Economics 212 Section A

Midterm Examination

October 29, 2009

Instructions: Please answer all questions in this exam booklet. If extra space is required, please continue your answer on the back of the previous page and indicate to graders that you have done so. The grade assigned to each question is indicated at the beginning of each section. You must show your work and calculations to receive full credit.

Name:

Student ID:

Section A: Each question is worth five marks.

Question One

Consider a low-income consumer who has \$1,000 to allocate between groceries, G, and transportation, T. The price of a unit of groceries, P_G, is \$10 and the price of a unit of transportation, P_T, is \$10. Draw and appropriately label this budget constraint. The government wishes to subsidize both groceries and transportation for low-income consumers. They devise a plan whereby the price of each good will fall to \$5, but only after the consumer has spent \$200 on each good. Draw and appropriately label the new budget constraint. Please put groceries on the horizontal axis.

Question Two

Suppose that quantity demanded is given by Q^D=12,000-180P, where P is the good's price, and quantity supplied is given by Q^S=20P. Determine the equilibrium price and quantity in the market and calculate the elasticity of supply at the equilibrium.

$25600

Question Three

I have been offered a bet: if I put up \$225 there is a 10% probability that I will end up with \$2,5600 and a 90% probability that I will end up with nothing. My risk preferences can be described by $U(I)=I^{1/2}$, where I is my income. Determine why I should accept this bet. The person offering the bet believes that I would still accept the bet if she offered me a winning amount lower than \$2,5600. How much can she lower the winning amount below \$2,5600 and still have me accept the bet? Explain.

Section B: Each of the following questions has three parts. Each part is worth five marks.

Question One

A consumer consumes two goods, X and Y, with prices given by P_X and P_Y. The consumer has an income, I, and preferences defined by $U(X,Y)=X^{1/3}Y^{2/3}$.

a) Derive the consumer's demand functions for god X and good Y.

b) Suppose the price of X is \$4, the price of Y is \$2, and the consumer's income is \$1,200. What amount of each good does the consumer choose?

c) Let the price of good X decrease to \$3. Determine the final consumption bundle of the consumer and calculate the income and substitution effects of the price change.

Question Two

Rick has 16 hours per day to divide between work and leisure, R. When he works Rick receives a wage of \$30 per hour. His earnings from work are spent on a consumption good, C, with a price of one. Rick's preferences over leisure and consumption are given by $U(R,C)=\text{Min}\{60R;3C\}$.

a) Determine Rick's optimal bundle and the amount of work he chooses.

b) Suppose Rick's boss informs him that he must work a 12 hour shift or he will not work at all. Which option will Rick choose? Show that Rick is worse off compared to part a).

c) Start from the optimal bundle in part a). Rick's boss used to offer Rick overtime at "time and a half", i.e., a wage of $45. Rick was free to work the overtime in any amount he chose or to refuse it. He always refused the opportunity to work overtime. Use a diagram to explain Rick's decision.

Question Three

Emily earns $4,000,000 while working (period 1) and nothing when retired (period 2). The interest rate between her working life and her retirement is 80%. Emily's preferences over consumption in period one, C_1, and period two, C_2, are given by $U(C_1,C_2)=C_1^{1/2}C_2^{1/2}$.

a) Determine Emily's optimal consumption bundle and her level of savings.

b) Suppose the government announces a pension plan whereby they give all retirees, including Emily, \$100,000 when they retire in period two. Determine the impact of this policy on Emily's optimal bundle and her savings behavior (Assume the benefit is financed by a tax that does not affect Emily).

c) As an alternative to the plan in part b), suppose the government instead decided to impose a tax on Emily while she is working in order to finance a retirement benefit for her when she retires. For every \$1 in tax collected the government promises to pay \$1.80 when Emily retires. (Effectively, this is a wage deferral plan). Show how this affects Emily's budget line. Which of the two pension plans would Emily prefer? Explain.

SECTION A.

