Economics 212

Section B

Midterm Exam

October 22, 2010

Student Number:

Section A: Three questions @ 5 marks. Total 15 marks.

1. [5 marks] Consider the utility function $U(X,Y)=X+2Y$, where X and Y are two goods. Draw and appropriately label two indifference curves for this consumer. Assume the price of X is \$4, the price of Y is \$4 and the consumer has an income of \$200. Derive the optimal consumption bundle for the consumer.

2. [5marks] Martin consumes bread, B, and cheese, C. The price of cheese is \$20 per unit and the price of bread is \$4 per unit. Martin has an income of \$200 to spend on the two goods. Draw and appropriately label his budget constraint. Now suppose the government imposes a tax equal to \$1 per unit on bread and offers a subsidy of \$4 per unit on cheese. Show how these actions affect Martin's consumption opportunities.

3. [5marks] Assume that market demand is given by Q^D=2000-2P+3I and market supply is given by Q^S=4P+200-10W, where Q is quantity, P is price, I is income and W is the wage paid to workers. Derive the equilibrium values of price and quantity (the expressions for P and Q will contain the terms I and W).

Section B: Three question @ 15 marks- 5 for each part of each question. Total 45 marks.

1. For entertainment, Wei consumes movies, M, and dinners, D, according to the utility function $U(M,D)=M^2D$. The price of a movie is P_M, the price of a dinner is P_D, and Wei's income is I.

   a) [5 marks] Derive Wei's demand functions for the two goods.

b) [5 marks] Assume the price of a movie is \$10, the price of a dinner is \$20 and Wei has an entertainment budget of \$600. Determine Wei's optimal bundle.

c) [5 marks] Assume that the price of a movie increases to \$20. Determine the new optimal bundle and the income and substitution effects of the price increase.

2. Alexa has 126 hours per week to divide between leisure, R, and work. When she works, Alex earns \$15 per hour. She values both leisure and consumption, C, according to the utility function U(R,C)=Min{15R;C}. The price of the consumption good is unity.
    a) [5marks] Derive Alexa's optimal bundle. How much does she work?

    b) [5 marks] The government tells Alexa that it will supplement her wage by \$5 per hour, but only if she works more than 30 hours per week and only on hours worked above 30 hours. Draw and appropriately label the new budget constraint. Explain how Alexa's choice is affected by the wage supplement.

c) If Alexa wins \$500 in the lottery, will she work more or less? Explain your answer.

3. [5 marks]Jonas is a music star who earns \$100,000,000 during his working life and nothing when he retires. The interest rate between his working life and retirement is 80%. His preferences over present consumption, C_P, and future consumption, C_F, are given by $U(C_P,C_F)=C_P^{1/2}C_F$.
   a) Derive Jonas optimal consumption bundle and his level of savings.

b) Draw and appropriately label Jonas budget constraint. Suppose the government decides to tax the interest earned on his savings at the rate of 20%. Draw and appropriately label the new budget constraint. Write the future value form of this new budget constraint.

c) [5 marks] How would your answer to part (a) change if Jonas had preferences given by $U(C_P, C_F) = 3C_P + C_F$? Explain your answer.

SECTION A.

QUESTION 1.

