Volatility in exchange rates and fundamentals*

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Casual observations and serious inquiries clearly show that exchange rates are excessively volatile in comparison to their underlying fundamentals. Any theory that claims to explain the volatility of exchange rates should be able to replicate this robust fact. Section 1 of this paper develops the canonical model of international finance. Section 2 shows the difficulties in reconciling its predictions with the evidence of excess volatility. Section 3 concludes with a discussion on bubbles and a brief account of a new approach that tries to address the deficiencies of the literature from a different angle.

1. The monetary model

The standard workhorse of international finance is the monetary model of the exchange rate\(^1\). It begins with two money-market equilibrium conditions, one for the domestic money market and one for the foreign money market. Expressed in logarithms,

\[
m_t - p_t = \beta y_t - \alpha i_t + \varepsilon_t
\]

(1.1)

\[
m_t^* - p_t^* = \beta y_t^* - \alpha i_t^* + \varepsilon_t^*
\]

(1.2)

where \(m_t\) denotes the stock of money at time \(t\), \(p\) denotes the price level, \(y\) denotes real income, \(i\) denotes the nominal interest rate, \(\varepsilon\) denotes a well-behaved shock to money demand, and \(\alpha\) and \(\beta\) are structural parameters. Requiring equilibrium in international capital markets amounts to imposing the uncovered interest rate parity condition:

\[
i_t - i_t^* = E_s s_{t+1} - s_t
\]

(1.3)

where \(s_t\) denotes the domestic price of a unit of foreign exchange. Finally, price levels and the exchange rate are related through purchasing power parity:

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\(^1\) Another “classic” model in international finance is the monetary model with sticky-prices. This model is not investigated in what follows as the thrust of our analysis is much more general. For an elaboration on this claim, see Flood and Rose (1995) and Frankel and Rose (1995).
By defining the fundamental as $f_t = (m_t - m_t^*) - \beta(y_t - y_t^*) - (\varepsilon_t - \varepsilon_t^*)$ and substituting equations (1.1), (1.2), and (1.3) into equation (1.4), the exchange rate will be given by the following difference equation:

$$s_t = f_t + \alpha(E_t s_{t+1} - s_t).$$

Solving for $s_t$ gives:

$$s_t = \gamma f_t + \psi E_t s_{t+1}$$

where $\gamma = 1/(1 - \alpha)$ and $\psi = \alpha/(1 - \alpha)$. Expression (1.6) is the basic first-order stochastic difference equation of the monetary model, with solution given by:

$$s_t = \gamma \sum_{j=0}^{k} \psi^j E_t f_{t+j} + \psi^{k+1} E_t s_{t+k+1}.$$  

We are obviously interested in deriving the case where $k \to \infty$. In doing so, we need to specify the behavior of the term $\psi^k E_t s_{t+k+1}$. Imposing the transversality condition

$$\lim_{k \to \infty} \psi^k E_t s_{t+k} = 0$$

we obtain the unique (no-bubbles) solution

$$s_t = \gamma \sum_{j=0}^{\infty} \psi^j E_t f_{t+j}$$

which expresses the exchange rate as an asset price. From this perspective, it will come as no surprise that the exchange rate is more volatile than fundamentals, just as stock prices are much more volatile than dividends.

To test the exchange rate equation (1.9), a process for fundamentals has to be specified. Keeping in mind that the model must be consistent with the excess volatility fact, the growth rate of the fundamentals is modeled as a persistent stationary process.
\[ \Delta f_t = \rho \Delta f_{t-1} + \epsilon_t \] with \( \epsilon_t \sim N(0, \sigma^2 \epsilon) \). The implied \( k \)-step-ahead prediction formula is

\[ E_t(\Delta f_{t+k}) = \rho^k \Delta f_t. \]

Converting to levels, \( E_t(f_{t+k}) = f_t + \sum_{i=1}^{k} \rho^i \Delta f_i = f_t + \frac{1}{(1 - \rho)} \sum_{i=1}^{k} \rho^i \Delta f_i \).

Notice that the use of these prediction formulae in equation (1.9) gives

\[
s_i = \gamma \sum_{j=0}^{\infty} \psi^j f_i + \gamma \sum_{j=0}^{\infty} \frac{\psi^j}{1 - \rho^2} \rho^j \Delta f_i - \gamma \sum_{j=0}^{\infty} \frac{1 - \psi^j}{1 - \rho} \rho^j \Delta f_i = f_i + \frac{\psi}{1 - \psi \rho} \Delta f_i
\]

and with some additional algebra,

\[ Var(\Delta s_i) = \frac{(1 - \rho \psi)^2 + 2 \rho \psi (1 - \rho)}{(1 - \rho \psi)^2} Var(\Delta f_i) > Var(\Delta f_i) \]

This variance inequality is in accordance with the excess volatility fact. However, the level expression of the fundamentals is not encouraging since the fundamentals are explosive. So without even testing the model, the predictions of this approach show some limitations. The next section provides further evidence against this model.

2. The (counter) evidence: volatility tests

2.1. The variance bounds test approach

A straightforward challenge to the present-value representation of the exchange rate is given by the variance bounds test. To illustrate this, equation (1.9) can be rewritten as

\[ s_i = E_t s^*_i \]

where \( s^*_i \) is the perfect foresight exchange rate. This representation implies that \( s^*_i - s_i \) is orthogonal to all information variables known at time \( t \). Therefore, \( s^*_i = s_i + u_i \) where \( u_i \) is the rational forecast error and, by definition, is uncorrelated with \( s_i \). Taking the variance of this expression reveals that

\[ \text{var}[s^*_i] = \text{var}[s_i] + \text{var}[s^*_i - s_i] \geq \text{var}[s_i] \]  

(2.1)
Equation (2.1) states that the fundamentals-based value of the exchange rate should be at least as volatile as the actual equilibrium rate. This implication is obviously at odds with the requirement of excess volatility.

