Solution section 1

Economics 212

Section 001

Midterm Exam

October 20, 2014

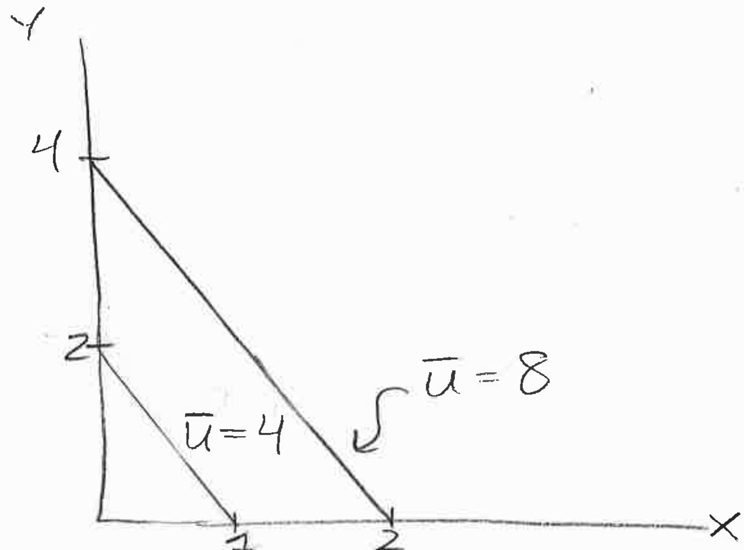
Student Number:

Section A: Three questions @ 5 marks. Total 15 marks.

1. [5 marks] Consider the utility function U(X,Y)=4X+2Y, where X and Y are two goods. Draw and appropriately label two indifference curves for this consumer. Assume the price of X is \$10, the price of Y is \$6 and the consumer has an income of \$500. Derive the optimal consumption bundle for the consumer.

$\bar{u} = 4X + 2Y \Rightarrow Y = \frac{\bar{u}}{2} - 2X$

Compare $MRS_{x,y}$ and $\frac{p_x}{p_y}$



$MRS_{x,y} = \frac{MU_x}{MU_y} = \frac{4}{2}$

$\frac{p_x}{p_y} = \frac{10}{6}$

$\frac{4}{2} > 10/6$

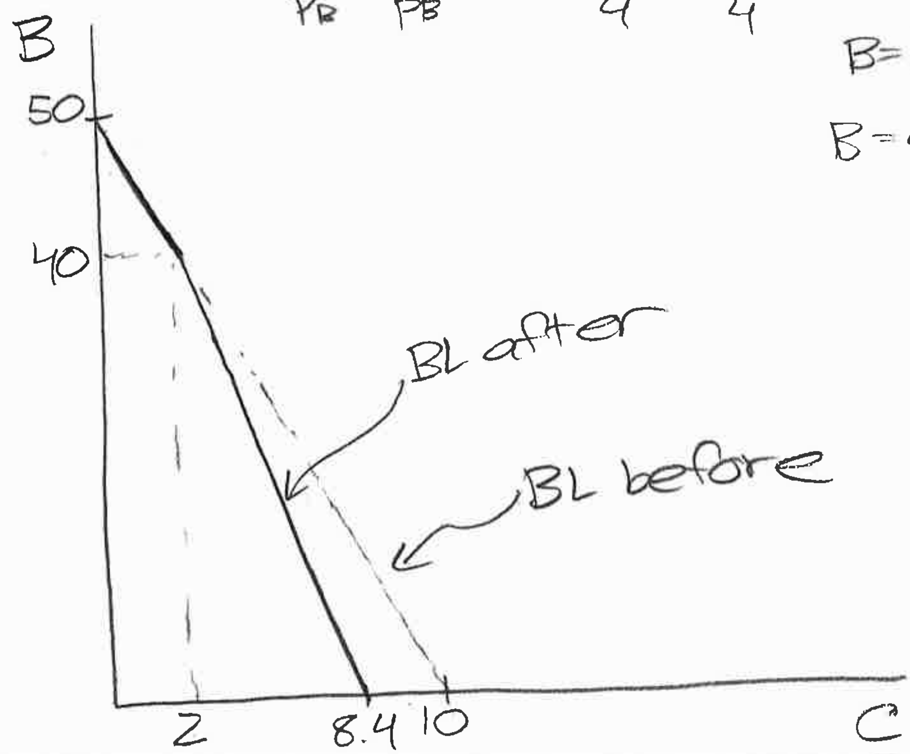
$\rightarrow$ buy all X

$x^* = 500/10$

$(x^*, y^*) = (50, 0)$

2. [5marks] Martin consumes bread, B, and cheese, C. The price of cheese is \$20 per unit and the price of bread is \$4 per unit. Martin has an income of \$200 to spend on the two goods. Draw and appropriately label his budget constraint. Now suppose the government imposes a tax equal to \$5 per unit on cheese, but only after Martin has consumed 2 units. Show how the tax affects Martin's consumption opportunities.

