The Dynamics of
Sectoral Labor Adjustment

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Abstract

This paper develops an equilibrium search and matching model to jointly study the aggregate, sectoral, and distributional impacts of labor adjustment. The model extends Pissarides (2000) to include multisector production and search and 'innovation' from investments that can potentially improve a match's productivity. These extensions deliver two mechanisms for inter-sectoral and intra-sectoral labor reallocation after productivity shocks. First, because workers search simultaneously in multiple sectors, changes in labor market conditions in one sector propagate to impact wages and hiring in the rest of the economy through a *reservation wage effect*. Second, a positive productivity shock causes firms to invest more resources in innovation. This *innovation effect* shifts production towards high-skill jobs and amplifies the impact of productivity shocks relative to the baseline model. I show that the model is useful for analyzing labor adjustments caused by a diverse set of factors including: technological change; persistent energy price and exchange rate shocks; and trade liberalization.

1 Introduction

Recent empirical evidence suggests a rise in the frequency and severity of sector-specific shocks and a pronounced increase in sectoral job changes.[1] Such job changes have become quite common in the U.S., for example, where more than 10 percent of workers change sectors each year and where workers today are more than twice as likely to change sectors, as compared to similar workers 30 years ago. The increase in sector-specific shocks raises several issues. For individuals, these shocks can necessitate an adjustment process involving large and persistent earnings losses — particularly for those with intervening unemployment spells.[2] In addition, the adjustment often has very different effects on low-skill and high-skill workers. At the sectoral level, labor reallocation can lead to equilibrium wage spillovers between sectors. And at the aggregate level, because sectors can differ significantly in their productive capacities, the resulting reallocation can impact output and productivity.[3]

Existing research fails to jointly capture these important features of sectoral labor reallocation in a unified, tractable, equilibrium framework that features explicit labor market frictions and considers transition dynamics. This paper attempts to fill this gap by developing a model to study the aggregate, sectoral, and distributional impacts of labor adjustment following unanticipated, sector-specific productivity shocks. The two central questions under study are: How does the economy reorganize production after sector-specific shocks? And what are the equilibrium implications of these shocks for other sectors?

To answer these questions, I develop a multi-sector equilibrium search and matching

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[1]See Kambourov and Manovskii (2008). For the purpose of this paper, the term 'sector' can represent industries or occupations. Also, a 'sector-specific shock' can be defined quite broadly as any unanticipated change in the relative profitability of production across sectors. This includes disturbances hitting a particular sector, but also aggregate developments that directly affect sectors differently. Examples include: demand shocks; technological change; persistent commodity price or exchange rate movements; new government regulation or policies such as trade liberalization.

[2]See *e.g.* McLaughlin and Bils (2001), Fallick (1996), Jacobson et al. (1993) and Topel (1993).

[3]As examples, for aggregate impacts see Lee and Wolpin (2006); for sectoral impacts, see Trefler (2004); Beaudry et al. (2007) find that changes in the sectoral composition can cause equilibrium wage spillovers.

model, derive the main analytical results and use simple quantitative examples to clearly illustrate the model's adjustment mechanisms. In doing so, I identify a relatively straightforward sectoral labor adjustment process. The salient property of the 'sector-specific shocks' is that they change the relative profitability of production across certain sectors. In sectors where production becomes more profitable, there is relatively more entry and over time employment increases. A positive shock makes high-skill production in these sectors more attractive, so additional resources are devoted to productivity-improving, innovative efforts to chase these new profits. This results in more high-skill production and rewards these high-skill workers with larger wage gains. In the model, sector-specific productivity shocks capture these effects by changing the relative match surpluses across and within sectors, generating a labor reallocation process that is consistent with those observed in sectoral labor adjustments caused by a variety of factors.

The model makes two key extensions to the baseline Pissarides (2000) labor search and matching model. The first extension is multi-sector production and search. This allows inter-sectoral labor reallocation to occur through a *reservation wage effect* that describes how changes in workers' outside options cause sector-specific shocks to spillover to other sectors and propagate across the economy. In the model, unemployed workers search simultaneously in multiple sectors. Therefore, when a shock changes labor market conditions in one sector, this affects workers' value of search, which is their 'outside option'. As a result, workers update their reservation wage, which ultimately impacts their bargained wages. This changes the cost of labor, impacts profitability and affects firms' incentives to hire workers in other sectors of the economy. The varied recruiting responses in different sectors change the sectoral composition of job postings and ultimately result in inter-sectoral labor reallocation. For example, a positive sector-specific productivity shock improves workers' outside option and, therefore, raises their reservation wages. This, in turn, raises the cost of labor and acts as a

negative spillover for hiring decisions in other sectors.

The second model extension is an 'innovation' process that allows matches to acquire skills and become more productive. This extension accomplishes two tasks. First, it parsimoniously models low-skill and high-skill workers in this environment to analyze how they might be impacted differently by sector-specific shocks, as suggested from empirical work by Keane and Prasad (1996), Autor et al. (1998), and Trefler (2004), among others. Second, the innovation process captures how resource reallocation within a sector between low and high-skill production can amplify productivity shocks through an *innovation effect* which works as follows: In the model, all new matches begin production as low-skill, but may become high-skill through a costly and uncertain productivity-enhancing investment, which could represent spending such as research and development (R&D) or on-the-job training. After a positive productivity shock in a sector, employing a high-skill worker becomes relatively more profitable, so firms expect a larger return from these investments. They respond by investing more resources into innovation with their low-skill workers. This accelerates skill acquisition, endogenously raises the share of high-skill production in the sector and amplifies the model's response to productivity shocks. Also because high-skill workers benefit disproportionately, wage dispersion increases in a sector after a positive shock.

Several studies relate to the model developed here. In the search and matching literature, Acemoglu (2001) and Albrecht et al. (2006a) consider two-sector production. However, while I focus on the impacts of productivity shocks for sectoral adjustment in the steady-state and transition, these papers study the impacts of labor market policies on the sectoral composition in the steady-state. In Acemoglu (2001), there are no productivity differences between workers within a sector, so distributional considerations are absent and the transition is not considered. Albrecht et al. (2006a) consider steady-state distributional effects, but not transition dynamics. Other approaches in the international trade literature model sectoral

reallocation.[4] These models typically ignore unemployment and labor market frictions to make long-run statements about the reallocation. In this paper, in addition to addressing the long-run steady-state impacts, I investigate the short-run adjustment, which can generate important costs for individual workers and is a relevant concern for policymakers.

Mortensen (2000) also develops a search model where firms make match-specific investments. A key difference between this paper and my work is how wage are determined. Mortensen uses wage-posting, whereas I use Nash Bargaining. He demonstrates that firms' varied wage offer strategies can result in endogenous productivity differences across jobs for ex-ante homogeneous workers. Overall, his main result is that including match-specific investments gives a more empirically-plausible shape for the equilibrium wage distribution. My focus is quite different. I restrict attention to the equilibrium where all firms in a sector make the same investment decision to investigate the model's adjustment to sector-specific shocks for employment, output and the relative wages of high and low-skill workers.

More generally, the model extensions here incorporate heterogeneous jobs and skill acquisitions to capture several facts identified by empirical labor studies which are absent in the baseline model: 1) separated workers commonly switch sectors; 2) separated workers can experience wage losses or gains in their subsequent job; 3) high wage earners are more likely to suffer wage losses in their subsequent job; and 4) wages rise with job tenure.

This rest of the paper is organized as follows: Section 2 presents some facts from sectoral adjustment episodes. Sections 3 and 4 describes the model and its transition dynamics. Section 5 quantitatively illustrates the model's mechanisms and Section 6 concludes. The Appendix contains proofs and derivations.

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[4]Melitz (2003) is a prominent example featuring intra-sectoral reallocation that can be contrasted with the innovation effect here. In Melitz's model increasing trade exposure improves a sector's productivity through selection effects. My model features within-firm productivity improvements, which aggregate to change the sectoral composition of production.

2 Facts from Sectoral Labor Adjustment Episodes

Sectoral labor adjustments can be caused by a variety of seemingly disparate factors. This section presents some new evidence and draws on existing findings from the experiences of several countries for: 1) persistent relative price shocks, such energy prices and exchange rates; 2) trade liberalization; and 3) broader technological change. I summarize three important common elements of these adjustments regarding inter-sectoral and intra-sectoral labor reallocation and relative wage effects between low and high-skill workers.

While initially these different episodes appear unrelated, the key uniting characteristic these events share is that they change the relative profitability between and within sectors. Typically, in sectors where production becomes more profitable, there is increased entry of new firms, increased employment in the sector and firms undertake costly productivity-enhancing investments to capture the new profit opportunities.

To be more concrete, consider some examples. Trade liberalization effectively improves market access for exporters. Firms respond to these new profit opportunities by entering and undertaking investments to improve their productivity. A large increase in energy prices makes resource sector jobs more profitable, spurring new investments and employment in the sector. Also, improvements in computing technologies disproportionately benefit information-intensive sectors, so there is entry and employment growth in these sectors. At the same time, because these new technologies increase the relative productivity differences between low and high-skill workers, firms invest in these technologies and increase their share of high-skill workers. Overall, in these episodes there is a general flow of workers towards the positively-affected sector, resources flow towards high-skill production within the sector, and real wage gains are concentrated on high-skill workers.

Fact 1: *Inter-Sectoral Labor Adjustment*

Consider the case of energy price shocks and the reallocation of labor across sectors that

results from them. These shocks are a particularly convenient way to study inter-sectoral labor reallocation, because they are relatively discrete episodes with some persistence. In addition, these shocks can reasonably be treated as unanticipated and exogenous from the point of view of the economies studied here. I analyze internationally-comparable employment data for the G7 countries (Canada, France, Germany, Italy, Japan, U.K. and U.S.) from the OECD's Structural Analysis Database.

