Economics 212

Section A

Midterm Exam

October 21, 2010

Student Number: Solution.

Section A: Three questions @ 5 marks. Total 15 marks.

1. [5 marks] Consider the utility function U(X,Y)=5X+5Y, where X and Y are two goods. Assume the price of X is $10, the price of Y is $10 and the consumer has an income of $2000. Derive the optimal consumption bundle for the consumer.

2. [5marks] A consumer has $1000 in income and purchases two goods, X, which has a price of $10 and Y, which has a price of $2. Draw and appropriately label the consumer's budget constraint. Now suppose the government imposes a tax on good X at the rate of $4 per unit, but the tax is levied only on units beyond the first twenty units purchased. Draw and appropriately label the new budget constraint.

3. [5marks]Each Sunday Bill sits down to watch football on television. Bill drinks two bottles of beer during each football game he watches. Write an equation that describes Bill's preferences over beer, B, and football games, F. Each Sunday Bill watches three football games. Draw and appropriately label Bill's indifference cuve.

Section B: Three question @ 15 marks- 5 for each part of each question. Total 45 marks.

1. Inge consumes two goods, X and Y, according to the utility function $U(X,Y)=XY^{1/2}$. Inge has an income, I, and faces prices for the two goods given by P_X and P_Y.
   a) [5 marks] Derive Inge's demand functions for the goods X and Y.

b) [5 marks] Assume that Inge's income is \$600, the price of X is \$4 and the price of Y is \$2. Calculate her demand for each good. What is the elasticity of demand for X at this bundle?

c) [5 marks]Suppose the price of X decreases to \$2. Determine the new demand for the goods and calculate the income and substitution effects of the price change.

2. Al has 126 hours per week to divide between leisure, R, and work. When he works, Al earns $30 per hour. He values both leisure and consumption, C, according to the utility function U(R,C)=Min{15R ; C}. The price of the consumption good is unity.
   a) [5marks] Derive Alex's optimal bundle. How much does he work?

   b) [5 marks] Explain Al's allocation of time between work and leisure in terms of the arguments in his utility function. [Hint: think about how R and C contribute to his well-being and how work and C are related].

SECTION A.

QUESTION 1

$$u(x,y) = 5x + 5y$$

$$p_x = \$10 \; ; \; p_y = \$10 \; ; \; I = \$2000$$

Budget constraint: $p_x x + p_y y \leq I$

$$\Leftrightarrow \; y \leq \frac{I - p_x x}{p_y} = \frac{2000 - 10x}{10}.$$

$$\Leftrightarrow \; y \leq 200 - x.$$

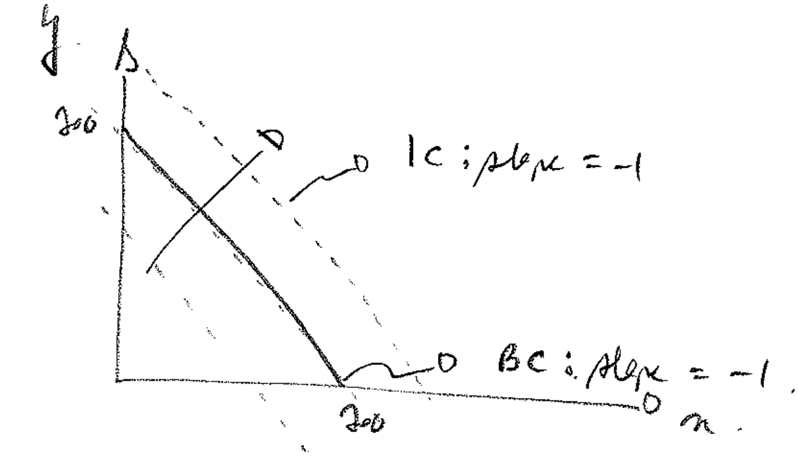
Indifference curve: $\bar{u} = 5x + 5y$

$$\Leftrightarrow \; y = \frac{\bar{u} - 5x}{5} = \frac{\bar{u}}{5} - x.$$

Note: slopes BC and IC are the same

$\Rightarrow$ infinite number of optimal consumption bundle on the line: $y = 200 - x$.

Graphically:



1.

