Economics 212

Section 001

Midterm Exam

March 6, 2017

Student Number: SOLUTION

Section A: Three questions @ 5 marks. Total 15 marks.

1. [5 marks] We know that a consumer maximizes utility by consuming 10 units of X and twenty units of Y. The price of X is four and the price of Y is two. The consumer's income is $80. Write a utility function that is consistent with this outcome and explain why it is consistent.

There are 3 utility functions that are consistent with the outcome:

1.) Perfect substitute case: $\frac{P_X}{P_Y} = \frac{4}{2} = 2$. So, $MRS_{X,Y} = \frac{MU_X}{MU_Y} = 2 \Rightarrow MU_X = 2$ and $MU_Y = 1 \Rightarrow U(X,Y) = 2X + Y$. Here, indifference curve overlaps budget constraint $\Rightarrow$ we have infinite bundles (X^*, Y^*) that each satisfy budget constraint. For examples, $(X^* = 10, Y^* = 20)$, $(20, 0)$, and $(0, 40)$.

2.) Perfect complement case: Consider $2X = Y$ from $U(X,Y) = \min\{2X, Y\}$. Substitute $Y = 2X$ into $P_X X + P_Y Y = I \Rightarrow (4)X + (2)2X = 80 \Rightarrow 8X = 80 \Rightarrow X^* = \frac{80}{8} = 10$, and $Y^* = 2X^* = 2(10) = 20$.

3.) Cobb-Douglas case: Consider $U(X,Y) = X^{\alpha} Y^{\alpha}$, $\alpha > 0$. $MRS_{X,Y} = \frac{MU_X}{MU_Y} = \frac{\alpha X^{\alpha-1} Y^{\alpha}}{\alpha X^{\alpha} Y^{\alpha-1}} = \frac{Y}{X} = \frac{P_X}{P_Y} = 2 \Rightarrow Y = 2X$. Thus, $P_X X + P_Y Y = (4)X + (2)(2)(X) = 8X = 80$. Thus, $X^* = \frac{80}{8} = 10$, and $Y^* = 2X^* = 2(10) = 20$.

We derive optimal bundles from the 3 utility functions given the budget constraint $\Rightarrow$ consistent

2. [5marks] A consumer is always willing to give up three units of good X in exchange for one unit of good Y. The price of X is nine and the price of Y is three. The consumer has an income of 900 dollars. Determine the consumer's optimal bundle.

"A consumer is always willing to give up three units of good X in exchange for one unit of good Y" implies:

1.) A utility function is for 2 goods that are perfect substitutes

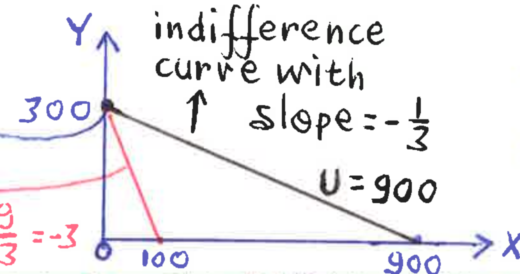
2.) $X = 3Y$. For example, if $Y = 1$, then $X = 3Y = 3(1) = 3$

Therefore, $U(X,Y) = X + 3Y$ We are given: $P_X = 9$, $P_Y = 3$, $I = 900$

At the optimum, $MRS_{X,Y} = \frac{MU_X}{MU_Y} = \frac{1}{3} < \frac{P_X}{P_Y} = \frac{9}{3} = 3$; therefore, the consumer consumes only good Y: optimal bundle: $X^* = 0$, $Y^* = \frac{I}{P_Y} = \frac{900}{3} = 300$

$U(X^* = 0, Y^* = 300) = 0 + 3(300) = 900$

optimal bundle $= (0, 300)$



budget constaint with slope $= -\frac{P_X}{P_Y} = -\frac{9}{3} = -3$

3. [5marks] Anthony has \$100 in income and is considering spending it on a lottery. The lottery gives Anthony a 60% probability of \$36 in final income, a 10% probability of \$144 in final income, and a 30% probability of \$400 in final income. Anthony is risk averse with a utility of income function given by $U = I^{1/2}$, where I is his income. Show that Anthony will choose the lottery. Calculate the risk premium associated with this lottery.