Figure 1 separates employment in the G7 economies into manufacturing and non-manufacturing sectors during these oil prices shocks, using the dates identified by Blanchard and Gali (2007).[5] The figure clearly shows the asymmetric negative impact on manufacturing employment following oil price shocks. In the four years following the oil shocks, there was a substantial drop in manufacturing employment, which fell by an average of 7.6 percent. These employment dynamics are a quite general phenomenon across all episodes and not restricted to, or driven by a single country. Figure 2 disaggregates the employment dynamics for each country's manufacturing sector before and after the oil price shocks. The results hold quite generally as the drop occurred in all countries except Italy, where employment rose a mere 0.6 percent.[6] Figure 3 disaggregates the non-manufacturing employment data by country. It shows that in the four years after the shocks, non-manufacturing employment continued to grow in all G7 economies, at or only slightly below trend. Not surprisingly, while there is a general increase in employment in the non-manufacturing sectors, using more detailed data, reveals that the largest employment gains occur in the resource sector — see Figure 4, which

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[5]Blanchard and Gali (2007) identify four oil price shocks: 1973, 1979, 1999, and 2002. They define a shock as an increase in the real oil price of more than 50 percent which persists longer than four quarters (the real price is the West Texas Intermediate price deflated by the U.S. GDP deflator). I normalize employment to 100 at each shock, so the relative changes are comparable. The reported results are averaged over the three shocks, since the same trends occur in each episode (1973, 1979, and 1999). Due to the lag in reporting internationally-comparable employment data, the results for the 2002 shock are not yet available.

[6]There are likely two effects at play here. First, manufacturers are the most energy-intensive producers so their input costs increase more than in other sectors. Second, there may also be an endogenous monetary policy response to raise interest rates and fight the inflationary impacts of the oil price shocks. Manufacturers are more sensitive to interest rates as their sales are often financed by borrowing.

shows the average response in the U.S. economy after the four oil price shocks.

Facts 2 and 3: *Intra-Sectoral Labor Adjustment; and Relative Wage Gains for High-Skill Workers*

Not only do sector-specific shocks lead to a movements of workers between sectors, but there is also often a shift from low to high-skill workers within the sectors made relatively more profitable by the shock. The general innovation and skill-upgrading responses after shocks are documented in several studies which use detailed microdata on individuals and firms. For instance, Keane and Prasad (1996) find such a shift following oil price shocks, as the relative employment and wages of high-skill workers increased. They use individual-level panel data from the NLSY covering 1966–1981 and control for individual fixed effects and sample selection bias. Tapp (2007) also finds that wage gains were concentrated in the upper end of the distribution in the Canadian resource sector following the most recent oil price shock.

Verhoogen (2008) studies another important relative price change: the exchange rate. In 1994, a rapid depreciation of the Mexican peso expanded opportunities for exporters by lowering the price of their output in international markets. Firms responded by increasing the quality of goods produced to export abroad. This quality upgrading, resulted in a relative increase in employment and wages of high-skilled workers in the Mexican manufacturing sector.

Other research in the international trade context provides similar results of so-called, 'skill-upgrading' or 're-tooling' following trade liberalizations. Using detailed plant-level data, Trefler (2004) finds a relative increase in the employment of high-skill relative to low-skill workers in Canadian manufacturing industries following the Canada-U.S. Free Trade Agreement. For this episode, the relative employment shift to high-skill workers is associated with investments that increased productivity within plants, particularly for those that entered ex-

port markets after trade liberalization (Lileeva and Trefler, 2007; and Lileeva, 2007). Several other recent papers for a variety of countries suggest that trade liberalization increases firms' incentives to invest in productivity-enhancing investments (e.g. on-the-job training, R&D, technological adoption) and raises productivity within plants.[7]

Finally, similar labor adjustments trends occurred for technological changes from computerization and general R&D, where considerable intra-sector employment shifts occurred towards high-skill labor. In the U.S., these effects were largest in computer-intensive industries (Autor et al., 1998). Also, in the U.K. most of the aggregate economy's skill upgrading was due to employing more skilled workers within continuing establishments and was related to computer usage (Haskel and Hayden, 1999). Finally, Machin and Reenen (1998) link the within-industry increases in the proportion of skilled-workers in several OECD countries to broader technological change through R&D intensity.

3   Multisector Search Model with Innovation

This section presents a general search and matching model of labor reallocation following unanticipated sector-specific productivity shocks. I add two key extensions relative to the baseline model of Pissarides (2000, Ch. 1): 1) multisector production and search; and 2) an innovation and skill acquisition process, motivated by the empirical evidence that suggests an upgrading process may be important part of the labor adjustment after sector-specific shocks. Unlike previous multisector versions of the model, I focus not only on the steady-state, but also on transition dynamics between steady-states. In addition, I allow for sector-specific separation rates and use sector-specific matching functions to capture the fact that job-finding and job-filling rates vary significantly by sector.

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[7]See Costantini and Melitz (2007); Aw et al. (2007); and Bustos (2005).

3.1 Environment and General Overview

This subsection provides a basic overview of the model's key ingredients and timing of events. The details are described in subsequent sections. I focus first on the model's steady-state; later sections consider sector-specific shocks and the model's transition dynamics.

Time is discrete with an infinite horizon. There are multiple sectors of the economy indexed by $i \in \{1, 2, \ldots, I\}$ that produce a non-storable good. The model features two types of agents: workers and firms. Each type of agent is *ex ante* identical, infinitely-lived and risk-neutral, discounting future payoffs at rate δ.[8] Agents are either matched and productive, or searching for a partner to begin production.

Figure 5 describes the timing of events in a given period for unmatched agents. A recruiting stage begins the period when unemployed workers collect unemployment benefits and search for jobs, and firms post vacancies in decentralized labor markets. The matching stage follows when a subset of firms with vacancies and unemployed workers are brought together in pairwise matches. Once matched, the pair bargain over the worker's wage and the firm declares its intended innovation level. If there is agreement, production begins next period as a low-skill match.

Figure 6 describes the timing for producing agents. Production begins the period and wage payments follow. Firms in low-skill matches can then attempt to innovate to improve their productivity. At the end of each period, some low-skill matches successfully acquire match-specific skills and become high-skill. Also at the end of the period, some low and high-skill matches terminate exogenously.

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[8]There will be heterogeneity *ex post* in the sectors in which agents work and their match skill levels, based on the luck associated with job search and skill acquisition.

3.2 Workers

The labor force consists of a measure one continuum of potential workers. At any point in time, a given worker is in one of the following $(2 \times I + 1)$ states: *Unemployed* — receiving unemployment benefits, z, and searching for a job; or, *working* — receiving a wage in sector i in a low-skill match of w_i^L or in high-skill match of w_i^H. The expected present values in these states are denoted U, W_i^L and W_i^H, respectively.

The unemployed search for jobs at no cost. As a result, their search is not directed to a particular sector, but rather simultaneous in all sectors. There is no on-the-job search or quits.[9] Workers do not value leisure. Therefore, when unemployed they allocate all their time to search and when employed they inelastically supply one unit of labor each period. There are no savings in the model; workers simply consume their current income.

3.3 Firms

There is a large measure of potential firms. Firms can be in one of the following $(3 \times I)$ states: posting a *vacancy* to recruit in sector i; or producing in sector i in a low or high-skill *job* match. The expected present values in these states are denoted V_i, J_i^L and J_i^H, respectively.

There is free entry and exit of vacancies and firms incur recruiting cost, c, each period their vacancy remains unfilled. In a low-skill matches, firms engage in innovation activities, $x_i \in \mathbb{R} \, [0, 1]$, at cost $\chi(x_i)$ each period, where $\chi(0) = 0$ and $\chi'(x_i) > 0$. Innovation is a costly and uncertain process, where firms make a match-specific investment in an attempt to improve the match's productivity. This can be interpreted in several ways. First, it can represent the lower productivity of a new worker while learning match-specific skills. Second,

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[9]Evidence from the U.S. and Canada finds a significant number of workers who change sectors experience an intervening unemployment spell. For the U.S., see Kambourov and Manovskii (2008) and for Canada, see Osberg (1991). The model abstracts from job-to-job transitions, so all movements are between employment and unemployment. The assumption ruling out quits simplifies things. It is also reasonable in the model environment, because in equilibrium, workers' wages compensate them for their value of search, so working in any sector is strictly preferred to unemployment.

it can represent an on-the-job training program, for which, the available empirical evidence suggests training costs can be substantial and are mainly paid by the firm.[10] Third, it can represent R&D to improve the production technology.

Innovation is beneficial because it makes skill acquisition more likely, reducing the expected time to become a high-skill match. Successful innovation transforms the match from low-skill into high-skill, and occurs with probability $\lambda_i x_i$, where λ_i is the exogenous skill arrival rate and x_i is innovative investment. High-skill matches are desirable because they produce more output and provide higher profits and wages.

The model captures the fact that labor adjustments can be costly for individual workers. Empirical work finds that following job loss, workers can suffer significant and persistent earning losses in their subsequent jobs, particularly those workers with longer tenure and/or those who endure intervening unemployment. These findings suggest that some skills which are accumulated are not easily transferred to new jobs. The model captures this in a stark and stylized way. Skills are match-specific and therefore are lost when the match terminates.

Matches produce output using only labor with constant returns to scale, skill-specific technologies. Each period sector i matches produce: $y_i^{SK} = A_i p_i^{SK} l_i^{SK}$, where y is output; $i \in \{1, 2, ..., I\}$ subscripts the sector; $SK \in \{L, H\}$ superscripts low and high-skill matches; A_i is a sector-wide productivity parameter, which is constant and normalized to one in the steady state, but will later serve as the source of the sector-specific shock; p is productivity; and l is labor. To simplify the exposition, I assume each firm employs one worker. Thus, in the steady-state, p_i is output per worker in sector i, where $p_i^H > p_i^L$.

Finally, each period all sector i matches face exogenous probability s_i of job destruction, where s_i is the sector i separation rate.

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[10]For estimates of training costs see, for example, Barron et al. (1989, 1999); and Dolfin (2006). Loewenstein and Spletzer (1998) analyze National Longitudinal Survey of Youth data and find employers pay the explicit cost of on-site training over 90 percent of the time.