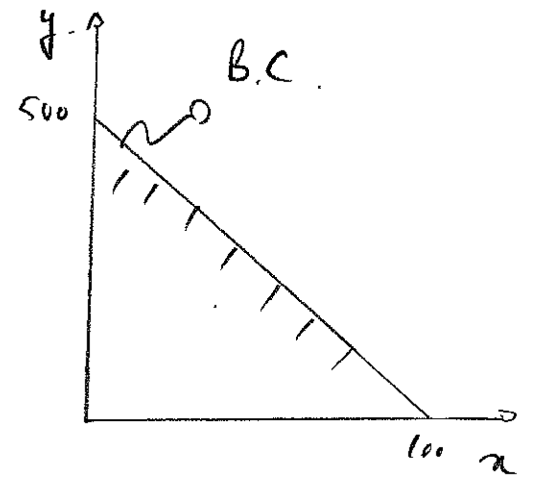
QUESTION 2

$I = \$1000$

$p_X = \$10$; $p_Y = \$2$

Budget const. : $p_X x + p_Y y \leqq I$

$$\Leftrightarrow y \leqq \frac{I - p_X x}{p_Y} = 500 - 5x.$$



Now, suppose tax on x at the rate \$4/unit for units after 20.

- Before 20 : same B.C. as above.
- After 20 : $10(20) + (p_X + t)z + p_Y y \leqq I$, where $z \equiv x - 20$

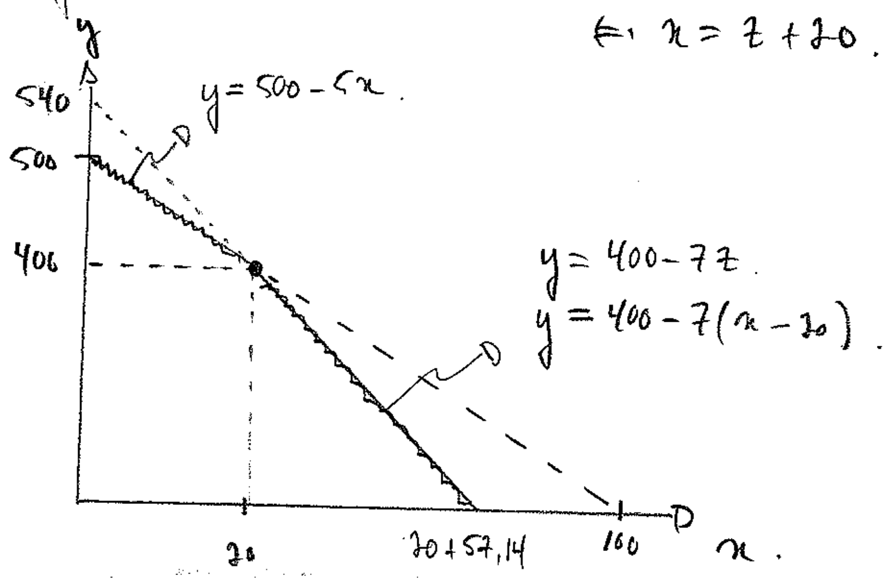
$$\Leftrightarrow (10+4)z + 2y \leqq 1000 - 200$$

$$\Leftrightarrow 7z + y \leqq 400.$$

$$\Leftrightarrow y \leqq 400 - 7z \quad \text{where } z \equiv x - 20.$$

$$\Leftrightarrow x = z + 20.$$

Graphically,



7.

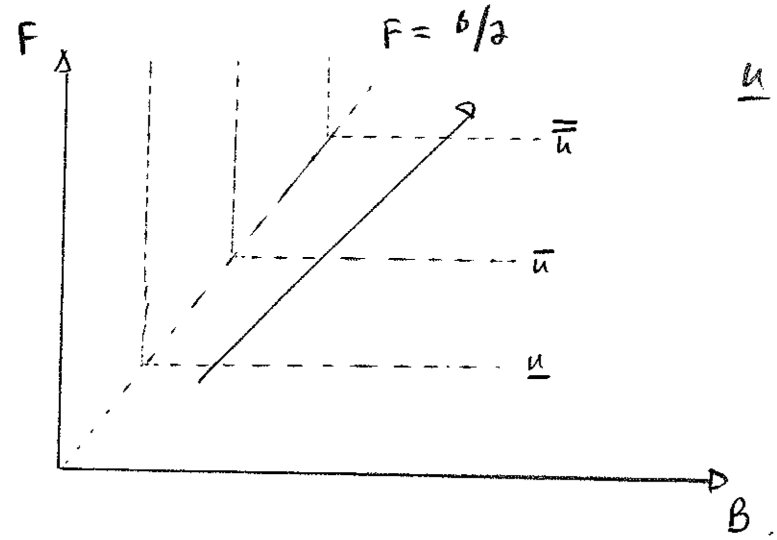
QUESTION 3.