Given: Sure income = 100

Lottery income: 36 with Prob = 0.6, 144 with Prob = 0.1, 400 with Prob = 0.3

$$EU(\text{lottery income}) = (0.6)(36)^{1/2} + (0.1)(144)^{1/2} + (0.3)(400)^{1/2}$$
$$= (0.6)(6) + (0.1)(12) + (0.3)(20) = 3.6 + 1.2 + 6 = 10.8$$

$U(\text{sure income}) = (100)^{1/2} = 10 \Rightarrow$ Since $10.8 > 10$, Anthony chooses lottery

Certainty equivalent (ce) of a lottery is the minimum amount an individual would accept in order to give up participating in the lottery.

$EU(\text{lottery income}) = 10.8 = (ce)^{1/2} \Rightarrow (10.8)^2 = \left((ce)^{1/2}\right)^2 = ce^{\frac{1}{2}\cdot 2} = ce^1 = ce$

$ce = (10.8)^2 = 116.64$

$E(\text{lottery income}) = (0.6)(36) + (0.1)(144) + (0.3)(400) = 156$ } Risk premium $= 156 - 116.64 = 39.36$

Section B: Three question @ 15 marks- 5 for each part of each question. Total 45 marks.

1. Art consumes two goods, X and Y, according to the utility function $U=X^{1/2}Y$. Art has an income, I, and faces prices for the two goods given by P_X and P_Y.
   a) [5 marks] Derive Art's demand functions for the goods X and Y.

At the optimum, $MRS_{X,Y} = \dfrac{MU_X}{MU_Y} = \dfrac{P_X}{P_Y}$

$$MRS_{X,Y} = \frac{MU_X}{MU_Y} = \frac{\frac{1}{2}X^{-\frac{1}{2}}Y}{X^{\frac{1}{2}}} = \frac{1}{2}\frac{Y}{X} \Rightarrow MRS_{X,Y} = \frac{1}{2}\frac{Y}{X} = \frac{P_X}{P_Y}$$

$$\Rightarrow Y = 2\frac{P_X}{P_Y}X \quad (i.)$$

$$\text{or } X = \frac{1}{2}\frac{P_Y}{P_X}Y \quad (ii.)$$

Suppose we work with (i.) $\Rightarrow$ We substitute (i.) into the budget constraint:

$$P_X X + P_Y Y = I \Rightarrow P_X X + \cancel{P_Y}\left(2\frac{P_X}{\cancel{P_Y}}X\right) = P_X X + 2P_X X = 3P_X X = I$$

Thus, $X^* = \dfrac{1}{3}\dfrac{I}{P_X}$ and $Y^* = 2\dfrac{P_X}{P_Y}X^* = 2\dfrac{\cancel{P_X}}{P_Y}\left(\dfrac{1}{3}\dfrac{I}{\cancel{P_X}}\right) = \dfrac{2}{3}\dfrac{I}{P_Y}$

b) [5 marks] Assume that Art's income is $1,000, the price of X is $2 and the price of Y is $2. Calculate his demand for each good.

Given: $P_X = 2$, $P_Y = 2$, $I = 1000$

In Part (a.), we obtain: $X^* = \frac{1}{3}\frac{I}{P_X}$ and $Y^* = \frac{2}{3}\frac{I}{P_Y}$

Therefore, $X^* = \frac{1}{3}\frac{1000}{2} = \frac{1}{3}(500) = 166.6667$

$$Y^* = \frac{\cancel{2}}{3}\frac{1000}{\cancel{2}} = \frac{1000}{3} = 333.3333$$

c) [5 marks]Suppose the price of X decreases to $1. Determine the new demand for the goods and calculate the income and substitution effects of the price change.

