Economics 212

Section 001

Midterm Exam

October 23, 2017

Student Number: Answers

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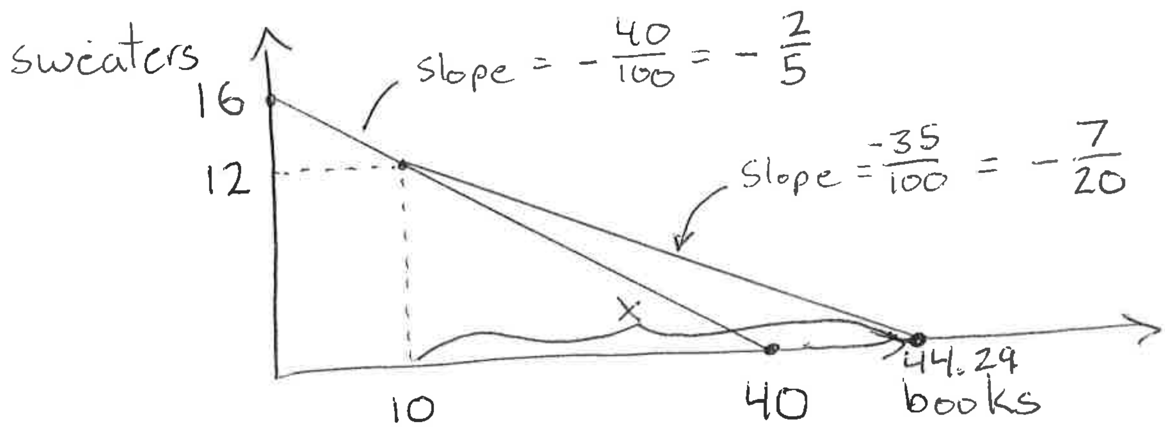
Section A: Three questions @ 5 marks. Total 15 marks.

1. [5 marks] Art has $1600 to spend on two goods, books, B, and sweaters, S. The price of a book is $40 and the price of a sweater is $100. Draw and appropriately label Art's budget constraint. The government has decided to encourage reading and offers a subsidy of $5 per unit on the price of a book, but only after Art buys 10 books. Draw and appropriately label Art's new budget constraint.

