Economics 212

Section 001

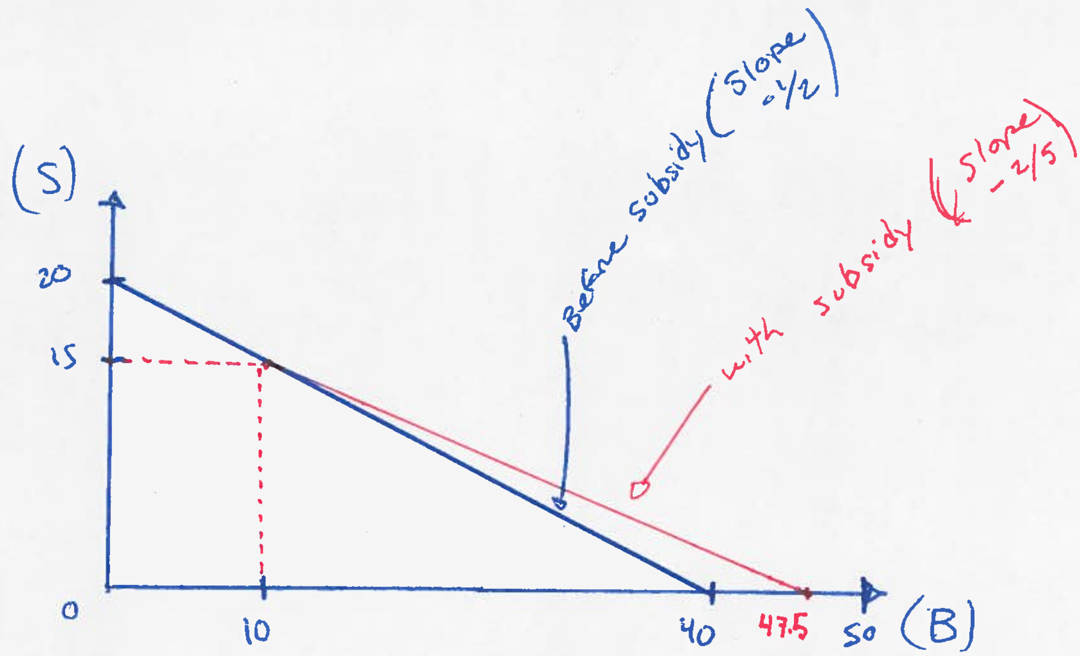
Midterm Exam

October 22, 2012

Student Number: Solution

Section A: Three questions @ 5 marks. Total 15 marks.

1. [5 marks] Art has $1000 to spend on two goods, books, B, and sweaters, S. The price of a book is $25 and the price of a sweater is $50. Draw and appropriately label Art's budget constraint. The government has decided to encourage reading and offers a 20% subsidy on the price of a book, but only after Art buys 10 books. Draw and appropriately label Art's new budget constraint.



2. [5marks] Jones has $100 in income and a utility of income function given by $U=I^{1/2}$. Jones is offered a bet with three possible outcomes: there is a 20% probability that he will finish with $0; a 50% probability that he finishes with $64; and a 30% probability that he finishes with $225. Will Jones accept or reject this bet? Explain.

Utility with no bet $U_0 = 100^{1/2} = 10$

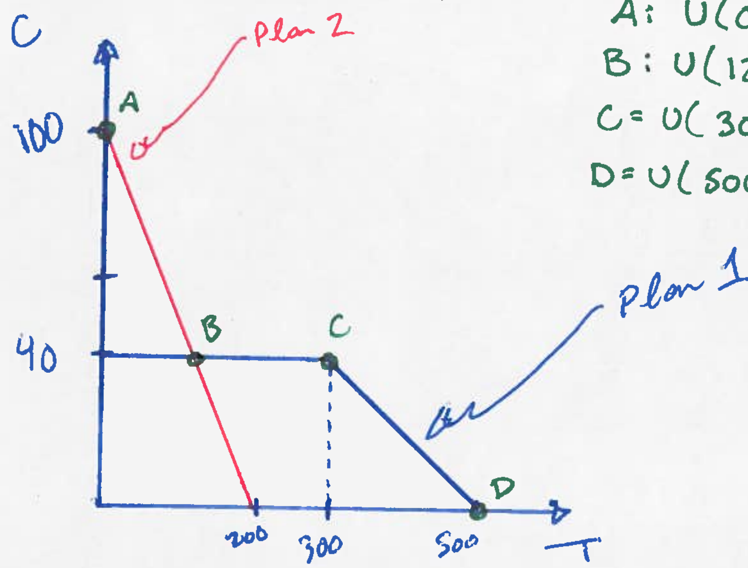
Expected utility with bet : $U_1 = 0.20(0)^{1/2} + 0.50(64)^{1/2} + 0.3(225)^{1/2}$

$= 8.5$

$U_0 > U_1$

will reject the bet since the expected utility is lower than the certainty case.