Given: $P_X' = 1$

If $P_X' = 1$ and $P_Y = 2$, Art's new optimal bundle:

$$X' = \frac{1}{3}\frac{I}{P_X'} = \frac{1}{3}\frac{1000}{1} = 333.3333$$

$$Y' = \frac{2}{3}\frac{I}{P_Y} = \frac{2}{3}\frac{1000}{2} = \frac{1000}{3} = 333.3333$$

At initial prices $P_X = 2$, $P_Y = 2$ with $X^* = 166.6667$ and $Y^* = 333.3333$,

$$U^*(X^*, Y^*) = (X^*)^{1/2}\, Y^* = (166.6667)^{1/2}(333.3333) = 4303.3315$$

From Part (a.), we have $MRS_{X,Y} = \frac{1}{2}\frac{Y}{X} = \frac{P_X}{P_Y}$.

Replace $P_X = 2$ with $P_X' = 1$, we obtain: $\frac{1}{2}\frac{Y}{X} = \frac{1}{2} \Rightarrow \frac{Y}{X} = 1 \Rightarrow Y = X$

We know $U^* = 4303.3315$ and $Y = X$, then we can calculate the decomposition quantity $\tilde{X}$, which arises from the movement along the indifference curve. Now, replace Y with X (from Y=X) in $U^* = 4303.3315 = X^{1/2}Y$ and solve for $\tilde{X}$

$$U^* = 4303.3315 = X^{1/2}Y = X^{1/2}X = X^{3/2} \Rightarrow X^{3/2} = 4303.3315$$

$$\Rightarrow \tilde{X} = (4303.3315)^{2/3} = 264.5675$$

substitution effect $(SE) = \tilde{X} - X^* = 264.5675 - 166.6667 = 97.9$

Income effect $(IE) = X' - \tilde{X} = 333.3333 - 264.5675 = 68.7658$

2. Al has 16 hours per day to divide between leisure, R, and work. When he works, Al earns $50 per hour. He values both leisure and consumption, C, according to the utility function U=Min{30R ; C}. The price of the consumption good is unity.

   a) [5marks] Derive Al's optimal bundle. How much does he work?

Given: $P_C = 1$ and $P_R = 50$ Budget constraint: $C = 50(16-R)$

Al always requires $30R = C$. Substitute $C = 30R$ into the budget constraint to obtain: $30R = 50(16-R)$

$$\Rightarrow 30R = 800 - 50R$$

$$\Rightarrow 80R = 800 \Rightarrow R^* = \frac{800}{80} = 10$$

$C^* = 50(16-R^*) = 50(16-10) = (50)(6) = 300$

Optimal bundle is: $(R^* = 10,\ C^* = 300)$

$U^* = \min\{30R^*, C^*\} = \min\{30(10), 300\} = \min\{300, 300\} = 300$

Al works for $(16-R^*) = (16-10) = 6$ hours

   b) [5 marks] Al's boss tells him that a reorganization of the workplace has occurred and that Al must work either longer or shorter hours. The longer shift is 12 hours and the shorter shift is 4 hours. Which shift would Al prefer? Explain your reasoning.

Case of longer shift:

If Al works for 12 hours, $R' = 16 - 12 = 4$

$C' = 50(16-R') = 50(16-4) = (50)(12) = 600$

$U' = \min\{30R', C'\} = \min\{30(4), 600\} = \min\{120, 600\} = 120$

Case of shorter shift:

If Al works for 4 hours, $R'' = 16 - 4 = 12$

$C'' = 50(16-R'') = 50(16-12) = (50)(4) = 200$

$U'' = \min\{30R'', C''\} = \min\{30(12), 200\} = \min\{360, 200\} = 200$

$\Rightarrow U'' = 200 > U' = 120 \Rightarrow$ Al prefers the shorter shift.

Al has a Leontief preference; therefore, the ratio of leisure to consumption is important. The optimal $R^* = 10$. $R' = 4$ is too few compared to $R^* = 10$. $R'' = 12$ is closer to $R^* = 10$. Thus U'' is larger than U'

c) Starting from the solution to part (a), assume Al's income is taxed at the rate of \$10 per hour and that Al's wage decreases by the full \$10. Calculate Al's new optimal bundle and amount of work and show that he is worse off because of the tax.