$$u(x,y) = x + 2y$$

$$p_x = \$4 \;;\; p_y = \$4 \;;\; I = \$200.$$

Budget constraint: $p_x x + p_y y \leq I$

$$\Leftrightarrow 4x + 4y \leq 200$$

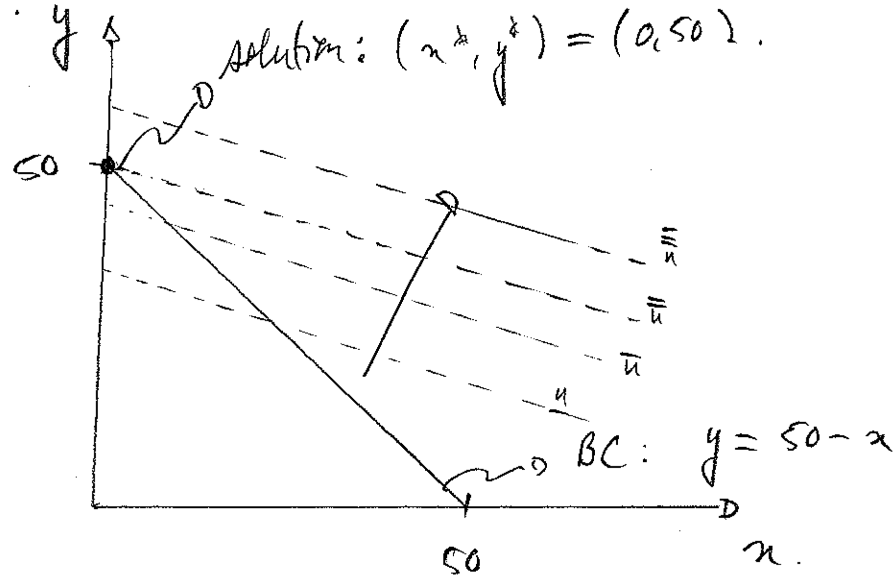
$$\Leftrightarrow y \leq 50 - x$$

Indifference curve: $u(x,y) = \bar{u} = x + 2y$.

$$\Leftrightarrow y = \frac{\bar{u}}{2} - \frac{1}{2}x$$

$$\underline{u} < \bar{u} < \bar{\bar{u}} < \bar{\bar{\bar{u}}}$$

Graphically: solution: $(x^*, y^*) = (0, 50)$.



QUESTION 2.

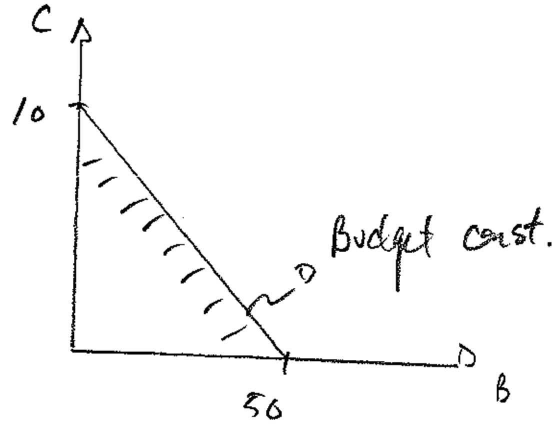
$p_C = \$20$; $p_B = \$4$; $I = \$200$

Budget constraint: $p_B B + p_C C \leq I$

$\iff 4B + 20C \leq 200.$

$\iff B + 5C \leq 50.$

$\iff C \leq 10 - \frac{1}{5}B$



With taxation: \$1/unit "t"

with subsidy: \$4/unit "s"

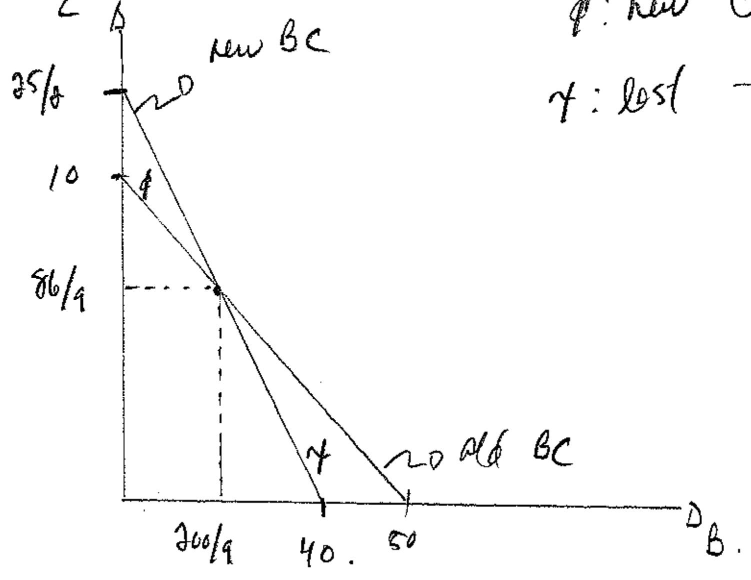
New budget constraint: $(p_B + t)B + (p_C - s)C \leq I.$

$\iff (4+1)B + (20-4)C \leq I$

$\iff 5B + 16C \leq 200$

$\iff C \leq \frac{200}{16} - \frac{5}{16}B. = \frac{25}{2} - \frac{5}{16}B.$

Graphically: old and new budget const.