$\boxed{A}$

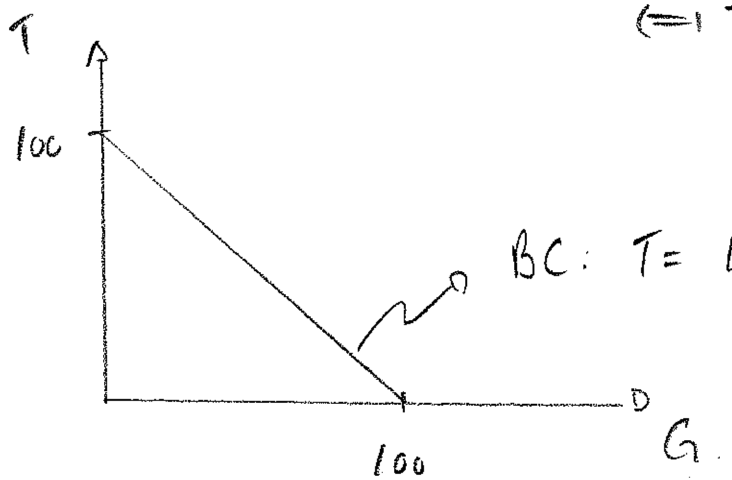
QUESTION 1

(i) $I = \$1000$ b/w G and T

$p_G = p_T = \$10$

Graphically, BC: $10G + 10T = 1000$

$\Leftrightarrow T = 100 - G$



(ii) Subsidize: when spent more than \$ 200 on a good, price drop to \$5

4 cases

① $G, T < 20 \Rightarrow T \leq 100 - G$

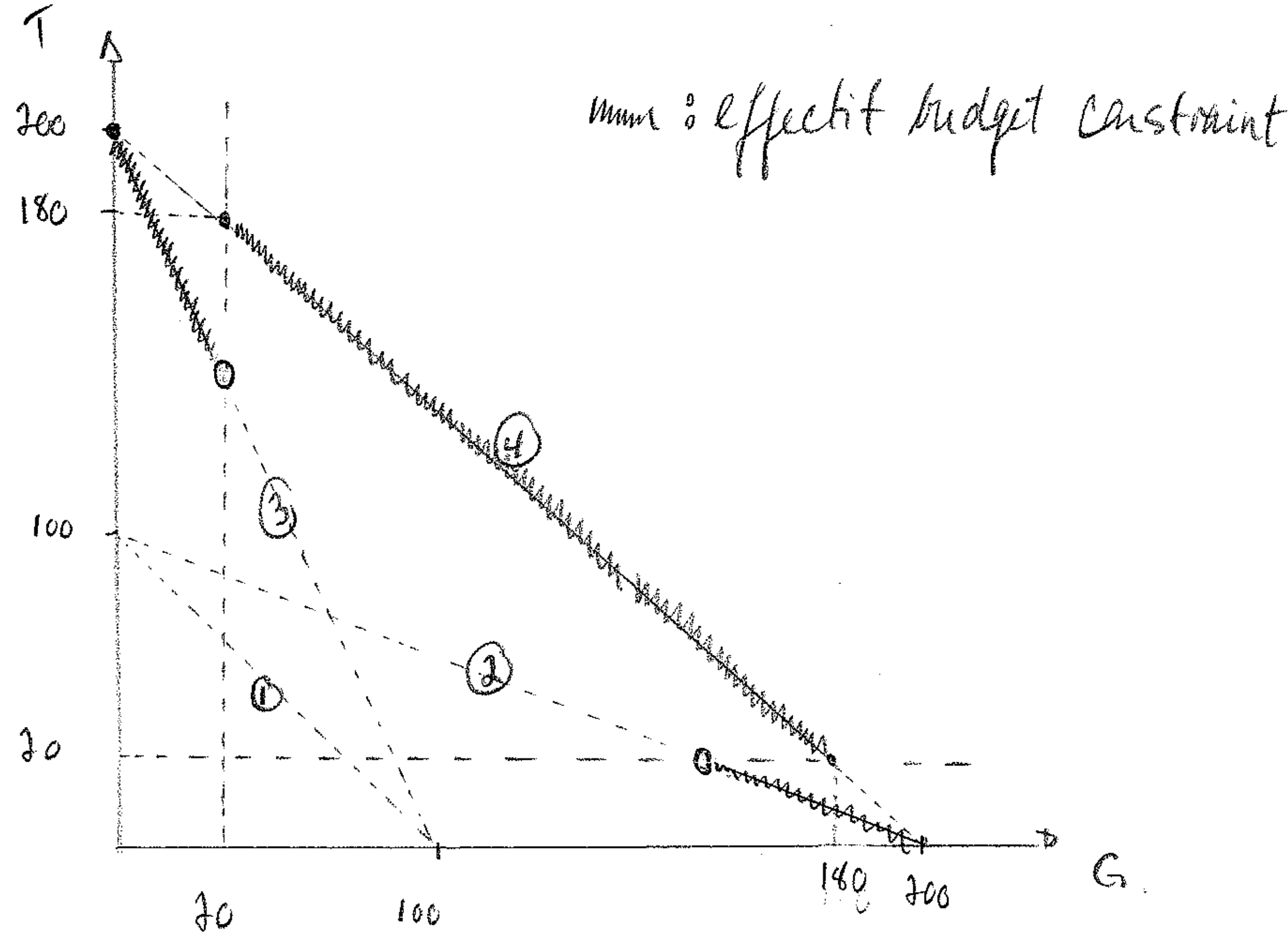
② $G \geq 20, T < 20 \Rightarrow 5G + 10T \leq 1000 \Leftrightarrow T \leq 100 - \frac{1}{2}G$