3.4 Matching Process

Unmatched firms post vacancies to attract unemployed workers in one of I sectors. The unemployed search simultaneously in all sectors. Sector-specific matching functions capture the time-consuming nature of search by determining the measure of pairwise matches per period in each sector. The matching functions have the Cobb-Douglas functional form:[11] $m_i(u, v_i) = \mu_i u^\alpha v_i^{1-\alpha}$, where m_i is the measure of sector i matches; u is the measure of unemployed workers; v_i is the measure of vacancies in sector i; μ_i is the recruiting effectiveness in sector i; and α is the elasticity of matches with respect to unemployment.

In previous multi-sector labor search models, such as Acemoglu (2001) and Davis (2001), matching occurs through an aggregate matching function. My formulation is more general. Sectors are allowed to vary in their recruiting effectiveness, μ_i's, because some sectors can assess applicants more easily than others. This means that market tightness, and therefore job-finding and job-filling rates, can vary by sector. This formulation brings the model closer to the data which feature clear differences in search outcomes across sectors.[12]

Because the model is set in discrete, rather than continuous time, this more general matching process implies that workers could potentially receive multiple offers in a period. This is an interesting and complex issue, which is explored in detail in several recent papers.[13] To keep the model's labor adjustment mechanisms transparent and comparable to the baseline Pissarides (2000) model, matching is determined in the following manner to avoid multiple offers. At the beginning of the matching stage the number of matches in each sector is determined. In each sector, these pairwise matches are randomly allocated. Once matched, the pair exits immediately to the bargaining stage. Define $\theta_i \equiv \frac{v_i}{u}$ as market tightness in

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[11]Petrongolo and Pissarides (2001) survey the empirical literature on estimating matching functions. They conclude that existing evidence generally supports the Cobb-Douglas specification.

[12]U.S. data from the Job Openings and Labor Turnover Survey and recent research by Davis et al. (2007), for instance, find significant heterogeneity in vacancy-filling rates across sectors.

[13]See Julien et al. (2006) and Albrecht et al. (2006b) among others.

sector i from the firm's perspective, so in a tighter labor market it is harder for a firm to find a worker; $f_i(\theta_i) = \frac{m_i}{u}$ denotes an unemployed worker's job-finding probability in sector i;[14] and $q_i(\theta_i) = \frac{m_i}{v_i}$ denotes the job-filling probability for a sector i vacancy, where $\sum_{i=1}^{I} f_i(\theta_i), q(\theta_i) \in [0,1] \quad \forall i$.

3.5 Value Functions in the Steady-State

In the steady-state, the worker's Bellman equations are as follows. The expected present value of being unemployed, U, is:

$$U = z + \delta[\sum_{i=1}^{I} f_i(\theta_i)W_i^L + (1 - \sum_{i=1}^{I} f_i(\theta_i))U] \tag{1}$$

In the current period the worker receives unemployment benefits. With probability $f_i(\theta_i)$ the worker matches with a firm and receives an offer in sector i. In equilibrium she accepts all job offers,[15] and thus will begin next period working as a low-skill match — the present value of which is W_i^L. Next period's payoffs are discounted by δ and the summation is over all sectors. With complementary probability the worker does not match and remains unemployed.

The expected present value of being a worker in a low-skill sector i match, W_i^L, is:

$$W_i^L = w_i^L + \delta[s_i U + \lambda_i x_i W_i^H + (1 - s_i - \lambda_i x_i)W_i^L] \tag{2}$$

The current return is the low-skill wage in sector i. With probability s_i, the match separates and the worker becomes unemployed next period. With probability $\lambda_i x_i$, the match acquires skill and produces next period as high-skill. With complementary probability the worker keeps his current job.

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[14]The probability of matching in sector i is the product of the probability of finding a job and the probability of that job being in sector i, $f_i = \frac{\sum m_i}{u} \times \frac{m_i}{\sum m_i} = \frac{m_i}{u}$.

[15]Section 3.9 derives the equilibrium wages, confirming this assertion.

The expected present value of working in a high-skill sector i match, W_i^H, is:

$$W_i^H = w_i^H + \delta[s_i U + (1 - s_i)W_i^H] \tag{3}$$

The worker receives the high-skill wage in the current period. The job terminates with probability s_i, leaving the worker unemployed next period, otherwise the job continues.

The value functions for the firm are given by the following: The expected present value of posting a sector i vacancy, V_i, is:

$$V_i = -c + \delta[q_i(\theta_i)J_i^L + (1 - q_i(\theta_i))V_i] \tag{4}$$

The firm incurs the recruiting cost in the current period. With probability $q_i(\theta_i)$, the job-filling rate, the firm matches with a worker and begins producing with a low-skill job next period, else the firm continues recruiting.

The expected present value for a firm in a low-skill match in sector i, J_i^L, is:

$$J_i^L = A_i p_i^L - w_i^L - \chi(x_i) + \delta[s_i V_i + \lambda_i x_i J_i^H + (1 - s_i - \lambda_i x_i)J_i^L] \tag{5}$$

The first term is the firm's current profit: the firm produces output $A_i p_i^L$, pays the worker wage w_i^L and provides investment of x_i at cost $\chi(x_i)$. The match separates with probability s_i, leaving the firm with a vacancy next period. With probability $\lambda_i x_i$, the match becomes high-skill next period. The expected present value for a firm in a high-skill match in sector i, J_i^H, is:

$$J_i^H = A_i p_i^H - w_i^H + \delta[s_i V_i + (1 - s_i)J_i^H] \tag{6}$$

The firm's current profit is its output less the wage, since the firm no longer invests in the worker, because output cannot be increased beyond the high-skill level. With probability s_i, the match terminates, otherwise high-skill production continues.

3.6 Wage Determination Through Nash Bargaining

When unmatched firms and workers first meet, they begin producing next period in a low-skill match only if they agree on how to split the expected surplus from their partnership. This is done by generalized Nash Bargaining with full information where the threat points are the continuation values from no-agreement — which leaves the worker unemployed, with value U, and the firm with a vacancy, valued at V_i. Agreement allows production to begin in a low-skill match giving the worker W_i^L and the firm J_i^L. The new match surplus, S_i, is what the pair gains from producing less what they give up, $S_i \equiv W_i^L - U + J_i^L - V_i$.

The wage paid each period to a worker in a low-skill match in sector i is set efficiently to split the weighted product of worker's and firm's net gains from the match:

$$w_i^L = \arg\max[W_i^L(w_i^L) - U]^\beta [J_i^L(w_i^L) - V_i]^{1-\beta}$$

where β is the worker's bargaining power and $\beta \in (0,1)$ so both sides have an incentive to produce. First order conditions for these maximization problems imply:

$$W_i^L - U = \beta S_i; \quad \text{and} \quad J_i^L - V_i = (1 - \beta)S_i. \tag{7}$$

Therefore, the low-skill wage in sector i, which I derive explicitly later, gives workers share β, and firms share $(1 - \beta)$, of the new match surplus.

If the match becomes high-skill, the pair once again splits the surplus via Nash Bargaining. The threat points are the values of continuing production as a low-skill match.[16] Define the sector i skill premium, SP_i, as the incremental surplus generated when moving from a low to high-skill match, where $SP_i \equiv W_i^H - W_i^L + J_i^H - J_i^L$. Similarly, the high-skill wage in sector

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[16]Since both agents strictly prefer participating in a low-skill match to being unmatched in equilibrium, threats to 'endogenously' separate the match by either side are not credible.

i is set so the worker receives share β of the skill premium and the firm receives the rest:

$$W_i^H - W_i^L = \beta SP_i; \quad \text{and} \quad J_i^H - J_i^L = (1 - \beta)SP_i. \tag{8}$$

3.7 Equilibrium

Definition: Given a set of constant exogenous parameters, $\{A_i, p_i^L, p_i^H, s_i, \lambda_i, \mu_i, \alpha, c, \delta, z, \beta\}_{i=1}^I$, a *symmetric steady-state rational expectations equilibrium* is a set of value functions $\{U, W_i^L, W_i^H, V_i, J_i^L, J_i^H\}_{i=1}^I$; transition probabilities $\{f_i(\theta_i)\}_{i=1}^I, \{q_i(\theta_i)\}_{i=1}^I$; wages $\{w_i^L, w_i^H\}_{i=1}^I$; investment policies $\{x_i\}_{i=1}^I$ and a labor allocation $\{e_i^L, e_i^H, u\}_{i=1}^I$, such that, in all sectors:

1. **Optimality**:

   (a) Taking job-filling probabilities and wages as given, firms maximize expected profit, (i.e. J_i^L, J_i^H and x_i solve the firm's problem).

   (b) Taking job-finding probabilities and wages as given, workers maximize expected income, (i.e. U, W_i^L and W_i^H solve the worker's problem).

2. **Free Entry and Exit of Vacancies:** In all sectors, zero profit conditions hold for the expected value of posting a vacancy (net of recruiting costs).

3. **Generalized Nash Bargaining:** splits the low and high-skill match surpluses.

4. **Rational Expectations:** Firms and workers correctly anticipate transition probabilities, wages and innovation investment.

5. **Stationary Labor Distribution:** There is a stationary distribution of workers over employment states.

A stationary distribution of labor has three requirements. First, in each sector, the flow of workers into unemployment equals the flow of workers out of unemployment. Second, the

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flow of workers into high-skill sector i matches equals the flow out. Finally, the labor force sums to one, the total measure of potential workers. These conditions are:

$$s_i(e_i^L + e_i^H) = f_i(\theta_i)u; \quad \text{and} \quad \lambda_i x_i e_i^L = s_i e_i^H; \quad \text{and} \quad \sum_{i=1}^{I}(e_i^L + e_i^H) + u = 1. \qquad (9)$$

An equilibrium solves for $\{x_i^*, \theta_i^*, w_i^{L*}, w_i^{H*}, e_i^{L*}, e_i^{H*}, u^*\}_{i=1}^{I}$. A representative firm in each sector makes two crucial decisions which drive the results. When unmatched, firms decide whether to post a vacancy; and once in a low-skill match, firms decide how much to invest in innovation. While these actions are sequential, in equilibrium, firms correctly anticipate the innovation policies offered once a meeting occurs. Since, the firm's vacancy posting decision takes the innovation decision into account, I discuss the innovation decision first.

