Bank money, asset prices and the financial liquidity channel of monetary policy

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Abstract

This paper extends the recent literature of “liquidity and asset prices” into monetary models by adding money-creating banks. We explain why the money creation function of banks is important to financial stability. We study an economy in which not all assets can be used to make payments, agents may have to sell assets when they need cash. Sale of assets can lead to low asset price because buyers have limited ability to buy assets. Banks can provide liquidity by creating and lending out new deposit. This will reduce the sale of assets and stabilize asset prices. We also compare two types of liquidity provision mechanisms: liquidity-risk sharing through coalitions and liquidity provision through money creation. We show that if people use mutual-fund-like non-bank coalitions to share liquidity risks, then the function of banks to relax the aggregate money constraint is important. Without banks, non-bank coalitions will not be able to insure against aggregate liquidity risks, they will only add more endogenous volatility to asset prices. We also model how the interest rate policy of the central bank is transmitted through the banking system to the financial market.

Journal of Economic Literature Classification: E4, E5, G12, G21

Keywords: banking, inside money, payment system, liquidity, asset prices, monetary policy

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1 Introduction

This paper explains why the function of banks to provide liquidity through money creation is important to financial stability. First, we model an economy where people need money to make payments. We show that new deposits created by banks can reduce the need for people to sell assets and help stabilize the economy. Second, we show that if people use mutual-fund-like coalitions to share aggregate liquidity risks, then the function of banks to relax the aggregate liquidity constraint is important. Without banks providing elastic aggregate money, non-bank coalitions cannot actually perform the risk-sharing function, they will only cause higher endogenous volatility in asset prices. Third, we model how the interest rate policy of the central bank is transmitted through the banking system to the financial market.

The main motivation of this paper is to extend the recent literature of “liquidity and asset prices” into monetary framework by adding money-creating banks. The ability of banks to provide liquidity through money creation is largely ignored in the standard banking literature. We think this function of banks has important effect on the stability of asset prices and the financial system.

The recent studies in “liquidity and asset prices” focus on how the limited abilities of the market to absorb sales of assets may cause assets to be sold at low prices. Our paper is closely related to the works of Franklin Allen and Douglas Gale. They argue that standard models usually ignore the role of liquidity. But in reality, not all assets can be used to make payments. If people have to sell assets, and if the amount of cash that buyers can use to buy assets is limited, then the market price of assets can deviate from the fundamental price, which will lead to the “cash-in-the-market-pricing” (Allen and Gale(2005)).

Most works in this area are still based on non-monetary models. For example, in Allen and Gale(1994, 2004b), people allocate their initial portfolio between short-term and long-term projects. If a large proportion of the agents need to consume in the short-term, then long-term assets will be sold at low prices relative to short-term consumption goods. Here, liquidity or “cash” is modelled as real consumption goods available for immediate delivery. Gale(2005) extends the framework into a monetary model with central bank money. He defines liquidity as means of payment. He shows that when both asset market and goods market are subject to the cash-in-advance constraint, liquidity shocks in the asset market can lead to relative price changes between assets and goods.

We follow Gale(2005) and define liquidity as means of payment. We add money-creating
banks and the payment system. This changes two fundamental aspects of the model. First, previous models usually use non-monetary loanable funds theory to model banks. Banks need to collect real resources from depositors before making loans to borrowers. We model banks using the *monetary loanable funds theory* in which loanable funds can come from newly-created bank money. In particular, we build a model for the interbank payment process, and show that the need for reserve in the settlement process can be much lower than the loan level. As long as banks can meet the settlement requirement, they can lend by creating new deposits without first collecting equivalent amount of money from depositors.\(^1\) In this case, banks are not simply middle-man who transfers money from depositors to borrowers, they also create liquidity by creating and lending liquid claims in the form of deposit. Second, in previous models, aggregate liquidity is usually limited to the liquid assets that people choose to hold in their initial portfolio. While in our model, because new bank money can be created, liquidity is no longer limited by existing liquidity or money issued by the central bank.

The above changes in the model have several important implications: First, if liquidity needs take the form of demand for means of payment, then since means of payment can be created by banks, the liquidity supply in a monetary economy will be more elastic than what implied by non-monetary models, and the financial system will also be more stable. Second, the soundness of the banking system is important, if the banking system is not healthy and their lending ability is limited, then liquidity supply will be less elastic and the asset price will be more volatile. We will show these results in our model.

We also analyze the relationship between two types of liquidity provision mechanisms: liquidity-risk sharing through a coalition and liquidity provision through money creation. Many previous works analyze how people can create a mutual-fund-like coalition to insure against liquidity risks. By creating a coalition, consumers who need to consume early can receive a higher consumption level than if there were no coalitions. In our model, we assume a group of mutual-fund-like non-bank intermediaries called “investment funds” will perform this function. We let non-banks to perform this function because this function does not depend on whether the coalition is a bank. In most previous models, the coalition is modelled as a mutual fund. For comparison purposes, we let banks only perform the function of creating and lending out inside money. The difference between banks and non-banks is that banks’ liability can be used to make payments while the share of non-banks can not.

\(^1\)Note that deposit is banks’ liability, it can be issued to depositors when banks collect new money, but new deposit can also be issued directly to borrowers during lending. The details will be shown in our model.
Instead of assuming that the liquidation price is exogenously fixed, we use a general equilibrium model to endogenously decide the asset price. Under this framework, people will not be able to sell unlimited amount of asset at a constant price. We show that the function of banks to relax the aggregate liquidity constraint is essential for the risk-sharing function of non-banks. The reason is that non-bank investment funds rely on the market for liquidity, they need to sell assets to other people to raise cash. But if the aggregate money is limited, then the cash that investment funds can raise will be limited. The attempt of investment funds to raise more cash will only cause asset price to decrease more. Thus, when aggregate liquidity is limited, risk-sharing coalitions can not actually improve the consumption of shareholders, their attempt to provide risk-sharing will only add more endogenous volatility to asset prices. But if we have banks to expand the aggregate liquidity, then shareholders who need liquidity early can indeed receive a higher payment than if there were no coalitions.

The basic structure of our model is an overlapping generations model with random relocation. There are households, investment funds, banks, and the central bank. Private agents can borrow from banks but not from the central bank. Private agents can use bank money to make payments. Banks are required to settle inter-bank payments using central bank money, banks can also borrow settlement loans from the central bank.

There are two locations in which households live and financial intermediaries operate. In each period some of the households must move to the other location. Households can only invest in local banks and non-bank investment funds. Investment funds collect resources, make real investments, and provide risk-sharing to households. The shares of investment funds can not be used as means of payment. Banks do not finance real investments in our model, as our focus is on banks’ function in creating and lending inside money. Movers must redeem their investment fund shares into bank deposit before they move to the other location. If investment funds raise additional money by selling assets, then higher sales can lead to lower prices. If banks lend new money to investment funds, investment funds will sell less assets and asset prices will be more stable.

Apart from the literature of liquidity and asset prices, this paper is also related to the literature of elastic money. For example, Freeman(1996a,b) formalize the traditional idea that elastic money can be good for the economy. In Freeman’s model, buyers do not have money when they purchase goods, they issue personal debt to sellers. Both buyers and sellers will travel to a central island where the debt will be settled with fiat money. But some sellers may have to leave before all buyers reach the central island. If those sellers have to sell their
debt, then the debt may be sold below its par value. Freeman then shows that welfare can be improved if we have a central bank or a clearinghouse that can provide discount services. For example, the central bank or a clearinghouse bank can issue banknotes to buy those debt, and later collect those debt from buyers.

Freeman focuses on the basic principle that elastic money can help stabilize asset prices. Freeman’s model is more about the function of the central bank. The central bank (and clearing-house banks) in his papers can provide liquidity by issuing banknotes and essentially face no liquidity constraints.

There are important differences in our paper. First, in our model, people can not directly borrow from the central bank, and when banks provide liquidity, they themselves will face liquidity risks caused by random interbank payments. Banks will take higher liquidity risks when they make more loans. We analyze how banks optimally supply loans when they face the liquidity constraint. We also consider the effect of capital constraint.