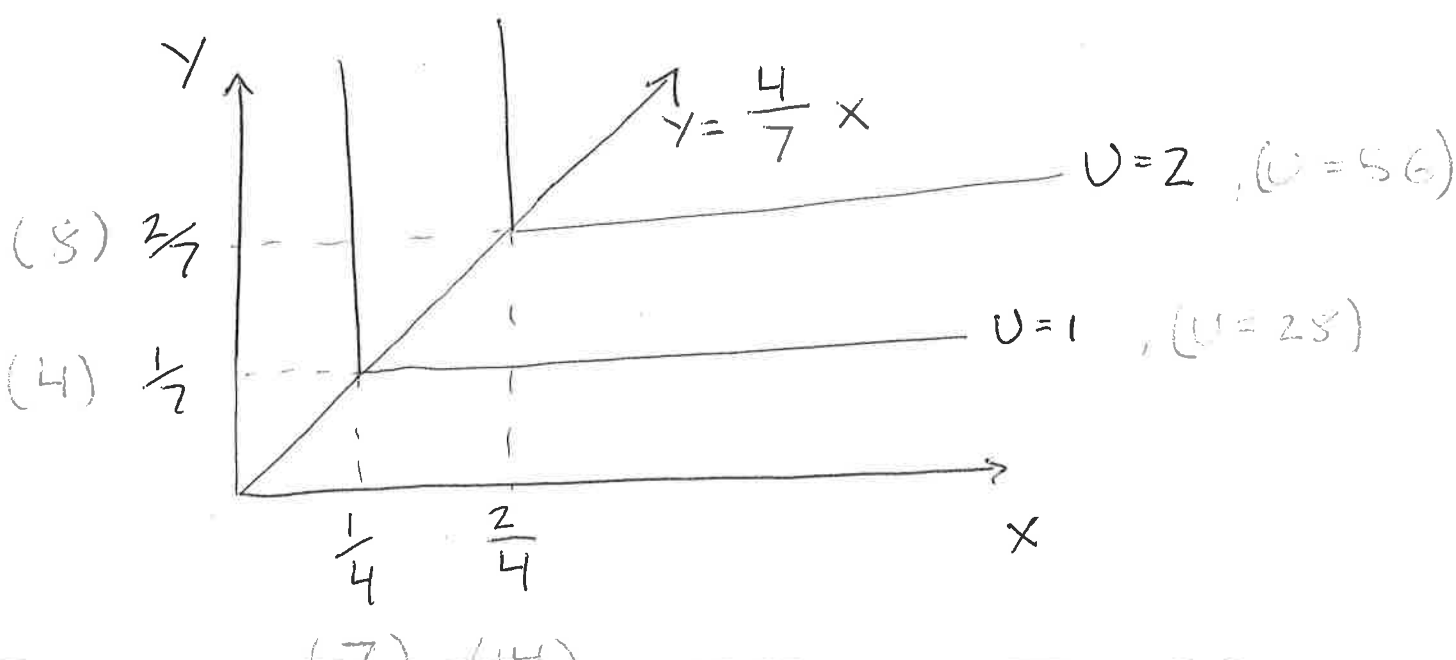


$$100S + 40B = 1600$$

$$\frac{12}{x} = \frac{7}{20} \Rightarrow x = (12)\frac{20}{7}$$

$$x = 34.29$$

2. [5marks]Consider the utility function U=Min{4X;7Y}. Draw and appropriately label two isoquants associated with this utility function.



3. [5marks]You are informed that an individual's demand function for the good X can be written as $X^D=2000/P_X$. The market consists of 4,000 identical individuals. Derive the market demand function for the good X. The market supply function for the good is given by $X^S=2P_X$. Find the equilibrium price of the good, the quantity purchased in the market and the quantity purchased by each individual.

mrkt demand: $X_M^D = \left(\frac{2{,}000}{P_X}\right)4{,}000 = \frac{8{,}000{,}000}{P_X}$

equilibrium: $\frac{8 \text{ mill}}{P_X} = 2P_X \Rightarrow P_X = \sqrt{4 \text{ mill}} = 2{,}000$

$\therefore X_M^D = \frac{8 \text{ mill}}{P_X} = \frac{8{,}000{,}000}{2{,}000} = 4{,}000$ ← mrkt demand

$$\boxed{X^D_{\text{individual}} = \frac{2{,}000}{P_X} = \frac{2{,}000}{2{,}000} = 1}$$

Section B: Three question @ 15 marks- 5 for each part of each question. Total 45 marks.

1. Anna consumes two goods, X and Y, according to the utility function $U(X,Y)=XY^{1/2}$. Anna has an income, I, and faces prices for the two goods given by P_X and P_Y.
   a) [5 marks] Derive Anna's demand functions for the goods X and Y.

$I = P_x \cdot X + P_y \cdot y$ & $\frac{MU_x}{MU_y} = \frac{P_x}{P_y}$

$\frac{y^{1/2}}{\frac{1}{2}xy^{-1/2}} = \frac{P_x}{P_y}$

$2\frac{y}{x} = \frac{P_x}{P_y}$

$y = \frac{1}{2}\frac{P_x}{P_y}x$

plug into BC

$I = P_x \cdot X + \cancel{P_y}\left(\frac{1}{2}\frac{P_x}{\cancel{P_y}}x\right)$

$I = \frac{3}{2}P_x \cdot X$

$$\therefore \boxed{x^* = \frac{2}{3}\frac{I}{P_x}}$$

$\therefore y^* = \frac{1}{2}\frac{P_x}{P_y}\left(\frac{2}{3}\frac{I}{P_x}\right)$

$$\boxed{y^* = \frac{1}{3}\frac{I}{P_y}}$$

b) [5 marks] Let Anna's income be \$1500, the price of X be \$5 and the price of Y be \$10. How much of each good should Anna consume?