$B = \frac{I}{P_B} - \frac{p_C}{P_B} C = \frac{200}{4} - \frac{20C}{4} = 50 - 5C, \; C \leq 2.$



$B = 50 - 5(2) = 40$ when $C = 2$

$B = 40 - \frac{(20+5)}{4}(C-2)$ when $C > 2$

$= 40 - \frac{25}{4}(C-2)$

when $B = 0, C = 8.4$

3. [5marks]A risk averse consumer has a utility of income function given by $U(I)=I^{1/2}$. The consumer has $100 and is asked to reject or accept the following bet: there is a probability of .6 that she will win and finish with an income of $144 and there is a probability of .4 that she will lose and finish with an income of $49. Will the consumer accept or reject the bet?

Expected utility of the consumer is

$$0.6(144)^{1/2} + 0.4(49)^{1/2} = 0.6\times12 + 0.4(7) = 7.2+2.8 = \underline{\underline{10}}$$

utility without the bet is $(100)^{1/2} = \underline{\underline{10}}$

So the consumer must be indifferent between accepting or rejecting the bet.

Section B: Three question @ 15 marks- 5 for each part of each question. Total 45 marks.

1. Eddie consumes two goods, X and Y, according to the utility function $U(X,Y)=XY$. The price of X is P_X and the price of Y is P_Y. Eddie has an income I.

a) [5 marks] Derive Eddie's demand functions for the two goods.

$$\frac{MU_x}{MU_y} = \frac{y}{x} = \frac{P_x}{P_y} \qquad P_y y = P_x X$$

$$BC \quad P_x X + P_y Y = I \Rightarrow \text{Sub. } P_y Y = P_x X \text{ in } BC$$

$$2P_x X = I$$

$$X^* = \frac{I}{2P_x} \qquad Y^* = \frac{I}{2P_y}$$

b) [5 marks] Assume the price of X is \$2, the price of Y is \$1 and that Eddie has an income of \$300. Determine Eddies's optimal bundle.

$X_0^* = \frac{I}{2P_x} = \frac{300}{4} = 75 \qquad Y_0^* = \frac{I}{2P_y} = \frac{300}{2} = 150$

BC $\quad 2 \times 75 + 1 \times 150 = 300$

c) [5 marks] Assume that the price of the good X increases to \$4. Determine the new optimal bundle and the income and substitution effects of the price increase.

$X_1^* = \frac{I}{2P_x^N} = \frac{300}{8} = 37.5$

$y_1^* = \frac{I}{2P_y} = \frac{300}{2} = 150$

Original utility $\quad U_0 = \cancel{75}\ X_0^* Y_0^* = 150 \times 75 = 11250$

at new price $\quad MRS_{x,y} = \frac{y^D}{x^D} = \frac{P_x}{P_y} \Rightarrow \frac{y_1^D}{x_1^D} = \frac{4}{1} \Rightarrow y_1^D = 4x_1^D$

So $\quad x_1^{D*} y_1^{D*} = 11250 \Rightarrow x_1^{D*}(4x_1^{D*}) = 11250$

$x_1^{D*2} = \frac{11250}{4}$

$x_1^{D*} = \sqrt{2812.5} = 53.033$

sub. effect $\quad \overbrace{53.033 - 75}^{x^D - X_0^*} = -21.966$

Inc. effect $\quad X_1^* - X^{*D} = 37.5 - 53.033 = -15.533$

2. Art has 24 hours per day to divide between leisure, R, and work, L. When he works, Art earns \$20 per hour. He values both leisure and consumption, C, according to the utility function U(R,C)=Min{10R;C}. The price of the consumption good is unity.

   a) [5marks] Derive Art's optimal bundle. How much does he work?

Optimal at $10R = C$ BC $C = 20(24 - R)$

$\Rightarrow 10R = 20(24-R) \Rightarrow 10R = 480 - 20R$

$30R = 480$

$R = \cancel{9.6}\ 16$

Art works $24 - 16 = \cancel{14.4}\ 8 = L$

   b) [5 marks]Art has received a raise and now earns \$25 per hour. Determine how the wage increase affects Art's optimal bundle. Is leisure a normal good? Explain.