3. [5marks] Hal is considering two cell phone plans. Plan one offers Hal up to 300 minutes of service, T, for a flat fee of \$60, with each additional minute of service priced at 20 cents. The second plan offers Hal a price per minute of 50 cents. Hal has \$100 per month to spend on cell phone service and some other composite good, C, with a price of \$1 per unit. Draw and appropriately label the two budget constraints. If Hal's utility function is given by U(T,C) = T + 10C, which plan will Hal choose? Explain.



$A: U(0,100) = 1000$

$B: U(120,40) = 520$

$C = U(300,40) = 700$

$D = U(500,0) = 500$

Will choose plan 2. Highest utility is at A. Corner solution.

Section B: Three question @ 15 marks- 5 for each part of each question. Total 45 marks.

1. Kate consumes two goods, X and Y, according to the utility function $U(X,Y)=XY^{1/2}$. Kate has an income, I, and faces prices for the two goods given by P_X and P_Y.
   a) [5 marks] Derive Kate's demand functions for the goods X and Y.

$$MRS = \frac{MU_X}{MU_Y} = \frac{Y^{1/2}}{\frac{1}{2}XY^{-1/2}} = \frac{2Y}{X} = \frac{P_x}{P_y} \Rightarrow X = 2Y\frac{P_y}{P_x} \quad (*)$$

$$B/C: \quad I = P_x X + P_y Y$$

$$\text{Plug } (*) \text{ in } B/C \Rightarrow I = \cancel{P_x}\left(2Y\frac{P_y}{\cancel{P_x}}\right) + P_y Y = 3P_y Y$$

$$\Rightarrow Y = \frac{I}{3P_y}$$

$$\text{And } X = 2\frac{\cancel{P_y}}{P_x}\left(\frac{I}{3\cancel{P_y}}\right) = \frac{2}{3}\frac{I}{P_x}$$

b) [5 marks] Let Kate's income be \$1000, the price of X be \$3 and the price of Y be \$2. How much of each good should Kate consume?

$I = 1000,\ P_x = 3,\ P_Y = 2$

$$x = \left(\frac{2}{3}\right)\frac{1000}{3} = \frac{2000}{9}$$

$$y = \left(\frac{1}{3}\right)\frac{1000}{2} = \frac{1000}{6} = \frac{500}{3}$$

c) [5 marks]Suppose the price of X increases to \$5. Determine the new demand for the goods and calculate the income and substitution effects of the price change.

Y unchanged ; $X = \frac{2}{3}\left(\frac{1000}{5}\right) = \frac{2000}{15} = \frac{400}{3}$

Original utility $U_0 = \left(\frac{2000}{9}\right)\left(\frac{500}{3}\right)^{1/2} \approx 2868.9$

From MRS : $X = 2Y\left(\frac{P_Y}{P_x}\right) = 2Y\left(\frac{2}{5}\right) \Rightarrow x = \frac{4}{5}Y$

we must find x, y s.t. $\left| \begin{array}{l} 2868.9 = XY^{1/2} \\ X = \frac{4}{5}Y \end{array} \right\}$ $2869.9 = \frac{4}{5}YY^{1/2}$

$\frac{5}{4}(2869.9) = Y^{3/2}$

$\left(\frac{5}{4}(2869.9)\right)^{2/3} = Y$

$$\boxed{\begin{array}{l} 234.34 = Y \\ X = \frac{4}{5}Y = \frac{4}{5} \cdot 234.34 = 187.47 \end{array}}$$

Income effect:

$187.47 - \frac{2000}{9} = -34.75$

inverse

Substitution effect.

