Economics 212

Section 002

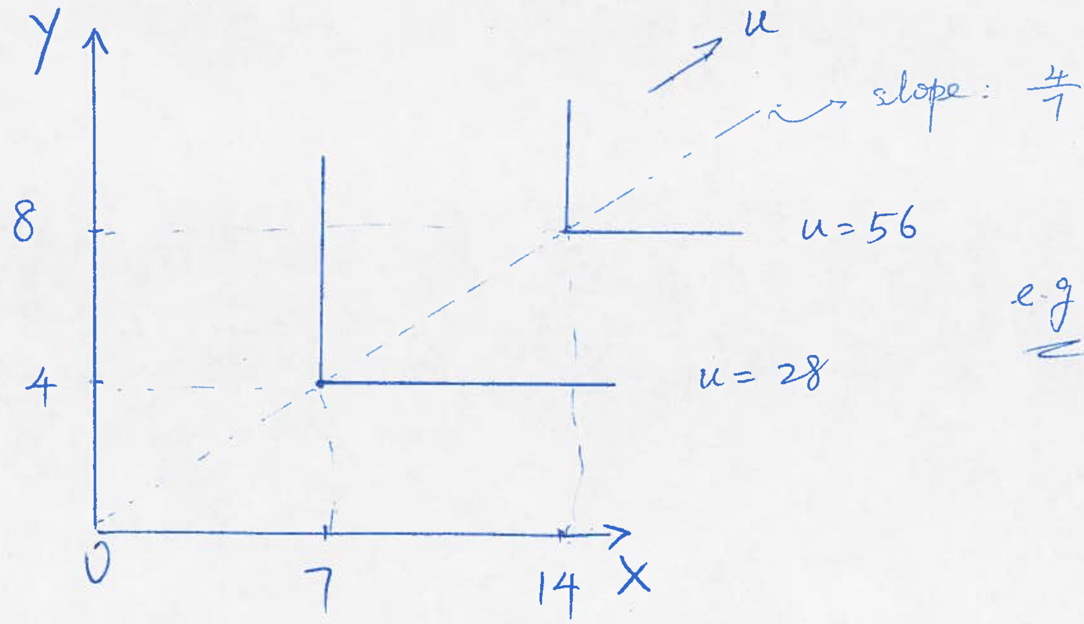
Midterm Exam

October 17, 2013

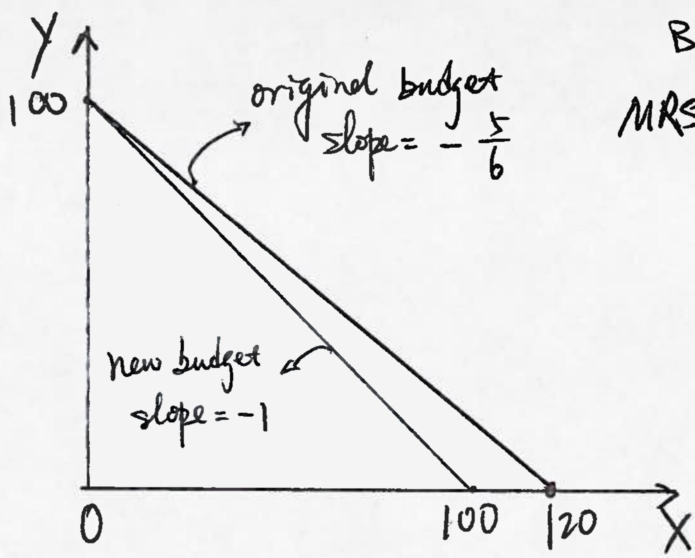
Student Number:

Section A: Three questions @ 5 marks. Total 15 marks.

1. [5 marks] Nouriel has a utility function defined over the goods X and Y that may be written as U(X,Y) = Min {4X ; 7Y}. Draw and appropriately label two indifference curves associated with Nouriel's preferences.



2. [5marks] Wei uses her income to purchase two goods, X and Y, with prices P_X=5 and P_Y=6. Wei's income, I, is \$600. Draw and appropriately label her budget constraint. If Wei's preferences are given by U=10X+10Y, determine her optimal bundle. If the government imposes a \$1 per unit tax on good X how does the optimal bundle change? Briefly explain.



B.C. $P_x \cdot X + P_y Y = I \Rightarrow 5X + 6Y = 600$

$MRS_{x,y} = \frac{MU_x}{MU_y} = \frac{10}{10} = 1$, $\frac{P_x}{P_y} = \frac{5}{6}$

$\frac{MU_x}{MU_y} > \frac{P_x}{P_y} \Rightarrow \frac{MU_x}{P_x} > \frac{MU_y}{P_y} \Rightarrow$

opt. to spend all on X $\Rightarrow$

$X^* = I/P_x = 120$, $Y^* = 0$.

with tax, P_x changes to $P_x^N = 6$. then $MRS_{x,y} = \frac{P_x^N}{P_y} = 1$.

all points alone the new budget line are opt. bundles .