Second, we analyze the transmission of interest rate policy. We assume there is no reserve requirement and all payments between non-banks are made with transfer of bank deposit. We show that in this type of “pure-credit” economy, the central bank can still control the interest rate by requiring banks to settle using central bank money. The money supply is endogenously decided by credit transactions and is no longer be directly controlled by the central bank. But the central bank can still use the interest rate to affect money supply by affecting the incentive of banks to provide loans. Third, the risk-sharing function of non-banks and its effect on asset price volatility is not analyzed in Freeman’s paper.

The basic environment (i.e., the overlapping generations model with random relocation) is taken from Champ, Smith and Williamson (1996). Their paper shows that the social welfare can be improved if banks can use private banknotes instead of central bank money to meet the random withdrawal needs of depositors. Our model focuses on how banks provide liquidity to borrowers instead of to depositors.

The rest of the paper is organized as follows. Section 2 describes the basic environment. Section 3 is the idealized case in which all assets can be used to make payments. Section 4 analyzes the case where payments must be made with money and investment funds can only raise money by selling assets. Section 5 looks at the case where investment funds can borrow from banks. Section 6 shows the numerical results for log utility function. Section 7 analyzes the relationship between the risk-sharing function of non-banks and the money
creation function of banks. Section 8 analyzes the case when the capital constraint is binding. Section 9 summarizes and concludes the paper.

2 The basic environment

In this part, we explain the basic environment of the model. Since there is no lending by commercial banks in section 3 and 4, the detailed environment for bank lending and the settlement process will be explained in section 5.

We consider an overlapping generations model with random relocation. Time is indexed by \( t = 1, 2, \ldots \). There are two locations in the economy. In each period, a new generation is born at each of the two locations. In each generation, there are three types of agents: “households”, “investment fund managers” and “bankers”. We normalize the size of each type of agents to one. There is no population growth. Each generation lives for two periods. The first old generation of households in each location at time \( t = 1 \) is endowed with outside money \( M \).

Agents only care about the consumption when they are old. There is a single good. Each household is endowed with \( e_h \) units of the good when young, and nothing when old. Young households save all their endowment. Households are risk-averse and they have a constant relative risk aversion (CRRA) utility function:

\[
U(c) = \frac{c^{1-\sigma}}{1-\sigma} \quad \sigma \geq 1
\]  

When \( \sigma = 1 \), \( U(c) = \ln c \).

Young investment fund managers can costlessly start new investment funds and young bankers can costlessly start new banks. There is free entry for investment funds and banks. Investment funds compete with each other by offering the best contract to shareholders and banks compete with each other by offering the best contract to depositors and borrowers.

Investment fund managers do not have endowments. And we assume away bankruptcy for banks using the following assumption: every banker has endowment \( e_b \) when old, which can be used to absorb the loss. Investment fund managers and bankers are risk neutral. Old fund managers and bankers consume all their wealth.

The good is non-storable but can be invested as capital to produce new goods in the next period. Real risky investments can only be made by investment funds. The gross return rate
for the risky project is

\[ R_k = A \]  \hspace{1cm} (2)

where \( A \) is the aggregate productivity. There are two values of \( A \): \( A_H \) (high), \( A_L \) (low) with equal probability. \( A \) is i.i.d. in each period.

The main events and the initial portfolio allocation process are shown in Figure 1. We assume that young households can only invest in local investment funds and banks. For simplicity, we assume that each young household can at most invest in one investment fund and one bank. At the end of period \( t \), young households sell part of their endowment to the old generation for money balance \( \frac{MP}{P} \). They invest the remaining endowment \( e_h - \frac{MP}{P} \) and part of the money \( \beta \frac{MP}{P} (\beta \in [0,1]) \) into investment funds. They deposit the remaining money \( (1 - \beta) \frac{MP}{P} \) into banks. Investment funds then make real investments and deposit the money \( \beta \frac{MP}{P} \) into banks. Banks will end up holding all the cash balance \( \frac{MP}{P} \). Note that investment funds can also choose to hold part of their assets in bank deposit, this happens when \( \beta > 0 \). If \( \beta = 0 \), investment funds only hold risky assets.

We focus on the symmetric case in which all investment funds and banks have the same size. We assume that the actual number of households is higher than the number of investment
funds and banks, so each investment fund has many shareholders and each bank has many
derivers. We assume that young households are equally distributed among all investment
funds and banks. We also assume that the actual number of investment funds is at least as
high as the number of banks, and the deposit account of new investment funds are equally
distributed among all banks. Deposit account holders are free to switch their account to other
banks.

After real investments are made, we enter period $t+1$. At the beginning of period $t+1$, the
productivity shock $A$ is publicly observed. At the same time, the Nature decides the liquidity
shock. A random fraction $\pi$ of the old households (denoted as “movers”) must move to the
other location and consume there. $\pi$ is distributed over $[0, \pi]$, where $\pi < 1$ is the upper bound
of the distribution. The distribution function is $F(\pi)$. $\pi$ is symmetric in the two locations.
$\pi$ is independently and identically distributed, so each old household has the same $ex$ ante
probability to be a mover.

Movers can not carry goods across locations. We assume there is a network between
banks, and the value of bank deposits in everyone’s banking account can be verified across
locations, so movers can use bank deposits to make payments. More specifically, we assume
that when movers move from location $i$ to location $j$, they still keep their deposits in the
banks in location $i$. And when they need to buy consumption goods in location $j$, they can
pay using their deposits.

The value of other assets can not be verified across locations. In particular, we assume
that people can not verify the value of investment fund shares, so investment fund shares are
not accepted as means of payment across locations. (Except in section 3 where we consider the
special case in which all assets can be used to make payments.) As a result, movers must use
money to buy consumption goods. We assume that movers must have their deposits ready in
their banking account when they move to the other location. As a result, movers must redeem
their investment fund shares into bank deposits before move.

In order to meet the redemption needs, investment funds may need to raise additional
money by selling assets or by borrowing from banks. When assets are sold, only the ownership
is transferred to the buyer, the production process is not stopped. The investment funds will
collect the return and pay it to the buyer at the end of the period.

The redemption process is as follows. After the shocks are realized, movers must send a
withdrawal notice to the investment fund. Then the financial market opens. If the investment
fund can not meet the withdrawal needs by its own riskless asset, then it can raise cash by selling assets to non-movers who have idle deposits. The fund can also choose to borrow from banks. The fund then pay movers by transferring bank deposits to them. For simplicity, we assume that movers only receive the payment after the transactions on financial market are completed, so the cash received from the fund can not be used to buy risky assets on the financial market.

After the redemption, movers move to the other location. At the end of $t+1$, risky projects are completed. Movers in each location use their bank deposits to buy consumption goods. Investment funds allocate returns to their shareholders and also repay the bank loan. Bankers consume the net income and old non-movers consume all their wealth.

For simplicity, we assume that non-bank agents always use bank deposits to make payments. Although people can choose to withdraw their deposits, in the equilibrium, people always keep their money in banks and there is no actual withdrawal of central bank currency.

We assume that banks must settle inter-bank balance with central bank money. Banks keep their reserves in the central bank deposit account, and the deposit rate paid by the central bank is normalized to zero. There is no official reserve requirement and banks can freely choose the reserve level.\(^2\) We assume that non-bank agents can not directly borrow from the central bank, they can only borrow from banks, but banks can borrow from the central bank.

We will explain the details of bank lending and inter-bank payments in section 5. In the next two sections, we will first analyze two cases without bank lending. In section 3, we consider a special case in which all assets can be used to make payments. In this case, there is no liquidity risks. In section 4, movers must use money to make payments and the investment fund can only raise cash by selling assets on the financial market, which may lead to volatile asset prices.