2.2. The regime-specific approach

Finally, a weaker volatility test has been suggested by Flood and Rose (1995). Their work originated from the classic finding by Mussa (1986) that nominal and real exchange rate volatility moved closely together, both being substantially lower during regimes of fixed rates. To explain this fact, Flood and Rose examined differences in the behavior of macroeconomic variables across-regimes of nominal exchange rates. They found that few macroeconomic variables have regime-specific volatility, and concluded that aggregate variables cannot be very important determinants of exchange rates volatility.

Recently Reinhart and Rogoff (2002) argued that these findings are overturned once exchange rate regimes are “accurately” classified. By relying on countries’ self-declared category of exchange rate regime, the official classification of the IMF often fails to describe the actual country practice. Reinhart and Rogoff looked at market-determined exchange rates and concluded that official historical groupings of exchange rates are misleading. In light of this reassessment of exchange rate regimes, they showed that Flood and Rose’s findings are no longer valid. However, in making this assertion, they did not test as Flood and Rose did different exchange rate models so their claim is still to be confirmed by further investigation.

3. Future research directions

3.1. Relaxing the no-bubble assumption
The evidence against the monetary model lead some economists to argue that volatility movements in the exchange rates arise from pure speculation. To address this possibility, the transversality assumption (1.8) is relaxed and a bubble with the following behavior is assumed:

$$b_t = \frac{b_{t-1}}{\psi} + \eta_t$$  \hspace{1cm} (2.2)

where $\eta_t \sim N(0, \sigma_\eta^2)$. The reason for this persistence form is inherent to the definition of a bubble: if it exists today then it is expected to be present next period. Adding $b_t$ to the solution (1.9), we obtain $s_t + b_t$. This solves the difference equation (1.7):

$$s_t + b_t = \gamma f_t + \psi E_t s_{t+1} + \psi E_t b_{t+1}$$
$$\Rightarrow s_t = \gamma f_t + \psi E_t s_{t+1}$$

so $s_t + b_t$ is a “bubble solution”. However, this solution violates the transversality condition as can be seen by substituting the bubble solution into equation (1.8):

$$\psi^{t+k} E_t (s_{t+k} + b_{t+k}) = \psi^{t+k} E_t s_{t+k} + \psi^{t+k} E_t b_{t+k} = b_t$$  \hspace{1cm} (2.3)

This implies that the bubble will eventually dominate the evolution of $s_t$ and will drive it far away from the fundamentals $f_t$. The volatility in exchange rates will then be entirely driven by self-fulfilling expectations.

A standard test for bubbles has been introduced by West (1987) and used by Meese (1986) across many exchange rates to find no evidence of their existence. More often, empirical findings rule out self-fulfilling prophecies. However, researchers are skeptical when it comes to testing for bubbles because these tests involve a composite null hypothesis of no bubbles and correctly specified market fundamentals. This latter property is hard to assert because some fundamentals are unobservable or are very hard to
forecast accurately. To illustrate this latter fact, suppose that the econometrician predicts constant future fundamentals, i.e., $f_t = f \forall t$. The implied exchange rate $s_t^E$ is then given by $s_t^E = f + \psi f + \psi^2 f + \cdots = f/(1-\psi)$. Now, suppose that the traders on the foreign exchange market expect a different pattern of evolution of the fundamentals. They anticipate that $f_t = f_{t+1} = f$ and $f_{t+i} = \bar{f} \forall i > 1$ where $\bar{f} > f$. Traders will compute a price of $s_t^T = f + \psi f + \psi^2 \bar{f} + \psi^3 \bar{f} + \cdots = (f - \psi^2 f + \psi^3 \bar{f})/(1-\psi)$. Thus, the “price” difference at time $t$ is given by $s_t^T - s_t^E = \beta^2 (\bar{f} - f) / (1-\psi) > 0$. Following the same reasoning, the price difference at time $t+1$ is given by $s_{t+1}^T - s_{t+1}^E = \psi (\bar{f} - f) / (1-\psi) > 0$. Therefore, $(s^T - s^E)_t < (s^T - s^E)_{t+1}$, i.e., the price difference is widening over time exponentially. The econometrician sees an unexplained price increase and thus wrongly concludes that there exists a bubble. So are bubbles a dead end? In a survey on nominal exchange rates measurements, Frankel and Rose (1995) suggest that further study of bubbles should be pursued: “the fact that exchange rate variation cannot be explained with any existing model of fundamentals is certainly intuitively consistent with the existence of bubbles.”

### 3.2. New avenues: the microstructure approach to the foreign exchange market

In this apparent disarray of many macroeconomic modeling strategies, a new perspective emerged: the market microstructure approach\(^2\). Drawing on this theory, Killeen et al. (2000) exploit the natural experiment that Europe experienced in 1999 of the switch of regimes from a target zone to a common currency. Using the concept of order flow (transaction volume accounting) and its implication as an informational vehicle from non-traders to traders, they claim that exchange rates are more volatile under flexible

\(^2\) A comprehensive account is given by Lyons (2001).
rates because of order flow. Under flexible rates, the elasticity of public demand is (endogenously) low, due to higher volatility and aversion to the risk this higher volatility entails. This allows order flow to convey more information and thereby to increase volatility. Therefore, order flow explains why exchange rates are more volatile under flexible rates. Thus, fundamentals play no role in explaining volatility of exchange rates just as Flood and Rose concluded.

To conclude, explaining exchange rates volatility is no easy task and will continue to be a lively challenge to research in international finance theory in the near future.

References