$\frac{400}{3} - 187.47 = -54.14$

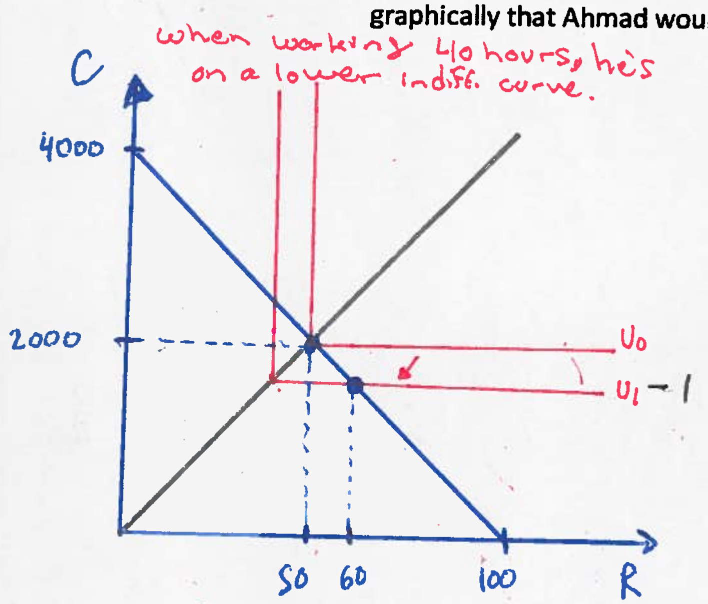
After 40 hours of work, the B/C become

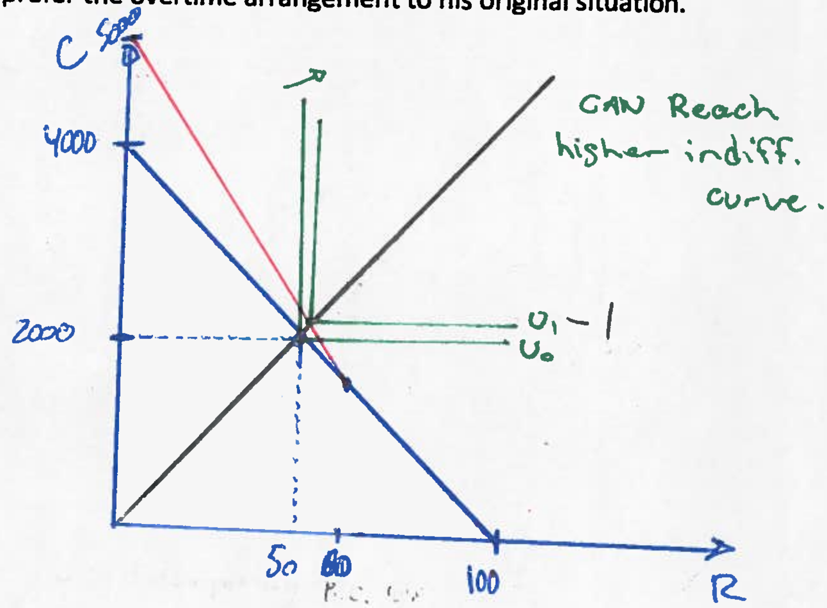
$C = (60 - R)50$

$= 3000 - 50R$

when $R = 0 \Rightarrow C = 3000$

c) Starting from the solution to part (a), assume Ahmad's boss tells him that he must work 40 hours per week. Show graphically that Ahmad is worse off. Now suppose that Ahmad can work overtime (beyond the 40 hours per week) at a wage of \$50 per hour. Show graphically that Ahmad would prefer the overtime arrangement to his original situation.





3. [5 marks] Emily works in the present period and earns an income of \$6,000,000. In the future period, Emily is retired and earns nothing. Her preferences over present consumption, C_P, and future consumption, C_F, are given by $U(C_P,C_F)=C_P^{1/2}\,C_F^{1/2}$. Emily's savings earn an interest rate of 50%.

a) Derive Emily's optimal consumption bundle and her level of savings.

