Economics 212

Section 001

Midterm Exam

October 18, 2013

Student Number:

Section:

Section A: Three questions @ 5 marks. Total 15 marks.

1. [5 marks] Consider the utility function U(X,Y)=2X+5Y, where X and Y are two goods. Assume the price of X is \$6, the price of Y is \$10 and the consumer has an income of \$1200. Derive the optimal consumption bundle for the consumer.

B.C. $P_x \cdot X + P_y \cdot Y = I \Rightarrow 6X + 10Y = 1200$

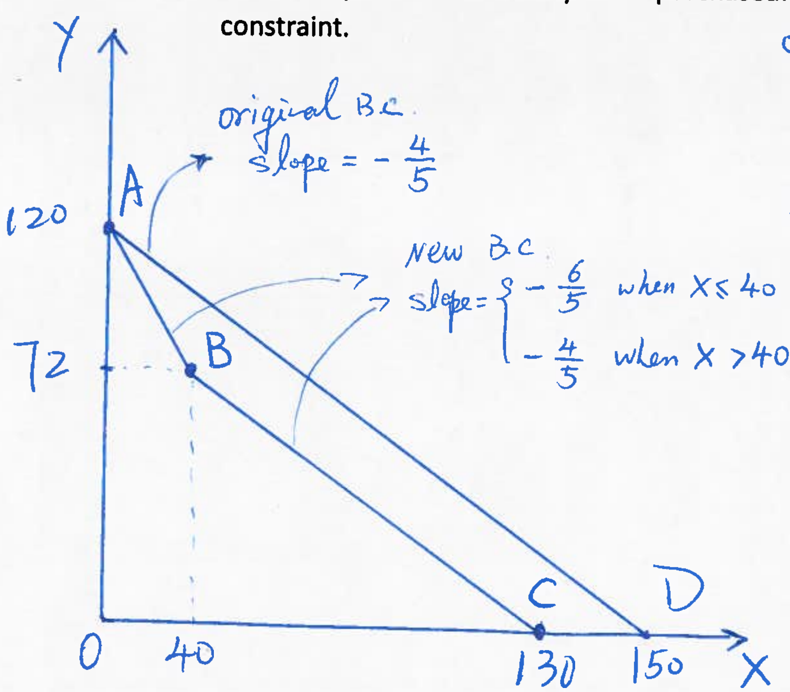
$$\left.\begin{array}{l} MRS_{x,y} = \dfrac{MU_x}{MU_y} = \dfrac{2}{5} \\ \dfrac{P_x}{P_y} = \dfrac{6}{10} = \dfrac{3}{5} \end{array}\right\} \Rightarrow \frac{MU_x}{MU_y} < \frac{P_x}{P_y} \Rightarrow \frac{MU_x}{P_x} < \frac{MU_y}{P_y}$$

$\Rightarrow$ consumer will spend all on Y.

$$\Rightarrow \quad X^* = 0$$

$$Y^* = \frac{1200}{10} = 120$$

2. [5marks] A consumer has \$600 in income and purchases two goods, X, which has a price of \$4 and Y, which has a price of \$5. Draw and appropriately label the consumer's budget constraint. Now suppose the government imposes a tax on good X at the rate of \$2 per unit, but the tax is levied only on the first forty units purchased. Draw and appropriately label the new budget constraint.



original B.C. $4X + 5Y = 600$

budget line: AD with slope $-\frac{4}{5}$.

New B.C. $\begin{cases} 4X + 5Y = 600 & \text{when } X > 40 \\ 6X + 5Y = 600 & \text{when } X \le 40 \end{cases}$