3 The benchmark case: all assets can be used to make payments

In this part, we consider a special case in which there exists an imagined “Walrasian Auctioneer”, who can verify the value of everything in the economy for free. As a result, movers can

\(^2\)Countries such as Canada, New Zealand and UK have already eliminated the reserve requirement.
carry investment fund shares with them and use the shares to buy goods in the other location. The transactions are then cleared by the “Walrasian Auctioneer”. Since $\pi$ is symmetric in the two locations, investment funds and movers will be willing to accept this arrangement.

In this case, people only hold money as a store of value. (Money still has value because it can be used to buy goods from the young generation.) All assets are equally liquid.

3.1 The optimal choice

Let $Z_p$, $Z_f$ and $Z_k$ denote the portfolio, riskless assets and risky assets of a representative investment fund. And let $\alpha \geq 0$ denote the share of riskless assets, we have $Z_f = \alpha Z_p$ and $Z_k = (1 - \alpha)Z_p$.

Let $s$ denote the savings of each household. Households save all endowment and so $s = e_h$. Let $d = \omega s$ and $z_p = (1 - \omega)s$ denote the bank deposit and investment fund shares held by each household. Let $v$ denote the value of the household’s portfolio (which is also the consumption level) in period $t + 1$. Movers and non-movers have the same $v$:

$$v = s[\omega + (1 - \omega)(\alpha + (1 - \alpha)R_k)] = s[\kappa + (1 - \kappa)R_k]$$

where

$$\kappa = \omega + (1 - \omega)\alpha$$

is the total share of bank deposit in household’s portfolio (i.e., both the deposit held directly by the household and the deposit held indirectly through the investment fund).

Because of competition, the contract of the investment fund should maximize the expected utility of households. Since the objective of the investment fund is the same as the objective of households, we can choose $\omega$ and $\alpha$ freely to maximize $EU$ of households. $\omega$ and $\alpha$ will not be unique because the household has the same portfolio as long as $\kappa$ is the same.

For simplicity, we use the example $[\omega = \kappa, \alpha = 0]$, that is, the investment fund only holds risky asset. We have $v = s(\omega + (1 - \omega)R_k)$ and the expected utility of the household is

$$EU = \frac{1}{2} \left[ s(\omega + (1 - \omega)R_{k,H}) \right]^{1-\sigma} + \frac{1}{2} \left[ s(\omega + (1 - \omega)R_{k,L}) \right]^{1-\sigma}$$

where $R_{k,H} = A_h$ and $R_{k,L} = A_L$ are high and low risky asset returns. The first order condition is

$$\frac{\partial EU}{\partial \omega} = \frac{1}{2} \frac{s(1 - R_{k,H})}{s(\omega + (1 - \omega)R_{k,H})^{\sigma}} + \frac{1}{2} \frac{s(1 - R_{k,L})}{s(\omega + (1 - \omega)R_{k,L})^{\sigma}} = 0$$
If $\frac{\partial E_U}{\partial \omega} < 0$ at $\omega = 0$, then we have the corner solution $\omega = 0$. In this case, people do not hold any money balance and the outside money $M$ is valueless in the equilibrium. If $\frac{\partial E_U}{\partial \omega} > 0$ at $\kappa = 1$, then households only hold bank deposits in the equilibrium, no risky investments are made, and we have a pure-exchange overlapping generations economy. If we have $\frac{\partial E_U}{\partial \omega} = 0$ for $\omega \in (0, 1)$, then money balance and real investments are both positive in the equilibrium.

Essentially, we have a standard portfolio choice problem with one risky and one riskless asset.

### 3.2 Equilibrium

We only focus on the stationary symmetric equilibrium. The equilibrium allocation is as follows. Households choose $\omega = \omega^*$ according to condition (5). Denote the aggregate saving as $S$(which is equals to $e_h$ in the equilibrium because the total size of young households is normalized to 1). The young generation sells $\omega^*S$ of goods to the old generation for money, and the remaining goods $(1 - \omega^*)S$ are invested in risky projects.

The consumption of each old household is $v = s(\omega^* + (1 - \omega^*)R_k)$, where $s(1 - \omega^*)R_k$ is the output from the risky projects and $s\omega^*$ is the goods purchased from the next young generation with money.

Since the aggregate value of the riskless asset chosen by the young generation is equal to the value of outside money spent by the old generation, we have

$$\omega^*S = \frac{M}{P}$$

(6)

Given $M$, this equation gives the equilibrium price $P^*$. In the stationary equilibrium, since $\omega^*S$ is the same in each period, and since $M$ is fixed, $P^*$ is the same in each period.

### 4 Liquidity provision through the financial market

In this section, we assume that there is no lending from the central bank or commercial banks. And we assume there is no cost for people to make payments using bank deposits. Since there is no bank loan and the central bank deposit rate is zero, the deposit rate for banks will also be zero. We assume investment funds can only raise cash by selling assets to non-movers. Since the amount of deposits is equal to the amount of reserve $M$, the money supply is essentially fixed at $M$. 


We only care about the symmetric equilibrium, and we will use the following method in our analysis. We assume that all investment funds and households make symmetric choices, and we then check whether an individual household or investment fund wants to deviate from the symmetric equilibrium.

We assume that non-movers incur zero transaction costs when they buy assets on the financial market. Note that in this model, there is no need for any bank to collect the idle money from non-movers and then lend them to the investment fund because non-movers can directly use their money to buy assets from the investment fund. So in order for banks to be useful, they must do more than collecting and lending out the idle cash.

4.1 Optimal choices

![Diagram](Image)

Figure 2: Household’s choice

![Diagram](Image)

Figure 3: The choice of the investment fund

The choices of households and the investment fund are shown Figure 2 and 3. Households choose $\omega$ at the end of $t$. Given $\omega$, the investment fund chooses $\alpha$ in $t$ and the optimal payout policy in $t+1$ to maximize the expected utility of households.

Let $r_m$ denote the return rate paid to movers at the beginning of $t+1$ and $r_n$ the return rate paid to non-movers at the end of $t+1$. First, we need to find out the optimal $r_m$ and $r_n$, then we need to find out the optimal $\omega$ and $\alpha$ in the initial portfolio.

Recall that $R_k$ is the fundamental price of risky assets. And let $Q_k$ denote the market price
of risky assets when investment funds sell assets to non-movers. First, we need to decide the 
or

optimal

m and

r

n

given

Q

k

, then we will use the result to derive the equilibrium distribution

distribution

of

Q

k

. In the following analysis, we will ignore

H

and

L

from the notations whenever the

analysis applies to both high and low productivity shocks.

Let

v

m

denote the value of movers’ portfolio and

v

n

the value of non-movers’ portfolio. We have

v

m

= s [\omega + (1 - \omega) r_m] \tag{7}

v

n

= s \left[ \frac{R_k}{Q_k} (1 - \omega) r_n \right] \tag{8}

For movers, the value of the deposit is

s \omega,

and the payment from the fund is

s (1 - \omega) r_m.

For non-movers, the payment from the investment fund is

s (1 - \omega) r_n.

The gross return for the initial deposit balance

s \omega

can be written as \frac{R_k}{Q_k}.

There are three situations. If investment funds do not sell assets, then the return for the initial deposit is simply 1, which can be written as \frac{R_k}{Q_k} because

R_k = Q_k.

Similarly, if investment funds sell some assets to non-movers but

R_k = Q_k,

then the return for the initial deposit is still 1. Finally, if

R_k > Q_k,

then non-movers will use all their initial deposit to buy assets, and the return is

\frac{R_k}{Q_k} > 1.

The expected utility of the household is

\frac{1}{2} \int_0^1 \left[ \pi \frac{(v_m, H)^{1-\sigma}}{1 - \sigma} + (1 - \pi) \frac{(v_n, H)^{1-\sigma}}{1 - \sigma} \right] dF(\pi) + \frac{1}{2} \int_0^1 \left[ \pi \frac{(v_m, L)^{1-\sigma}}{1 - \sigma} + (1 - \pi) \frac{(v_n, L)^{1-\sigma}}{1 - \sigma} \right] dF(\pi) \tag{9}

where

H

and

L

are productivity shocks. Since

r

m

is chosen after the shocks are realized, the fund can choose the best

r

m

for each level of the shock. So the fund maximizes

\pi \frac{(v_m)^{1-\sigma}}{1 - \sigma} + (1 - \pi) \frac{(v_n)^{1-\sigma}}{1 - \sigma} \tag{10}

subject to its budget constraints and the constraint

r_m \leq r_n

(payment to movers can not be higher than non-movers, otherwise non-movers will pretend to be movers and withdraw.) The details of the optimal payout policy is shown in the Appendix. We find that when

Q_k = R_k,

it is optimal to set

r_m = r_n.

