Economics 212 Section B

Midterm Examination

October 29, 2009

Instructions: Please answer all questions in this exam booklet. If extra space is required, please continue your answer on the back of the previous page and indicate to graders that you have done so. The grade assigned to each question is indicated at the beginning of each section. You must show your work and calculations to receive full credit.

Name:

Student ID:

Section A: Each question is worth five marks.

Question One

Consider a consumer who has \$200 to allocate between cell phone service, C, and a composite consumption good, G, with a price of one. The consumer faces a choice between two cell phone plans. Plan A has a flat fee of \$50 that includes 250 minutes of service, with each additional minute available at a price of \$.50. Plan B simply charges the consumer \$.25 per minute. Draw and appropriately label the budget constraints for each of the plans. If the consumer has a utility function given by U(C,G)=C+4G, which plan will the consumer choose and what is his optimal bundle?

Question Two

Suppose that quantity demanded is given by Q^D=1,000-8P, where P is the good's price, and quantity supplied is given by Q^S=4P-200. Determine the equilibrium price and quantity in the market and calculate the elasticity of demand at the equilibrium.

Question Three

I have been offered a bet: if I put up \$100 there is a 20% probability that I will end up with \$400 and an 80% probability that I will end up with nothing. My risk preferences can be described by $U(I)=2I^{1/2}$, where I is my income. Determine why I should reject this bet. The person offering the bet would really like me to accept the bet. How much does she have to raise the winning amount above \$400 to just induce me to accept the bet? Explain.

Section B: Each of the following questions has three parts. Each part is worth five marks.

Question One

A consumer consumes two goods, X and Y, with prices given by P_X and P_Y. The consumer has an income, I, and preferences defined by $U(X,Y)=2X^{1/2}Y^{1/2}$.

a) Derive the consumer's demand functions for good X and good Y.

b) Suppose the price of X is 5, the price of Y is five, and the consumer's income is $2,000. What amount of each good does the consumer choose?

c) Let the price of good X increase to 8. Determine the final consumption bundle of the consumer and calculate the income and substitution effects of the price change.

Question Two

Aidan has 120 hours per week to divide between work and leisure, R. When he works Aidan receives a wage of \$20 per hour. His earnings from work are spent on a consumption good, C, with a price of one. Aidan's preferences over leisure and consumption are given by $U(R,C)=Min\{20R;C\}$.

a) Determine Aidan's optimal bundle and the amount of work he chooses.

b) Suppose Aidan's wage falls to \$15 per hour. Calculate his new optimal bundle and new amount of work. What can we say about the shape of Aidan's labour supply curve over this wage range?

c) Show diagrammatically how the change in leisure because of the wage decrease can be divided into an income effect and a substitution effect. Explain your diagram.

Question Three

Katherine earns $5,000,000 while working (period 1) and nothing when retired (period 2). The interest rate between her working life and her retirement is 100%. Katherine's preferences over consumption in period one, C_1, and period two, C_2, are given by $U(C_1,C_2)=C_1^{1/2}C_2^{1/2}$.

a) Determine Katherine's optimal consumption bundle and her level of savings.

b) Suppose the government announced a pension plan whereby they give all retirees, including Katherine, \$250,000 when they retire in period two. Determine the impact of this policy on Katherine's optimal bundle and her savings behavior (Assume the benefit is financed by a tax that does not affect Katherine).

c) As an alternative to the plan in part b), suppose the government instead decided to impose a tax on Katherine while she is working in order to finance a retirement benefit for her when she retires. For every \$1 in tax collected the government promises to pay \$1.50 when Katherine retires. (Effectively, this is a wage deferral plan). Show how this affects Katherine's budget line. Which of the two pension plans would Katherine prefer? Explain.

SECTION A

B

QUESTION 1

Plan A: \$50 for 250 minutes with additional minute available at \$0.50.

Plan B: \$0.25/minute

Budget: \$200 to be allocated b/w C and G (price = \$1)

Pref.: $u(C,G) = C + 4G$.