3. [5marks] Al has 24 hours per day to divide between leisure, R, and work. When he works, Al earns \$30 per hour. He values both leisure and consumption, C, according to the utility function U(R,C)= $R^{1/2}C$. The price of the consumption good is unity. Derive Al's optimal bundle. How much does he work?

B.C.: $P_c \cdot C + P_R \cdot R = 24w$ with $P_c = 1$ $P_R = w = 30 \Rightarrow$

$c + 30R = 720$. and $P_R / P_c = w = 30$

$MRS_{R,C} = \frac{MU_R}{MU_c} = \frac{\frac{1}{2}R^{-1/2}C}{R^{1/2}} = \frac{c}{2R}$, opt at: $\frac{c}{2R} = 30 \Rightarrow$

$C = 60R$ substitute into B.C. $\Rightarrow 90R = 720 \Rightarrow R^* = 8$

$C^* = 60R^* = 480 \Rightarrow$ opt. bundle $(R^*, C^*) = (8, 480)$

Al works: $T - R = 24 - 8 = 16$ hrs

Section B: Three question @ 15 marks- 5 for each part of each question. Total 45 marks.

4. Katie consumes two goods, X and Y, according to the utility function U(X,Y)=X $Y^{1/2}$. Katie has an income, I, and faces prices for the two goods given by P_X and P_Y.
   a) [5 marks] Derive Katie's demand functions for the goods X and Y.

B.C.: $P_x X + P_y Y = I$

$MRS_{x,y} = \frac{MU_x}{MU_y} = \frac{Y^{1/2}}{\frac{1}{2}XY^{-1/2}} = \frac{2Y}{X}$, opt at: $\frac{2Y}{X} = \frac{P_x}{P_y} \Rightarrow$

$Y = \frac{X}{2}\left(\frac{P_x}{P_y}\right)$ substitute into B.C. $\Rightarrow$

$P_x \cdot X + P_y \cdot \frac{X}{2}\left(\frac{P_x}{P_y}\right) = I \Rightarrow X^* = \frac{2I}{3P_x}$

$\Rightarrow Y^* = \frac{X^*}{2}\left(\frac{P_x}{P_y}\right) = \frac{I}{3P_y}$

b) [5 marks] Assume that Katie's income is $1,200, the price of X is $6 and the price of Y is $3. Calculate her demand for each good. What is the elasticity of demand for Y at this bundle?

$$X^* = \frac{2I}{3P_x} = \frac{2400}{18} = \frac{400}{3} \approx 133.33$$

$$Y^* = \frac{I}{3P_Y} = \frac{1200}{9} = \frac{400}{3} \approx 133.33$$

$$\varepsilon_Y = \frac{\partial Y}{\partial P_Y} \cdot \left(\frac{P_Y}{Y}\right) = -\frac{I}{3P_Y^2} \cdot \left(\frac{P_Y}{\frac{I}{3P_Y}}\right) = -1$$

c) [5 marks]Suppose the price of X increases to $9. Determine the new demand for the goods and calculate the income and substitution effects of the price change.

New demands: $\begin{cases} X_N^* = \dfrac{2I}{3P_x^N} = \dfrac{2400}{27} = \dfrac{800}{9} \approx 88.89 \\ Y_N^* = \dfrac{I}{3P_Y} \approx 133.33 \end{cases}$

need original utility level to find decomposition bundle:

$U_0 = X^*(Y^*)^{1/2} = 133.33(\sqrt{133.33}) \approx 1539.60$

$MRS_{X,Y} = \frac{MU_X}{MU_Y} = \frac{2Y}{X}$ with new price ratio: $\frac{P_X}{P_Y} = \frac{9}{3} = 3$

the decomposition bundle must be $\frac{2Y}{X} = 3 \Rightarrow Y = \frac{3X}{2}$

to get original utility level: $U_0 = 1539.60 = X_D \cdot \left(\sqrt{\frac{3X_D}{2}}\right) = \sqrt{1.5} \cdot (X_D)^{3/2} \Rightarrow$

$X_D \approx 116.48$

$\Rightarrow$ substitution effect: $X_D - X^* = 116.48 - 133.33 = -16.85$

income effect: $X_N^* - X_D = 88.89 - 116.48 = -27.59$

note: negative sign means the price change causes the demand of good X decreases.

5. The market for bushels of squash is characterized by a demand function of the form $Q^D = 24000 - 100P$ and a supply function of the form $Q^S = 2400P - 1000$, where Q is quantity and P is price.

a) [5 marks] Calculate the equilibrium price and quantity in the market and the elasticity of demand at the equilibrium.

at eq^{m}: $Q^D = Q^S \Rightarrow 24000 - 100P = 2400P - 1000$

$$\Rightarrow \begin{cases} P^* = 10 \\ Q^* = 24000 - 100(P^*) = 23000 \end{cases}$$

$$\varepsilon_D = \frac{\partial Q^D}{\partial P}\left(\frac{P}{Q^D}\right) = -100\left(\frac{10}{23000}\right) = -\frac{1}{23} \approx -0.043$$

b) [5 marks] If market demand is the result of 100 identical individuals in the market, write the demand function for one individual consumer. How many bushels of squash would the consumer purchase at a price of twenty?

individual demand: $Q^d = \frac{Q^D}{100} = 240 - P$

with $P = 20$, $Q^d = 220$

c) Suppose that the demand for squash can be written more fully as $Q^D = 3I + 5P_P - 100P_S$, where I is income, P_P is the price of potatoes and P_S is the price of squash. Determine some positive values of I and P_P that make this extended demand function agree with the one provided in the introduction to this question. Given these correct values, calculate the cross price elasticity of squash and potatoes. Are the goods substitutes or complements?