Two bottles of beer during each football game and 3 games/Sunday.

$\Rightarrow$ Perfect complement: 2 bottles of beer for a game.

$\Rightarrow u(B,F) = 3\min\{B/2, F\}$.

Graphically,



$\underline{u} < \bar{u} < \bar{\bar{u}}$

SECTION B.

QUESTION 1

$u(x,y) = x y^{1/2}$

(a) $\max_{x,y} u(x,y)$ s.t.: $p_x x + p_y y \leq I$.

Cobb-Douglas $\Rightarrow$ interior solution, at optimality $MRS = p_x/p_y$.

$$MRS = \frac{\partial u(\cdot)/\partial x}{\partial u(\cdot)/\partial y} = \frac{y^{1/2}}{\frac{1}{2} x y^{-1/2}} = \frac{2y}{x}$$

At optimality $MRS = p_x / p_y$

$$\Leftrightarrow \quad \frac{2y}{x} = \frac{p_x}{p_y}$$

$$\Leftrightarrow \quad y = \frac{x p_x}{2 p_y}$$

Using budget constraint (at optimality with Cobb-Douglas, solution is on the BC).

$$\Rightarrow \quad p_x x + p_y y = I$$

$$\Leftrightarrow \quad p_x x + p_y \left[\frac{x p_x}{2 p_y} \right] = I.$$

$$\Leftrightarrow \quad 3 x p_x = 2I.$$

$$\Leftrightarrow \quad x^* = \frac{2I}{3 p_x} \Rightarrow y^* = \frac{p_x}{2 p_y} \left[x^I \right] = \frac{p_x}{2 p_y} \left[\frac{2I}{3 p_x} \right] = \frac{I}{3 p_y}$$

(b) $x^I = \dfrac{2I}{3 p_x} = \dfrac{2(600)}{3(4)} = 100$

$y^I = \dfrac{I}{3 p_y} = \dfrac{600}{3(2)} = 100$

$\eta = \dfrac{dx}{dp} \cdot \dfrac{p_x}{x^I} = \dfrac{-2I}{3 p_x^2} \dfrac{(4)}{(100)} = \dfrac{-2(600)}{3(16)} \cdot \dfrac{4}{100} = -1$

5/.

(c) $p_x' = 2$.

Final basket: $x^F = \frac{2I}{3p_x'} = \frac{2(600)}{3(2)} = 50.$

$$y^F = \frac{I}{3p_y} = \frac{600}{3(2)} = 100.$$

1. Initial basket: $(x^I, y^I) = (100, 100)$

2. Final basket: $(x^F, y^F) = (50, 100)$.

3. Decomposition basket: x^D

(i): same utility as initial value.

$$u(x^I, y^I) = 100\,(100)^{1/2} = 1000 \equiv \bar{u}^I$$

(ii): tangency new price and initial utility level.

$$MRS = \frac{p_x'}{p_y} \Leftrightarrow y = \frac{x p_x'}{2 p_y}$$

| Income : $x^F - x^D$
| Substitution : $x^D - x^I$

$$\Rightarrow u(x, y(x)) = \bar{u}^I$$

$$\Leftrightarrow x\left[\frac{x p_x'}{2p_y}\right]^{1/2} = \bar{u}^I$$

$$\Leftrightarrow x^D = 1000^{2/3}\left(\frac{2(2)}{2}\right)^{1/3} = 100 \cdot 2^{1/3}$$

1.

QUESTION 2

126 hours/week $\Rightarrow$ R or W

\$30/h

$u(R,C) = \min\{15C; C\}$.