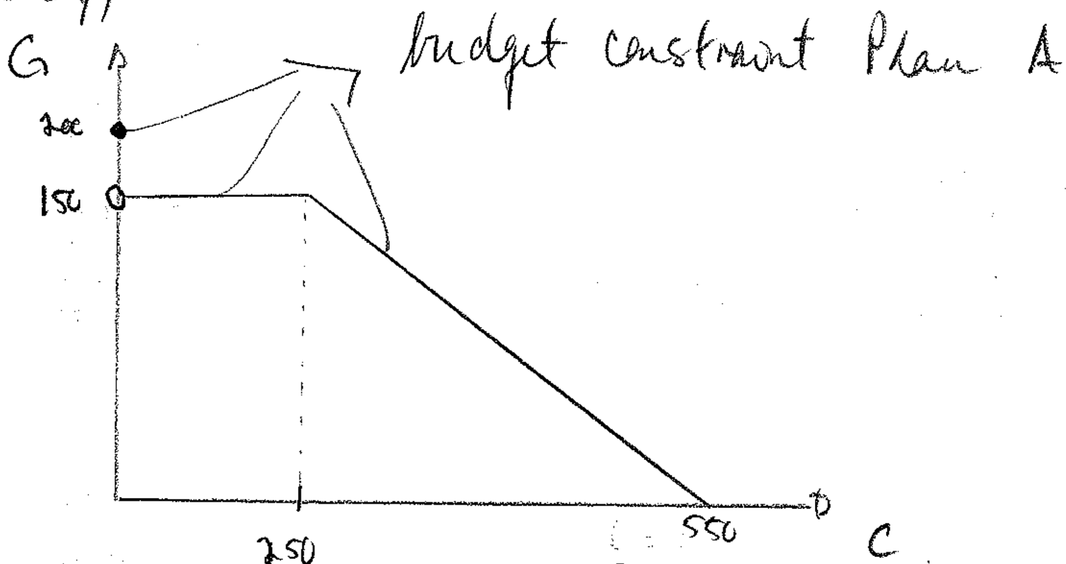
PLAN A:

* If no consumption of C: G = \$200
* If no more than 250 minutes: cost = \$50
* If more than 250 minutes: \$50 + \$0.50 C

$$\Rightarrow [50 + 0.50C] + G = 200$$

$$\Leftrightarrow G = 150 - 0.50C$$

Graphically,



$G = 0$

$\Rightarrow C = 150(2) + 250$

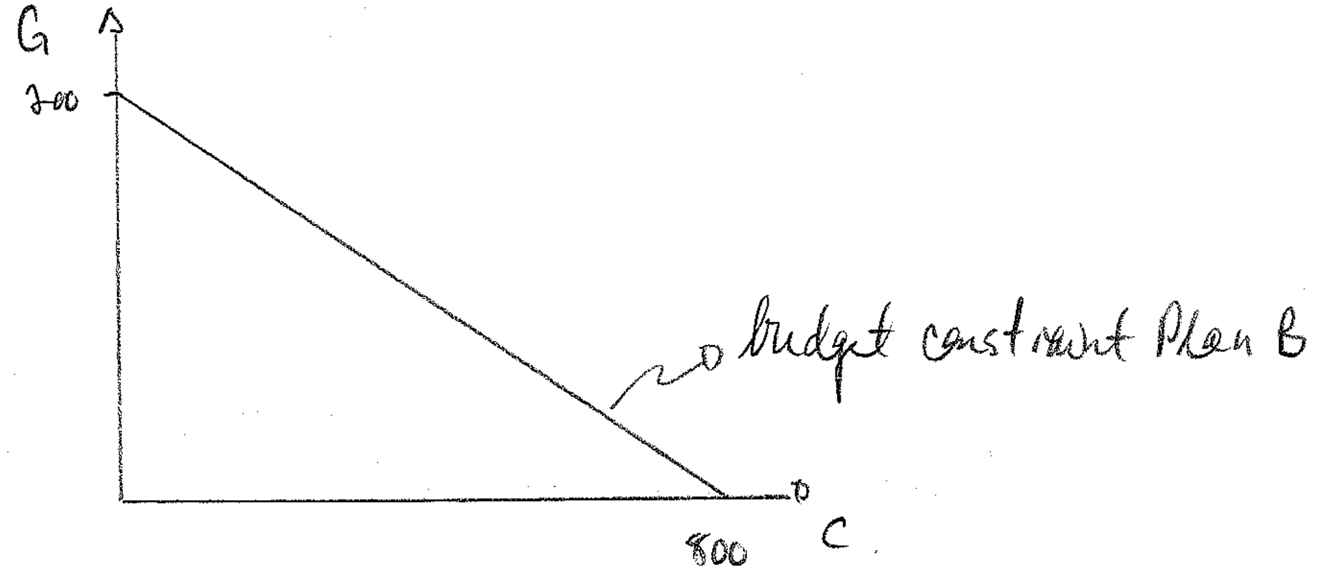
PLAN B.

