x + p_y \left[2x \frac{p_x}{p_y} \right] = I$$

$$\iff x^* = \frac{I}{3p_x} \quad \text{and} \quad y^* = \frac{2I}{3p_y}$$

✓

5/.

(b) $I = \$2000$; $p_x = \$2$; $p_y = \$4$

(i) For demand ... plug in the values.

$$x^I = \frac{I}{3p_x} = \frac{2000}{3(2)} = \frac{1000}{3}$$

$$y^I = \frac{2I}{3p_y} = \frac{4000}{3(4)} = \frac{1000}{3}$$

(ii) Elasticity of demand at (x^I, y^I)

$$\eta - \frac{dy}{dp_y} \cdot \frac{p_y}{y^I} = \frac{-2I}{3p_y^2} \cdot \frac{p_y}{y^I}$$

$$= \frac{-2I}{3p_y y^I}$$

$$= \frac{-2(2000)}{3(4)\left(\frac{1000}{3}\right)} = -1$$

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(c) $p_x = \$4$ from $p_x = \$2$

Final basket: $x^F = \dfrac{I}{3p_x'} = \dfrac{2000}{3(4)} = \dfrac{500}{3}$

$$y^F = \frac{2I}{3p_Y} = \frac{1000}{3}$$

(i) Initial basket: $(x^I, y^I) = (1000/3, 1000/3)$

(ii) Final basket: $(x^F, y^F) = (500/3, 1000/3)$.

(iii) Decomposition basket: (x^D, y^D)

- same utility as initial basket: $u(x^I, y^I) = \bar{u}^I = \left(\frac{1000}{3}\right)^{1/2}\left(\frac{1000}{3}\right)$
- tangency new price and initial utility level:

$$MRS_{x,y} = \frac{p_x'}{p_Y} \Leftrightarrow y = \frac{2p_x' x}{p_Y}$$

$$\Rightarrow u(x, y(x)) = \bar{u}^I$$

$$\Leftrightarrow x^{1/2}\left[\frac{2p_x' x}{p_Y}\right] = \bar{u}^I$$

$$\Leftrightarrow x^D = \left[\frac{p_Y \bar{u}^I}{2p_x'}\right]^{2/3}$$

(iv) Income effect $= x^F - x^D$

Substitution effect $= x^D - x^I$

QUESTION # 2.

126 hours b/w R and L (work)

Wage $\equiv w = \$40/h$

$u(R,C) = \min\{20R, C\}$.

(a) (1) $C \leq 40(126-R) = 5040-40R$

$\Rightarrow C + 40R \leq 5040$ (budget constraint). $\Leftarrow C \leq 5040-40R$

(2) With perfect complement: solution is on the ray $20R = C$

(3) Plugging $20R = C$ in budget constraint (at equality)

$C + 40R = 5040$

$\Rightarrow R^* = \frac{5040}{60} = 84 \rightarrow L^* = 126 - 84 = 42$ hours.

(4) $C^* = 20(84) = 1680 = u^*$

(b). $u(R,C) = \min\{20R, C\}$.

Perfect complements goods are goods the consumer always wants in fixed proportion to each other, thus $\Delta R = 1 \Rightarrow \Delta C = 20$.

Using the BC. $C = 40(126-R)$, this implies that by utility fct; we have $20R = 40(\underbrace{126-R}_{L}) \rightarrow \Delta R = 1 \Rightarrow \Delta L = +\frac{1}{2}$

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ⓒ In ⓐ $R^* = 84$; $L^* = 42$