① when $X \le 40$ slope $= -\frac{6}{5}$

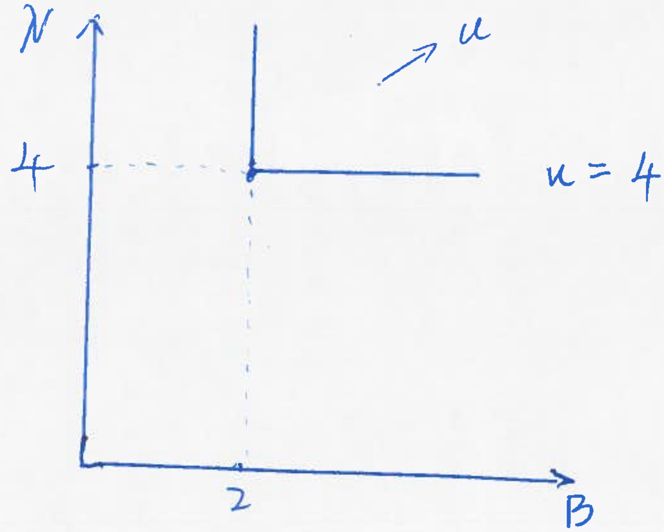
with $X = 40 \Rightarrow Y = 72$.

budget line: AB with slope $-\frac{6}{5}$

② $X > 40$: with slope $-\frac{4}{5}$.

budget line: BC.

3. [5marks]Each Sunday Bill sits down to watch football on television. Bill drinks one bottle of beer and eats two bags of nachos during each football game he watches. Write an equation that describes Bill's preferences over beer, B, and nachos, N. Each Sunday Bill watches two football games. Draw and appropriately label Bill's indifference curve.



$$U(B,N) = \min\{2B, N\}$$

Section B: Three question @ 15 marks- 5 for each part of each question. Total 45 marks.

1. Inge consumes two goods, X and Y, according to the utility function $U(X,Y)=X^{1/2}Y^2$. Inge has an income, I, and faces prices for the two goods given by P_X and P_Y.
   a) [5 marks] Derive Inge's demand functions for the goods X and Y.

$$MRS_{x,y} = \frac{MU_x}{MU_y} = \frac{\frac{1}{2}X^{-1/2}Y^2}{2X^{1/2}Y} = \frac{Y}{4X}$$

opt: $MRS_{x,y} = \frac{P_x}{P_y} \Rightarrow Y = 4X\left(\frac{P_x}{P_y}\right)$

substitute into budget $P_x X + P_y Y = I \Rightarrow$

$$P_x X + P_y \cdot 4X\left(\frac{P_x}{P_y}\right) = I \Rightarrow X^* = \frac{I}{5P_x}$$

$$\Rightarrow Y^* = 4X^*\left(\frac{P_x}{P_y}\right) = \frac{4I}{5P_y}$$

b) [5 marks] Assume that Inge's income is \$1000, the price of X is \$4 and the price of Y is \$2. Calculate her demand for each good. What is the elasticity of demand for X at this bundle?

$$X^* = \frac{I}{5P_x} = \frac{1000}{20} = 50$$

$$Y^* = \frac{4I}{5P_y} = \frac{4000}{10} = 400$$

$$\varepsilon_D = \frac{\partial X^* / \partial P_x}{X^* / P_x} = \frac{\frac{I}{5}\cdot(-P_x^{-2})}{X^*/P_x} = -\frac{I}{5}\cdot\frac{1}{P_x}\cdot\frac{1}{X^*} = -\frac{1000}{5(4)(50)}$$

$$= -1$$

c) [5 marks]Suppose the price of X increases to \$6. Determine the new demand for the goods and calculate the income and substitution effects of the price change.

New opt. bundle: $X_N^* = \frac{I}{5P_{xN}} = \frac{1000}{5(6)} = \frac{100}{3}$

$Y_N^* = \frac{4I}{5P_y} = 400$ (no change on P_y).

need to find original utility level to find decomposition bundle:

$U_0 = X^{1/2}Y^2 = \sqrt{50}\cdot(400^2)$

$MRS_{x,y} = \frac{Y}{4X}$ with new price ration: $\frac{P_{xN}}{P_y} = \frac{6}{2} = 3$

at decomp. bundle must have: $\frac{Y}{4X} = 3 \Rightarrow Y = 12X$

to get original utility level: $U_0 = \sqrt{50}(400^2) = X^{1/2}(12X)^2 \Rightarrow$

$X^D \approx 36.15$.

sub. effect: $X^D - X^* \approx 36.15 - 50 = -13.86$ (decreasing).

income effect: $X_N^* - X^D \approx \frac{100}{3} - 36.15 \approx -2.82$ (decreasing).