When

Q_k < R_k,

since it is costly to raise cash by selling assets, the fund may not provide full insurance. If the constraint

r_m \leq r_n

is not binding, then the fund tends to set a higher \frac{r_m}{r_n} when

\sigma

is higher. That is, the fund will provide more risk sharing when people are more risk averse.

Since \sigma = 1 is the simplest case, for the remaining part of this section, we will show the result of \sigma = 1. The details of the general case (\sigma \geq 1) are shown in the Appendix.
Proposition 1. For $U(c) = \ln c$, the optimal payout policy is

\begin{align*}
    r_m &= \alpha + (1 - \alpha)Q_k \\
    r_n &= \alpha \frac{R_k}{Q_k} + (1 - \alpha)R_k = r_m \frac{R_k}{Q_k}
\end{align*} \tag{11} \tag{12}

Proof: see the Appendix. ■

We can see that when $U = \ln c$, the optimal $r_m$ is to pay the market value of the fund’s asset. Given this payout policy, we have the following result:

Proposition 2. When $\sigma = 1$, the equilibrium is defined by $\kappa$, and the solution for $\omega$ and $\alpha$ is not unique. As long as $\kappa$ is the same, the equilibrium result is the same.

Proof: We need to show that first, given $Q_k$, the value of the household’s portfolio only depends on $\kappa$. Second, the same $\kappa$ also gives the same distribution of $Q_k$. For the first part, substituting the optimal $r_m$ and $r_n$ (11 and 12) into (7) and (8)

\begin{align*}
    v_m &= s [\omega + (1 - \omega)(\alpha + (1 - \alpha)Q_k)] = s [\kappa + (1 - \kappa)Q_k] \\
    v_n &= s \left[ \omega \frac{R_k}{Q_k} + (1 - \omega)(\alpha \frac{R_k}{Q_k} + (1 - \alpha)R_k) \right] = s \left[ \kappa \frac{R_k}{Q_k} + (1 - \kappa)R_k \right]
\end{align*}

So given $Q_k$, the value of the portfolio only depends on $\kappa$. The second part of the proof is shown in the Appendix. ■

In the remaining part of the section, we use the example $(\omega = \kappa, \alpha = 0)$, that is, investment funds only hold risky assets. The payment to movers (11) becomes $r_m = Q_k$.

4.2 The distribution of $Q_k(\pi)$

Let $\pi_1$ denote the value of $\pi$ above which the cash of non-movers is binding. We have the following result:

- If $\pi < \pi_1$, then after the investment funds raise enough cash by selling assets, the non-movers still have positive cash balance left, and we have $Q_k = R_k$.
- If $\pi \geq \pi_1$, non-movers will use all their deposits to buy assets. $Q_k = R_k$ at $\pi = \pi_1$, and $Q_k < R_k$ when $\pi > \pi_1$.

We first derive $\pi_1$, then we derive $Q_k$ for $\pi > \pi_1$.

Recall that $Z_k$ is the level of risky assets held by every investment fund. It is also the total investment made by the fund because we assume investment funds only hold risky assets.
Let $D_0$ denote the value of deposits held by the households who belong to a representative investment fund. Since the choices of investment funds and households are symmetric, we can also use $Z_k$ and $D_0$ to denote the level of aggregate risky assets and deposit balance.

At $\pi_1$, the total cash raised from non-movers is $(1 - \pi_1)D_0$, which is equal to the total redemption, and $Q_k$ is still equal to $R_k$. Since $r_m$ is equal to the market price of the fund’s asset, we should have

$$\text{Total redemption by movers} = \pi \ast NV (\text{Total market value of the fund's asset})$$

$$\Rightarrow \pi_1 = \frac{\text{Redemption}}{NV} = \frac{(1 - \pi_1)D_0}{Z_kR_k}$$

Using the relationship $D_0 = \omega S$ and $Z_k = (1 - \omega)S$, we have

$$\pi_1 = \frac{\omega}{\omega + (1 - \omega)R_k}$$

(13)

When $\pi > \pi_1$, $Q_k(\pi)$ can be decided according to

$$\pi = \frac{\text{Redemption}}{NV} = \frac{(1 - \pi)D_0}{Z_kQ_k}$$

$$\Rightarrow Q_k(\pi) = \frac{\omega(1 - \pi)}{\pi(1 - \omega)}$$

(14)

So the distribution of $Q_k(\pi)$ is

$$Q_k(\pi) = \begin{cases} R_k : \pi \leq \pi_1 \text{ (Non-movers' cash is not binding)} \\ \frac{\omega(1 - \pi)}{\pi(1 - \omega)} : \pi > \pi_1 \text{ (Non-movers' cash is binding)} \end{cases}$$

(15)

From (14), we can see that when $\pi$ is higher, the cash raised from non-movers, $(1 - \pi)D_0$, will be lower, but the share of movers $\pi$ is higher, so the asset price $Q_k$ must decrease in order for (14) to hold.

### 4.3 Optimal choice of $\omega$

Let $\omega_i$ denote the $\omega$ chosen by an individual household $i$. Household $i$ chooses $\omega_i$ by taking $\omega_j$ chosen by all other agents ($j \neq i$) as given.

The value of household’s portfolio is

$$v_m = s \left[ \omega_i + (1 - \omega_i)Q_k \right]$$

$$v_n = s \left[ \omega_i \frac{R_k}{Q_k} + (1 - \omega_i)R_k \right] = \frac{R_k}{Q_k} v_m$$
And the expected utility of household $i$ is

$$EU_i = \frac{1}{2} \int_0^1 \left\{ \pi \ln \left[ s \left( \omega_i + (1 - \omega_i)Q_{k,H} \right) \right] + (1 - \pi) \ln \left[ \frac{Q_{k,H}}{Q_k} (\omega_i + (1 - \omega_i)Q_{k,H}) \right] \right\} dF(\pi)$$

$$+ \frac{1}{2} \int_0^1 \left\{ \pi \ln \left[ s \left( \omega_i + (1 - \omega_i)Q_{k,L} \right) \right] + (1 - \pi) \ln \left[ \frac{Q_{k,L}}{Q_k} (\omega_i + (1 - \omega_i)Q_{k,L}) \right] \right\} dF(\pi)$$

$$\frac{\partial EU_i}{\partial \omega_i} = \frac{1}{2} \int_0^1 \frac{1 - Q_{k,H}}{\omega_i + (1 - \omega_i)Q_{k,H}} + \frac{1 - Q_{k,L}}{\omega_i + (1 - \omega_i)Q_{k,L}} dF(\pi) = 0 \quad (16)$$

In the symmetric equilibrium, given $\omega_j = \omega$ chosen by all other agents, household $i$ should find it is optimal to set $\omega_i = \omega_j = \omega$.

### 4.4 General equilibrium

Similar to section 3.2, the equilibrium is still a repeated one-period portfolio choice problem.

The equilibrium is defined by $\omega = \omega^*$. Given the expected distribution of $Q_k(\pi)$, the equilibrium $\omega$ maximizes the expected utility of each household according to (16). Given the equilibrium $\omega$, $Q_k(\pi)$ is decided according to equation (15).

The consumption of movers is $v_m$ and the consumption of non-movers is $v_n$. The total real investment is $Z_k = (1 - \omega^*)S$. The total goods sold by the young generation to the old generation is $D_0 = \omega^*S$. The nominal price level on the goods market is $P = \frac{M}{\omega^*S}$.

