Economics 212

Section 002

Midterm Exam

October 23, 2012

Student Number: Solution

Section A: Three questions @ 5 marks. Total 15 marks.

1. [5 marks] Art has $600 to spend on two goods, books, B, and sweaters, S. The price of a book is $30 and the price of a sweater is $60. Draw and appropriately label Art's budget constraint. The government has decided to tax sweaters and levies a 25% tax on the price of a sweater, but only after Art buys 2 sweaters. Draw and appropriately label Art's new budget constraint.

Starting at $(2,16)$: $480 = 30B + \frac{5}{4}(60)S$

when $B=0 \Rightarrow S = 6.4$

2. [5marks] Jones has $125 in income and a utility of income function given by $U=I^{1/3}$. Jones is offered a bet with three possible outcomes: there is a 30% probability that he will finish with $27; a 40% probability that he finishes with $64; and a 30% probability that he finishes with $343. Will Jones accept or reject this bet? Explain.

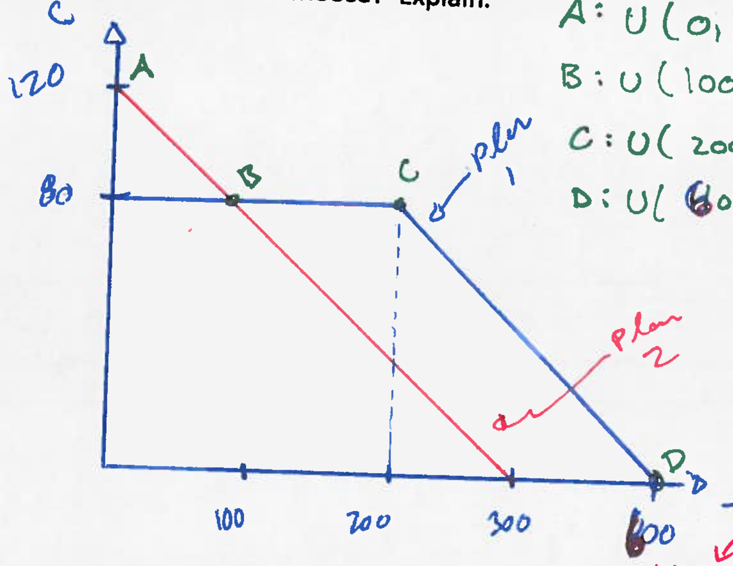
Utility with no bet $U_0 = (125)^{1/3} = 5$

Expected utility with bet, $U_1 = 0.3(27)^{1/3} + 0.4(64)^{1/3} + 0.3(343)^{1/3} = \frac{1}{3}(3) + \frac{1}{4}4 + \frac{1}{3}7$

$= 2 + \frac{7}{3} = 4.6 < 5$

He will reject the bet since $U_0 > U_1$

3. [5marks] Hal is considering two cell phone plans. Plan one offers Hal up to 200 minutes of service, T, for a flat fee of $40, with each additional minute of service priced at 20 cents. The second plan offers Hal a price per minute of 40 cents. Hal has $120 per month to spend on cell phone service and some other composite good, C, with a price of $1 per unit. Draw and appropriately label the two budget constraints. If Hal's utility function is given by U(T,C) = 5T + 5C, which plan will Hal choose? Explain.



A: U(0,120) = 600

B: U(100, 80) = 900

C: U(200, 80) = 1400

D: U(600, 0) = 3000

He will choose plan 1

Highest utility is at D.

Corner solution.

flat fee mins

80/0.2

200+400 = 600

mins total

Section B: Three question @ 15 marks- 5 for each part of each question. Total 45 marks.

1. Kate consumes two goods, X and Y, according to the utility function $U(X,Y)=X^{1/2}Y^{1/2}$. Kate has an income, I, and faces prices for the two goods given by P_X and P_Y.
   a) [5 marks] Derive Kate's demand functions for the goods X and Y.

$$MRS = \frac{MU_X}{MU_Y} = \frac{\frac{1}{2}X^{-1/2}Y^{1/2}}{\frac{1}{2}X^{1/2}Y^{-1/2}} = \frac{Y}{X} = \frac{P_x}{P_y} \Rightarrow X = Y\frac{P_Y}{P_X} \quad (*)$$

B/C : $I = P_x X + P_y Y$

Plug (*) in B/C $\Rightarrow I = \cancel{P_x}\left(Y\frac{P_Y}{\cancel{P_X}}\right) + P_y Y = I = 2P_y Y$

$\Rightarrow Y = \frac{I}{2P_y}$

$$X = \frac{I}{2P_y}\,\frac{P_Y}{P_X} = \frac{I}{2P_X}$$

b) [5 marks] Let Kate's income be \$600, the price of X be \$5 and the price of Y be \$4. How much of each good should Kate consume?