Notice: total effect $\Rightarrow$ decreasing $13.86 + 2.82 \approx 16.68$

which is the total decreasing $X_N^* - X^* = \frac{100}{3} - 50 \approx 16.67$

2. Al has 126 hours per week to divide between leisure, R, and work. When he works, Al earns $24 per hour. He values both leisure and consumption, C, according to the utility function U(R,C)=Min{12R ; C}. The price of the consumption good is unity.

   a) [5marks] Derive Al's optimal bundle. How much does he work?

B.C: $C = w(T-R)$ with $T=126$, $w=24$, $P_c=1$.

$\Rightarrow C = 126(24) - 24R$

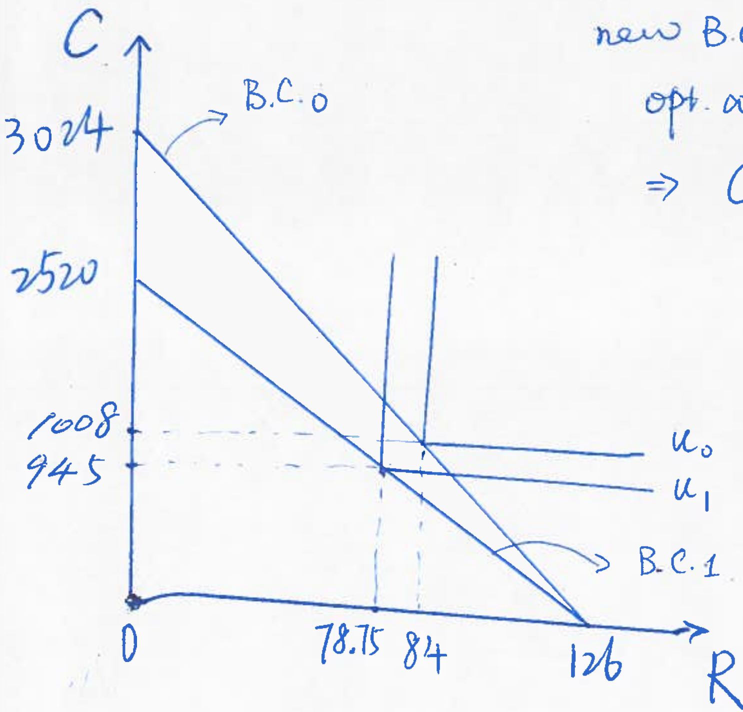
at opt: $C = 12R$

$\Rightarrow 36R = 126(24) \Rightarrow R^* = 84$

$\Rightarrow C^* = 12R^* = 1008$ → opt. bundle at utility level $=1008$ u_0^*

Al works $T - R = 126 - 84 = 42$ hrs.

   b) [5 marks] Explain using a diagram how Al's allocation of time between work and leisure changes if Al is taxed on his earnings at the rate of $4 per hour.



new B.C. $C = w(T-R)$ with $T=126$, $w=20$, $P_c=1$

opt. at $C=12R$ $\Rightarrow R_1^* = 78.75$

$\Rightarrow C_1^* = 12R_1^* = 945$. $\Rightarrow u_1^* = 945$ at $w=20$

with $u = \min\{12R ; C\}$

the decrease in wage will decrease Al's ~~total~~ income, in response Al will work more and consume less

i.e., work: $126 - 78.75 = 47.25$ hrs

consume: 945

and new utility level $= 945 < 1008$

c) Starting from the solution to part (a), assume Al's boss tells him that new rules mean Al must work 48 hours per week. Calculate Al's utility level under the new rules and show that he is worse off because of the new rules.

under new rule: $U_N^* = \min\{12R\ ;\ C\}$ fix $R = 126 - 48$

$= \min\{12(126-48)\ ;\ C\}$

at opt: $12(126-48) = C \Rightarrow U_N^* = 12R^* = C^* = 12(126-48) = 936$

$U_N^* = 936 < 1008 = U_0^*$

Al is worseoff.