φ: new consumption opportunities

ψ: lost ———

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QUESTION 3.

$$Q^D = 2000 - 2P + 3I$$

$$Q^S = 4P + 200 - 10W$$

At equilibrium : $Q^D = Q^S$

$$\Leftrightarrow 2000 - 2P + 3I = 4P + 200 - 10W$$

$$\Leftrightarrow \frac{1800 + 3I + 10W}{6} = P^*$$

$$\Rightarrow Q^* = 2000 + 3I - 2\left[\frac{1800 + 3I + 10W}{6}\right]$$

$$\Leftrightarrow Q^* = \frac{12\,000 + 18I - 3600 - 6I - 20W}{6}.$$

$$\Leftrightarrow Q^* = \frac{8400 + 12I - 20W}{6}.$$

Solution: $Q^* = \dfrac{8400 + 12I - 20W}{6}.$

$$P^* = \frac{1800 + 3I + 10W}{6}.$$

3/.

SECTION B.

QUESTION 1.

$u(m,d) = m^2 d$

(a) With Cobb-Douglas utility function: interior solution given by the condition $MRS = \frac{p_M}{p_D}$

$$MRS = \frac{\partial u(\cdot)/\partial m}{\partial u(\cdot)/\partial d} = \frac{2md}{m^2} = \frac{2d}{m}.$$

at optimality: $\frac{2d}{m} = \frac{p_M}{p_D} \Leftrightarrow d = \frac{m p_M}{2 p_D}.$

Using budget constraint: $p_M m + p_D d \leq I$.

$\Rightarrow$ at optimality $p_M m + p_D d = I$.

$$\Rightarrow p_M m + p_D \left[\frac{m p_M}{2 p_D} \right] = 2I$$

$$\Leftrightarrow 3 p_M m = 2I.$$

$$\Leftrightarrow m^* = \frac{2I}{3p_M} \Rightarrow d^* = \frac{I}{3 p_D}.$$

4.

(b). $m^I = \frac{2I}{3p_M} = \frac{2(600)}{3(10)} = 40.$

$d^I = \frac{I}{3p_D} = \frac{600}{3(20)} = 10.$

(c) Final basket:

$m^F = \frac{2I}{3p_M'} = \frac{2(600)}{3(20)} = 20$ with $p_M' = 20$.

$d^F = \frac{I}{3p_D} = 10.$

From (b): initial basket $(m^I, d^I) = (40, 10)$.

Intermediate decomposition basket:

(i) same initial utility

$\bar{u}_I = u(m^I, d^I) = 40^2 \cdot 10 = 16\,000$

(ii) tangency new price and initial utility level

$MRS = \frac{p_M'}{p_D} \Leftrightarrow d = \frac{m\, p_M'}{2p_D}$

| Income effect: $m^F - m^0$
| Substitution effect: $m^0 - m^I$

$\Rightarrow m^2 \left[\frac{m\, p_M'}{2p_D}\right] = \bar{u}_I$

$\Leftrightarrow m^0 = \left[\frac{2p_D\, \bar{u}_I}{p_M'}\right]^{1/3} = \left[\frac{2(20)(16000)}{20}\right]^{1/3} \hat{=} 31{,}75$

7.

QUESTION 2.

