Economics 212

Section 002

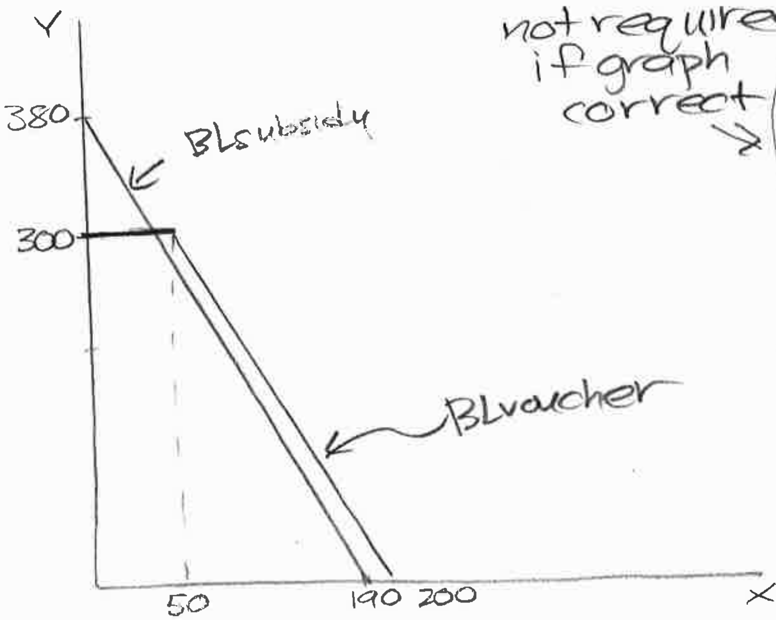
Midterm Exam

March 7, 2017

Student Number: Solutions

Section A: Three questions @ 5 marks. Total 15 marks.

1. [5 marks] A consumer consumes a composite good, Y, and food, X. The government can offer an income subsidy of \$80 to be spent on either good, or a food voucher of \$100 to be spent on food only. Suppose the consumer's original income is \$300, the price of food is \$2, and the price of the composite good is \$1. Plot the budget line under the income subsidy program and the voucher program using a single graph. Please label your diagrams appropriately.



not required if graph correct

Original BC:

$Y = \frac{I}{P_Y} - \frac{P_X}{P_Y} X$

$\rightarrow Y = 300 - 2X$

Under subsidy

$Y = \frac{I+S}{P_Y} - \frac{P_X}{P_Y} X$

$Y = 380 - 2X$

Under voucher:

Can buy $\frac{\$100}{2} = 50$ X with voucher

$Y = 300$ if $X \leq 50$

$Y = 400 - 2X$ if $X > 50$

Max X is increased by 50.

2. [5marks] A consumer must decide whether to risk \$100 buying a lottery ticket. The lottery has three outcomes: a probability of .5 that the consumer finishes the lottery with \$16, a probability of .4 that the consumer finishes with \$100, and a probability of .1 that the consumer finishes with \$324. Explain whether the consumer will accept or reject the lottery given a utility of income function $U=I^{1/2}$. Now determine how the largest prize (\$324) must be changed such that the consumer is indifferent between rejecting and accepting the lottery.

$EU(\text{keep money}) = (100)^{\frac{1}{2}} = 10$

$EU(\text{play}) = 0.5(16)^{\frac{1}{2}} + 0.4(100)^{\frac{1}{2}} + 0.1(324)^{\frac{1}{2}}$

$= 0.5 \cdot 4 + 0.4 \cdot 10 + 0.1 \cdot 18$

$= 7.8$

Expected utility is higher without playing
$\rightarrow$ the consumer will not play

(Note: lottery must be very generous for a risk-averse consumer such as this one to play)

Need $EU(\text{play}) = 10$

$\rightarrow 0.5 \cdot 4 + 0.4 \cdot 10 + 0.1\sqrt{x} = 10$

$\rightarrow 6 + 0.1\sqrt{x} = 10 \rightarrow 0.1\sqrt{x} = 4 \rightarrow \sqrt{x} = 40$

$\rightarrow x^* = 1600.$

The largest prize must increase to 1600.

