Economics 212

Section 002

Midterm Exam

March 7, 2017

Student Number: Answers

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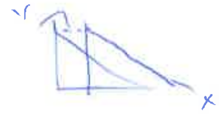

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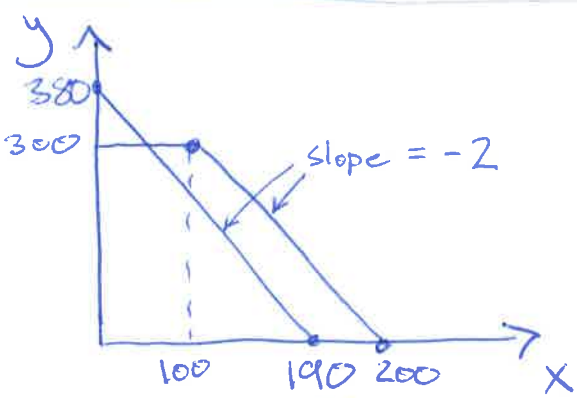
Section A: Three questions @ 5 marks. Total 15 marks.

1. [5 marks] A consumer consumes a composite good, Y, and food, X. The government can offer an income subsidy of $80 to be spent on either good, or a food voucher of $100 to be spent on food only. Suppose the consumer's original income is $300, the price of food is $2, and the price of the composite good is $1. Plot the budget line under the income subsidy program and the voucher program using a single graph. Please label your diagrams appropriately.

$y + 2X = 300$ with no program

$\Rightarrow y + 2X = 380$ with income subsidy

$y = -2x + 380$



with voucher

$y + 2x = 400$ for $x \geq 100$

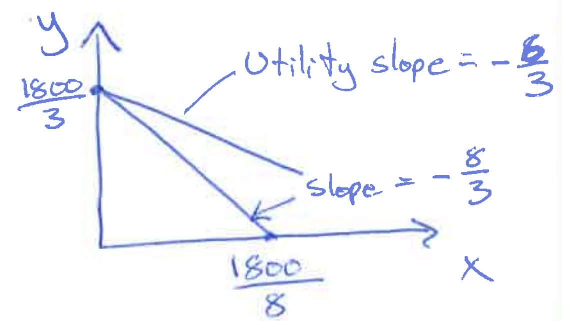
$y = 300$ for $x < 100$

2. [5marks] A consumer must decide whether to risk $100 buying a lottery ticket. The lottery has three outcomes: a probability of .5 that the consumer finishes the lottery with $16, a probability of .4 that the consumer finishes with $100, and a probability of .1 that the consumer finishes with $324. Explain whether the consumer will accept or reject the lottery given a utility of income function $U=I^{1/2}$. Now determine how the largest prize ($324) must be changed such that the consumer is indifferent between rejecting and accepting the lottery.

3. [5marks]Given the utility function U=6X+3Y and parameter values where P_X=8, P_Y=3 and income, I=$1800, determine the consumer's optimal bundle of goods X and Y.

will be a corner solution

$$\frac{MU_x}{MU_y} = \frac{6}{3} < \frac{8}{3} = \frac{P_x}{P_y}$$



$$\Rightarrow y^* = \frac{1800}{3} = 600, \; x^* = 0$$

could also check U of each bundle

Section B: Three question @ 15 marks- 5 for each part of each question. Total 45 marks.

1. Kai consumes two goods, X and Y, according to the utility function $U=X^{1/3}Y$. Kai has an income, I, and faces prices for the two goods given by P_X and P_Y.
   a) [5 marks] Derive Kai's demand functions for the goods X and Y.

$$I = P_x \cdot X + P_y \cdot y \quad \& \quad \frac{MU_x}{MU_y} = \frac{P_x}{P_y}$$

$$\frac{\frac{1}{3} x^{-\frac{2}{3}} y}{x^{1/3}} = \frac{P_x}{P_y}$$

$$y = 3\frac{P_x}{P_y} X$$

Plug in

$$I = P_x \cdot X + P_y\left(3\frac{P_x}{P_y} y\right)$$

$$I = 4 P_x X$$

$$\therefore \boxed{x^* = \frac{I}{4P_x}}$$

$$\therefore \boxed{y^* = 3\frac{P_x}{P_y} x^* = \frac{3}{4}\frac{I}{P_y}}$$

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b) [5 marks] Let Kai's income be \$1600, the price of X be \$1 and the price of Y be \$3. How much of each good should Kai consume?