3. [5 marks] Emily works in the present period and earns an income of \$6,000,000. In the future period, Emily is retired and earns nothing. Her preferences over present consumption, C_P, and future consumption, C_F, are given by $U(C_P,C_F)=\text{Min}\{2C_P\ ;\ 4\ C_F\}$. Emily's savings earn an interest rate of 100%.

a) Derive Emily's optimal consumption bundle and her level of savings.

B.C. $C_P + \frac{C_F}{1+r} = I_P + I_F$ with $r=1$, $I_P = 6M$, $I_F = 0$

opt. at: $2C_P = 4C_F \Rightarrow C_P = 2C_F \Rightarrow$

$2C_F + \frac{C_F}{2} = 6M \Rightarrow C_F^* = 2.4M$ and $C_P^* = 2C_F^* = 4.8M$

saving $= I_P - C_P^* = 6M - 4.8M = 1.2M$

b) Suppose that Emily had treated present and future consumption as perfect substitutes rather than perfect complements. Write an equation for Emily's preferences (perfect substitutes) that would lead Emily to consume all of her present income in the future period (no, she will not starve or die if she has zero present consumption).

perfect substitutes pref. can be expressed as: $u(C_P, C_F) = aC_P + bC_F$

with: $MU_{C_P} = a$, $MU_{C_F} = b$.

if we want $C_P = 0$, must have $\frac{MU_{C_P}}{MU_{C_F}} = \frac{a}{b} < \frac{P_{C_P}}{P_{C_F}} = 1 + r = 2$

so any a and b satisfied $\frac{a}{b} < 2$ will do.

e.g. $u(C_P, C_F) = 2C_P + 4C_F$ with $\frac{a}{b} = \frac{2}{4} = \frac{1}{2} < 2$.

c) Explain and illustrate how Emily's original budget line would change if the government taxed both her present earnings at the rate of 20% and the interest earned on her savings at the rate of 10%.

original B.C. $C_P + \frac{C_F}{1+r} = I_P \Leftrightarrow C_P + \frac{C_F}{2} = 6M$

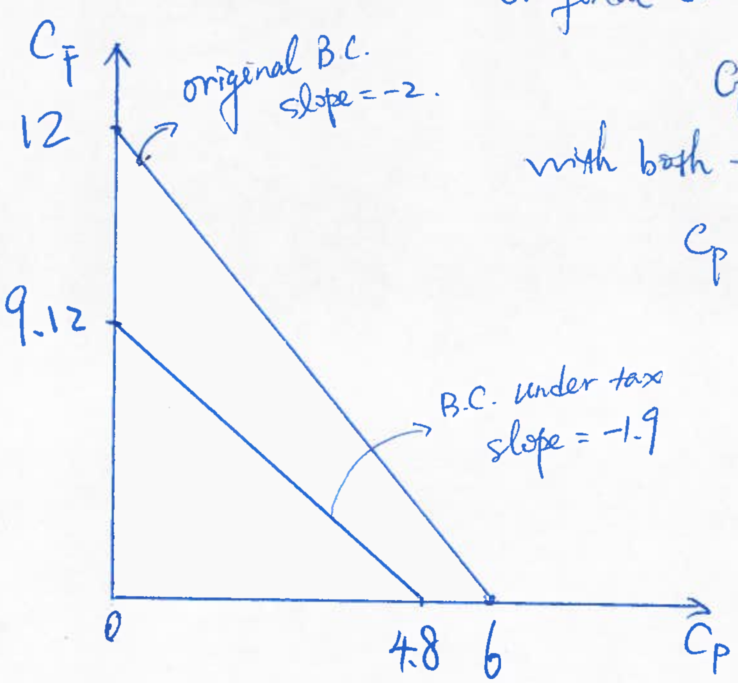
$C_{P,max} = 6M$ $\quad C_{F,max} = 12M$

with both tax: B.C becomes.

$$C_P + \frac{C_F}{1 + r(1-0.1)} = I_P(1-0.2) \Rightarrow$$

$$C_P + \frac{C_F}{1.9} = 4.8M$$

$C_{P,max} = 4.8M$, $C_{F\,max} = 9.12M$



because of the tax, total wealth decrease, new budget line shrink towards origin and slope changed to -1.9.