3. [5marks]Given the utility function U=6X+3Y and parameter values where P_X=8, P_Y=3 and income, I=$1800, determine the consumer's optimal bundle of goods X and Y.

$$\frac{MU_X}{P_X} = \frac{6}{8} < \frac{3}{3} = \frac{MU_Y}{P_Y}$$

$\Rightarrow$ buy all Y

$$Y^* = \frac{I}{P_Y} = \frac{1800}{3} = 600$$

$$(X^*, Y^*) = (0, 600)$$

Section B: Three question @ 15 marks- 5 for each part of each question. Total 45 marks.

1. Kai consumes two goods, X and Y, according to the utility function $U=X^{1/3}Y$. Kai has an income, I, and faces prices for the two goods given by P_X and P_Y.
   a) [5 marks] Derive Kai's demand functions for the goods X and Y.

not required

$\max U = X^{1/3}Y$

s.t. $p_x X + p_y Y = I$

Tangency cond$^{\underline{n}}$

$$MRS_{x,y} = \frac{p_x}{p_y}$$

$$\Leftrightarrow \frac{MU_x}{MU_y} = \frac{p_x}{p_y}$$

$$\Leftrightarrow \frac{\frac{1}{3}X^{-2/3}Y}{X^{1/3}} = \frac{p_x}{p_y}$$

$$\rightarrow \frac{1Y}{3X} = \frac{p_x}{p_y} \rightarrow Y = \frac{3p_x}{p_y}X \quad (1)$$

Budget line:

$$p_x X + p_y Y = I \quad (2)$$

Sub (1) into (2)

$$p_x X + p_y \frac{3p_x X}{p_y} = I \rightarrow 4p_x X = I$$

$$X^* = \frac{I}{4p_x} \qquad Y = \frac{3p_x}{p_y}\left(\frac{I}{4p_x}\right) \rightarrow Y^* = \frac{3I}{4p_y}$$

b) [5 marks] Let Kai's income be \$1600, the price of X be \$1 and the price of Y be \$3. How much of each good should Kai consume?

$$X = \frac{I}{4p_X} = \frac{1600}{4(1)}$$

$$X^* = 400$$

$$Y = \frac{3I}{4P_Y} = \frac{3(1600)}{4(3)}$$

$$Y^* = 400$$

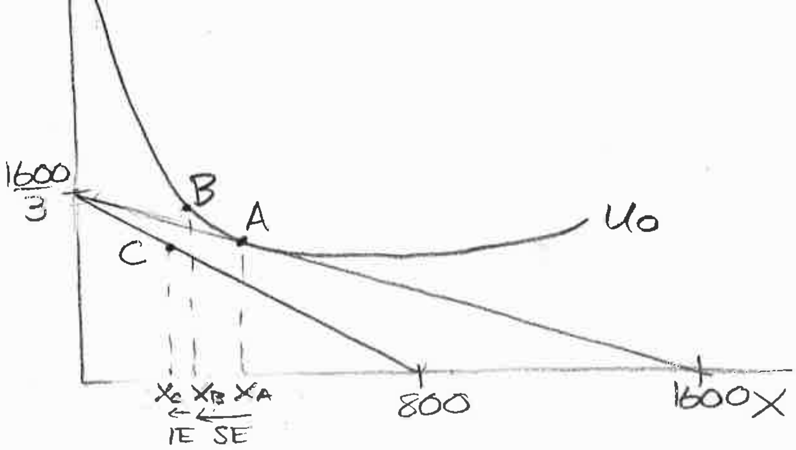
c) [5 marks]Suppose the price of X increases to \$2. Determine the new demand for the goods and calculate the income and substitution effects of the price change.

Initial basket $(x_a, y_a) = (400, 400)$

Final basket $(x_c, y_c) = \left(\frac{1600}{4 \cdot 2}, 1600\right) = (200, 400)$

Need decomposition x, x_b, a bundle on the original indifference curve, but on the compensated budget line.