126 hours / week b/w R and W

\$15 / h

$u(R,C) = \min\{15R, C\}$ and $p_C = 1$

(a). $C = 15(126 - R) = 1890 - 15R \Leftarrow C + 15R = 1890$

For perfect complement : $15R = C$

Using budget constraint : $(15R) + 15R = 1890$

$\Leftarrow R^* = \frac{1890}{30} = 63$

$\Rightarrow W^* = 63$

$\Rightarrow C^* = 15(63) = 945$

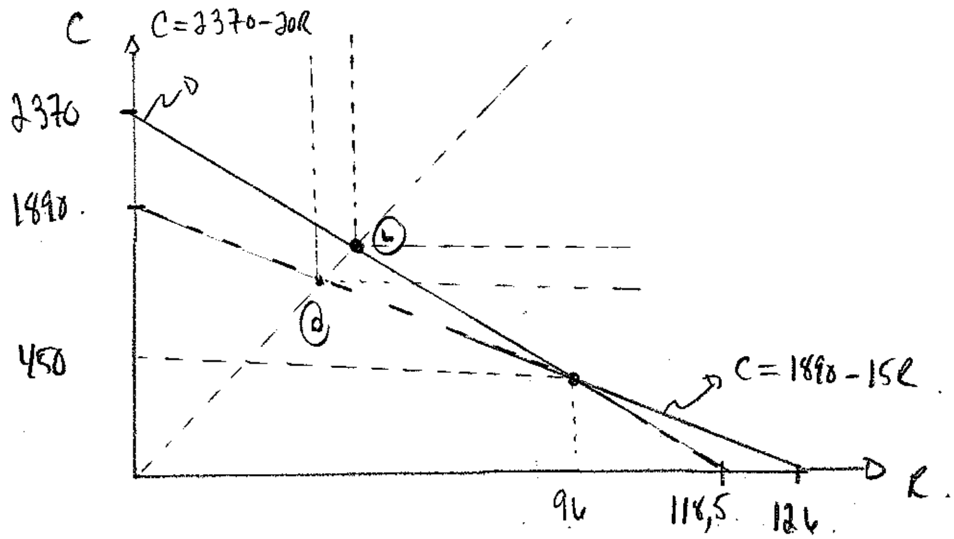
4/.

(b) On hours worked under 30 hours ($R \geq 96$), same situation as part (a).

On hours worked above 30 hours ($R < 96$), the budget const.:

$$C = 15(30) + 20(96 - R) = 2370 - 20R.$$

Graphically, the complete picture:



With the wage supplement: new feasible possibilities

As for the optimal choice: $(15R) + 20R = 2370$

$$\Rightarrow R^* = \frac{2370}{35} = 67,7$$

$$\Rightarrow W^* = 58,3$$

$$\Rightarrow C^* = 1015,71.$$

$\Rightarrow U_{(a)}(.) < U_{(b)}(.)$.

(c) To be compared to part (a).

$$C = 15(126 - R) + 500 = 2390 - 15R. \iff C + 15R = 2390.$$

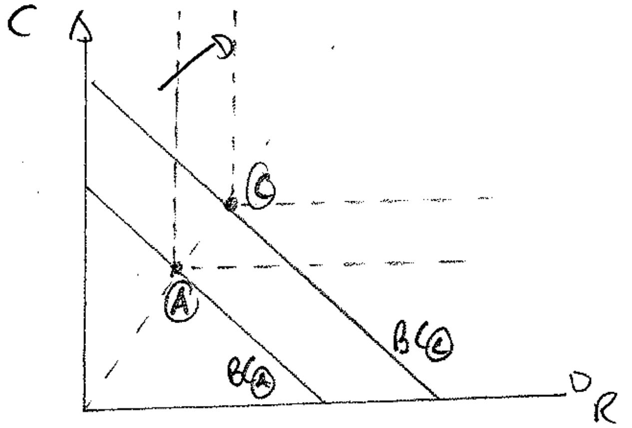
At optimality: $15R = C$

$$\Rightarrow \text{In BC}: \quad R^*_{(c)} = \frac{2390}{30} = \frac{239}{3} \approx 79.\overline{6}$$

$$\Rightarrow W^*_{(c)} = 46.\overline{3}$$

where $W^*_{(c)} < W^*_{(a)} = 63$.

Graphically,



hence, lower level of work with a flat increase in income.

QUESTION 3.

$I_p = \$100,000,000$

$r = 0.8$

$u(c_p, c_F) = c_p^{1/2} c_F$.

ⓐ. present period: $c_p + S = I_p$

future period: $c_F = S(1+r)$.

Intertemporal budget constraint using $S = \frac{c_F}{1+r}$.

$\Rightarrow c_p + \frac{c_F}{1+r} = I_p \Leftrightarrow c_F = I_p(1+r) - (1+r)c_p$

Since Cobb-Douglas utility function: at optimality the condition $MRS = \frac{p_p}{p_F}$ holds.

$$MRS = \frac{\partial u(\cdot)/\partial c_p}{\partial u(\cdot)/\partial c_F} = \frac{\frac{1}{2} c_p^{-1/2} c_F}{c_p^{1/2}} = \frac{c_F}{2c_p}$$

$$\Rightarrow \frac{c_F}{2c_p} = \frac{1}{1/(1+r)} = 1+r. \Leftrightarrow c_F = 2(1+r)c_p.$$