③ $T \geq 20, G < 20 \Rightarrow 10G + 5T \leq 1000 \Leftrightarrow T \leq 200 - 2G$

④ $T, G \geq 20 \Rightarrow 5G + 5T \leq 1000 \Leftrightarrow T \leq 200 - G$

where "20" comes from $10G \geq 200$ & $10T \geq 200$

Graphically,



Note that ① BC is not effectif because "" curve embed the region.

QUESTION 2

$$Q^D = 12\,000 - 180P$$
$$Q^S = 20P$$

Equilibrium price where $Q^D = Q^S$

$$\Rightarrow 12\,000 - 180P = 20P \Leftrightarrow P^* = 60$$
$$\Rightarrow Q^* = 1200$$

as for elasticity of supply at equilibrium:

$$\eta = \frac{dQ}{dP} \cdot \frac{P^*}{Q^*} = 20\,\frac{(60)}{1200} = 1$$

QUESTION 3.

if put \$225 bet 10% end up with \$25600

$u(I)=\sqrt{I}$ 90% ——— \$0

Sure bet: $u(225)=\sqrt{225}=15$.

Taking the bet yield: $Eu = \frac{1}{10}u(25600)+\frac{9}{10}u(0)$

$= \frac{1}{10}\cdot 160 = 16$.

Since $Eu > u(225) \Rightarrow$ take the bet.

Minimum value \$25600 can be reduce to still accept the bet:

$$\frac{1}{10}u\left[25600-\Delta\right]+\frac{9}{10}u(0) = u(225)$$

$$\Leftrightarrow \frac{1}{10}\sqrt{25600-\Delta} = 15$$

$$\Leftrightarrow \Delta = 25600 - 150^2 = 3100$$

SECTION B

QUESTION 1

$u(x,y) = x^{1/3} y^{2/3}$

ⓐ $\max_{x,y} \; u(x,y)$ p.t. $p_X x + p_Y y = I$

Cobb-Douglas utility function $\Rightarrow$ interior solution
$MRS = p_X / p_Y$

$$MRS = \frac{\partial u(\cdot)/\partial x}{\partial u(\cdot)/\partial y} = \frac{1/3 \cdot (y/x)^{2/3}}{2/3 \, (x/y)^{1/3}} = \frac{y}{2x}$$