- \$ 0.25/minute

$\Rightarrow 0.25C + G = 200$

$\Leftrightarrow G = 200 - 0.25C$

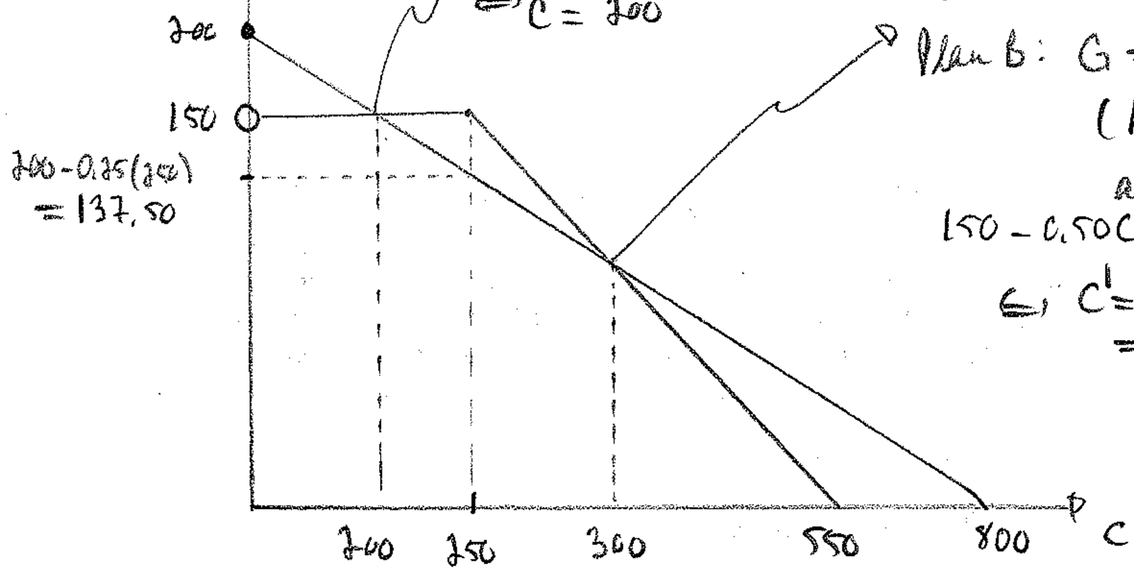
Graphically,



Plan A and B on the same graph,

$150 = 200 - 0.25C$

$\Leftrightarrow C = 200$



Plan A: $G = 150 - 0.50C'$

Plan B: $G = 137.50 - 0.25C'$

(to have both start at 250 minutes)

$150 - 0.50C' = 137.50 - 0.25C'$

$\Leftrightarrow C' = 50$

$\Rightarrow C = C' + 250 = 300$

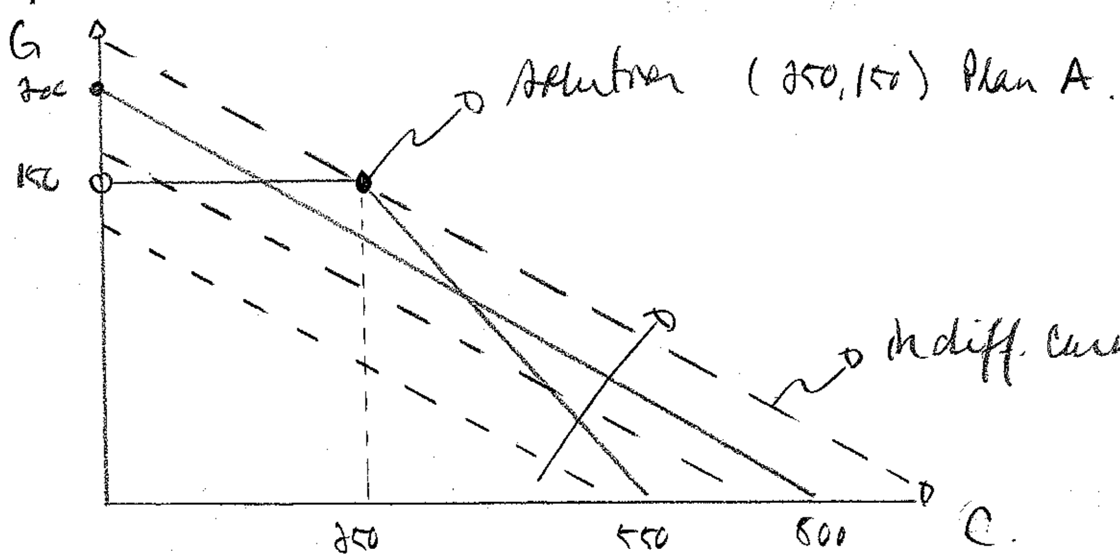
With $u(C,G) = C + 4G$ perfect substitutes, we solve graphically.