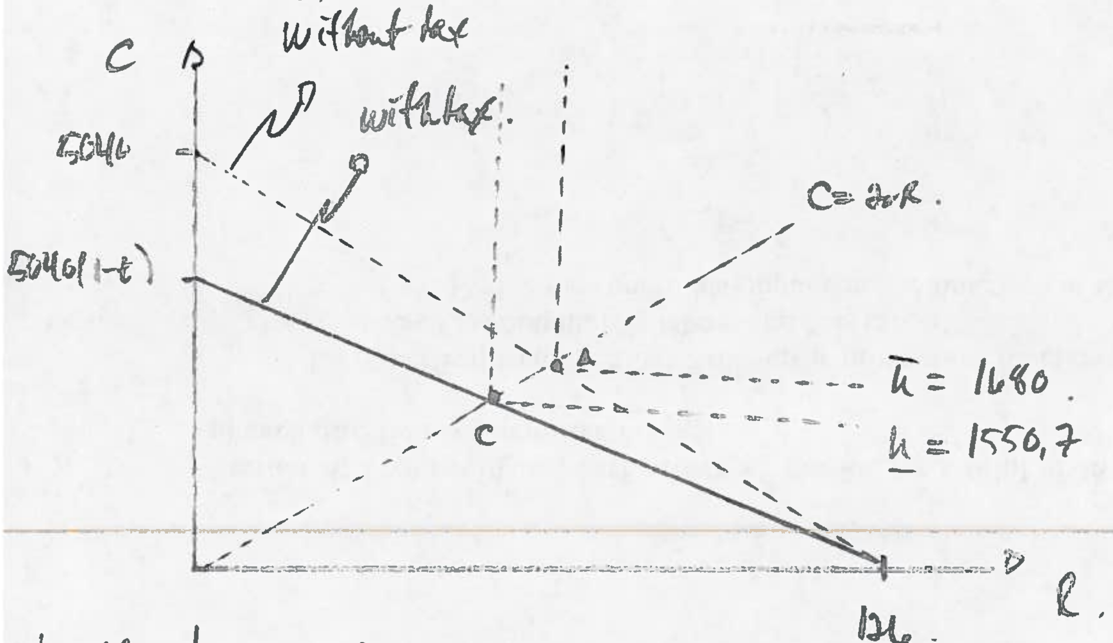
$C^* = 1680 = u^*$

New regulation: income is taxed at the rate $80\% = t$

$\Rightarrow C \leq 40(1-t)(126-R)$.

$\Leftrightarrow C + 40R(1-t) \leq 5040(1-t) \Leftrightarrow C \leq 5040(1-t) - 40R(1-t)$.

Graphically,



With tax: ① $C = 20R$.

② $C + 40R(1-t) = 5040(1-t)$

$\Leftrightarrow 20R + 40R(1-t) = 5040(1-t)$.

$\Leftrightarrow R^* = \dfrac{5040(1-t)}{60-40t} \cong 77,5 \Rightarrow L^* = 48,5$.

$\Rightarrow C^* \cong 1550,7 \cong u^*$

And $u_A = 1680 > u_C = 1550,7$.

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QUESTION #3

$I_p = \$10{,}000{,}000$; $I_F = \$0$

$u(C_P, C_F) = C_P C_F$

$r = 100\%$

(a) ① Present period B.C : $C_P + S \leq I_P$

② Future period B.C : $C_F \leq S(1+r)$

③ ? Intertemporal B.C. ?

From future period B.C : $S = \frac{C_F}{1+r}$

④ $S = \frac{C_F}{1+r}$ in present period B.C. (present time version intertemporal B.C)

$C_P + \frac{C_F}{1+r} \leq I_P \iff C_F \leq I_P(1+r) - (1+r)C_P$

⑤ Solving : Cobb-Douglas. at optimality $MRS_{P,F} = \frac{P_1}{P_2} = \frac{1}{\frac{1}{1+r}} = 1+r$

⑥ $MRS_{P,F} = \frac{\partial u(\cdot)/\partial C_P}{\partial u(\cdot)/\partial C_F} = \frac{C_F}{C_P}$

⑦ At optimality : $MRS_{P,F} = 1+r \iff C_F = C_P(1+r)$

⑧ $C_F = C_P(1+r)$ in intertemporal budget constraint :

$C_P + \frac{C_F}{1+r} = I_P \iff C_P^* = \frac{I_P}{2} = \$5M \Rightarrow C_F^* = (1+r)\frac{I_P}{2} = \$10M$

$\Rightarrow S^* = \frac{C_F^*}{1+r} = \$5M$

10/

(b) $I_F = \$ 3M$.

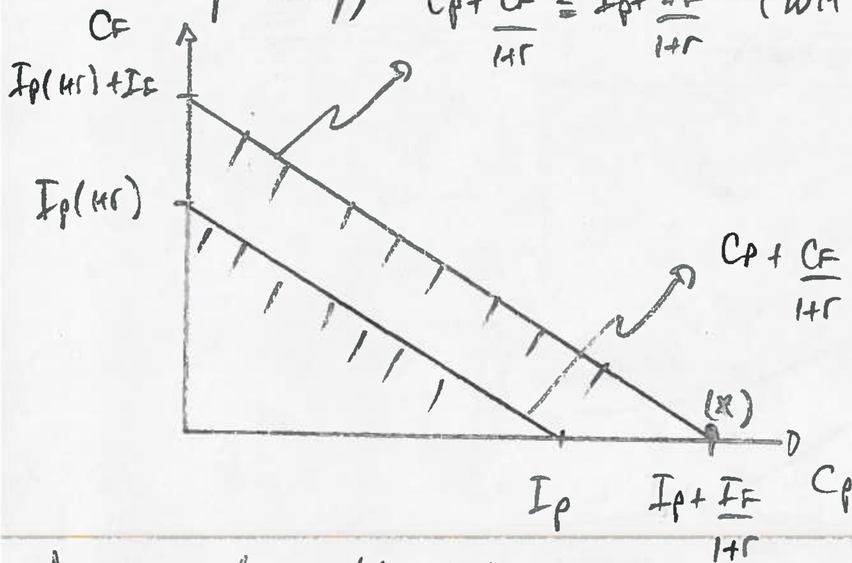
(1) Present period B.C.: $C_p + S \leq I_p$

(2) Future period B.C.: $C_F \leq I_F + S(1+r)$.