$$\Rightarrow MRS = \frac{p_X}{p_Y} \Leftrightarrow \frac{y}{2x} = \frac{p_X}{p_Y} \Leftrightarrow y = \frac{2x p_X}{p_Y}$$

$$\Rightarrow \text{In BC: } p_X x + p_Y \left[\frac{2x p_X}{p_Y} \right] = I$$

$$\Leftrightarrow x^* = \frac{I}{3 p_X} \Rightarrow y^* = \frac{2I}{3 p_Y}$$

(b) $p_x = 4$; $p_y = 2$; $I = 1200$

$$x_I^* = \frac{I}{3p_x} = \frac{1200}{3(4)} = 100$$

$$y_I^* = \frac{2I}{3p_y} = \frac{2(1200)}{3(2)} = 400$$

(c) $p_x = 4 \rightarrow p_x' = 3$.

Final consumption bundle: $x_F^* = \dfrac{I}{3p_x'} = \dfrac{1200}{3(3)} \approx 133{,}33$

$$y_F^* = y_I^* = 400$$

Intermediate decomposition basket:

* same initial utility: $u_I^* = u(x_I^*, y_I^*) = 100^{1/3}\, 400^{2/3} \approx 251{,}98$
* tangency new price and initial utility level

Tangency: $MRS = \dfrac{p_x'}{p_y} \Leftrightarrow y = \dfrac{2xp_x'}{p_y}$

$$\Rightarrow x^{1/3} \left[\frac{2xp_x'}{p_y}\right]^{2/3} = u_I^* \Leftrightarrow x_D^* = u_I^* \left[\frac{p_y}{2p_x'}\right]^{2/3}$$

$$= u_I^* \left[\frac{2}{2(3)}\right]^{2/3}$$

$$= \frac{u_I^*}{3^{2/3}} \approx 121{,}14$$

$\Rightarrow$ Income eff.: $x_F^* - x_D^* = 12{,}19$

Subs. eff.: $x_D^* - x_I^* = 21{,}14$

QUESTION 2.

16 hours divide b/w work and leisure (R)

wage = \$30/h

$u(R,C) = \min\{60R, 3C\}$.

ⓐ Budget constraint: $C = 30(16-R) \Leftrightarrow C + 30R = 480$

With Perfect Complements the solution is on the ray: $60R = 3C$

$\Leftrightarrow 20R = C$

$\Rightarrow$ highest utility is obtained on the budget constraint: $(20R) + 30R = 480$