Plug into demand functions

$x^* = \left(\frac{1500}{5}\right)\left(\frac{2}{3}\right) = 200 \quad y^* = \left(\frac{1500}{10}\right)\left(\frac{1}{3}\right) = 50$

c) [5 marks]Suppose the price of X increases to \$10. Determine Anna's new demand for the goods and calculate the income and substitution effects of the price change.

new demands: $\tilde{x}^* = \frac{1500}{10}\frac{2}{3} = 100 \quad \tilde{y}^* = \left(\frac{1500}{10}\right)\left(\frac{1}{3}\right) = 50$

old utility is $\bar{U} = 200 \cdot 50^{1/2} = 1414.21$

decomposition bundle solves $\tilde{x}\tilde{y}^{1/2} = \bar{U}$ & $\frac{MU_{\tilde{x}}}{MU_{\tilde{y}}} = \frac{10}{10}$

$\Rightarrow \tilde{x} = \frac{\bar{U}}{\tilde{y}^{1/2}}$ & $\tilde{y} = \frac{1}{2}\tilde{x} \Rightarrow \tilde{x} = \frac{\bar{U}}{\frac{1}{\sqrt{2}}\sqrt{\tilde{x}}}$

$\tilde{x}^{3/2} = \sqrt{2} \cdot \bar{U}$

$\tilde{x} = (\sqrt{2} \cdot \bar{U})^{2/3} = 158.74$

$\therefore \tilde{y} = \frac{1}{2}\tilde{x} = 79.37$

Sub. effect:

$\Delta x^S = \tilde{x} - x^* = 158.74 - 200 = -41.26$

$\Delta y^S = \tilde{y} - y^* = 79.37 - 50 = 29.37$

Income effect:

$\Delta x^I = \tilde{x}^* - \tilde{x} = 100 - 158.74 = -58.74$

$\Delta y^I = \tilde{y}^* - \tilde{y} = 50 - 72.37 = 29.37$

2. Nouriel has 120 hours per week to divide between leisure, R, and work. When he works, Nouriel earns \$20 per hour. He values both leisure and consumption, C, according to the utility function $U(C,R)=C^{1/2}R^{1/2}$. The price of the consumption good is unity.

   a) [5marks] Derive Nouriel's optimal bundle. How much does he work?

$$BC: 120 = \left(\frac{1}{20}\right)C + R$$

optimality condition: $\dfrac{\frac{1}{2}C^{-1/2}R^{1/2}}{\frac{1}{2}C^{1/2}R^{-1/2}} = \dfrac{\frac{1}{20}}{1} \Rightarrow \dfrac{R}{C} = \dfrac{1}{20}$