Not required



$$X^{\frac{1}{3}}Y = U_0 = (400)^{\frac{1}{3}}400 \quad (1)$$

$$\frac{Y}{3X} = \frac{MU_X}{MU_Y} = \frac{p_X'}{P_Y} = \frac{2}{3} \quad (2)$$

$$(2) \rightarrow Y = 2X \quad (3)$$

Put (3) into (1)

$$X^{\frac{1}{3}}2X = 400^{\frac{1}{3}}400$$

$$X^{4/3} = 400^{\frac{1}{3}}400$$

$$X_b = \left(\frac{400^{\frac{1}{3}}400}{2}\right)^{3/4}$$

$$X_b = 237.84$$

$$SE = X_b - X_a = 237.84 - 400$$
$$= -162.16$$

$$IE = X_c - X_b = 200 - 237.84$$
$$= -37.84$$

2. Martha has 120 hours per week to divide between leisure, R, and work, L. When she works, Martha earns \$10 per hour. She values both leisure and consumption, C, according to the utility function U=Min{10R ; C}. The price of the consumption good is unity.

   a) [5marks] Derive Martha's optimal bundle. How much does she work? Calculate her level of utility.

$\max U = \min\{10R, C\}$

s.t. $C = w(\text{hours} - R) \Rightarrow C = 10(120 - R)$

optimality cond$^{\underline{n}}$

$10R = C$

sub into BL:

$10R = 1200 - 10R$

$20R = 1200$

$R^* = 60$

$C = 10R = 10 \cdot 60$

$C^* = 600$

$(R^*, C^*) = (60, 600)$

$U_0 = \min\{10(60), 600\} = 600$

   b) [5 marks] The government decides that Martha's wage is too low and offers a subsidy at the rate of \$5 per hour on Martha's earnings (assume her wage rises to \$15/hour). Find Martha's new optimal bundle, including the amount of work and leisure chosen. In words, explain this outcome in terms of the income and substitution effects of the ~~tax.~~ subsidy.

BL $\rightarrow$ $C = 15(120 - R)$

$10R = 1800 - 15R$

$25R = 1800$

$R^* = 72$

$C^* = 720$

$(R^*, C^*) = (72, 720)$

WT makes leisure expensive
$\rightarrow$ substitution t/w work

but income effect (more income per hours worked) causes substitution t/w leisure

Leisure increased, so income effect of the tax must be larger than the substitution effect of the raise

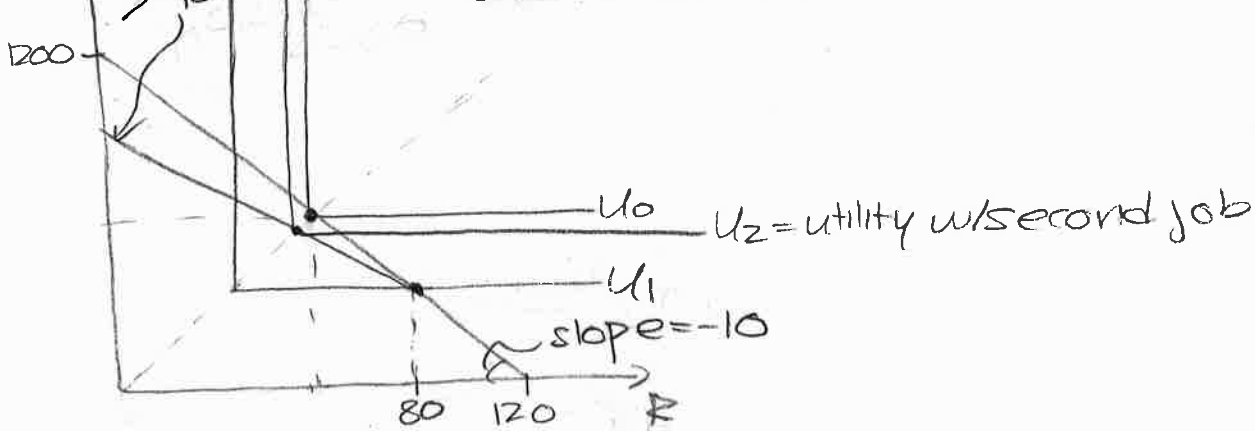
c) Starting from the solution to part (a), assume Martha's boss tells her that she must work 40 hours per week. Show that Martha is worse off. Starting from the situation where Martha works only 40 hours per week, show graphically that Martha would be willing to accept a second job at a wage lower than her current wage.

Now $R = 120 - 40 = 60$

$C = 40 \cdot 10 = 400$

$U_1 = \min\{10(40), 400\} = 400 < U_0$

She is worse off.