Given: $P_C = 1$ and $P_R^N = 40$ New budget constraint: $C^N = 40(16 - R^N)$

Al always requires $30R = C$, then: $30R^N = 40(16 - R^N)$

$$\Rightarrow 30R^N = 640 - 40R^N \Rightarrow 70R^N = 640$$

$$\Rightarrow R^N = \frac{640}{70} = 9.143$$

$C^N = 40(16 - R^N) = 40(16 - 9.143) = 274.28$

$U^N = \min\{30R^N, C^N\} = \min\{30(9.143), 274.28\} = \min\{274.28, 274.28\} = 274.2$

Al works for $(16 - R^N) = 16 - 9.143 = 6.857$ hours

Al works longer hours $(6.857 > 6)$; therefore, leisure drops $(9.143 < 10$

However, consumption does not increase $(274.28 < 300)$ even if Al works longer hours. Thus, Al's utility drops from 300 to 274.28; and therefore, Al is worse off because of the tax.

3. [5 marks] Maria works in the present period and earns an income of \$4,000,000. In the future period, Maria is retired and earns nothing. Her preferences over present consumption, C_P, and future consumption, C_F, are given by $U=2C_PC_F$. Maria's savings earn an interest rate of 100%.

a) Derive Maria's optimal consumption bundle and her level of savings.

Budget constraint: $\underbrace{(1)}_{=P_{C_P}} C_P + \underbrace{\frac{1}{1+r}}_{=P_{C_F}} C_F = I_P \Rightarrow C_P + \frac{C_F}{1+r} = I_P$

$P_{C_P} = 1$, $P_{C_F} = \frac{1}{1+r}$

At the optimum, $MRS_{C_P, C_F} = \frac{MU_{C_P}}{MU_{C_F}} = \frac{P_{C_P}}{P_{C_F}}$

$$\Rightarrow MRS_{C_P, C_F} = \frac{MU_{C_P}}{MU_{C_F}} = \frac{2C_F}{2C_P} = \frac{C_F}{C_P} = \frac{P_{C_P}}{P_{C_F}} = \frac{1}{\frac{1}{1+r}} = 1 + r, \text{where } r = 1$$

Thus, $\frac{C_F}{C_P} = 1 + r \Rightarrow C_F = (1+r)C_P \Rightarrow$ substitute into budget constraint

$$C_P + \frac{C_F}{1+r} = C_P + \frac{\cancel{(1+r)}C_P}{\cancel{(1+r)}} = 2C_P = I_P \Rightarrow C_P^* = \frac{1}{2} I_P$$

$C_F^* = (1+r)C_P^* = (1+r)\frac{1}{2}I_P = \frac{1}{2}(1+1)I_P = (\frac{1}{2})(2)I_P = I_P$

$C_F^* = I_P = 4{,}000{,}000$ and $C_P^* = \frac{1}{2}I_P = \frac{1}{2}(4{,}000{,}000) = 2{,}000{,}000$. Thus, savings $= I_P - C_P^* = 2{,}000{,}00$

b) Write a perfect complements utility function that will yield the same optimal bundle and level of savings as correctly derived in part a). Show that this utility function does yield the same optimal bundle and level of savings.

In Part (a.), we know $\left.\begin{array}{l} C_P^* = 2{,}000{,}000 \\ C_F^* = 4{,}000{,}000 \end{array}\right\}$ ratio : 1 to 2