$10R = C$ BC $\rightarrow C = 25(24-R)$

$\Rightarrow 10R = 25(24-R)$

$10R = 600 - 25R$

$35R = 600$

$R = 17.1428$

$C = 25(24 - 17.1428)$
$= 171.43$

$L = 24 - 17.1428$
$= 6.8572$

As price of work (wage) increases, Art reduces his number of hours of work from 8 to 6.8572

~~As Art~~ Art's new income is $6.8572 \times 25 = 171.43$
" original income was $8 \times 20 = 160$

As art's income increases, he consumed more leisure $\Rightarrow$ hence, leisure is the normal good

c) After the raise, Art is now told that he must work 8 hours per day. Show that Art is made worse off by this restriction on his freedom of choice of hours of work.

utility ~~when~~ at raise $u(R,C) = \min(10R, C)$
$= \min(10(17.1428), 171.43)$
$= \min(171.43, 171.43)$
171.43

If art works 8 hours
$C = 25 \times 8 = 200$

$u = \min(10(16), 200)$ ← 24-8
$= \min(160, 200)$
$= 160$ ⇒ so art is worse off

3. [5 marks]Amy is a teacher who earns $5,000,000 during her working life and $2,000,000 when she retires. The interest rate between her working life and retirement is 100%. Her preferences over present consumption, C_P, and future consumption, C_F, are given by $U(C_P,C_F)=C_P^{1/2}\, C_F^{1/2}$.

a) Derive Amy's optimal consumption bundle and her level of savings.

present income = present cons. + saving ⇒ $I_P = C_P + S$
future ~~wealth~~ future income = future ~~income~~ cons. + saving(1+r) = $I_F = C_F + S(1+r)$

~~5Mill = $C_P + S$ ⇒ 5Mill − C_P~~
~~2Mill = $C_F + S(1+r)$~~
~~2Mill = $C_F + (5\text{Mill} - C_P)(1+r)$~~

(1) $I_P = C_P + S$ (2) $C_F = I_F + S(1+r)$

$S = \frac{C_F - I_F}{1+r}$

$I_P = C_P + \frac{C_F - I_F}{1+r}$ ⇒ present value BC

$C_P + \frac{C_F}{1+r} = I_P + \frac{I_F}{1+r}$ ⇒ $C_P + \frac{C_F}{1+1} = 5\text{mill} + \frac{2\text{mill}}{1+1}$

⇒ $C_P + \frac{C_F}{2} = 6\text{ mill}$

$MRS_{C_P C_F} = \frac{\frac{1}{2} C_P^{-1/2} C_F^{1/2}}{\frac{1}{2} C_P^{1/2} C_F^{-1/2}} = \frac{C_F}{C_P} = \frac{1+r}{1} = C_F = (1+r)C_P$ ⇒ $C_F = 2C_P$, $C_P + \frac{2C_P}{2} = 6\text{mil}$

⇒ $2C_P = 6\text{ mill}$
$C_P = 3\text{ mill}$

$C_P + S = I_P$
$3\text{mill} + S = 5\text{mill}$
$S = 2\text{mill}$

$C_F = 2C_P$
$= 6\text{mill}$

b) Now suppose the interest rate decreases to 50%. Show how this affects Amy's optimal bundle and level of savings. Explain (in words) the income and substitution effects of the interest rate decrease.

$$MRS_{C_P, C_F} = \frac{C_F}{C_P} = 1.5 \Rightarrow C_F = 1.5 C_P$$

$$PVBC = C_P + \frac{C_F}{1+r} = I_P + \frac{I_F}{1+r} \Rightarrow C_P + \frac{C_F}{1.5} = I_P + \frac{I_F}{1.5}$$

$$\Rightarrow C_P + \frac{C_F}{1.5} = 5mill + \frac{2mill}{1.5} \Rightarrow 6.3333$$

$$2C_P = 6.33333$$

$$C_P = 3.16665 \Rightarrow$$

Because the opportunity cost of present consumtion decreased, Amy consumes more in the present. ~~She~~ Decrease in future income (benefit of saving) makes her substitute future consumption by present one.

c) [5 marks] How would your answer to part (a) change if Amy had preferences given by $U(C_P, C_F) = C_P^2 C_F^2$? Explain your answer using words and reasoning. Do not solve for the optimal bundle.

$$MRS_{C_P, C_F} = \frac{2C_P^1 C_F^2}{2C_P^2 C_F^1} = \frac{C_F}{C_P} \Rightarrow$$

MRS does not change hence the answer is the same. The difference is only in the rescaling of the utility function which does not matter