On the financial market, when $\pi < \pi_1$, the total cash used by non-movers to buy assets is equal to the redemption of movers $\pi Z_k R_k = \pi (1 - \omega^*)SR_k$. When $\pi > \pi_1$, all cash of non-movers, which is $(1 - \pi)D_0 = (1 - \pi)\omega^*S$, is used to buy assets.

### 5 Liquidity provision through inside money creation

In this section, we analyze how banks provide liquidity through inside money creation. We also analyze the transmission mechanism of the central bank’s interest rate policy.

#### 5.1 Basics about credit money creation

The key steps for bank lending

Let $D_0$ denote the initial deposit and reserve balance of a representative bank. The main steps for bank lending in our model are as follows.

First, when bank $i$ makes loan $L$, it creates an equivalent amount of new deposit $L$. 
Bank \( i \) makes new loan \( L \)

\[ \begin{array}{|c|c|}
\hline
\text{Reserve:} & D_0 \\
\text{Initial deposit:} & D_0 \\
\text{Loan:} & +L \\
\text{New deposit owned by the borrower:} & +L \\
\hline
\end{array} \]

The bank makes the loan by adding a number \( L \) into the borrower’s deposit account. The new inside money (i.e., new bank deposit) \( L \) is created and ready for use. Note that we cannot have an increase in “Loan” by \( L \) on the asset side without an equivalent increase in bank deposit by \( L \) on the liability side, otherwise the balance sheet will not be balanced.

Second, borrowers spend the money. If payments are made to depositors in other banks, then the inter-bank payments will create a liquidity constraint for the bank. The details will be analyzed later.

Third, when the borrower uses his deposit to repay the loan, we have

\[ \begin{array}{|c|c|}
\hline
\text{Loan:} & -L \\
\text{Deposits owned by the borrower:} & -L(1 + r^l) \\
\text{Bank Equity (interest income):} & +Lr^l \\
\hline
\end{array} \]

where \( r^l \) is the lending rate. The outstanding loan is reduced by \( L \) and the outstanding deposit is reduced by \( L(1 + r^l) \). The interest income is \( Lr^l \).

In our model, the borrower are the investment funds. After the shocks are realized, if bank loan is needed, then banks create deposits and lend them to the investment funds, the investment funds then use the deposits to meet the redemption of movers. Movers then hold the deposits. At the end of period \( t + 1 \), movers use their deposits to buy goods from the investment funds. Then the investment funds use the deposits to repay the bank loan, and the deposits are returned to banks and are destroyed. The main steps are shown in Figure 4.

\[ \text{Figure 4: The flow of inside money.} \]

### 5.2 The environment for bank lending and settlement

This part describes the environment for bank lending and inter-bank settlement. We focus on the case in which banks lend to the investment funds.
At the end of period $t$, depositors are equally distributed among the banks, and the deposit account of investment funds are also equally distributed among the banks. Initially, each investment fund only has deposit account in one of the banks. The shareholders of each investment fund are equally distributed among all banks. There is no interest payment to depositors for holding deposits between the end of $t$ and the beginning of $t + 1$.

At the beginning of period $t+1$, the productivity shock and the liquidity shock are realized. And then banks announce their deposit rate and lending rate. We assume that banks are competitive on the lending side and they take the market lending rate as given. And we assume that banks are also competitive on the deposit side and the resulting deposit contract maximizes the expected utility of depositors. In our model, there is no uncertainty in the payment of deposits, so the deposit rate that maximizes the expected utility of depositors is simply the highest deposit rate. We only focus on the symmetric equilibrium, and we assume that in the equilibrium, all banks simultaneously offer the same deposit rate $r^d$ that gives zero expected profit to all banks.

Let $\pi_2$ denote the liquidity shock above which investment funds will borrow from banks. When $\pi \leq \pi_2$, investment funds will only raise money by selling assets. In this case, the deposit rate can only be zero because there is no income for banks. (We assume there is no cost for managing the deposits and allowing the depositors to use the payment facility). Since there is no bank loan, the level of deposits is the same as the level of reserves. This means all deposits are backed by reserves, so banks can never run out of reserves during the settlement process because the maximum outflow of payment is equal to the level of deposits.

Now suppose $\pi > \pi_2$ and bank loan is needed. We assume that investment funds can use the risky assets as collateral to borrow from banks. Each unit of bank loan incurs a management cost $\delta$ to the bank. Denote the net real lending rate as $r^l$, where $r^l \geq \delta$, and denote the gross lending rate as $R = 1 + r^l$.

The investment fund will borrow from the bank only when the borrowing cost is less than or equal to the cost for selling assets on the financial market. If the fund sells the asset, for each unit of asset with value $R_k$, the fund can get $Q_k$. If the fund borrows from the bank, for each unit of loan with future payment $R_k$, the fund can borrow $\frac{R_k}{1 + r^l}$. In the equilibrium, we must have

$$Q_k = \frac{R_k}{1 + r^l}$$

(17)

that is, investment funds will only borrow from banks when the market price decreases to
1. Investment funds sell assets to non-movers
2. Banks make loans.
Investment funds pay movers

Settlement 1

Settlement 2

Movers move to the other location

Final Settlement

Figure 5: The lending and settlement process

\[ \frac{R_k}{1 + r^l} \] Since \( r^l \geq \delta \), so \( R \geq 1 + \delta \) and we must have \( Q_k \leq \frac{R_k}{1 + \delta} \) when bank loan is needed.

The main steps for lending and settlement are as follows (see Figure 5).

Recall that \( D_0 \) is the deposit and reserve balance for each bank at the beginning of \( t + 1 \). After the shocks, the financial market opens, and the investment funds sell assets to non-movers. When bank loan is needed, \( \frac{R_k}{Q_k} \) is equal to the lending rate \( 1 + r^l \), which is higher than the deposit rate, so non-movers will use all their deposits to buy assets. So after the transactions, non-movers transfer all their deposits to the investment funds. (This is “settlement 1” in Figure 5). Because all bank deposits are still backed by reserves, banks will not face a liquidity constraint no matter how the inter-bank settlement process is carried out. In the symmetric case, each fund sells the same amount of asset, and after the payment, the deposit balance in each bank is still \( D_0 \).

After all payments by non-movers are completed, the financial market closes. We then enter step 2 in which banks make loans to the investment funds. If an investment fund has its deposit account in bank \( i \), then it will first choose to borrow from bank \( i \). The investment fund can also borrow from other banks if their lending rate is lower. In the symmetric case, every investment fund borrows from its own bank. If an investment fund borrows from bank \( i \), it also keeps the newly borrowed money with bank \( i \) before making payments to movers.

Investment funds then use all their deposits to pay movers. The process is “settlement 2” in Figure 5. Since banks have created new deposits during lending, the payment may not be fully covered by initial reserves and banks may face a binding liquidity constraint during the settlement process. The details will be explained later.

We assume that after each mover receives all of his money, he can also switch his deposit account to a new bank if that bank offers a higher deposit rate. We use this assumption to force banks to be competitive on the deposit side. If the deposit rate offered by a bank were lower than the market rate, then all its depositors would switch to other banks and this bank would run out of reserves in the settlement process. In the symmetric case, all banks offer the
same deposit rate and no mover would switch banks. As a result, in “settlement 2”, we only need to consider the inter-bank payments caused by investment funds paying their movers.

After the redemption process is completed, movers move to the other location.

For simplicity, we ignore the liquidity constraint for banks during the transactions at the end of $t+1$. We assume that after the transactions are completed, there is a settlement between banks based on net balance. As we will show later, after all transactions are completed, the deposits in each bank will be fully backed by reserves. So the liquidity constraint for banks will not be binding during the “final settlement” because in the worst case, all deposits in a bank are paid to other banks and the reserve level is reduced to zero.

As a result, we only need to consider the liquidity constraint in “settlement 2”.

The settlement method
Since our paper is not about the optimal design of the settlement process, we will use a simple model to capture the liquidity constraint generated by the settlement requirement.