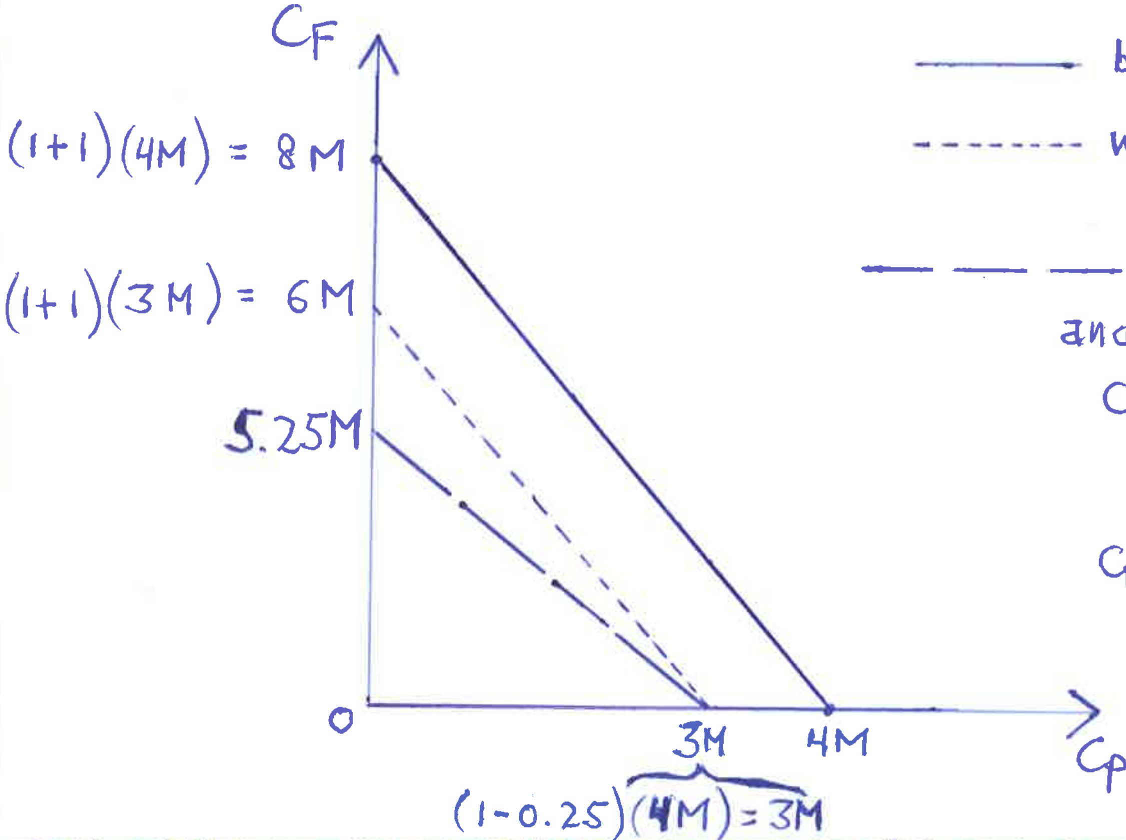
Thus, for every \$1 of C_P^*, Maria has to consume $C_F^* = \$2$.
This implies $C_F = 2C_P$; and consequently, Maria has the following perfect complement utility function: $\min\{2C_P, C_F\}$.
Substitute $C_F = 2C_P$ into budget constraint with $r = 1$:

$$C_P + \frac{C_F}{1+r} = I_P \Rightarrow C_P + \frac{2C_P}{1+1} = C_P + \frac{\cancel{2}C_P}{\cancel{2}} = 2C_P = I_P \Rightarrow C_P^* = \frac{1}{2} I_P$$

$$C_F^* = 2C_P^* = 2\,\frac{1}{2}\,I_P = I_P.$$

$$\left.\begin{array}{l} C_P^* = \frac{1}{2} I_P = \frac{1}{2}(4{,}000{,}000) = 2{,}000{,}000 \\ C_F^* = I_P = 4{,}000{,}000 \end{array}\right\} \text{savings} = I_P - C_P^* = 4{,}000{,}000 - 2{,}000{,}000 = 2{,}000{,}000$$

c) Draw and appropriately label Maria's budget line from part (a). Show how the budget line would change if the government taxed both her earnings and the interest earned on her savings at the rate of 25%. Please label the new budget line. Let $M = 1{,}000{,}000$



——— before-tax (pre-tax)

- - - - - - with 0.25 tax on I_P only

— — — with 0.25 tax on I_P and 0.25 tax on savings

$$C_P = (1-0.25)\,I_P = (0.75)(4M) = 3M$$

$$C_F = C_P\,(1 + 1(1-0.25)) = 3M(1+0.75) = 5.25M$$