Indifference curves are given by: $G = \frac{\bar{u}}{4} - \frac{C}{4}$

<u>Note that</u>

Plan B and the indifference curves have the same slope, therefore, subject to budget constraint, we maximize utility at $C^* = 250$ and $G^* = 150$
$\Rightarrow$ Choose Plan A.

Graphically,

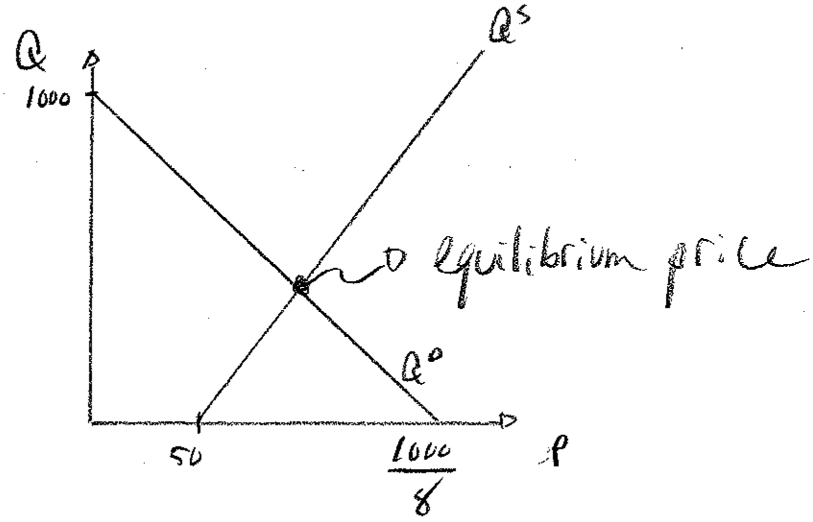


QUESTION 2

Demand: $Q^D = 1000 - 8P$

Supply: $Q^S = 4P - 200$

Graphically,



Solving: $1000 - 8P = 4P - 200 \Leftrightarrow P^* = 100 \Rightarrow Q^* = 200$

Elasticity of demand at equilibrium:

$$\eta^* = -\frac{dQ}{dP} \cdot \frac{P^*}{Q^*} = -(-8)\frac{(100)}{(200)} = 4$$

Or $\eta^* = \frac{dQ}{dP} \cdot \frac{P^*}{Q^*} = -4$ (depending on the def. used).

QUESTION 3.

A bet: \$100 VS 20% with \$400
80% with \$0

$u(I) = 2\sqrt{I}$

Sure situation is represented by $u(100) = 2\sqrt{100} = 20$

Lottery, Expected utility: $Eu = \frac{2}{10}\left[2\sqrt{400}\right] + \frac{8}{10}(0)$

$$= \frac{1}{5} \cdot 2 \cdot 20 = 8$$

To accept the bet, the minimum \$400 should be raised to is given by Δ in:

$$Eu(\Delta) = u(100) = 20$$

$$\Leftrightarrow \frac{1}{5}\left[2 \cdot \sqrt{400+\Delta}\right] = 20$$

$$\Leftrightarrow \sqrt{400+\Delta} = 50$$

$$\Leftrightarrow \Delta = 2500 - 400 = 2100$$

SECTION B

QUESTION 1

* prices : p_X + p_Y ; income: I
* $u(x,y) = 2x^{1/2}y^{1/2}$

ⓐ. $\max_{x,y} \; u(x,y) \;\; \text{s.t.} \;\; p_X x + p_Y y = I$

Utility function is a scaled Cobb-Douglas, therefore interior solution : $MRS = p_X / p_Y$