3.8 Intra-Sectoral Labor Reallocation: The Innovation Effect

Firms in low-skill matches in sector i optimally choose their innovation policies taking as given wages, the skill arrival rate, and other firms' innovation decisions:

$$J_i^L = \max_{0 \leq x_i \leq 1} A_i p_i^L - w_i^L - \chi(x_i) + \delta[s_i V_i + \lambda_i x_i J_i^H + (1 - s_i - \lambda_i x_i)J_i^L]$$

The first order condition for an interior solution is (The Appendix considers corner solutions):

$$\begin{aligned} \chi'(x_i^*) &= \delta\lambda_i(J_i^H - J_i^L) \\ &= \delta\lambda_i(1 - \beta)SP_i \end{aligned}$$

The LHS is the marginal cost and the RHS is the expected discounted marginal benefit of increasing innovation. The second equality uses the Nash Bargaining solution, equation (8). The benefit of innovating is the increase in the arrival rate λ_i, multiplied by the firm's share $(1 - \beta)$ of the skill premium — the increased production from becoming a high-skill match plus the investment savings, because high-skill matches require no further investment.

I assume a linear innovation cost function, which yields a simple closed-form solution for firms' optimal innovation investment, given in the following proposition:

Proposition 3.1 (Optimal innovation policies) *When the innovation investment cost function is linear, $\chi(x_i) = k_i x_i$, a threshold skill arrival rate, $\overline{\lambda}_i$, characterizes firms' innovation decisions. The optimal symmetric innovation policy in sector i is:*[17]

$$x_i^* = \begin{cases} 0 & \text{if } \lambda_i \leq \overline{\lambda}_i \\ \min\{\frac{(1-\beta)}{k_i\beta} A_i(p_i^H - p_i^L) - \frac{r+s_i}{\lambda_i\beta}, 1\} & \text{if } \lambda_i > \overline{\lambda}_i \end{cases} \tag{10}$$

where: $\overline{\lambda}_i = \frac{k_i(r+s_i)}{(1-\beta)A_i(p_i^H - p_i^L)}$

Firms innovate only if the skill arrival rate is sufficiently high, $\lambda_i > \overline{\lambda}_i$. Innovation is increasing in the skill arrival rate and the difference between high and low-skill productivity. Innovation is also increasing in the sector-specific productivity shock, A_i. Therefore, when a sector's productivity rises, firms innovate more. These actions accelerate skill acquisition and endogenously increase the share of high-skill matches in the sector. As a result, the output response to the productivity shock is amplified relative to the baseline model. This is the *'innovation effect'*.

3.9 Inter-Sectoral Labor Reallocation: The Reservation Wage Effect

Now consider the firm's entry decision of whether to post a vacancy. In equilibrium, free entry drives the expected value of posting a vacancy to zero, $V_i = 0$, which implies:

$$\frac{c}{q_i(\theta_i^*)} = \frac{\pi_i^L}{(r + s_i + \lambda_i x_i^*)} + \lambda_i x_i^* \frac{\pi_i^H}{(r + s_i + \lambda_i x_i^*)(r + s_i)} \tag{11}$$

where π is current period profit. The LHS is the total expected recruiting cost: the per-period cost, c, times the expected number of periods to fill the vacancy, $\frac{1}{q_i(\theta_i^*)}$. The RHS is

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[17]I consider only the symmetric innovation equilibrium. Equilibria may exist where some firms in a sector offer lower starting wages, but innovate more, or higher wages and innovate less. See Mortensen (2000).

the expected discounted accounting profits earned in a match. Notice this anticipates the expected gain in value if the match becomes high-skill, which occurs with probability $\lambda_i x_i^*$, for the optimal innovation choice x^*.

I derive equilibrium wages using the value functions, equations (1) - (6), the Nash bargaining solutions, equations (7) and (8), and the zero profit conditions $V_i = 0 \ \forall i$, giving:

$$w_i^{L*} = \overline{w} + \beta(A_i p_i^L - \chi(x_i^*) - \overline{w}) \tag{12}$$

$$w_i^{H*} = \overline{w} + \beta(A_i p_i^H - \overline{w}) \tag{13}$$

where $\overline{w} \equiv z + \delta \sum_{i=1}^{I} f_i(\theta_i^*)(W_i^L - U)$

Each period, workers receive their reservation wage, $\overline{w}$, plus their bargaining power share β of the low and high-skill per-period match values respectively. The reservation wage, as defined above, is a key concept in the model. It is the worker's outside option — the value of continuing to search while unemployed, or equivalently, what the worker foregoes by accepting the job since there is no on-the-job search. The option value of search is the unemployment benefits the worker would collect, z, plus the expected gain in value from accepting a job in a given sector, $(W_i^L - U)$, weighted by the probabilities of receiving offers in these sectors, $f_i(\theta_i)$, summed over all sectors and discounted because production begins next period.

A key difference relative to the basic one-sector model, is that with multisector search, the outside option includes the possibility of working in other sectors. As a result, the worker's reservation wage updates when market conditions change in other sectors. Sectoral spillovers occur through this feature of the model, which effectively creates equilibrium linkages in labor market conditions across different sectors.

Equilibrium profits are:

$$\pi_i^{L*} = (1 - \beta)(A_i p_i^L - \overline{w}) + \beta\chi(x_i^*) \tag{14}$$

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$$\pi_i^{H*} = (1 - \beta)(A_i p_i^H - \overline{w}) \tag{15}$$

Substituting equilibrium profits into the vacancy posting equation (11), illustrates that increasing the worker's value of search, $\overline{w}$, discourages entry:

$$\frac{c}{q_i(\theta_i^*)} = \frac{(1 - \beta)(A_i p_i^L - \overline{w}) + \beta \chi(x_i^*)}{(r + s_i + \lambda_i x_i^*)} + \lambda_i x_i^* \frac{(1 - \beta)(A_i p_i^H - \overline{w})}{(r + s_i + \lambda_i x_i^*)(r + s_i)} \tag{16}$$

This *'reservation wage effect'* leads to the sectoral spillovers and shock propagation. For example, positive developments in one sector raise workers' reservation wage. As wages are bid up, labor becomes more expensive, new jobs become less profitable, and job creation falls in other sectors. This intuition is formalized in the following proposition:

Proposition 3.2 (Sector-Specific Shocks and Equilibrium Market Tightness) *A positive sector-specific productivity shock in sector i, A_i, causes equilibrium market tightness to rise in sector i, θ_i^*, and fall in the other sectors, $\{\theta_j^*\}_{j \neq i}^I$. Conversely, a negative shock in sector i, reduces market tightness in that sector and increases market tightness in the other sectors.*

4 Transition Dynamics

The previous section establishes the model's steady-state properties. This section characterizes the model's transition dynamics between steady-states.

To illustrate, assume the economy is in a steady-state and consider an unanticipated sector-specific productivity shock, denoted $\hat{A}_{i,t}$, that occurs in sector i, at the beginning of period t, where the hat superscript denotes an updated value. As in the baseline Pissarides (2000) model, labor contracts are costlessly renegotiated whenever shocks hit the economy. Therefore, prior to production in period t, existing matches renegotiate low and high-skill wages using the Nash bargaining solutions and firms update their innovation policies, as described above. In addition, prior to recruitment, unmatched firms optimally update their vacancy decisions. Because there is free entry and free disposal of vacancies, the value of a

vacancy is zero for all sectors at all points in time. In Pissarides' terminology, wages, innovation investment and market tightness (vacancies) are 'jump variables' updating immediately in the period the shock hits, prior to production and search. Their new values are:

$$\hat{x}_{i,t}^* = \begin{cases} 0 & \text{if } \lambda_i \leq \hat{\overline{\lambda}}_i \\ \min\{\frac{(1-\beta)}{k_i\beta}\hat{A}_{i,t}(p_i^H - p_i^L) - \frac{r+s_i}{\lambda_i\beta}, 1\} & \text{if } \lambda_i > \hat{\overline{\lambda}}_i \end{cases}$$

$$\frac{1+r}{q(\hat{\theta}_{i,t}^*)} = \frac{(1-\beta)}{c}[\hat{A}_{i,t}p_i^L - \chi(\hat{x}_{i,t}^*) - z + \lambda_i\hat{x}_{i,t}^* \cdot \frac{\hat{A}_{i,t}(p_i^H - p_i^L) + \chi(\hat{x}_{i,t}^*)}{r + s_i + \lambda_i\hat{x}_{i,t}^*}] + E_{\hat{t}}\{\frac{1-s_i}{q(\hat{\theta}_{i,t+1}^*)} - \beta\sum_{i=1}^{I}\hat{\theta}_{i,t+1}^*\}$$

$$\hat{w}_{i,t}^{L*} = \hat{\overline{w}}_t + \beta(\hat{A}_{i,t}p_i^L - \chi(\hat{x}_{i,t}^*) - \hat{\overline{w}}_t); \qquad \hat{w}_{i,t}^{H*} = \hat{\overline{w}}_t + \beta(\hat{A}_{i,t}p_i^H - \hat{\overline{w}}_t)$$

where $\hat{\overline{\lambda}}_i = \frac{k_i(r+s_i)}{(1-\beta)\hat{A}_{i,t}(p_i^H - p_i^L)}$ and $\hat{\overline{w}}_t = z + \delta E_{\hat{t}}\{\sum_{i=1}^{I} f_i(\hat{\theta}_{i,t}^*)(\hat{W}_{i,t}^L - \hat{U}_t)\}$

Notice these variables can jump to their new values because they do not depend directly on employment and unemployment levels. Given these new wages and equilibrium transition probabilities, the value functions also discretely update in period t. For example, in period t prior to the shock, the present value of being unemployed is:

$$U_t = z + \delta E_t[\sum_{i=1}^{I} f_i(\theta_{i,t+1}^*)(W_{i,t+1}^L - U_{t+1}) + U_{t+1}]$$

After the shock in period t, the value of unemployment updates immediately to:

$$\hat{U}_t = z + \delta E_{\hat{t}}[\sum_{i=1}^{I} f_i(\hat{\theta}_{i,t+1}^*)(\hat{W}_{i,t+1}^L - \hat{U}_{t+1}) + \hat{U}_{t+1}]$$