$\Leftrightarrow R^* = \frac{48}{5} \simeq 9.6$

$\Rightarrow 16 - R^* = 6.4$

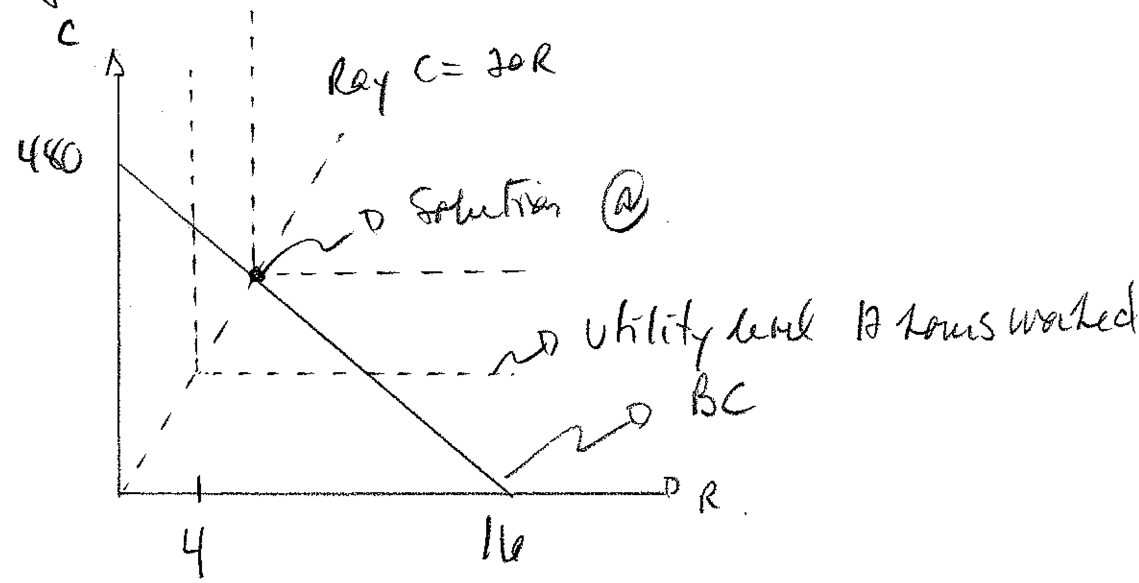
$\Rightarrow C^* = 192$

(b) Must work 12 hours or not work at all.

If working 12 hours $\Rightarrow R = 4$.

If not working $\Rightarrow C = 0$ & $R = 16$.

Graphically,



With restriction hours worked: $u(4, c) = 60(4) = 240$

if not working: $u(16, 0) = 0$

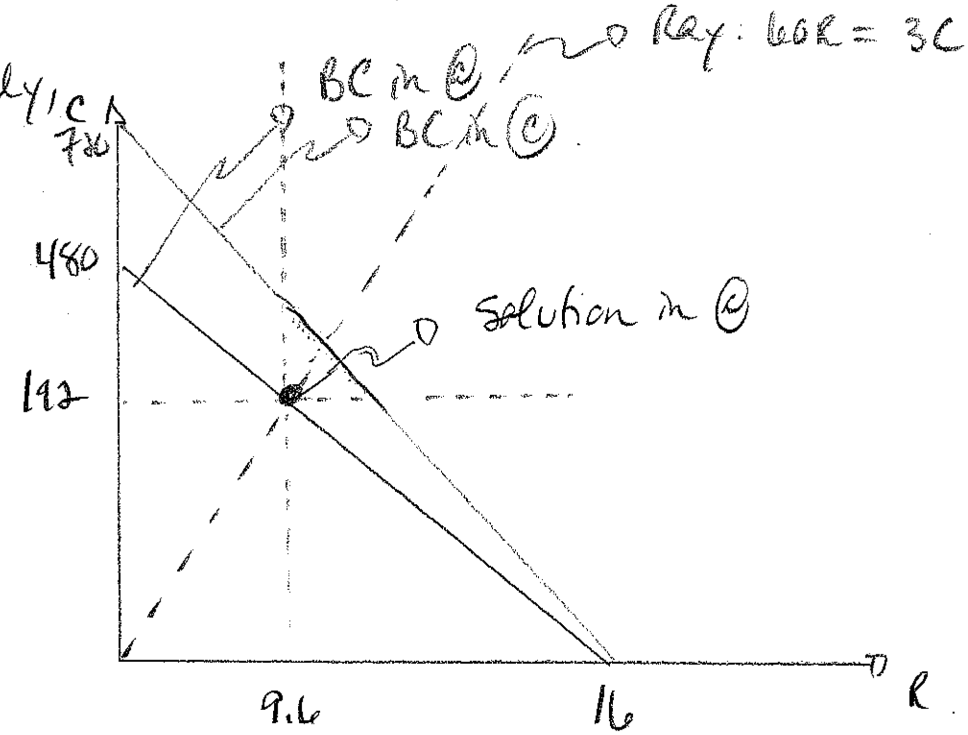
from @: $u(9.6, 192) = 576$

$\Rightarrow \quad u(16, 0) < u(4, c) < u(9.6, 192)$.

ⓒ Overtime \$45/h.

New BC: $C = 45(16 - R) \Leftrightarrow C + 45R = 720$

Graphically,



To induce Rick to work more, the boss must create an incentive such that R^* is now less than 9.6. This is not compatible with the proposed overtime since preferences are perfect complements.