Plug values into demand functions

$$x^* = \frac{1600}{4} = 400 \quad \& \quad y^* = \frac{3}{4}\,\frac{1600}{3} = 400$$

c) [5 marks]Suppose the price of X increases to \$2. Determine the new demand for the goods and calculate the income and substitution effects of the price change.

new demands: $\tilde{x}^* = \frac{1600}{4 \cdot 2} = 200 \qquad \tilde{y}^* = \frac{3}{4}\,\frac{1600}{3} = 400$

old utility is $\bar{U} = 400^{1/3} \cdot 400 = 2947.23$

decomposition bundle solves $\tilde{x}^{1/3}\tilde{y} = \bar{U}$ & $\frac{MU_{\tilde{x}}}{MU_{\tilde{y}}} = \frac{2}{3}$

$\Rightarrow \tilde{x}^{1/3}\tilde{y} = \bar{U}$ & $\tilde{y} = 2\tilde{x} \Rightarrow \tilde{x} = (\frac{1}{2}\bar{U})^{3/4} = 237.84$

$\tilde{y} = 2 \cdot 237.84 = 475.68$

Sub. effect: $\Delta x^s = \tilde{x} - x^* = 237.84 - 400 = -162.16$

$\Delta y^s = \tilde{y} - y^* = 475.68 - 400 = 75.68$

Income effect:

$\Delta x^I = \tilde{x}^* - \tilde{x} = 200 - 237.84 = -37.84$

$\Delta y^I = \tilde{y}^* - \tilde{y} = 400 - 475.68 = -75.68$

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2. Martha has 120 hours per week to divide between leisure, R, and work, L. When she works, Martha earns $10 per hour. She values both leisure and consumption, C, according to the utility function U=Min{10R ; C}. The price of the consumption good is unity.
   a) [5marks] Derive Martha's optimal bundle. How much does she work? Calculate her level of utility.

(2) $10R = C$ & $w120 = C + wR$

$120 = \frac{1}{w} * C + R$

(plug in)

$120 = \left(\frac{1}{10}\right)(10R) + R \Rightarrow \boxed{R^* = \frac{120}{2} = 60}$ (1)

(1) $\Rightarrow \boxed{C^* = 10R^* = 600}$

$\boxed{U = \min\{10 \cdot 60, 600\} = 600}$ (1)

$L = 120 - R^* = 60$

   b) [5 marks] The government decides that Martha's wage is too low and offers a subsidy at the rate of $5 per hour on Martha's earnings (assume her wage rises to $15/hour). Find Martha's new optimal bundle, including the amount of work and leisure chosen. In words, explain this outcome in terms of the income and substitution effects of the tax.

now $120 = \left(\frac{1}{15}\right)(10R) + R \Rightarrow R^* = 120 \cdot \frac{3}{5} = 72$ (1.5)

and $C^* = 10R^* = 720$ $L = 120 - 72 = 48$ (0.5)

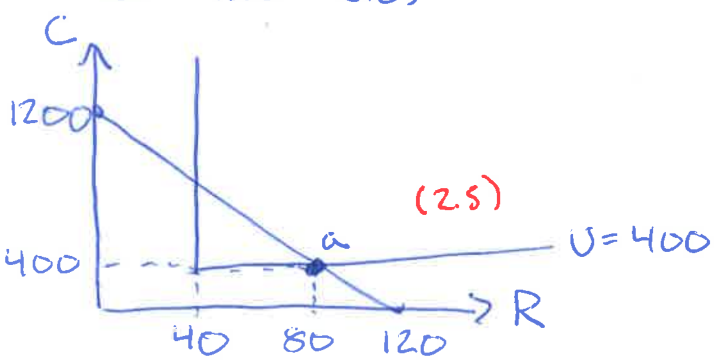
with a higher wage, the income effect (1) causes her to consume more of both goods, but the relative price of consumption $\left(\frac{1}{w}\right)$ decreased encouraging her to (1) substitute into more (1) consumption. Here the income effect dominates and she consumes more of both.

c) Starting from the solution to part (a), assume Martha's boss tells her that she must work 40 hours per week. Show that Martha is worse off. Starting from the situation where Martha works only 40 hours per week, show graphically that Martha would be willing to accept a second job at a wage lower than her current wage.