Similarly, the other value functions update to:

$$\hat{W}_{i,t}^L = \hat{w}_{i,t}^{L*} + \delta E_{\hat{t}}[s_i(\hat{U}_{t+1} - \hat{W}_{i,t+1}^L) + \lambda_i\hat{x}_{i,t+1}^*(\hat{W}_{i,t+1}^H - \hat{W}_{i,t+1}^L) + \hat{W}_{i,t+1}^L]$$

$$\hat{W}_{i,t}^H = \hat{w}_{i,t}^{H*} + \delta E_{\hat{t}}[s_i(\hat{U}_{t+1} - \hat{W}_{i,t+1}^H) + \hat{W}_{i,t+1}^H]$$

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$$\hat{V}_{i,t} = -c + \delta E_{\hat{t}}[q_i(\hat{\theta}^*_{i,t+1})(\hat{J}^L_{i,t+1} - \hat{V}_{i,t+1}) + \hat{V}_{i,t+1}]$$

$$\hat{J}^L_{i,t} = \hat{A}_{i,t}p^L_i - \hat{w}^{L*}_{i,t} - \chi(\hat{x}^*_{i,t}) + \delta E_{\hat{t}}[s_i(\hat{V}_{i,t+1} - \hat{J}^L_{i,t+1}) + \lambda_i\hat{x}^*_{i,t+1}(\hat{J}^H_{i,t+1} - \hat{J}^L_{i,t+1}) + \hat{J}^L_{i,t+1}]$$

$$\hat{J}^H_{i,t} = \hat{A}_{i,t}p^H_i - \hat{w}^{H*}_{i,t} + \delta E_{\hat{t}}[s_i(\hat{V}_{i,t+1} - \hat{J}^H_{i,t+1}) + \hat{J}^H_{i,t+1}]$$

Free entry and exit of vacancies imply $\hat{V}_{i,t} = \hat{V}_{i,t+1} = 0$. Nash Bargaining implies $\hat{J}^L_{i,t} = (1-\beta)\hat{S}_{i,t}$ and $\hat{W}^L_{i,t} - \hat{U}_{i,t} = \beta\hat{S}_{i,t}$, so one can succinctly write the updated joint value of a low-skill match in sector i as $\hat{S}_{i,t} = \frac{c}{(1-\beta)\delta q_i(\hat{\theta}^*_{i,t})}$ or equivalently:

$$\hat{S}_{i,t} = \hat{A}_{i,t}p^L_i - \chi(\hat{x}^*_{i,t}) - z + \delta E_{\hat{t}}\{\lambda_i\hat{x}^*_{i,t+1}\hat{SP}_{i,t+1} + [1 - s_i - f_i(\hat{\theta}^*_{i,t+1})]\hat{S}_{i,t+1} - \sum_{j\neq i}^{I} f_j(\hat{\theta}^*_{j,t+1})\hat{S}_{j,t+1}\}$$

Other variables, such as employment and unemployment, evolve more slowly to their new steady-state values according to the following difference equations:

$$\hat{e}^L_{i,t+1} = f_i(\hat{\theta}^*_{i,t})u_t + (1 - s_i - \lambda_i\hat{x}^*_i)e^L_{i,t}$$

$$\hat{e}^H_{i,t+1} = \lambda_i\hat{x}^*_i e^L_{i,t} + (1 - s_i)e^H_{i,t}$$

$$\hat{u}_{t+1} = \sum_{i=1}^{I} s_i(e^L_{i,t} + e^H_{i,t}) + [1 - \sum_{i=1}^{I} f_i(\hat{\theta}^*_{i,t})]u_t$$

Finally, output moves along with changes in employment during the transition:

$$\hat{Y}_t = \sum_{i=1}^{I}\sum_{SK=L}^{H} \hat{A}_{i,t}p^{SK}_i e^{SK}_{i,t}$$

A stable transition requires that each sector's market tightness updates immediately to its new steady-state value, $\hat{\theta}^*_{i,t}$. However, since market tightness is $\theta_{i,t} \equiv \frac{v_{i,t}}{u_t}$, vacancies overshoot their steady-state level and move in the same direction as unemployment so that market tightness remains constant at its new steady-state value during the transition. See Pissarides (1985) or (2000, Ch. 1.7).

5 Inspecting the Model Mechanisms: General versus Sector-Specific Productivity Shocks

5.1 Quantitative Approach

This section presents simple quantitative examples to illustrate the key model mechanisms through the innovation and reservation wage effects. I compare the model economy's response to an equal-sized productivity shock in two scenarios. The first scenario is a general shock that affects all sectors equally. This results in intra-sectoral labor reallocation in both sectors through the innovation effect. The second scenario is a sector-specific shock, which *directly* affects only one sector. This results in intra-sectoral reallocation in the sector where the shock occurs through the innovation effect as well as inter-sectoral reallocation between sectors through the reservation wage effect.

To keep the results transparent and emphasize the model's adjustment mechanisms, I parameterize a benchmark economy consisting of two perfectly symmetric sectors. Each sector uses the same production technologies and each has half of the economy's employed workers, of which half are in low-skill and half are in high-skill matches. Table 1 reports the parameter values for the benchmark model. In these examples, the only parameters that change are the sector-specific productivity terms, A_1 and A_2.

To quantify a reasonable size for the productivity shocks, Table 2 reports summary statistics using Canadian data for sectoral and aggregate output per worker, expressed in log deviations from their HP-filtered trends.[18] The table shows that productivity is considerably more volatile at the sectoral level than the aggregate level. In the resource and manufacturing sectors, productivity is often 3-4 percent or more away from its trend growth. Furthermore, these deviations from trend are quite persistent with autocorrelations of 0.86 and higher. In the numerical example, I use 3 percent for the sector-specific shock. The equivalent-sized

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[18]Output per worker proxies productivity here because labor is the only factor of production in the model.

general productivity shock in the two-sector economy is 1.5 percent, since the 3 percent shock directly affects half of the economy. The shock is unanticipated and permanent.

5.2   Quantitative Results

Table 3 reports the percentage change in the new steady-states, relative to the benchmark economy, following the general productivity shock and the equal-sized, sector-specific productivity shock.

While the aggregate differences are reasonably small, there are important distinctions for the sectoral and skill composition of production, aggregate productivity and wages.

First, consider the model economy's response to the general productivity shock. This case isolates the innovation effect and demonstrates that firms' endogenous innovation responses amplify the impacts of productivity shocks. The productivity shock was an increase of 1.5 percent, however, aggregate output rises by 2.4 percent because the economy invests more resources in innovation to substitute toward high-skill production (whose share of overall production increases from 50 percent to 51.2 percent after the shock).

The economy's response to the sector-specific productivity shock is quite different due to the asymmetric nature of the shock. The sector-specific shock raises aggregate output and output per worker more than the general shock (2.7 percent rather than 2.4 percent). The reason is that the economy concentrates production in high-skill jobs in the more productive sector, through inter-sectoral and intra-sectoral labor reallocation. While the shock directly affects Sector 1, there are equilibrium wage spillovers which discourage job posting in Sector 2 through the reservation wage effect.[19]

The mechanism here works as follows: Firms post vacancies and increase investment in

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[19]This result is consistent with recent empirical findings by Beaudry et al. (2007) who show that changes in the sectoral composition in U.S. cities have equilibrium spillovers on the level of wages, after controlling for observable characteristics.

Sector 1 to take advantage of the now-more-productive workers. The increase in Sector 1 vacancies changes the composition of job postings, making matches with Sector 1 firms more likely. These firms are now more productive and invest more resources in becoming high-skill to take advantage of the improved productivity. Therefore, workers in Sector 1 generally receive higher starting wages and also expect to earn high-skill wages sooner because, on average, they will acquire skills faster in this sector. The value of search for the unemployed rises because of the improved probability of getting these better paying jobs, pushing up the reservation wage.

The increase in the reservation wage has equilibrium effects. Wages are re-bargained in Sector 2 to reflect workers' improved outside option. With more expensive labor in Sector 2 and no change in the productivity of their workers, these jobs become less profitable so recruiting falls in this sector. Thus in the new steady-state, the asymmetric recruiting responses — vacancies rise in Sector 1 and fall in Sector 2 — lead to inter-sectoral reallocation, shifting labor into the more productive sector. These productivity-enhancing labor movements between sectors are reinforced by the shift within the more productive sector to high-skill matches due to a larger innovation effect after the sector-specific shock.

Finally, the sector-specific shock has larger distributional consequences for wages. Relative to the general shock scenario, high-skill workers in Sector 1 are the major winners and high-skill workers in Sector 2 are the major losers (as wages rise by 1 percent and fall by 0.9 percent respectively).

Theses effects are steady-state comparisons. Figure 7 shows the transition dynamics to illustrate the sectoral employment responses. After the sector-specific shock, the composition of vacancies shifts immediately and a larger proportion of new hires work in Sector 1 each period. Over time, employment rises in Sector 1 and falls in Sector 2.

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5.3 Discussion

Consider the model's labor adjustment after sector-specific shocks relative to the general process described from the empirical facts presented in Section 2.

Fact 1 is that labor flows towards the sector which experienced a positive sector-specific shock and away from the sector which experienced a negative shock. Compare Figure 1, the employment levels after the oil price shocks and Figure 7, the model's employment response following a relative sector-specific shock. The results in Figure 1 are driven by the increased price of oil, which raises the profitability of non-manufacturing production relative to manufacturing production. Given that manufacturing production is more energy-intensive, its production costs are more adversely affected. Over time, as the incentives to hire shift toward non-manufacturing jobs, labor is reallocated.[20] In the model, after Sector 1 receives a positive shock, employment flows to this sector and rises by over 18 percent in the new steady-state (see Table 3).

The model's predictions are also consistent with empirical Facts 2 and 3 — that there is a relative increase in the employment and real wages of high-skill workers in sector which receives a positive shock. After Sector 1 receives the positive shock, high-skill employment rises from 50 percent to 52.3 percent. Wages gains are also concentrated for high-skill workers in this sector, rising 2.8 percent after the shock.