(3) Intertemporal budget constraint:

$$C_p + \frac{C_F}{1+r} \leq I_p + \frac{I_F}{1+r} \iff C_F \leq I_p(1+r) + I_F - (1+r)C_p.$$

Graphically, $C_p + \frac{C_F}{1+r} \leq I_p + \frac{I_F}{1+r}$ (with inheritance)



$C_p + \frac{C_F}{1+r} \leq I_p$. (without inheritance).

Note that the two curves are parallel (same slope only higher wealth with inheritance)

Now, is it possible that for some utility fct Emily might borrow against this future income? YES

An example is Emily consuming in present period and nothing in future period (*).

As we did in Quiz#2, set $u(C_p, C_F) = C_p C_F + A C_p$ (Quasi-linear utility function)

$$MRS_{P,F}\Big|_{(I_p + \frac{I_F}{1+r}, 0)} = \frac{C_F + A}{C_p}\Big|_{(I_p + \frac{I_F}{1+r}, 0)} = \frac{A}{I_p + \frac{I_F}{1+r}}$$

Finally if $MRS_{P,F}\Big|_{(I_p + \frac{I_F}{1+r}, 0)} > 1+r$, that is $A > I_p(1+r) + I_F$, we get that Emily consume everything in Present period, therefore we can say that she is borrowing against the inheritance. (next page) //

In fact, note that in this example

$$S = \frac{C_F - I_F}{1+r} = \frac{0 - I_F}{1+r} = \frac{-I_F}{1+r}.$$

Note that, the important part of the "borrowing against inheritance" is to mention that quasi-linear utility function can get you the point $\left(I_P + \frac{I_F}{1+r}, 0\right)$.

Note that, we can obtain the same result using Perfect Substitute utility function: $u(C_P, C_F) = AC_P + BC_F = \bar{u}$

$$\Leftrightarrow C_F = \frac{\bar{u}}{B} - \frac{A}{B} C_P.$$

then if $-\frac{A}{B} < -(1+r)$, we get desired result, that is the optimality of the point $\left(I_P + \frac{I_F}{1+r}, 0\right)$.

ⓒ Gvt taxes present earnings and interest earned on saving at 25% (compared to original budget line), $t = 0.25$

① Present period budget constraint:

$$C_p + S \leqq I_p(1-t)$$

② Future period budget constraint:

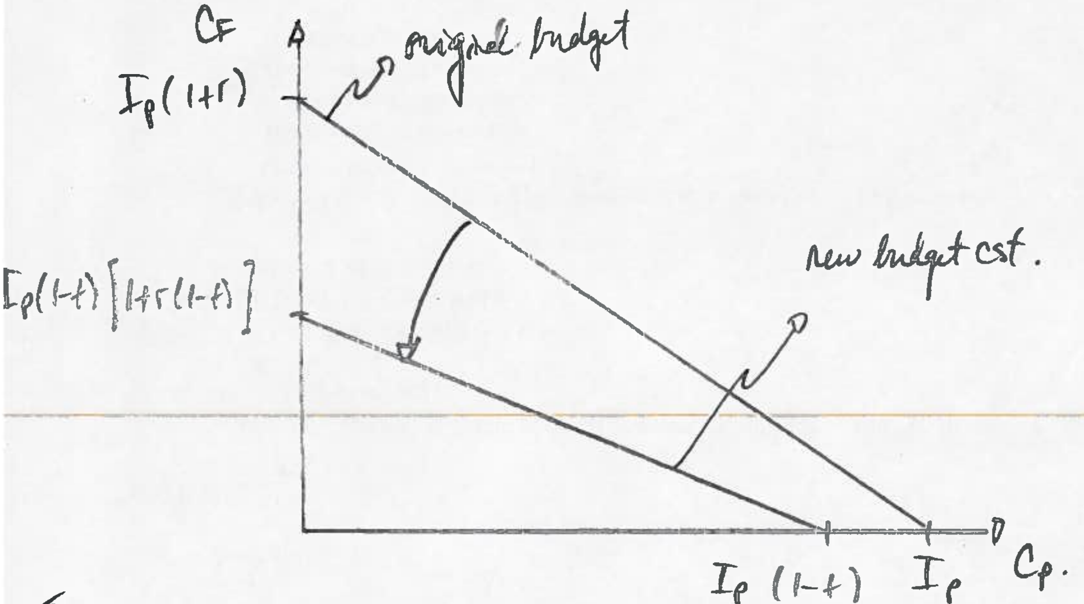
$$C_F \leqq S\left[1 + r(1-t)\right]$$

Intertemporal budget cst (present version).

$$C_p + \frac{C_F}{1+r(1-t)} \leqq I_p(1-t).$$

$$\Leftrightarrow C_F \leqq I_p(1-t)\left[1+r(1-t)\right] - \left[1+r(1-t)\right]C_p$$

Graphically,



Two effects: - Wealth effect: less revenue because of earning tax $I_p(1-t) < I_p$.

- Substitution effect: future consumption is now more costly because $\frac{1}{1+r(1-t)} > \frac{1}{1+r}$, (higher cost of saving)