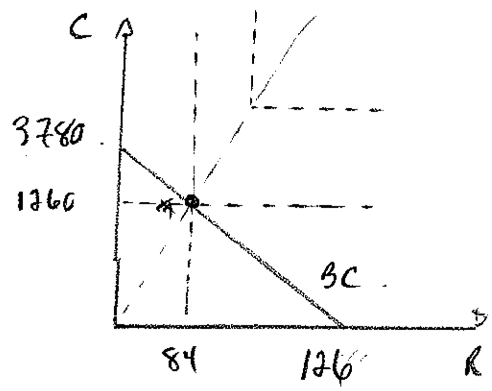
$p_C = 1$

(a) Consumption is bounded by revenue:

$$C = 30(\overbrace{126 - R}^{W}) = 3780 - 30R.$$

$$\Leftrightarrow C + 30R = 3780$$

And with perfect complement: solution is on the ray $15R = C$



$\Rightarrow$ In the BC: $(15R) + 30R = 3780$

$\Leftrightarrow R^* = 84 \Rightarrow W^* = 84$

$\Rightarrow C^* = 1260$

(b) $u(R,C) = \min\{15R; C\}$.

$\Rightarrow$ Perfect complements are goods the consumer always wants in fixed proportion to each other.

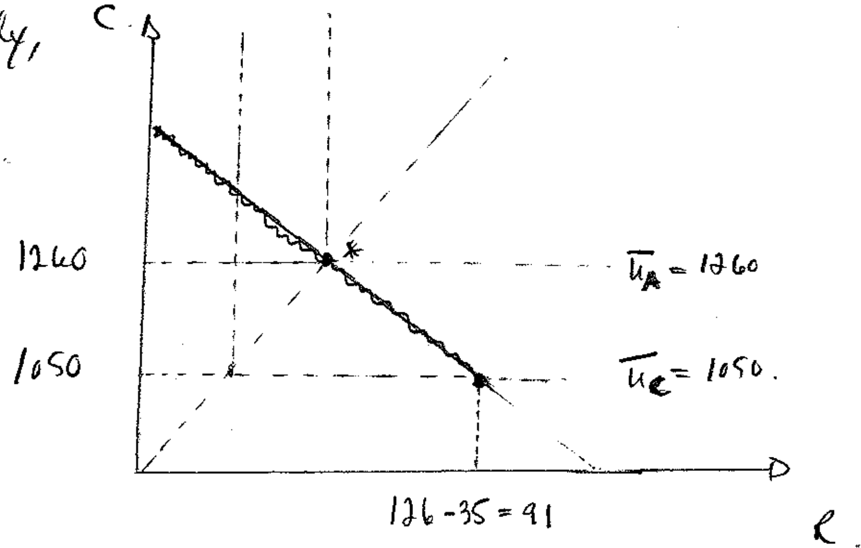
$\Rightarrow$ Here $15R = C$, that is, $\Delta R = 1 \Rightarrow \Delta C = 15 \Rightarrow \Delta u = 15$

$\Rightarrow$ Similarly $15(126 - W) = C$, that is, $\Delta W = -1 \Rightarrow \Delta C = 15$.

$\Rightarrow \Delta u = 15$.

(c). At most work 35 hours per week.

Graphically,



In part (a): $\bar{u}_A(84, 1260) = \min\{15(84); 1260\} = 1260.$

In part (c): $\bar{u}_C(91, 1050) = \min\{15(91); 1050\} = 1050.$

for (c), the solution comes from $u(R, C) = \min\{15R; C\}$ and the budget constraint now limited to the part where $W \geq 35$, that is $R \leq 91$.

QUESTION 3.

$I_p = \$ 4,000,000$

$I_F = 0$

$h(C_P, C_F) = \min\{C_P ; 3C_F\}$

Interest rate on saving of $r = 0.5$

ⓐ $C_P + S = I_P$: present period

$C_F = S(1+r)$: future period.

$\Rightarrow$ isolating "s" in future period budget constraint:

$$S = \frac{C_F}{1+r}.$$

$\Rightarrow$ With perfect complement, solution is on the ray: $C_P = 3C_F$.

$\Rightarrow$ Using Intertemporal budget constraint:

$$C_P + \left\{\frac{C_F}{1+r}\right\} = I_P.$$

$$\Rightarrow (3C_F) + \frac{C_F}{1+r} = I_P.$$

$$\Leftarrow (3(1+r)+1)C_F = I_P(1+r).$$

$$\Leftarrow C_F^* = \frac{I_P(1+r)}{4+3r} = \frac{4m(1.5)}{4+3(0.5)} = \frac{6m}{5.5} \simeq 1.09m$$

$$\Rightarrow C_P^* = \frac{3I_P(1+r)}{4+3r} \cong 3.27m.$$

9/.