With wage increase we graphically see that Rick likes having more leisure.

QUESTION 3.

$I_1 = 4m \quad ; \quad I_2 = 0$

$r = 0.8 \quad ; \quad u(c_1, c_2) = c_1^{1/2} c_2^{1/2}$

ⓐ. Budget constraint:

$t=1: \quad c_1 + b_1 = I_1$

$t=2: \quad c_2 = b_1(1+r) + I_2 \Rightarrow b_1 = \frac{c_2 - I_2}{1+r}$

$\Rightarrow$ Intertemporal BC in present term:

$$c_1 + \frac{c_2}{1+r} = I_1 + \frac{I_2}{1+r}$$

Cobb-Douglas $\Rightarrow$ interior solution: $MRS = \frac{p_1}{p_2}$

$$\Rightarrow MRS = \frac{\partial u(\cdot)/\partial c_1}{\partial u(\cdot)/\partial c_2} = \frac{c_2}{c_1} \quad \& \quad \frac{p_1}{p_2} = \frac{1}{\left(\frac{1}{1+r}\right)} = 1+r$$

$$\Rightarrow c_2 = (1+r)\, c_1$$

$$\Rightarrow \text{In IBC}: \quad c_1 + \frac{(1+r)c_1}{1+r} = I_1 + \frac{I_2}{1+r}$$

$$\Leftarrow c_1^* = \frac{1}{2}\left[I_1 + \frac{I_2}{1+r} \right].$$

$$\Rightarrow c_2^* = \frac{1+r}{2}\left[I_1 + \frac{I_2}{1+r} \right].$$

$$\Rightarrow b_1^* = \frac{1+r}{2(1+r)}\left[I_1 + \frac{I_2}{1+r} \right] - \frac{I_2}{1+r} = \frac{I_1}{2} - \frac{I_2}{2(1+r)}.$$

$$\Rightarrow C_1^* = \frac{1}{2} \cdot 4m = 2m.$$

$$C_2^* = 1{,}8\, C_1^* = 3{,}6\, m$$

$$b_1^* = \frac{4m}{2} = 2m.$$

(b) From BC general form in (a), replace I_2 by $R = \$100\,000$

$$\Rightarrow C_1^* = \frac{1}{2}\left[I_1 + \frac{R}{1+r}\right] = \frac{1}{2}\left[4m + \frac{100\,000}{1{,}8}\right] \approx 2\,006\,250$$

$$C_2^* = \frac{1+r}{2}\left[I_1 + \frac{R}{1+r}\right] = \frac{1{,}8}{2}\left[4m + \frac{100\,000}{1{,}8}\right] \approx 3\,611\,250$$

$$b_1^* = \frac{I_1}{2} - \frac{R}{2(1+r)} = \frac{4m}{2} - \frac{100\,000}{2(1{,}8)} = 1\,972\,222{,}22$$

(c) \$1 tax collected in $t=1 \Rightarrow$ get \$1,8 $t=2$ per \$1 collected.
Assume t: % income collected

$t=1$: $C_1 + b_1 = I_1(1-t)$

$t=2$: $C_2 = b_1(1+r) + 1{,}8 I_1 t \Rightarrow b_1 = \dfrac{C_2 - 1{,}8 I_1 t}{1+r}.$

$$\Rightarrow C_1 + \frac{C_2}{1+r} = I_1(1-t) + \frac{1{,}8 I_1 t}{1+r}$$

Since both pension plans have the same slope (BC), the only difference is the total income. Therefore, if a plan allows for more disposable income, it will be prefered.

Prefer B if $I_1 + \frac{R}{1+r} > I_1(1-t) + \frac{1.8 I_1 t}{1+r}$

Since $r = 0.8$ and after canceling I_1 on both sides

$$\Rightarrow \frac{R}{1.8} > -I_1 t + I_1 t = 0$$

And since $R > 0$ all the time, we have that disposable income of (b) is higher, therefore the plan is prefered.