We assume that inter-bank payments are settled according to “Real-Time Gross Settlement” method. More specifically, we assume that there are $N$ banks in the economy, with $N$ being a very large number. And the redemption process will be separated into $N$ subperiods. We normalize the total time length of the redemption process to 1. The time length of each subperiod is $\frac{1}{N}$. In each subperiod, a bank is randomly chosen by the Nature to make payments to other banks, and the inter-bank balance is settled right away (i.e., the transfer of reserve happens right away). In the next subperiod, another bank is randomly chosen. During this process, any negative balance of reserve must be met by borrowing from the central bank.

For simplicity, we assume there is no inter-bank loan market, and banks can use the collateral collected from investment funds as collateral when borrowing from the central bank. Also, we assume that the central bank consumes the interest by purchasing consumption goods, so the outside money is always restored to $M$ at the end of each period.

5.3 The portfolio choice
The value of household’s portfolio are

\begin{align*}
v_m &= s \left[ \omega + (1 - \omega) r_m \right] (1 + r^d) \\
v_n &= s \left[ \omega \frac{R_k}{Q_k} + (1 - \omega) r_n \right]
\end{align*}

(18)  
(19)
In this case, \( r^d \) can be positive. The deposit interest is paid at the end of the period. The portfolio of non-movers is not affected by the deposit rate. As we will show below, when \( r^d \) is positive, \( \frac{R_k}{Q_k} \) is equal to \( 1 + r^d \), which is higher than \( 1 + r^d \). So non-movers will use all initial deposits to buy assets and earn the return \( \frac{R_k}{Q_k} \).

We derive the optimal payout policy in the Appendix. We find that it is similar to the no-bank-loan case. When \( Q_k = R_k \), we have \( r_m = r_n \). When \( Q_k < R_k \), the investment fund tends to set higher \( \frac{r_m}{r_n} \) when people are more risk averse.

In the case of \( U = \ln c \), we find that \( r_m \) is still equal to the market price of the fund. Also, the best equilibrium is for households to hold all the riskless assets (\( \alpha = 0 \)). The basic reason is that investment funds need to transfer the riskless assets back to movers during the redemption process. Lower \( \alpha \) can reduce the payment and so banks are less likely to borrow from the central bank. In the following analysis, we will show the result for \( [U = \ln c, \alpha = 0] \). The results for the general case are shown in the Appendix. We will also discuss some results of the general case in section 7.

5.4 Bank’s problem

5.4.1 The expected borrowing from the central bank

In this part, we compute the expected central bank loan during the settlement process.

Let \( L_i \) denote the loan made by bank \( i \) and \( L_j = L \) denote the loan made by all other banks \( j \neq i \). Right after the loan is made, the deposit balance for bank \( i \) is \( D_0 + L_i \) and the deposit balance for all other banks \( j \neq i \) is \( D_0 + L_j \), where \( L_i \) and \( L_j \) are new deposits created during lending.

Then investment funds use all their deposits to pay the movers. We use \( X \) to denote the payment made by each bank, then

\[
X_i = (1 - \pi)D_0 + L_i \quad (20)
\]

\[
X_j = (1 - \pi)D_0 + L_j \quad (21)
\]

where \( (1 - \pi)D_0 \) is the deposits raised by selling assets to non-movers.

Recall that the settlement process is divided into \( N \) subperiods and in each subperiod a bank is chosen to make the payment to other banks. Since we assume the shareholders of each fund are evenly distributed among all banks, when bank \( i \) is chosen to make the payment, the payment made to the shareholders in the same bank is \( \frac{1}{N} X_i \), and the payment made to each
of the other \( N - 1 \) banks is also \( \frac{1}{N} X_i \), and the total payment outflow is \( \frac{N-1}{N} X_i \). The pattern is symmetric for all other banks \( j \neq i \).

Let \( n \) denote the subperiod in which bank \( i \) is chosen to make the payment. And let \( k = 1, 2, ..., N \) denote the subperiod. The pattern of payment flow is shown in Table 1.

\[
\begin{pmatrix}
\text{Out} & \text{In} & \text{In} & \cdots \\
\text{In} & \text{Out} & \text{In} & \cdots \\
\text{In} & \text{In} & \text{Out} & \cdots \\
\text{In} & \text{In} & \text{In} & \cdots \\
\vdots & \vdots & \vdots & \ddots \\
\text{Bank 1} & \text{Bank 2} & \text{Bank 3} & \cdots
\end{pmatrix}
\]

Table 1: “Out” is payment outflow and “in” is payment inflow. Column \( n \) shows the result if a bank makes its payment in subperiod \( n \). The bank receives payments from other banks for subperiods \( k < n \) and \( k > n \).

\[
\begin{pmatrix}
\frac{(N-1)X_i}{N} & -\frac{X_j}{N} & -\frac{X_j}{N} & \cdots & -\frac{X_j}{N} \\
\frac{(N-1)X_i}{N} & -\frac{X_j}{N} & -\frac{X_j}{N} & \cdots & -\frac{X_j}{N} \\
\frac{(N-1)X_i}{N} & -\frac{2X_j}{N} & -\frac{2X_j}{N} & \cdots & -\frac{2X_j}{N} \\
\frac{(N-1)X_i}{N} & -\frac{3X_j}{N} & -\frac{3X_j}{N} & \cdots & -\frac{3X_j}{N} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
\frac{(N-1)X_i}{N} & -\frac{(N-2)X_j}{N} & -\frac{(N-2)X_j}{N} & \cdots & -\frac{(N-1)X_j}{N} \\
\frac{(N-1)X_i}{N} & -\frac{(N-1)X_j}{N} & -\frac{(N-1)X_j}{N} & \cdots & -\frac{(N-1)X_j}{N}
\end{pmatrix}
\]

Table 2: Accumulated flow of payments. The accumulated flow in subperiod \( k \) (row \( k \)) is the total outflow minus the total inflow up to that subperiod. Column \( n \) shows the accumulated flow if bank \( i \) makes the payment in subperiod \( n \).

Table 2 shows the accumulated flow of payments. For example, column 1 shows the result if bank \( i \) is chosen to make the payment in subperiod 1. In subperiod 1 (row 1), the outflow of payment is \( \frac{(N-1)X_i}{N} \). In each subperiod \( k \) > 1, bank \( i \) receives \( \frac{X_j}{N} \). Similarly, in column 2, bank \( i \) receives \( \frac{X_j}{N} \) in \( k = 1 \), makes the payment \( \frac{(N-1)X_i}{N} \) in \( k = 2 \), and receives \( \frac{X_j}{N} \) in each of the subperiods \( k \) > 2.

Let \( FL(k, n) \) denote the accumulated flow in row \( k \) column \( n \). Banks are required to borrow from the central bank as long as \( FL(k, n) > D_0 \). Let \( b(k, n) \) denote the central bank
loan.

\[ b(k, n) = \max(0, FL(k, n) - D_0) \]  

(22)

The value of \( FL(k, n) \) depends on \( X_i \). Since \( X_i = (1 - \pi)D_0 + L_i \), higher \( L_i \) will lead to higher payment \( X_i \). Given \( L_j \), higher \( X_i \) has two effects: it will increase the level of \( b(k, n) \) when \( b(k, n) \) is positive, and it will also increase the probability for \( b(k, n) \) to be positive.

Suppose bank \( i \) makes the payment in period \( n \), we define the accumulated borrowing as

\[ b_n = \frac{1}{N} \sum_{k=1}^{N} b(k, n) \]

And the interest cost for central bank loan is \( r^c b_n \), where \( r^c \) is the central bank lending rate.

Before the settlement process starts, the expected future borrowing is defined as

\[ Eb = \frac{1}{N} \sum_{n=1}^{N} b_n \]

When \( N \) is large, we can get a closed-form solution for \( Eb \), the result is as follows:

**Proposition 3.** Suppose \( N \) is very large. Given \( L_j \), when \( Eb > 0 \), it can be written as

\[ Eb(L_i) = \frac{1}{6} \frac{(X_i - X_j - D_0)^3}{X_j^2} + \frac{1}{2} \frac{(X_i - X_j - D_0)^2}{X_j} + \frac{1}{2} (X_i - X_j - D_0) + \frac{1}{6} X_j \]  

(23)

In the symmetric case \( (L_i = L_j = L) \), \( Eb(L) > 0 \) when \( L > \pi D_0 \) (when \( X > D_0 \)).