$I = 600, \quad P_x = 5, \quad P_y = 4$

$$y = \frac{600}{2(4)} = \frac{600}{8} = \frac{300}{4} = 75$$

$$x = \frac{600}{2(5)} = \frac{600}{10} = 60$$

c) [5 marks]Suppose the price of X decreases to \$3. Determine the new demand for the goods and calculate the income and substitution effects of the price change.

$$x = \frac{600}{2(3)} = 100 \quad ; y \text{ unchanged}$$

$$U_0 = (60)^{1/2}(75)^{1/2} \approx 67.1$$

From MRS $\quad x = y\frac{P_y}{P_x} = y\frac{4}{3}$

we must find x, y s.t. $\left| \begin{array}{l} x^{1/2}y^{1/2} = 67.1 \\ x = y\frac{4}{3} \end{array} \right\}$ $\quad y\left(\frac{4}{3}\right)^{1/2} = 67.1$

$y = 58.1$

$\Rightarrow x = 77.5$

Income effect:

$77.5 - 60 = 17.5$

Substitution effect

$100 - 77.5 = 22.5$

2. Ahmad has 112 hours per week to divide between leisure, R, and work. When he works, Ahmad earns \$25 per hour. He values both leisure and consumption, C, according to the utility function U(R,C)=Min{30R ; C}. The price of the consumption good is unity.

a) [5marks] Derive Ahmad's optimal bundle. How much does he work?

↳ $30R = C$

B/C : $C = (112 - R) \cdot 25$

$30R = 112(25) - 25R$

$55R = 112 \cdot (25)$

$R = \frac{112 \cdot 25}{55} \approx 50.1$

And work $112 - 50.1 = 61.9$

$C = 1547.5$

b) [5 marks] The government decides to levy a tax at the rate of \$5 per hour on Ahmad's earnings. Find Ahmad's new optimal bundle, including the amount of work and leisure chosen. In words, explain this outcome in terms of the income and substitution effects of the tax.

$30R = C$

B/c is now : $C = (112 - R)\,20$

$30R = 112(20) - 20R$

$50R = 112(20)$

$R = \frac{112(20)}{50} = 44.8$

work : $112 - 44.8 = 67.2$

$C = 1344$

Subsitution effect

↓ wage makes leisure cheaper relatively to work.
→ increase leisure

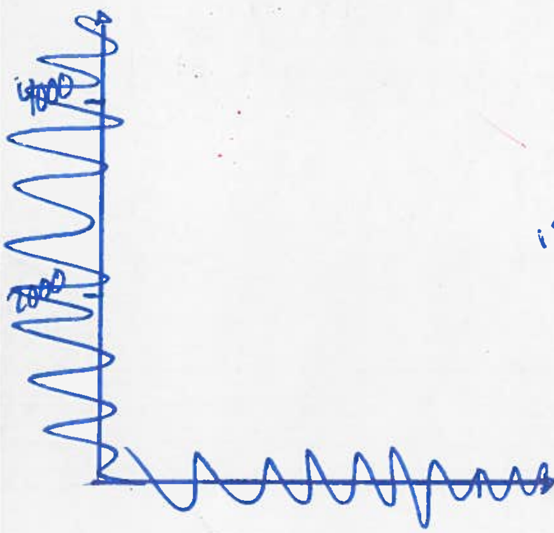
Income effect

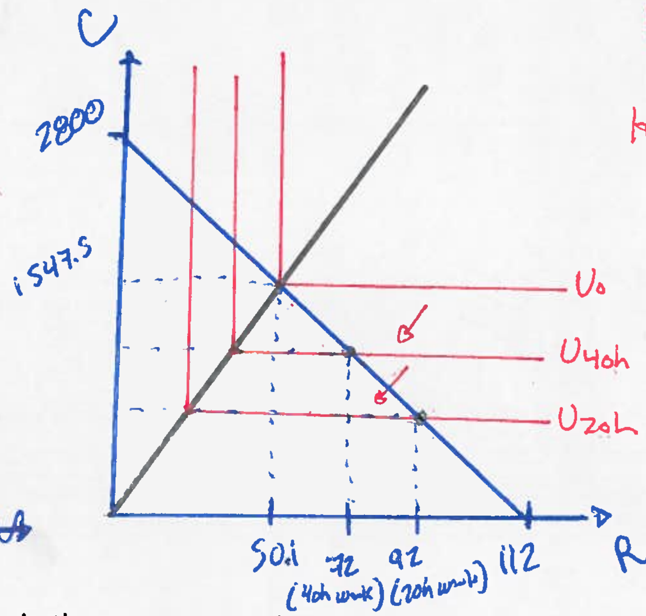
↓ wage makes income lower. So ↓ leisure and ↑ work if we assume that leisure is a normal good.