$\Rightarrow C = 20R$ — Plug into BC

$120 = \left(\frac{1}{20}\right)(20R) + R \Rightarrow \boxed{R^* = 60}$

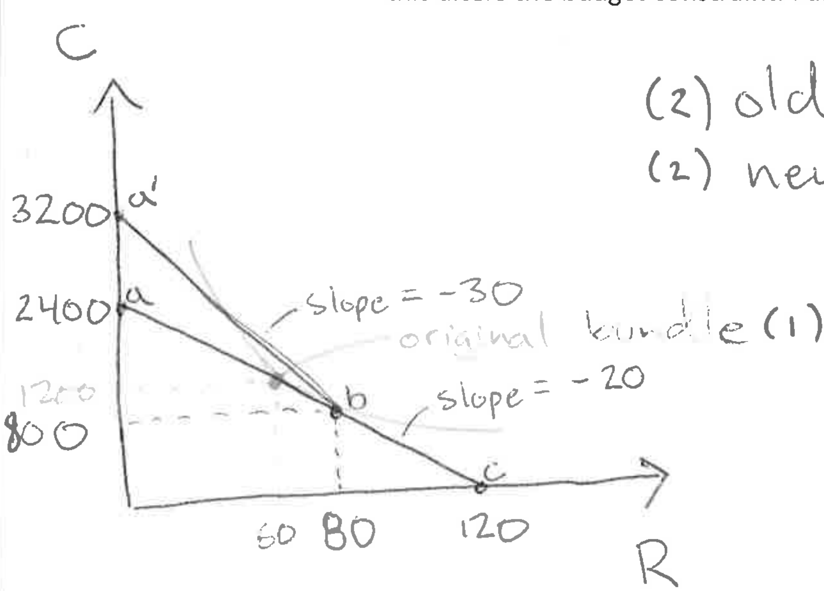
$\therefore \boxed{C^* = 20R^* = 1200}$

$L = 120 - R^* = 60$

   b) [5 marks]Draw and appropriately label Nouriel's budget constraint. Show the optimal bundle as determined in part a). Now assume that Nouriel receives an overtime wage equal to \$30 per hour for all hours worked beyond 40. Illustrate on your diagram how this alters the budget constraint. Fully label the new budget constraint.

(2) old BC is line ac

(2) new BC is a'bc



c) Given the overtime wage, will Nouriel choose to work more or less? Explain your reasoning.

when w↑, the cost of C in terms of R falls causing Nouriel to substitute into more (i) C. But her income increases causing her to consume (i) more of both.
In this case she will work more:

new BC: $30R + C = 3200$ if $R < 80$ (3)
$20R + C = 2400$ if $R \geq 80$

new optimality condition: $C = 30R$ if $R < 80$
$C = 20R$ if $R \geq 80$

$C^* = \frac{3200}{2} = 1600$
$R^* = \frac{1600}{30} = 53.3$
"need to check $R^* < 80$"

3. [5 marks] Emily works in the present period and earns an income of $5,000,000. In the future period, Emily works part-time and earns $2,000,000. Her preferences over present consumption, C_P, and future consumption, C_F, are given by $U(C_P,C_F)=Min\{4C_P;C_F\}$. Emily's savings and borrowing are done at an interest rate of 100%.

a) Derive Emily's optimal consumption bundle and her level of savings.

r=1

$$C_P + \frac{C_F}{1+1} = 5\text{mill} + \frac{2\text{mill}}{1+1} = 6\text{mill} \quad \text{(BC)}$$

$$4C_P = C_F \quad \text{(optimal condition)}$$

$$C_P + \frac{1}{2}(4C_P) = 6\text{mill} \Rightarrow \boxed{C_P^* = 2\text{mill}}$$

$$\therefore \boxed{C_F^* = 4C_P^* = 8\text{mill}}$$

Savings = 5mill − 2mill = 3mill

b) If I_F were to increase by \$2,000,00, Emily would choose to increase C_F by \$1,600,000 and C_P by \$400,000. With regard to the utility function, explain why this must be the way Emily divides the increase in future income.

She has a perfect complement utility function. ∴ she will always consume in the ratio $\frac{C_F}{C_P} = 4$. Any new income will be consumed at this ratio.

c) Using the information given in the introduction to this question, write a perfect complements utility function that would make Emily consume exactly all of her income in each period, i.e. Emily is neither a borrower nor a saver. Explain your answer.

we want to find a utility of the form $U = \min\{xC_P, C$
in this case we have the optimal condition $xC_P = C_F$
no borrowing means $C_P = 5$ mill. Plug these into the BC:

$$(5)_{mill} + \frac{(x\,5)_{mill}}{i+1} = 5\,mill + \frac{2\,mill}{1+1} \Rightarrow x = \frac{2}{5}$$

$$\therefore\ U = \min\left\{\frac{2}{5}C_P, C_F\right\}$$