$$MRS = \frac{\partial u(\cdot)/\partial x}{\partial u(\cdot)/\partial y} = \frac{\sqrt{y/x}}{\sqrt{x/y}} = \frac{y}{x}.$$

$$\Rightarrow MRS = \frac{p_X}{p_Y} \Leftrightarrow \frac{y}{x} = \frac{p_X}{p_Y} \Leftrightarrow y = \frac{x \cdot p_X}{p_Y}$$

In the budget constraint:

$$p_X \cdot x + p_Y \left[\frac{x p_X}{p_Y} \right] = I$$

$$\Leftrightarrow x^* = \frac{I}{2 p_X} \Rightarrow y^* = \frac{I}{2 p_Y}.$$

(b) $p_X = 5$; $p_Y = 5$; $I = 2000$

from (a)

$$x_I^* = \frac{2000}{2(5)} = 200 = y_I^*$$

(c) $p_X = 5 \longrightarrow p_X' = 8$

<u>Final basket</u>: $x_F^* = \dfrac{2000}{2(8)} = \dfrac{2000}{16} = 125$

$$y_F^* = y_I^* = 200$$

<u>Intermediate Decomposition basket</u>

* initial utility level

$$u_I^* = u(x_I^*, y_I^*) = 2 \cdot 200^{1/2} \cdot 200^{1/2} = 400$$

* tangency condition with new price

$$y = \frac{x \cdot p_X'}{p_Y}$$

$\Rightarrow$ in initial utility level:

$$2\, x^{1/2} \left[\frac{x \cdot p_X'}{p_Y}\right]^{1/2} = u_I^*$$

$$\Leftrightarrow x_D^* = \frac{u_I^*}{2} \cdot \sqrt{\frac{p_Y}{p_X'}} = \frac{400}{2}\sqrt{\frac{5}{8}} \approx 158{,}11$$

$\Rightarrow$ Substitution effect: $x_D^* - x_I^* = -41,89$

Income effect: $x_F^* - x_D^* = -33,11$

QUESTION 2

120 hours divide b/w work and leisure (R)

wage = \$20/h

$p_c = 1$

$u(R,C) = \min\{20R, C\}$.

ⓐ

$$\max_{R,C} \min\{20R, C\} \text{ s.t. } C = (120 - R) \cdot 20$$

$$\Leftrightarrow C = 2400 - 20R.$$

At optimum : $20R = C$ (on the ray)

Plug in budget constraint: $20R = 2400 - 20R$

$$\Leftrightarrow R^* = 60$$

$$\Rightarrow C^* = 1200$$

ⓑ wage = \$15/h.

Budget constraint: $C = (120 - R) \cdot 15 = 1800 - 15R$

At optimum : $20R = C$ (on the ray)