- compare $Q^D = 3I + 5P_P - 100P_S$ with $Q^D = 24000 - 100P$
- 24000 captures the influence of I and P_P on demand for squash
- $\Rightarrow 3I + 5P_P = 24000$, to have positive I and P_P, must be:
- $I = \frac{1}{3}(24000 - 5P_P) > 0 \Rightarrow 0 < P_P < 4800$
- $P_P = \frac{1}{5}(24000 - 3I) > 0 \Rightarrow 0 < I < 8000$
- with $\frac{\partial Q^D}{\partial P_P} = 5 > 0 \Rightarrow \varepsilon_{Q^D, P_P} = \frac{\partial Q^D}{\partial P_P} \cdot \frac{P_P}{Q^D} > 0 \Rightarrow$

the goods are substitutes.

6. Emily works in the present period and earns an income of $10,000,000. In the future period, Emily is retired and earns nothing. Her preferences over present consumption, C_P, and future consumption, C_F, are given by $U(C_P, C_F) = \text{Min}\{C_P ; 3C_F\}$. Emily's savings earn an interest rate of 50%.

a) [5 marks] Derive Emily's optimal consumption bundle and her level of savings.

B.C. $C_P + \frac{C_F}{1+r} = I_P + \frac{I_F}{1+r}$ with $I_P = 10M$, $I_F = 0$, $r = 0.5$

$\Rightarrow C_P + \frac{C_F}{1.5} = 10M$

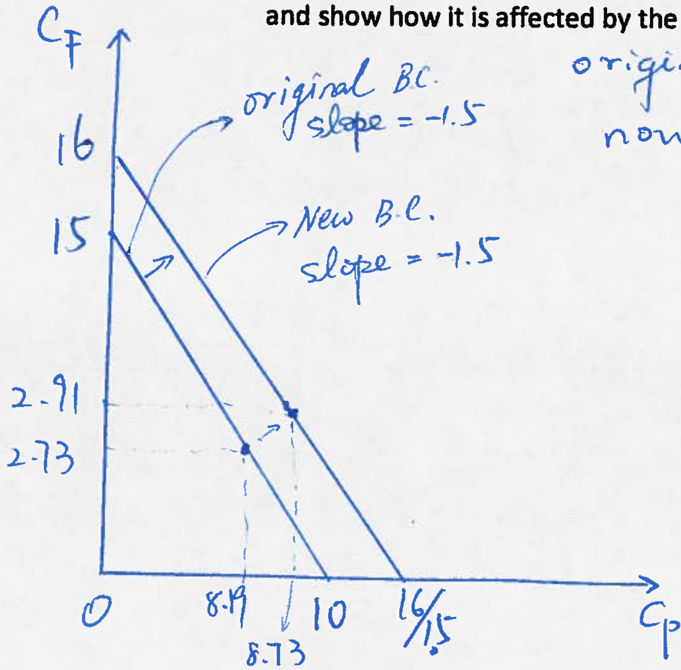
opt. at $C_P = 3C_F$ substitute into B.C. $\Rightarrow$

$3C_F + \frac{C_F}{1.5} = 10M \Rightarrow C_F^* \approx 2.73M$

$C_P^* = 3C_F^* = 8.19M$

saving $S = I_P - C_P^* = 1.81M$

b) [5 marks] Suppose Emily learns that she will inherit $1,000,000 at the start of the future period and that she can borrow against it if she wishes. Draw Emily's original budget line and show how it is affected by the inheritance.



original: $C_P + \frac{C_F}{1.5} = 10M$.

now: $C_P + \frac{C_F}{1.5} = 10M + \frac{1M}{1.5}$

opt at: $C_P = 3C_F$

$\Rightarrow C_F^N \approx 2.91M$

$C_P^N \approx 3C_F^N = 8.73M$

saving $= I_P - C_P^N = 1.27M$

Both C_P and C_F increasing b/c inheritance

saving decreasing b/c inheritance

c) [5 marks] Explain and illustrate how Emily's budget line from part (b) would change if the government taxed both her earnings and the interest earned on her savings at the rate of 40%.

with tax B.C. $C_P + \frac{C_F}{1+r(1-0.4)} = I_P(1-0.4) + \frac{I_F}{1+r(1-0.4)}$

with $r = 0.5$, $I_P = 10M$, $I_F = 1M$ $\Rightarrow$

$C_P + \frac{C_F}{1.3} = 6M + \frac{1M}{1.3}$

the budget line will shrink towards origin.

$C_{P,max} = 6M + \frac{1M}{1.3} \approx 6.77M$

$C_{F,max} = 6M \times 1.3 + 1M \approx 8.8M$

slope changed from -1.5 to -1.3