~~[illegible]~~

Here the income effect is stronger than the subs. effect since he worked more.

c) Starting from the solution to part (a), assume Ahmad's boss tells him that he must work 40 hours per week or 20 hours per week. Show graphically that Ahmad is worse off with either choice. Which option would he choose? Explain.





Both on Lower indiff. curves

He will choose 40h since higher utility than 20h.

3. [5 marks] Emily works in the present period and earns an income of $8,000,000. In the future period, Emily is retired and earns nothing. Her preferences over present consumption, C_P, and future consumption, C_F, are given by $U(C_P,C_F)=C_P C_F^{1/2}$. Emily's savings earn an interest rate of 80%.

a) Derive Emily's optimal consumption bundle and her level of savings.

B/C: $8,000,000 = C_P + s$; $s(1+r) = C_F \Rightarrow \frac{C_F}{1+r} = s$ $\Big\}$ $8,000,000 = C_P + \frac{C_F}{1+r}$

$$MRS = \frac{MU_{C_P}}{MU_{C_F}} = \frac{C_F^{1/2}}{\frac{1}{2}C_P C_F^{-1/2}} = \frac{2C_F}{C_P} = (1+r) = 1.8 \Rightarrow C_F = \frac{1.8\,C_P}{2}$$

BACK in B/C: $8,000,000 = C_P + \frac{1.8\,C_P}{2}\left(\frac{1}{1.8}\right) = \frac{3C_P}{2}$

$$C_P = \frac{16,000,000}{3}$$

$$C_F = \frac{9}{10}\,\frac{16,000,000}{3} = \left(\frac{3}{10}\right)16,000,000$$

$$s = \frac{24,000,000 - 16,000,000}{3} = \frac{8,000,000}{3}$$

Normal good since an increase in income increased C_F.

b) How would Emily's optimal bundle and her level of savings be affected by an inheritance of $2,000,000 in the future period? Is future consumption a normal good? Explain.

B/c $8,000,000 = C_P + s$

$2,000,000 + (1+r)s = C_F \Rightarrow s = \dfrac{C_F - 2,000,000}{1+r}$

in B/c : $8,000,000 = C_P + \dfrac{C_F - 2,000,000}{1+r}$ And $C_F = \dfrac{1.8}{2} C_P$

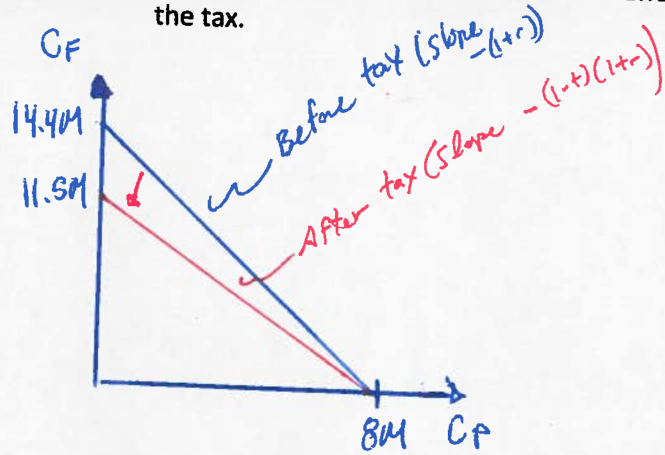
$\Rightarrow 8,000,000 = C_P + \dfrac{C_P}{2} - \dfrac{2,000,000}{1.8}$

$\dfrac{82,000,000}{9} = \dfrac{3C_P}{2} \Rightarrow \left(\dfrac{2}{3}\right)\dfrac{82,000,000}{9} = C_P = 6,074,074.1$

$C_F = 5,466,666,66$

$s = 1,925,925,9$

c) Explain and illustrate how Emily's budget constraint [from part a)] would be affected by a government tax at the rate of 20% on the interest earned on her savings. Use words to describe the substitution and income effects with respect to Emily's potential response to the tax.



Max C_F is now lower and the slope less steeper.

Substitution effect

↓ interest rate → ↓ rel. price of current consumption

⇒ increase C_P

Income effect

↓ interest rate → ↓ income (total earnings)

↓ C_P since C_P is a normal good.