$R^* = 120-40 = 80$ (0.5) $\therefore C^* = (120-R^*)10 = 400$ (0.5)

$\Rightarrow U = \min(800, 400) = 400$ (1.5) so she is worse off

Plot $120 = (\frac{1}{10})C + R$



from her current bundle at point a, she would give up 40R for any small amount of C. $\therefore$, for any wage greater than zero she would choose to take a second job.

3. [5 marks] Kevin works in the present period and earns an income of $8,000,000. In the future period, Kevin is partly retired and earns $2,000,000. His preferences over present consumption, C_P, and future consumption, C_F, are given by $U(C_P,C_F)=C_P C_F$. Kevin can save or borrow at an interest rate of 100%.

$r=1$

a) Derive Kevin's optimal consumption bundle and his level of savings or borrowing.

$$C_P + \frac{C_F}{1+1} = 8\text{mill} + \frac{2\text{mill}}{1+1} = 9\text{mill} \quad (1) \quad (BC)$$

$$\frac{C_F}{C_P} = \frac{1}{\frac{1}{1+1}} = 2 \quad (2) \qquad \text{(optimal condition)}$$

$$C_P + \frac{1}{2}(2C_P) = 9\text{mill} \quad (1) \Rightarrow C_P^* = 4,500,000$$

$$\therefore C_F^* = 9,000,000$$

(1) Savings = 8mill − 4.5mill = 3,500,000

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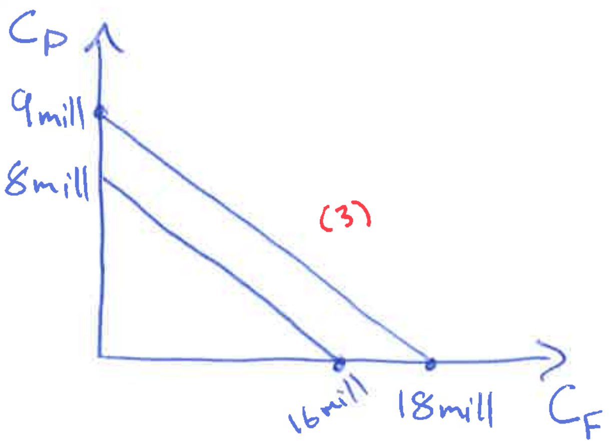
b) Draw and appropriately label Kevin's budget constraint from part a). On the same graph, draw Kevin's new budget line on the assumption that he fully retires in the future period and earns zero future income. If moving from the first budget constraint to the second represents a pure income effect, what should happen to Kevin's level of savings? Briefly explain.

(1)

Plot $C_P + \frac{1}{2}C_F = 9\text{mill}$ (original), & $C_P + \frac{1}{2}C_F = 8\text{mill}$ (new)



Kevin always wants to consume at a constant ratio $\left(\frac{C_F}{C_P} = 2\right)$. Thus, with no substitution he will adjust savings to keep this ratio constant. Now $C_P + \frac{1}{2}(2C_P) = 8\text{mill}$

$\therefore C_P^* = 4\text{mill}$, $\therefore$ Savings $= 4\text{mill}$. (1)

c) Write a perfect complements utility function that ensures Kevin is neither a saver nor a borrower. Explain why your utility function answers the question.

we want to find a utility of the form $U = \min\{xC_P, C_F\}$. The optimal condition will be $xC_P = C_F$, and no saving means $C_P = 8\text{mill}$.

(2)

Plug these into the BC:

(1)

$$8\text{mill} + \frac{x\,8\text{mill}}{1+1} = 9\text{mill} \Rightarrow x = \frac{1\cancel{9}}{4}$$

(2)

$$\therefore U = \min\left\{\frac{1\cancel{9}}{4}C_P, C_F\right\}$$