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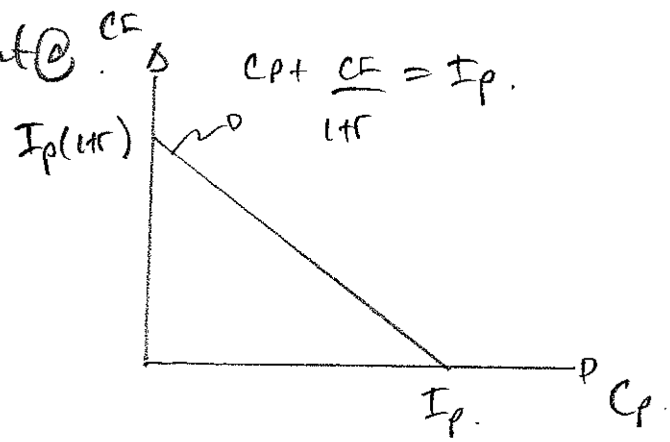
Using BC: $C_P + \frac{1}{1+r}\left[2(1+r)C_P\right] = I_P$

$$\Leftrightarrow C_P^* = \frac{I_P}{3} = \frac{100m}{3}$$

$$\Rightarrow C_F^* = \frac{2(1+r)I_P}{3} = \frac{2(1.8)(100m)}{3} = \frac{360m}{3} = 120m$$

$$\Rightarrow S = \frac{C_F^*}{1+r} = \frac{2I_P}{3} = \frac{2(100m)}{3} = \frac{200m}{3}.$$

(b) From part (a)



With tax $t = 0.2$ on interest earned:

present period: $C_P + S = I_P$.

future period: $C_F = S\left(1 + r(1-t)\right)$

Intertemporal budget constraint:

$$C_P + \frac{C_F}{1+r(1-t)} = I_P.$$

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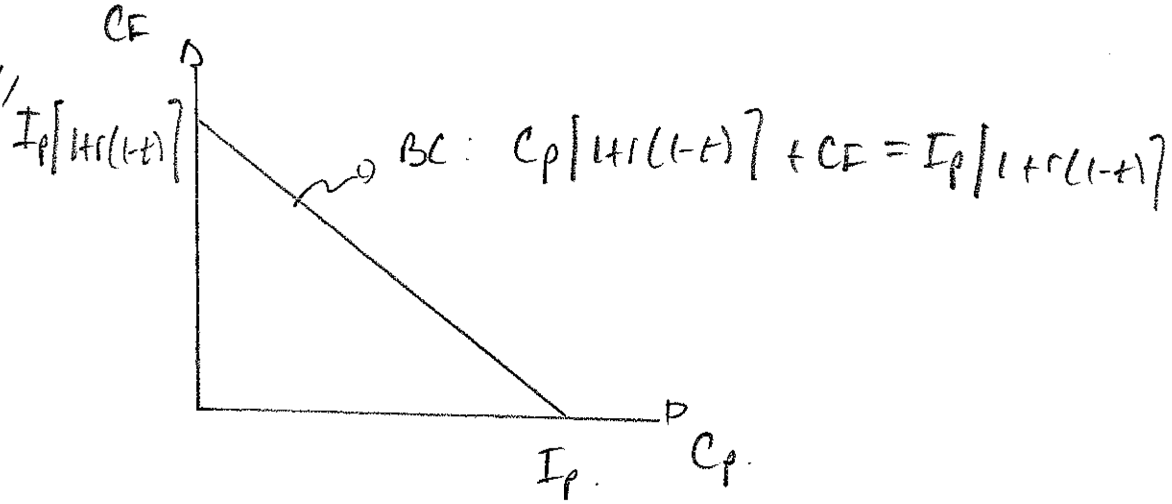
In future form: $C_P[1+r(1-t)] + C_F = I_P[1+r(1-t)]$.

Graphically,



(c). With $u(C_P, C_F) = 3C_P + C_F$: perfect substitutes.

From (a) $C_F = I_P(1+r) - (1+r)C_P$

$\leftrightarrow C_F = 100(1.8) - (1.8)C_P = 180 - 1.8C_P$.

$\Rightarrow$ slope $= -1.8$.

From indifference curve: $C_F = \bar{u} - 3C_P$.

$\Rightarrow$ slope $= -3$.

$\Rightarrow$ willing to give more of CF for an additional C_P than the market

$-3 < -1.8$.

Solution: $C_P = I_P = \$100m$

$C_F = 0$.

Graphically,