The quantitative results in this particular example show that increasing a sector's productivity leads to increased innovation investment, an increased share of high-skill workers, and larger wage gains for these workers. In fact, it is straight-forward to show these general results analytically. The intra-sectoral reallocation result is the following: In the steady-state, from equation (9), in sector i the flow of workers into high-skill jobs equals the flow

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[20]This is consistent with findings by Davis and Haltiwanger (2001) who analyze plant-level data within the manufacturing sector. They find larger employment reductions in more energy-intensive plants following oil price increases.

out: $\lambda_i x_i^* e_i^{L*} = s_i e_i^{H*}$. Rearranging this equation shows that a sector's optimal innovation investment in the steady-state is:

$$x_i^* = [\frac{s_i}{\lambda_i}] \cdot [\frac{e_i^{H*}}{e_i^{L*}}]$$

From Proposition 3.1, innovation investment, x^*, increases with a sector's productivity, A_i. This implies that higher sector-specific productivity increases innovation investment, which in turn, raises the steady-state ratio of high-to-low skill workers in sector i.[21] Furthermore, the relative wages of high-skill to low-skill workers rise as a sector's productivity increases. The wage differential can be expressed as: $w_i^{H*} - w_i^{L*} = \beta[A_i(p_i^H - p_i^L) + \chi(x_i^*)] > 0$. This expression is directly increasing in a sector's productivity A_i, which in turn, also increases innovation costs, $\chi(x_i)$, and causes further wage dispersion.

6   Conclusions

This paper presents a general model of sectoral labor reallocation. I demonstrate that the model's key adjustment mechanisms — of entry, innovation and wage spillovers — are consistent with the results from several labor adjustment episodes after sector-specific shocks. The model is reasonably parsimonious and tractable, which facilitates taking it to the data to study particular labor adjustment episodes.[22]   This analysis suggests that the widely-used search and matching framework is well-suited to tackle, not only the aggregate and distributional issues to which it is generally applied, but also to study sectoral issues. Examples include: the welfare costs of sectoral labor reallocation and the potential role of labor market policies to facilitate structural adjustment, such as addressing the secular decline in manufacturing employment in developed economies.

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[21]As A_i increases, the LHS of the above equation increases. The first fraction on the RHS is an exogenous constant, therefore the second fraction on the RHS must increase in the steady-state.

[22]Tapp (2007) is an example that applies this model to Canadian data to quantify the aggregate costs of labor adjustment following a global commodity price shock.

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Tables

Table 1: **Parameter Values for the Benchmark Model**

Variable	Parameter	Value	Rationale
Real Interest Rate	r	0.33%	4 percent annual
Discount Factor	δ	0.997	$\delta = \frac{1}{1+r}$
Separation Rate, Sector 1	s_1	3.4%	Shimer (2005)
Separation Rate, Sector 2	s_2	3.4%	Shimer (2005)
Low-Skill Productivity, Sector 1	p_1^L	1.0	
Low-Skill Productivity, Sector 2	p_2^L	1.0	
High-Skill Productivity, Sector 1	p_1^H	2.0	
High-Skill Productivity, Sector 2	p_2^H	2.0	
Recruiting Cost	c	0.1	Tapp (2007)
Investment Cost Scale, Sector 1	k_1	1.0	
Investment Cost Scale, Sector 2	k_2	1.0	
Productivity Shock, Sector 1	A_1	1.0	Steady-State
Productivity Shock, Sector 2	A_2	1.0	Steady-State
Matching Fx. Scale, Sector 1	μ_1	0.13	Sector 1 Employment Share $= \frac{1}{2}$
Matching Fx. Scale, Sector 2	μ_2	0.13	Sector 2 Employment Share $= \frac{1}{2}$
Skill Arrival Rate, Sector 1	λ_1	0.11	High-Skill Employment Share $= \frac{1}{2}$
Skill Arrival Rate, Sector 2	λ_2	0.11	High-Skill Employment Share $= \frac{1}{2}$
Unemployment Income	z	0.6	40% Replacement Rate $= 0.4(\frac{p_1^L + p_2^L}{2})$
Matching Function Elasticity	α	0.6	Petrongolo and Pissarides (2001)
Workers' Bargaining Power	β	0.5	Equal split of surplus
Model Period = 1 Month			

Table 2: **Summary Statistics, Sectoral and Aggregate Output per Worker, Canada 1987Q1–2001Q4**

	Resources	Manufacturing	Aggregate Economy
Standard Deviation	0.043	0.030	0.012
Quarterly Autocorrelation	0.86	0.89	0.95
Correlation with Resources	1	0.52	0.30
Correlation with Manufacturing		1	0.53
Correlation with Aggregate Economy			1

Note: All variables are reported in logs as deviations from an HP trend with smoothing parameter of 10^5. All data are from Cansim. Aggregate, Resource and Manufacturing output series are seasonally adjusted at annual rates expressed in 1997 constant dollars: v2036138; v2036146; and v2036171. Aggregate, Resource and Manufacturing Employment series are: v13682073; v13682076; and v13682079.

Table 3: **Steady-State Impacts of General versus Sector-Specific Shocks**

Aggregate Impacts	Benchmark	General Shock	Sector-Specific Shock	Sector-Specific Effect
Output	100	102.4	102.7	0.3
Unemployment Benefits	100	98.4	100.6	2.2
Innovation Investment Costs	100	102.5	102.7	0.2
Recruiting Costs	100	102.8	105.5	2.7
Employment	100	100.1	100.0	-0.1
% High-Skill	50.0	51.2	51.4	0.2
Output per Worker	100	102.3	102.7	0.4
Reservation Wage	100	102.2	102.5	0.3
Unemployment	100	98.4	100.6	2.2
Sectoral Impacts				
Output - Sector 1	100	102.4	124.1	21.7
Sector 2	100	102.4	81.2	-21.2
Employment - Sector 1	100	100.1	118.7	18.6
Sector 2	100	100.1	81.2	-18.9
% High Skill - Sector 1	50.0	51.2	52.3	1.1
Sector 2	50.0	51.2	50.0	-1.2
Profits - Sector 1	100	102.7	128.2	25.5
Sector 2	100	102.7	72.5	-30.2
Market Tightness - Sector 1	100	104.5	151.3	46.8
Sector 2	100	104.5	58.5	-46.0
Distributional Impacts				
Low-Skill Wages - Sector 1	100	101.4	101.6	0.2
Sector 2	100	101.4	101.6	0.2
High-Skill Wages - Sector 1	100	101.8	102.8	1.0
Sector 2	100	101.8	100.9	-0.9

Note: Steady-state comparison following unanticipated, permanent productivity shocks which are general versus sector-specific. The relevant variables in the benchmark steady-state are normalized to 100. In the Benchmark model $A_1 = A_2 = 1$; General Shock $A_1 = A_2 = 1.015$; Sector-Specific Shock for Sector 1: $A_1 = 1.03, A_2 = 1$. The sector-specific effect of the shock is the specific shock minus the general shock.
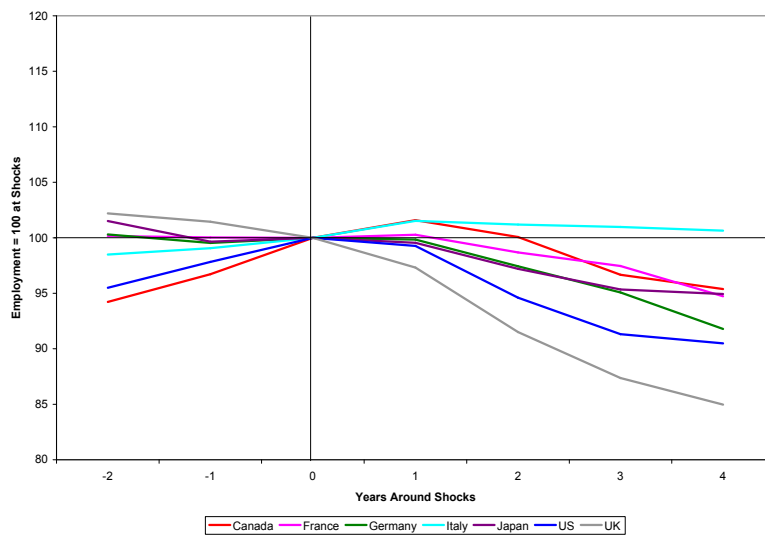
Figures

Figure 1: **Average Employment Response After Oil Shocks, G7 Economies**
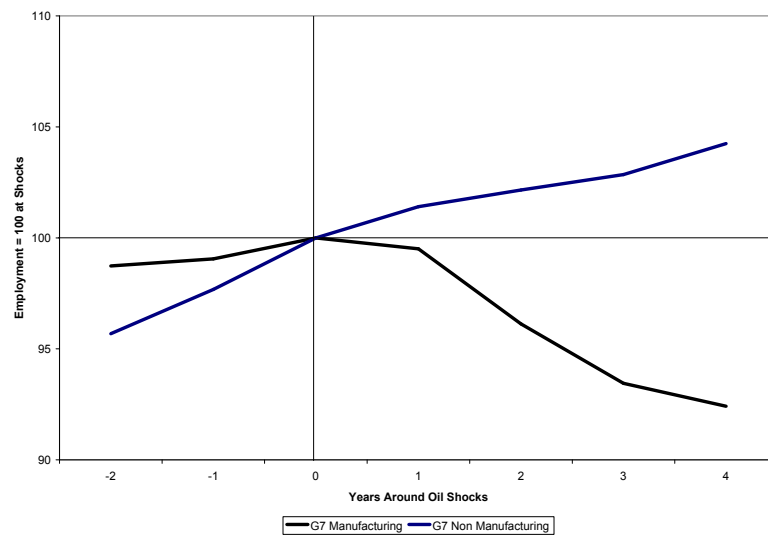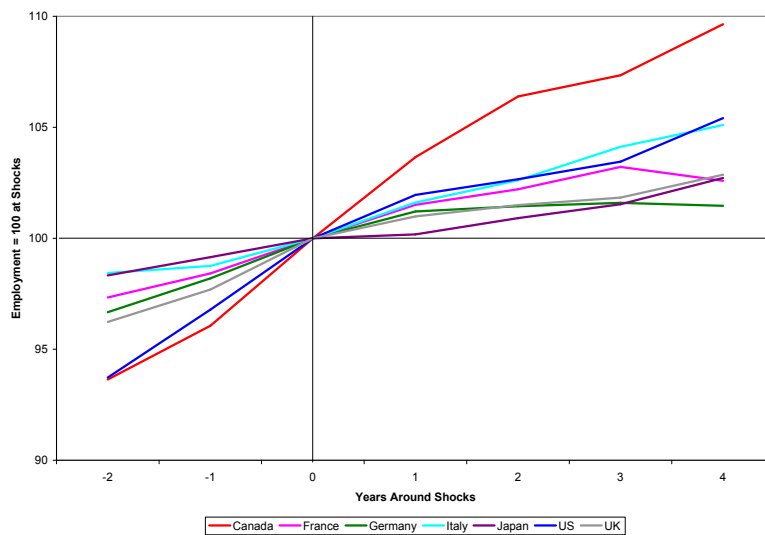

Data Source: OECD Structural Analysis (STAN) Database.