Proof: See the Appendix ■

In the symmetric case, the payment \( X \) is \( (1 - \pi)D_0 + L \). If \( L > \pi D_0 \), then \( X > D_0 \) and the payments will not be fully covered by reserves.

### 5.4.2 The loan supply curve

In this part, we derive the bank loan supply curve.

After all inter-bank payments in “settlement 2” are completed, bank \( i \)'s deposit balance is

\[
\text{deposit after loan making} = \text{payment outflow} + \text{payment inflow} \\
= (D_0 + L_i) - \frac{N-1}{N} X_i + \frac{N-1}{N} X_j \approx (D_0 + L_i) - X_i + X_j = D_0 + L_j
\]

(24)

And the reserve balance is

\[
\text{initial reserve} = \text{payment outflow} + \text{payment inflow} \\
= D_0 - \frac{N-1}{N} X_i + \frac{N-1}{N} X_j \approx D_0 - (X_i - X_j) = D_0 - (L_i - L_j)
\]

22
We assume that banks pay the interest of central bank loan $r^c b_n$ at the end of “settlement 2”. For simplicity, we focus on the case in which the reserve balance after paying the interest, $D_0 - (L_i - L_j) - r^c b_n$, is positive. That is, $D_0$ is high enough to cover marginal increases in $L_i$ and the interest cost $r^c b_n$. (Since we focus on the symmetric case, $L_i - L_j$ means small marginal deviations of $L_i$ from $L_j$.) So banks do not need to borrow any central bank loan after the settlement process is completed.

The profit of the bank is

$$\Pi = [D_0 - (L_i - L_j) - r^c b_n] + L_i (R - \delta) - (1 + r^d)(D_0 + L_j)$$

(25)

The first term is the remaining reserves, the second term is the value of bank loan, and the third term is the gross payment to deposits.

The expected profit is

$$E\Pi = [D_0 - (L_i - L_j)] - r^c Eb + L_i (R - \delta) - (1 + r^d)(D_0 + L_j)$$

(26)

If no central bank loan is needed and $Eb = 0$, then (26) becomes

$$E\Pi = [D_0 - (L_i - L_j)] + L_i (R - \delta) - (1 + r^d)(D_0 + L_j)$$

(27)

In the equilibrium, bank $i$ should not be able to increase the profit by changing $L_i$. The first order condition is

$$\frac{\partial E\Pi}{\partial L_i} = -1 + R - \delta = 0$$

(28)

which gives

$$R = 1 + \delta \quad (\text{loan supply curve when } Eb=0)$$

(29)

This is the bank loan supply curve when liquidity shock is low and $Eb = 0$. In the symmetric case we have $L_i = L_j = L$. The deposit rate can be computed by applying the zero expected profit condition ($E\Pi = 0$) to equation (27), the result is $r^d = 0$.

If $Eb$ is positive, then the first order condition is

$$\frac{\partial E\Pi}{\partial L_i} = -1 - r^c \frac{\partial Eb(L_i)}{\partial L_i} + (R - \delta) = 0$$

(30)

Using (20) and (23), we have

$$\frac{\partial Eb(L_i)}{\partial L_i} = \frac{1}{2} \left(1 + \frac{X_i - X_j - D_0}{X_j}\right)^2$$

(31)
and (30) becomes

\[ R = 1 + \delta + r^c \frac{1}{2} \left( 1 + \frac{X_i - X_j - D_0}{X_j} \right)^2 \]

This is the loan supply curve of bank \( i \) given \( L_j \). In the symmetric case \((L_i = L_j = L)\), the supply curve becomes:

\[
R = 1 + \delta + r^c \frac{1}{2} \left( 1 - \frac{D_0}{(1 - \pi)D_0 + L} \right)^2 \quad (\text{loan supply curve when } Eb>0) (32)
\]

\( R \) is increasing in \( L \). From (32), we can see that when \( L > \pi D_0, \frac{D_0}{(1 - \pi)D_0 + L} < 1 \) and so \( R > 1 + \delta \). When \( L \) is very large, \( R \) approaches \( 1 + \delta + \frac{1}{2} r^c \).

5.5 The bank loan equilibrium

In this part, we derive the loan demand curve and then solve the bank loan equilibrium.

The demand curve for bank loan can be derived from the payout policy of the investment fund, which is

\[
\pi = \frac{\text{Redemption}}{NV} = \frac{(1 - \pi)D_0 + L(\pi)}{Z_k Q_k(\pi)} \quad (33)
\]

And the market price for asset is

\[
Q_k(\pi) = \frac{R_k}{R(\pi)} \quad (34)
\]

where \( R(\pi) \) is the bank lending rate when the liquidity shock is \( \pi \). Substitute \( Q_k(\pi) \) into equation (33) and we have the demand curve for bank loan

\[
R(\pi) = \frac{\pi Z_k R_k}{(1 - \pi)D_0 + L(\pi)} = \frac{\pi Z_k R_k}{X} \quad (\text{loan demand curve}) \quad (35)
\]

\( R \) is decreasing in \( L \), so when the lending rate is lower, the borrowing is higher.

Bank loan equilibrium

When \( Eb = 0 \), the loan supply curve is \( R = 1 + \delta \), and \( Q_k = \frac{R_k}{1 + \delta} \). Using (33), the equilibrium loan level is

\[
L = \text{Total redemption} - \text{cash raised from nonmovers}
\]

\[
= \pi Z_k Q_k - (1 - \pi)D_0 = \pi Z_k \frac{R_k}{1 + \delta} - (1 - \pi)D_0 = S \left[ \pi(1 - \omega) \frac{R_k}{1 + \delta} - (1 - \pi)\omega \right] \quad (36)
\]

When \( Eb > 0 \), (32) and (35) give the following result:
Proposition 4. The equilibrium $L(\pi)$ and $R(\pi)$ are

\[
L^*(\pi) = S \left( \frac{r^e \omega + \pi (1 - \omega) R_k + \sqrt{(r^e \omega + \pi (1 - \omega) R_k)^2 - 2(1 + \delta + \frac{r^e}{2})r^e \omega^2}}{2(1 + \delta + \frac{r^e}{2})} - (1 - \pi) \omega \right) \tag{37}
\]

\[
R^*(\pi) = \frac{2(1 + \delta + \frac{r^e}{2}) \pi (1 - \omega) R_k}{r^e \omega + \pi (1 - \omega) R_k + \sqrt{(r^e \omega + \pi (1 - \omega) R_k)^2 - 2(1 + \delta + \frac{r^e}{2})r^e \omega^2}} \tag{38}
\]

Proof: Eliminating $R$ from (32) and (35), we get a quadratic equation for $X$.

\[
X^2 \left( 1 + \delta + \frac{r^e}{2} \right) - X \left( \frac{r^e}{2} D_0 + \pi Z_k R_k \right) + \frac{r^e D_0^2}{2} = 0
\]

After we solve for $X$, we have $L^*(\pi) = X - (1 - \pi) D_0$. $R^*(\pi)$ is decided according to (35). Finally, we can simplify the results using the relationship $D_0 = \omega S$ and $Z_k = (1 - \omega) S$.

Given the equilibrium $L(\pi)$ and $R(\pi)$, we can solve for $Eb(\pi)$ from equation (23) by setting $L_i = L_j = L^*(\pi)$, and then solve for the equilibrium deposit rate $r^d(\pi)$ from equation (26) by setting $E\Pi = 0$. The result is as follows:

Proposition 5. When $Eb \geq 0$, the expected central bank loan is

\[
Eb = \frac{-D_0^3}{6X^2} + \frac{D_0^2}{2X} - \frac{D_0}{2} + \frac{X}{6} \tag{39}
\]

and the equilibrium deposit rate is\(^3\)

\[
r^d(\pi) = \frac{Lr^e}{2} \left( 1 - \frac{D_0}{X} \right)^2 - r^e Eb \left( 1 - \frac{D_0}{X} \right)^2 - r^e \left( \frac{D_0^3}{6X^2} + \frac{D_0^2}{2X} - \frac{D_0}{2} + \frac{X}{6} \right) \left( 1 - \frac{D_0}{X} \right)
\]

where $X = (1 - \pi) D_0 + L$, and $L = L^*(\pi)$ is the equilibrium loan level.