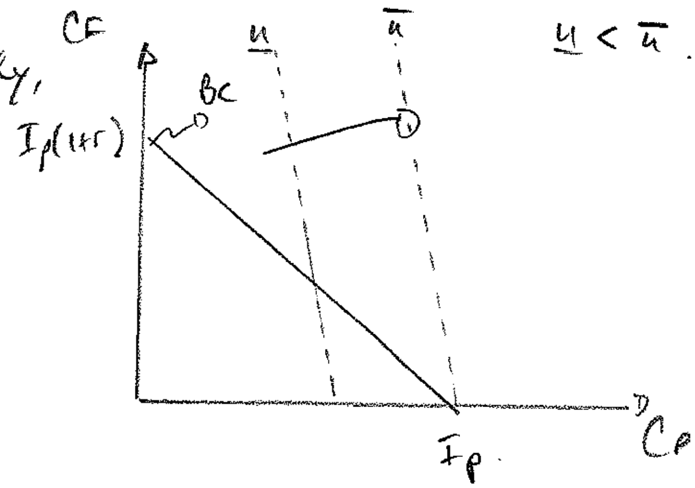
(b). Perfect substitutes: $u(C_P, C_F) = A C_P + B C_F$, $A, B > 0$.

Remember the intertemporal BC:

$$C_P + \frac{C_F}{1+r} = I_P \Leftrightarrow C_F = I_P(1+r) - (1+r)C_P.$$

Finally, to consume all of her present income in the present period, it must be that Emily is willing to give more of C_F for an additional C_P, than what the market asks for. Said otherwise, it must be that the slope of the indifference curve is steeper than the slope of the budget constraint: $-\frac{A}{B} < -(1+r)$.

Graphically, $\underline{u} < \overline{u}$.



ⓒ: present period: $C_P + S = I_P(1-t)$

future period: $C_F = S\left(1 + r(1-t)\right)$

where t: government tax: equal to 0.2

The intertemporal budget constraint is now:

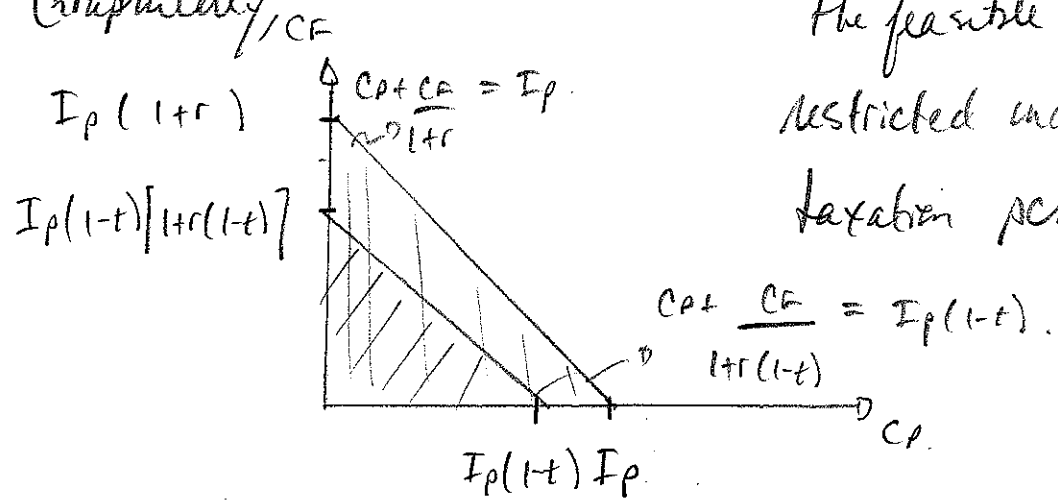
$$C_P + \frac{C_F}{1+r(1-t)} = I_P(1-t).$$

① Income is lower $I_P(1-t) < I_P$.

② Price ratio is different $1+r(1-t) < 1+r$

that is the market now gives less of C_F units for an additional unit of C_P.

Graphically



The feasible bundles are restricted under government taxation scheme.