3. [5 marks] Kevin works in the present period and earns an income of \$8,000,000. In the future period, Kevin is partly retired and earns \$2,000,000. His preferences over present consumption, C_P, and future consumption, C_F, are given by $U(C_P,C_F)=C_P C_F$. Kevin can save or borrow at an interest rate of 100%.

   a) Derive Kevin's optimal consumption bundle and his level of savings or borrowing.

$\max U(C_P, C_F) = C_P C_F$

$\text{s.t. } C_P + \frac{C_F}{(1+r)} = I_P + \frac{I_F}{(1+r)} \rightarrow C_P + \frac{C_F}{2} = 8m + \frac{2m}{2} = 9m$

Tangency:

$\frac{MU_{CP}}{MU_{CF}} = (1+r)$

$\rightarrow \frac{C_F}{C_P} = 2 \rightarrow C_F = 2C_P$

Sub into BL

$C_P + \frac{2C_P}{2} = 9m$

$2C_P = 9m$

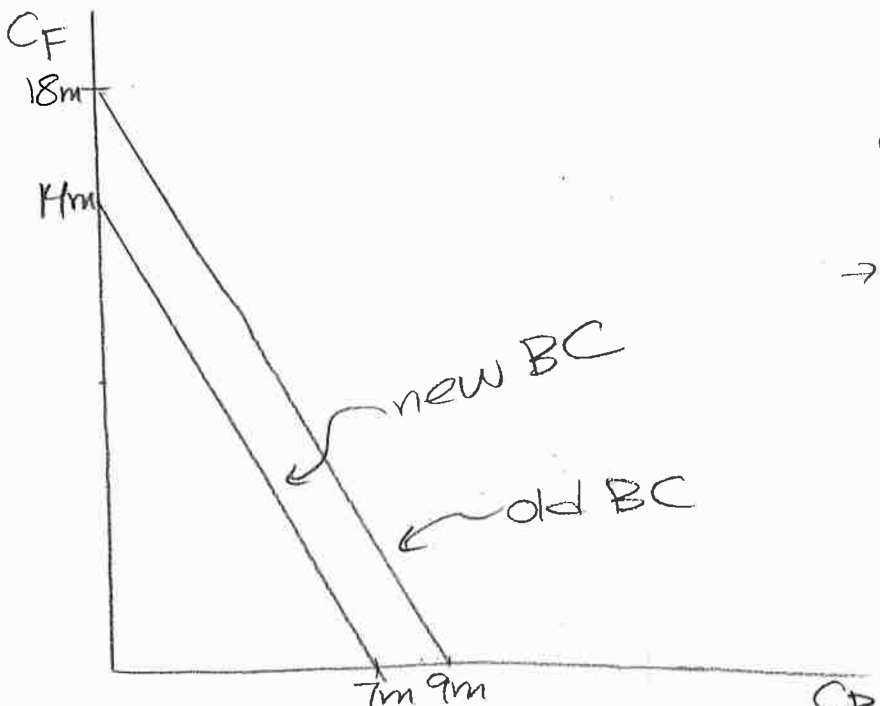
$C_P^* = 4.5m$

$C_F^* = 9m$

$S = I_P - C_P = 8m - 4.5m$

$S^* = 3.5m$

b) Suppose that the government introduces a temporary income tax, where they will tax Kevin's current income at a rate of 25%, but will not tax his future income. Write down Kevin's new budget line. Illustrate Kevin's original budget line and his new budget line on the same graph.



Kevin keeps 75% of current income

$$C_P + \frac{C_F}{2} = 8(1-.25) + \frac{2m}{2}$$

$$\rightarrow C_P + \frac{C_F}{2} = 8 \cdot 0.75m + 1m$$

$$C_P + \frac{C_F}{2} = 6m + 1m$$

$$C_P + \frac{C_F}{2} = 7m$$

c) Return to the situation in part a) where there is no income tax. Write a perfect complements utility function that ensures Kevin is neither a saver nor a borrower. Explain why your utility function answers the question.

$$U(C_P, C_F) = \{C_P, 4C_F\}$$

We want Kevin to consume 4 times as much income in the current period, so that $C_P = I_P = 8$ and $C_F = I_F = 2$.

The f^n above is for a person that consumes 4 units of C_P for each unit of C_F.

Other answers:

$$U(C_P, C_F) = \{2C_P, 8C_F\}$$

$$U(C_P, C_F) = \{\tfrac{1}{4}C_P, C_F\}$$

etc.