B/C: $6{,}000{,}000 = C_P + s$

$s(1+r) = C_F \Rightarrow \frac{C_F}{1+r} = s$

$\Rightarrow 6{,}000{,}000 = C_P + \frac{C_F}{1+r}$

$$MRS = \frac{MU_{C_P}}{MU_{C_F}} = \frac{\frac{1}{2} C_P^{-1/2} C_F^{1/2}}{\frac{1}{2} C_P^{1/2} C_F^{-1/2}} = \frac{C_F}{C_P} = (1+r) = 1.5 \Rightarrow C_F = 1.5\,C_P$$

BACK in budget constraint: $6{,}000{,}000 = C_P + \frac{1.5\,C_P}{1.5}$

$\Rightarrow C_P = 3{,}000{,}000$

$s = 3{,}000{,}000$

$C_F = 4{,}500{,}000$

b) How would Emily's optimal bundle and her level of savings be affected by a lottery win of \$2,000,000 in the current period? Is future consumption a normal good? Explain.

$8,000,000 = 2C_P \Rightarrow C_P = 4,000,000$

$s = 4,000,000$

$C_F = 6,000,000$

NORMAL good. Increase of income $\Rightarrow$ increase in the future consumption

c) Explain and illustrate how Emily's budget constraint [from part a)] would be affected by a government policy that required Emily to put all of her savings in a government pension plan that delivered only a 40%interest rate. Briefly explain why Emily is worse off because of this government action.

$6,000,000 = 2C_P$

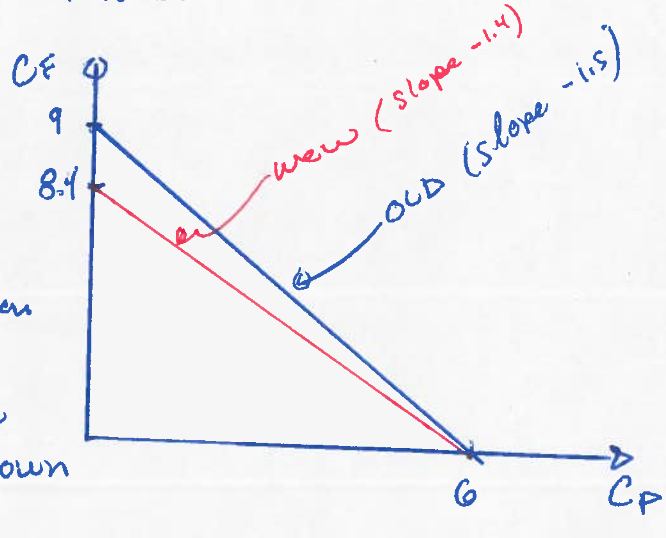
$\Rightarrow C_P = 3,000,000$ (unchanged)

$s = 3,000,000$

$C_F = 4,200,000$

Worse off because he consume the same amount in the current period but consume 300,000 less in the future.

worse since we can't achieve the same level of utility. Only way is corner solution at (6,0) but with Cobb-Douglas we have interior solution as shown in part (a)



2. Ahmad has 100 hours per week to divide between leisure, R, and work. When he works, Ahmad earns $40 per hour. He values both leisure and consumption, C, according to the utility function U(R,C)=Min{80R ; 2C}. The price of the consumption good is unity.

a) [5marks] Derive Ahmad's optimal bundle. How much does he work?

$80R = 2C \Rightarrow C = 40R$

B/C: $C = (100-R)\cdot 40$

$40R = 100(40) - 40R$

$80R = 100(40)$

$R = \underline{50}$

$w = \underline{50}$

$C = 50 \cdot 40 = \underline{2000}$

b) [5 marks] The government decides to levy a tax at the rate of $5 per hour on Ahmad's earnings. Find Ahmad's new optimal bundle, including the amount of work and leisure chosen. In words, explain this outcome in terms of the income and substitution effects of the tax.

Still have $C = 40R$

But B/C is now $C = (100-R)35$

$40R = 100(35) - 35R$

$75R = 100(35)$

$R = \frac{100}{75}(35) = 46.\overline{66} < 50$

0.5 - R*

5600/3 → 1866.67

140/3 → 46.67

160/3 → 53.33

work more $(100 - 46.\overline{66})$

Less leisure $46.\overline{66}$

Consume less : $(100-46.\overline{66})35$ < 2000

<u>Substitution effect</u>

↓ in wage makes leisure cheaper (relatively)

⇒ increase leisure

<u>Income effect</u>

↓ wage makes income lower. So ↓ leisure and ↑ work if we assume that leisure is a normal good.

~~[illegible] , he must work more.~~

Here the income effect is stronger than the substitution effect.