Figure 2: **Average Manufacturing Employment Response After Oil Shocks, G7 Economies**
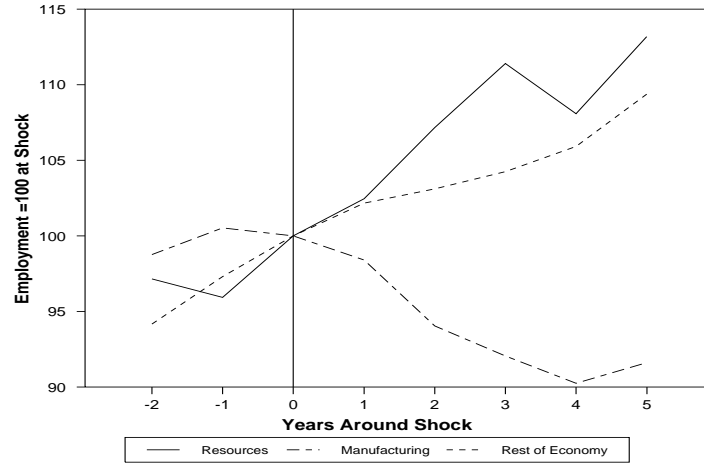

Data Source: OECD Structural Analysis (STAN) Database.

Figure 3: **Average Non-Manufacturing Employment Response After Oil Shocks, G7 Economies**

Data Source: OECD Structural Analysis (STAN) Database.

Figure 4: **Average Employment Response After Oil Shocks, U.S.**

Data Source: Current Employment Statistics (CES) Survey, Resources = Natural Resources and Mining, CES1000000001; Manufacturing CES3000000001; Rest of Economy = Total Nonfarm Employment, CES0000000001, less Resources and Manufacturing Employment.
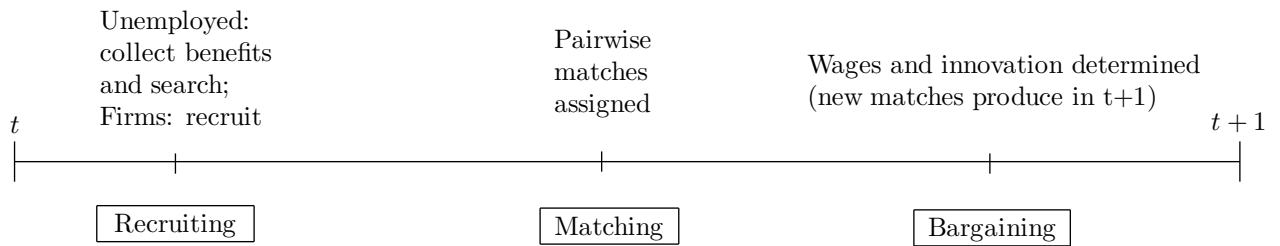

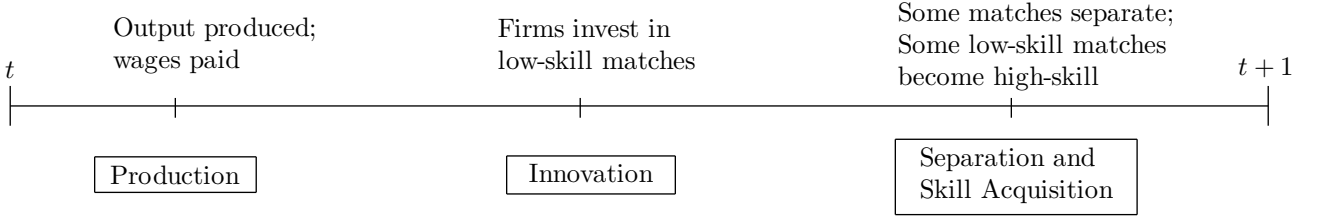Figure 5: **Model Timing: Unmatched Workers and Firms**

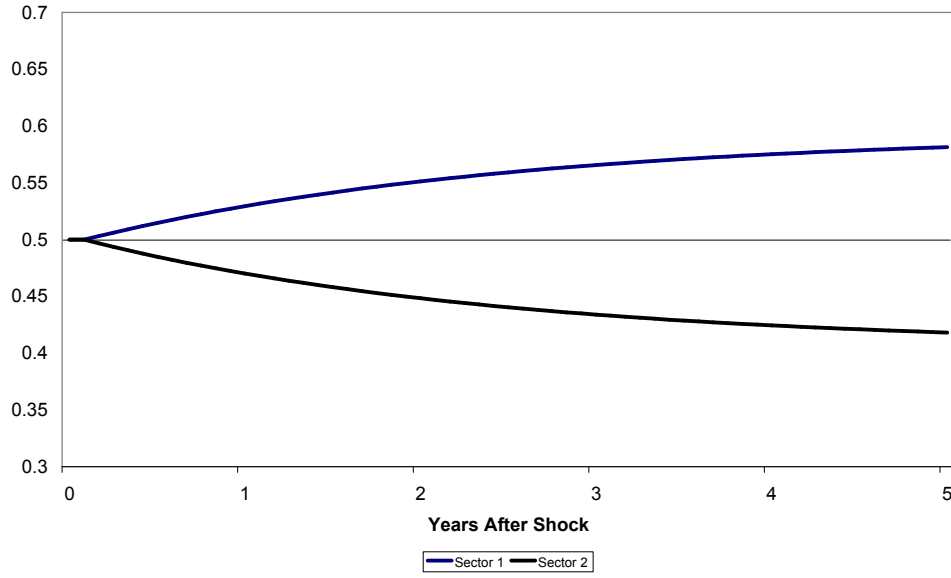Figure 6: **Model Timing: Producing Workers and Firms**

Figure 7: **Model's Employment Response After Sector-Specific Productivity Shock**

Benchmark model's dynamic response of sectoral employment shares to a permanent productivity shock to sector 1, $A_1 = 1.03; A_2 = 1$.

Model Derivations and Solution Method, Proofs of Propositions

Model Derivations

Derivation of Equilibrium Wages:

Low-Skill Wage in Sector i**:** The social present value of a low-skill match is $S_i = W_i^L - U + J_i^L - V_i$. The skill premium of a high-skill relative to a low-skill match is $SP_i = W_i^H - W_i^L + J_i^H - J_i^L$. Using the worker's and firm's value functions, equations (1) – (6), and the free entry/zero profit condition, $V_i = 0$, gives an expression for low-skill match surplus:

$$S_i = A_i p_i^L - \chi(x_i) - z - \delta \sum_i^I f_i(\theta_i)(W_i^L - U) + \delta \lambda_i x_i SP_i + \delta(1 - s_i)S_i \qquad \text{(A.1)}$$

Using equation (5) and $V_i = 0$ gives:

$$J_i^L = A_i p_i^L - w_i^L - \chi(x_i) + \delta \lambda_i x_i (J_i^H - J_i^L) + \delta(1 - s_i)J_i^L$$

Substituting in $J_i^L = (1 - \beta)S_i$, and $J_i^H - J_i^L = (1 - \beta)SP_i$ from the Nash Bargaining solutions, equations (7) and (8), gives another expression in the low-skill surplus:

$$(1 - \beta)S_i = A_i p_i^L - w_i^L - \chi(x_i) + \delta \lambda_i x_i (1 - \beta)SP_i + \delta(1 - s_i)(1 - \beta)S_i \qquad \text{(A.2)}$$

Multiplying (A.1) by $(1 - \beta)$ gives:

$$(1 - \beta)S_i = (1 - \beta)[A_i p_i^L - \chi(x_i) - z - \delta \sum_i^I f_i(\theta_i)(W_i^L - U) + \delta \lambda_i x_i SP_i + \delta(1 - s_i)S_i] \quad \text{(A.3)}$$

Equating the RHS of (A.2) and (A.3) and simplifying gives the equilibrium low-skill wage in sector i, equation (12) in the paper:

$$w_i^{L*} = \overline{w} + \beta(A_i p_i^L - \chi(x_i^*) - \overline{w})$$

where $\overline{w} = z + \delta \sum_i^I f_i(\theta_i^*)(W_i^L - U)$

High-Skill Wage in Sector i:

Subtracting the worker's value functions, equations (3) from (2) gives:

$$W_i^H - W_i^L = w_i^H - w_i^L + \delta[(1 - s_i - \lambda_i x_i)(W_i^H - W_i^L)]$$

Using the fact that $\delta = \frac{1}{1+r}$ and simplifying gives:

$$\delta(r + s_i + \lambda_i x_i)(W_i^H - W_i^L) = w_i^H - w_i^L$$

Substituting in $W_i^H - W_i^L = \beta SP_i$ from the Nash Bargaining solution, (8) gives:

$$\delta(r + s_i + \lambda_i x_i)\beta SP_i = w_i^H - w_i^L \tag{A.4}$$

Then explicitly solve for the skill premium using the worker's and firm's value functions, equations (2) and (3) and (5) and (6) and $\delta = \frac{1}{1+r}$:

$$SP_i = \frac{A_i(p_i^H - p_i^L) + \chi(x_i)}{\delta(r + s_i + \lambda_i x_i)} \tag{A.5}$$