Plug in budget constraint : $20R = 1800 - 15R$

$$\Leftrightarrow R^* = 1800/35 \simeq 51{,}43$$

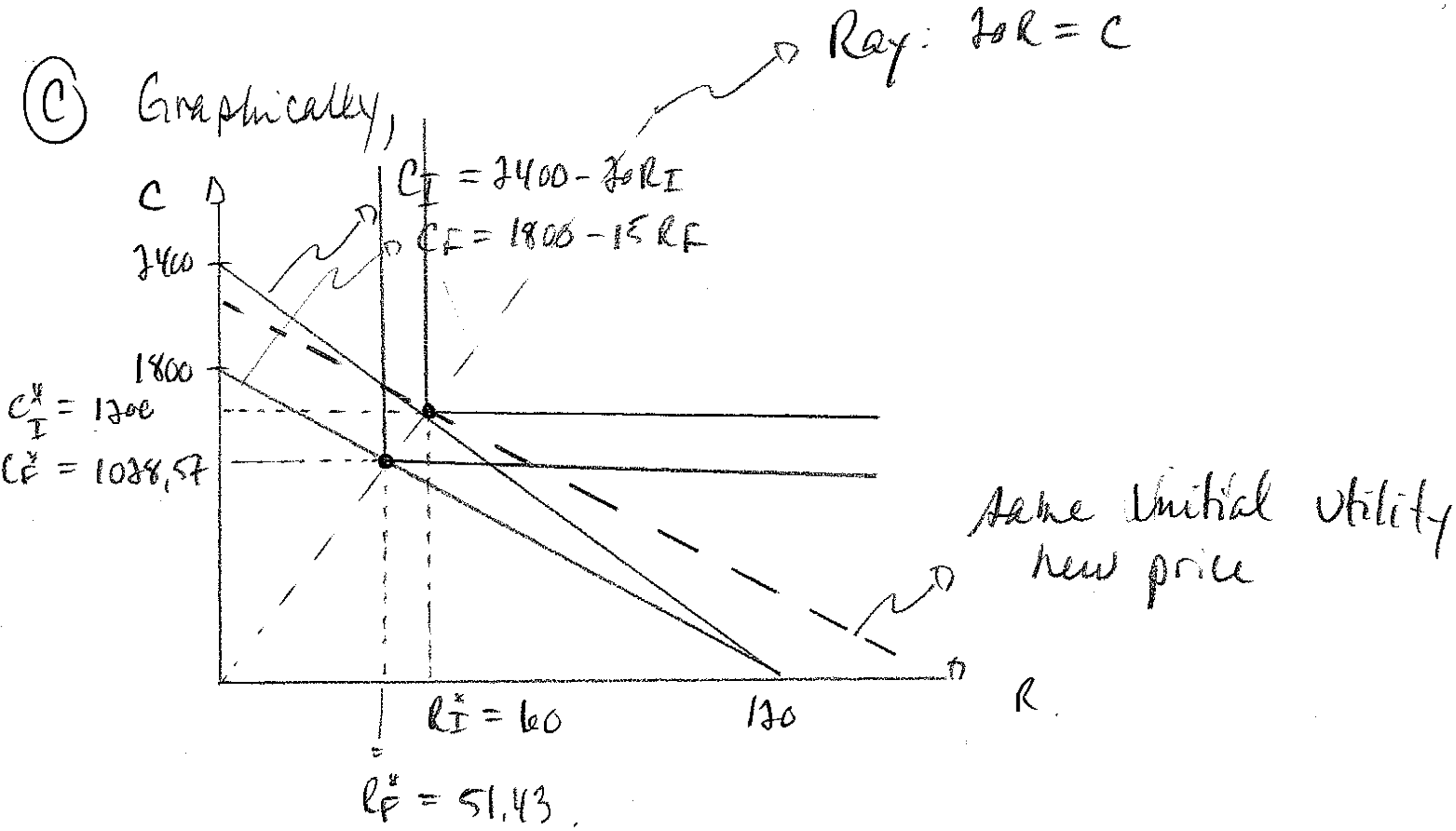
$$\Rightarrow C^* = 20R^* = 1028{,}57$$

$$\Rightarrow 120 - R^* = 68{,}57.$$

With $w \nearrow \Rightarrow$ work $\downarrow$

Decreasing labour supply curve.

ⓒ Graphically,



With Perfect Complements, substitution is null. Therefore leisure component for decomposition intermediare basket (R_D^*) equals R_I^*

Substitution effect : $R_D^* - R_I^* = 0$

Income effect : $R_F^* - R_D^* = R_F^* - R_I^* = -8,57$

QUESTION 3.

$I_1 = 5m$; $I_2 = 0$

$r = 100\%$; $u(c_1, c_2) = c_1^{1/2} c_2^{1/2}$

(a) $t=1$: $c_1 + b_1 = I_1$ b_1 -

$t=2$: $c_2 = I_2 + b_1(1+r)$

We get the intertemporal budget constraint by substituting $b_1 = \frac{c_2 - I_2}{1+r}$ in $BC(t=1)$:

Present value version: $c_1 + \frac{c_2}{1+r} = I_1 + \frac{I_2}{1+r}$

Future value version: $(1+r)c_1 + c_2 = (1+r)I_1 + I_2$

Problem: $\max_{c_1, c_2} u(c_1, c_2)$ s.t. $c_1 + \frac{c_2}{1+r} = I_1 + \frac{I_2}{1+r}$

Cobb-Douglas utility fct $\Rightarrow$ Interior solution:

$MRS = p_1/p_2$

where $MRS = \frac{\partial u(\cdot)/\partial c_1}{\partial u(\cdot)/\partial c_2} = \frac{\sqrt{c_2/c_1}}{\sqrt{c_1/c_2}} = \frac{c_2}{c_1}$

$\frac{p_1}{p_2} = \frac{1}{\left[\frac{1}{1+r}\right]} = 1+r$.

$$\Rightarrow MRS = \frac{p_1}{p_2} \Leftrightarrow \frac{C_2}{C_1} = 1+r \Leftrightarrow C_2 = (1+r)C_1$$

$\Rightarrow$ in IBC

$$C_1 + \frac{\left[(1+r)C_1\right]}{1+r} = I_1 + \frac{I_2}{1+r}$$

$$\Leftrightarrow C_1^* = \frac{1}{2}\left[I_1 + \frac{I_2}{1+r}\right].$$

$$\Rightarrow C_2^* = \frac{1+r}{2}\left[I_1 + \frac{I_2}{1+r}\right].$$

$$\Rightarrow b_1^* = \frac{\left(\frac{1+r}{2}\left[I_1 + \frac{I_2}{1+r}\right]\right) - I_2}{1+r}$$

$$= \frac{I_1}{2} + \frac{I_2}{2(1+r)} - \frac{I_2}{1+r}$$

$$= \frac{I_1}{2} - \frac{I_2}{2(1+r)}.$$

$$\Rightarrow C_1^{A*} = \frac{1}{2}\left[5m + \frac{0}{2}\right] = 2{,}5m$$

$$C_2^{A*} = \frac{2}{2}\left[5m + \frac{0}{2}\right] = 5m$$

$$b_1^{A*} = \frac{5m}{2} - \frac{0}{2(2)} = \frac{5m}{2} = 2{,}5m.$$

(b) in $t=2$: 250 000 and tax does not affect Katherine.

$t=1$: $c_1 + b_1 = I_1$

$t=2$: $c_2 = b_1(1+r) + R$, $R = 250\,000$

Note that, from (a) general formulation, $I_2^G = R$, where I_2^G denote I_2 in the general formulation of (a).

$$\Rightarrow c_1^{B*} = \frac{1}{2}\left[I_1 + \frac{R}{1+r}\right] = \frac{1}{2}\left[5\,000\,000 + \frac{250\,000}{2}\right] = 2562500$$

$$c_2^{B*} = \frac{1+r}{2}\left[I_1 + \frac{R}{1+r}\right] = \frac{2}{2}\left[5\,000\,000 + \frac{250\,000}{2}\right] = 5\,125\,000$$

$$b_1^{B*} = \frac{I_1}{2} - \frac{R}{2(1+r)} = \frac{5\,000\,000}{2} - \frac{250\,000}{2(2)} = 1\,187\,500$$

IBC: $c_1 + \frac{c_2}{1+r} = I_1 + \frac{R}{1+r}$

(c) tax on Katherine received in $t=2$

every \$1 tax collected $t=1 \Rightarrow$ \$1.50 given $t=2$.

$t=1$: $C_1 + b_1 = I_1(1-t)$

$t=2$: $C_1 = b_1(1+r) + 1.5tI_1$, where t is tax

Again, consider @ general formulation $I_2^G = 1.5tI_1$, $I_1^G = I_1(1-t)$.

$\Rightarrow$ Present value budget constraint:

$$C_1 + \frac{C_2}{1+r} = \underbrace{I_1(1-t)}_{\text{less today}} + \underbrace{\frac{1.5tI_1}{1+r}}_{\text{more tomorrow}}$$

Since both pension plans have the same slope (bc), the only difference is the total income. Therefore, if a plan allows for more disposable income it will be prefered.

Prefer B if $I_1 + \frac{R}{1+r} > I_1(1-t) + \frac{1.5tI_1}{1+r}$

$$\Leftrightarrow I_1(1+r) + R > I_1(1-t)(1+r) + 1.5tI_1$$

$$\Leftrightarrow R > -tI_1(1+r) + 1.5tI_1$$

$$\Leftrightarrow R > tI_1[1.5 - (1+r)]$$

$$\Leftrightarrow \frac{-R}{I_1(1+r-1.5)} < t$$ which is always true since $r=1$ and $t>0$ by definition.