Substituting into (A.4) for the skill premium and the low-skill wage and simplifying gives the high-skill wage in sector i, equation (13) in the paper:

$$w_i^{H*} = \overline{w} + \beta(A_i p_i^H - \overline{w})$$

where $\overline{w} = z + \delta \sum_i^I f_i(\theta_i^*)(W_i^L - U)$

Equilibrium System of Equations in Market Tightness:

Using the zero profit condition, $V_i = 0$ in the firm's value of a vacancy equation, (4), gives $J_i^L = \frac{c}{\delta q_i(\theta_i)}$. From the firm's Nash Bargaining, (7), $(1 - \beta)S_i = J_i^L$. So,

$$(1 - \beta)S_i = \frac{c}{\delta q_i(\theta_i)}$$

Substitute in for $(1 - \beta)S_i$ using (A.3):

$$(1 - \beta)[A_i p_i^L - \chi(x_i) - z - \delta \sum_i^I f_i(\theta_i)(W_i^L - U) + \delta \lambda_i x_i SP_i + \delta(1 - s_i)S_i] = \frac{c}{\delta q_i(\theta_i)}$$

Use the worker's Nash Bargaining solution, (7), $\beta S_i = W_i^L - U$ and use $S_i = \frac{c}{\delta(1-\beta)q_i(\theta_i)}$,

$$(1-\beta)[A_i p_i^L - \chi(x_i) - z - \delta \sum_i^I f_i(\theta_i)\frac{\beta c}{\delta(1-\beta)q_i(\theta_i)} + \delta \lambda_i x_i SP_i + \delta(1-s_i)\frac{c}{\delta(1-\beta)q_i(\theta_i)}] = \frac{c}{\delta q_i(\theta_i)}$$

Use the fact that $f_i(\theta_i) = \theta_i q_i(\theta_i)$ and $\frac{1}{\delta} = 1 + r$ to get:

$$(1 - \beta)[A_i p_i^L - \chi(x_i) - z - \sum_i^I \frac{\beta c \theta_i}{(1 - \beta)} + \delta \lambda_i x_i SP_i] = \frac{c(r + s_i)}{q_i(\theta_i)}$$

Divide both sides by c, substitute in for the skill premium, SP_i, from (A.5) and rearrange to get the equilibrium system of equations in $\{\theta_i\}_1^I$:

$$\frac{r+s_i}{q(\theta_i)} + \beta \sum_i \theta_i = \frac{(1-\beta)}{c}[A_i p_i^L - \chi(x_i^*) - z + \lambda_i x_i^* \cdot \frac{A_i(p_i^H - p_i^L) + \chi(x_i^*)}{r + s_i + \lambda_i x_i^*}] \qquad \text{(A.6)}$$

Solving the Model

The model's steady-state is solved in stages. First, I find the optimal innovation policies, $\{x_i^*\}_{i=1}^I$, using equations (10). These solutions are independent of market tightness. Given these innovation policies, I solve for equilibrium market tightness, $\{\theta_i^*\}_{i=1}^I$, using equations (A.6). A key feature of the model is the interdependence of labor market conditions. For example, the decision to post a vacancy in sector i depends on the expected ease of finding a worker, which in turn, depends on the vacancy posting decisions in other sectors. The model must therefore be solved simultaneously. Fortunately, the model can be distilled into the following system of I simultaneous non-linear equations in $\{\theta_i\}_{i=1}^I$: given by (A.6).

This expression provides a straight-forward generalization of the basic one-sector model without aggregate uncertainty and innovation investment (e.g., Shimer (2005) equation 6):

$$\frac{r+s}{q(\theta)} + \beta\theta = \frac{(1-\beta)}{c}(p-z)$$

Solving the system given by (A.6), using the Newton-Raphson method to find its roots, yields equilibrium market tightness. Equilibrium wages, profits and employment shares are found using equations (9) and (12) — (15).

Proofs of Propositions

Proof of Proposition 3.1:

$$J_i^L = \max_{x_i} A_i p_i^L - w_i^L - \chi(x_i) + \delta[s_i V_i + \lambda_i x_i J_i^H + (1 - s_i - \lambda_i x_i)J_i^L(x_i)] \;\; s.t. \; 0 \le x_i; \; x_i \le 1$$

The associated optimization problem is:
$$\mathcal{L} = A_{i,t} p_{i,t}^L - w_{i,t}^L - \chi(x_{i,t}) + \delta E_t[s_i V_{i,t+1} + \lambda_i x_{i,t+1} J_{i,t+1}^H + (1 - s_i - \lambda_i x_{i,t+1})J_{i,t+1}^L(x_{i,t+1})]$$
$$-\gamma_1(-x_{i,t}) - \gamma_2(x_{i,t} - 1)$$

I focus on stationary innovation policies, where $x_{i,t} = x_{i,t+1}$. The Kuhn-Tucker conditions are:
$$\frac{\partial \mathcal{L}}{\partial x_{i,t}} = -\chi'(x_{i,t}) + \delta E_t[\lambda_i(J_{i,t+1}^H - J_{i,t+1}^L) + (1 - s_i - \lambda_i x_{i,t+1})\frac{\partial J_i^L}{\partial x_i^*}] + \gamma_1 - \gamma_2$$
$$\frac{\partial \mathcal{L}}{\gamma_1} = x_i$$

$\frac{\partial \mathcal{L}}{\gamma_2} = 1 - x_i$

There are three cases to consider: the two corner solutions $x_i^* = 0, x_i^* = 1$ and interior solutions $x_i^* \in (0, 1)$.

Case 1: $x_i^* = 0$. If the first constraint holds, $x_i^* = 0$, so $\gamma_1 > 0$ by the complementary slackness condition. The second constraint is satisfied, so $\gamma_2 = 0$. Collecting terms on the first order condition for investment gives: $\frac{\partial \mathcal{L}}{\partial x_i^*} [1 - \delta(1 - s_i)] = -\chi'(x_{i,t}) + \delta \lambda_i E_t[(J_{i,t+1}^H - J_{i,t+1}^L)] + \gamma_1$. Given the boundary solution, this expression is non-positive so: $\chi'(x_{i,t}) \geq \delta \lambda_i E_t[(J_{i,t+1}^H - J_{i,t+1}^L)] + \gamma_1$. Since γ_1 is positive, this implies the marginal investment cost exceeds the expected marginal benefit at $x_i^* = 0$.

Case 2: $x_i^* = 1$. If the second constraint holds, $x_i^* = 1$, so $\gamma_2 > 0$ by the complementary slackness condition. The first constraint is satisfied, so $\gamma_1 = 0$. Collecting terms on the first order condition for investment gives: $\frac{\partial \mathcal{L}}{\partial x_i^*} [1 - \delta(1 - s_i - \lambda_i)] = -\chi'(x_{i,t}) + \delta \lambda_i E_t[(J_{i,t+1}^H - J_{i,t+1}^L)] - \gamma_2$. Given the boundary solution, this expression is non-negative so: $\delta \lambda_i E_t(J_{i,t+1}^H - J_{i,t+1}^L) \geq \chi'(x_{i,t}) + \gamma_2$. Since γ_2 is positive, the expected marginal benefit of investment exceeds the marginal cost at $x_i^* = 1$.

Case 3: $x_i^* \in (0, 1)$. Both constraints are satisfied so $\gamma_1 = \gamma_2 = 0$. By the envelope theorem, $\frac{\partial J_i^L}{\partial x_{i,t}^*} = 0$, so the first order condition for investment simplifies to:

$$\frac{\partial \mathcal{L}}{\partial x_{i,t}^*} = -\chi'(x_{i,t}) + \delta E_t[\lambda_i(J_{i,t+1}^H - J_{i,t+1}^L)] = 0$$

For interior solutions, the marginal benefit of investment equals the marginal cost. Substituting into the first order necessary condition for investment using $J_{i,t+1}^H - J_{i,t+1}^L = (1 - \beta)SP_{i,t+1} = (1 - \beta)\frac{A_{i,t+1}(p_i^H - p_i^L) + k_i x_i}{\delta(r + s_i + \lambda_i x_i)}$ from the Nash Bargaining solution, equation (8), and using the properties of investment cost function, $\chi(x_i) = k_i x_i$, gives: $k_i = \lambda_i(1 - \beta)\frac{A_{i,t+1}(p_i^H - p_i^L) + k_i x_{i,t+1}}{(r + s_i + \lambda_i x_{i,t+1})}$, which after lagging one-period, simplifies to:

$$x_{i,t}^* = \frac{(1 - \beta)}{k_i \beta} A_{i,t}(p_i^H - p_i^L) - \frac{r + s_i}{\lambda_i \beta}$$

Finally, when $\lambda_i \leq \overline{\lambda}_i$, the skill arrival rate is sufficiently low so no investment is offered.

Proof of Proposition 3.2: The sector that received the positive shock is now more productive, so its surplus from a new match increases. This in turn, means jobs in this sector are more profitable, so vacancy posting and market tightness increase in this sector.

Now, assume unemployed workers' reservation wage falls. With cheaper labor, jobs in all other sectors also become more profitable. Therefore, vacancy posting increases, raising market tightness in these other sectors, $\{\theta_j^*\}_{j \neq i}^I$. The reservation wage can be expressed as $\overline{w} = z + \frac{\beta c}{1 - \beta} \sum_i \theta_i$. Therefore, because z, c, β are fixed, the reservation wage would increase. However, this contradicts the original assumption that the reservation wage falls.

Thus, it must be the case that following a positive productivity shock in sector i, workers' reservation wage increases. Jobs in the other sectors are therefore less profitable at the higher wage, so from the zero profit conditions, the RHS of equation (16) falls. For the zero profit condition to hold in the new equilibrium, firms expected recruiting costs must also fall — the LHS of equation (16). Given the cost of a vacancy, c, is fixed, the job filling rates in these other sectors must increase, $\{q(\theta_j)\}_{j \neq i}^{I}$, which requires that market tightness fall in the other sectors, $\{\theta_j^*\}_{j \neq i}^{I}$.

The same argument applies after a negative shock in sector i, but in the opposite direction.