We can see that if $L$ is very high compared to $D_0$, then $X \approx L$ and $\frac{D_0}{X} \approx 0$, and we have $Eb \approx \frac{L}{6}$, and $r^d \approx \frac{r^e}{2}$.

Once the equilibrium loan level is decided, then the aggregate deposit level in the economy is also decided. The aggregate deposit (i.e., money supply) is $D_0 + L(\pi)$, where $D_0$ is the initial deposit balance, and $L(\pi)$ is the deposits that are created by banks during lending. Note that since banks can lend by creating new credit money, the aggregate bank loan level is elastic and is not directly limited by the monetary funds saved by depositors. (For example, $L$ can certainly be higher than $D_0$.) Instead, it is the lending activities of banks that decide

\(^3\)We replace $R$ in (26) using the loan supply curve (32).
the aggregate deposits in the economy. By creating and lending out deposits \( L(\pi) \), banks increase the aggregate money that people can use to make payments, and thus help to relax the aggregate financial constraint imposed by the existing money.

In most standard banking models, depositors are the people who provide liquidity to banks. It is usually assumed that the depositors have some idle cash that they will not use in the near future, and they lend those money to banks, and banks then lend those money to borrowers. In our model, the movers will end up holding all the deposits \( D_0 + L \). So the final deposit holders are actually the people who need liquidity. It would be inappropriate to say that the movers provide liquidity to banks. In our model, the liquidity provider is the bank, it creates and lends out new deposit, which is then used by people to make payments. By doing so, banks take more liquidity risks associated with payment outflows.

**A summary of the steps for solving the loan equilibrium**

The steps for solving the loan equilibrium can be summarized as follows.

First, we build a general equilibrium model for inter-bank settlement process, we use it to decide \( E_b \) and then derive the loan supply curve. Then we derive the bank loan demand curve using the optimal payout policy of the investment fund. Then we solve the equilibrium loan level \( L \) and the lending rate \( R \).

Once \( L \) is solved, the aggregate deposit is also decided because banks create equivalent amount of deposit during lending, and so the total deposit is \( D_0 + L \). Finally, we use the zero expected profit condition to solve for the deposit rate.

**The distribution of \( Q_k \):** The distribution of \( Q_k \) is

\[
Q_k(\pi) = \begin{cases} 
R_k & : \pi = \pi_1 = \frac{\omega}{\omega + (1-\omega) R_k} \\
\frac{\omega(1-\pi)}{\pi(1-\omega)} & : \pi_1 < \pi < \pi_2 = \frac{\omega}{\omega + (1-\omega) \frac{R_k}{1+\delta}} \quad \text{(Nonmovers’ cash is binding)} \\
\frac{R_k}{1+\delta} & : \pi_2 \leq \pi \leq \pi_3 = \frac{\omega}{(1-\omega) \frac{R_k}{1+\delta}} \quad \text{(Bank loan } L > 0, E_b = 0) \\
\frac{R_k}{R(\pi)} & : \pi > \pi_3 \quad \text{(Bank loan } L > 0, E_b > 0) 
\end{cases}
\]

The definition of \( \pi_1 \) is still the same as in the no-bank-loan case. Non-movers use all their deposits to buy assets if \( \pi \geq \pi_1 \). \( \pi_2 \) is the \( \pi \) above which the investment funds borrow positive bank loans. Below \( \pi_2 \), the investment funds only raise cash by selling assets on the financial market. The distribution of \( Q_k \) for \( \pi < \pi_2 \) is the same as when there is no bank lending(equation 15). For example, below \( \pi_1 \), the asset is sold for its fundamental value. Between \( \pi_1 \) and \( \pi_2 \), \( Q_k \) is lower than \( R_k \), but since \( Q_k > R_k \), it is not worthwhile for the
investment funds to borrow from banks. At $\pi_2$, $Q_k$ decreases to $\frac{R_k}{1+\delta}$, and the funds start to borrow from banks. $\pi_3$ is the level of $\pi$ above which the expected central bank loan $Eb$ is positive. So between $\pi_2$ and $\pi_3$, the lending rate is $R = 1 + \delta$ (equation 29), and $Q_k = \frac{R_k}{1+\delta}$. Above $\pi_3$, $Eb > 0$, and $Q_k = \frac{R_k}{R(\pi)}$, where $R(\pi)$ is defined in (38).

$\pi_2$ and $\pi_3$ can be decided as follows. At $\pi_2$, $Q_k = \frac{R_k}{1+\delta}$, non-movers use all their deposits to buy assets and the bank loan is still zero, so we have

$$\pi_2 = \text{Redemption} = \frac{(1 - \pi_2)D_0}{Z_kQ_k} = \pi_2 = \frac{\omega}{\omega + (1 - \omega)\frac{R_k}{1+\delta}}$$

Since $Eb > 0$ when $L > \pi D_0$, at $\pi_3$, we have $L(\pi_3) = \pi_3 D_0$. Since $Q_k(\pi_3)$ is still $\frac{R_k}{1+\delta}$, we have

$$\pi_3 = \text{Redemption} = \frac{(1 - \pi_3)D_0 + L(\pi_3)}{Z_kQ_k} = \frac{D_0}{Z_kQ_k} = \frac{\omega}{(1 - \omega)\frac{R_k}{1+\delta}}$$

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**The effects of the central bank lending rate $r^c$**

Please note that $r^c$ does not need to be constant, the central bank can actively change $r^c$ contingent on $\pi$. When $r^c$ is a function of $\pi$, the solutions for $R(\pi)$ and $L(\pi)$ are still the same. $r^c$ affects the equilibrium by affecting the loan supply curve. When $Eb = 0$, $r^c$ does not affect the equilibrium. When $Eb > 0$, lower $r^c$ reduces the slope of the supply curve (32). Since the demand curve is downward sloping and the supply curve is upward sloping, a lower slope of the supply curve will lead to higher $L^*(\pi)$ and lower $R^*(\pi)$.

5.6 The equilibrium

Using $v_m$ and $v_n$ from (18) and (19), the expected utility for a representative household $i$ is

$$EU_i = \frac{1}{2} \int_0^1 \left\{ \pi \ln \left[ (1 + r^d)s(\omega_i + (1 - \omega_i)Q_{k,H}) \right] + (1 - \pi) \ln \left[ \frac{s}{Q_{k,H}} \omega_i + (1 - \omega_i)Q_{k,H} \right] \right\} dF(\pi)$$

$$+ \frac{1}{2} \int_0^1 \left\{ \pi \ln \left[ (1 + r^d)s(\omega_i + (1 - \omega_i)Q_{k,L}) \right] + (1 - \pi) \ln \left[ \frac{s}{Q_{k,L}} \omega_i + (1 - \omega_i)Q_{k,L} \right] \right\} dF(\pi)$$

Because of the log utility function, $(1 + r^d)$ does not affect the first order condition for $\omega_i$. So the first order condition is the same as equation (16).
5.7 Endogenous money supply and the transmission mechanism of the central bank interest rate policy

In our model, the central bank can not directly control the money supply, the money supply is not decided exogenously according to the money multiplier, it is endogenously decided by the credit transactions of private agents. The loan supply and loan demand are decided together in the general equilibrium. The money supply is “endogenous” in the sense that it is greatly affected by the money demand. For example, when the demand for bank loan is low, the supply will also be low because banks can not lend when people do not want to borrow. And when the demand is high, the equilibrium supply will also tend to be high.

Although money supply is not directly controlled by the central bank, the central bank can still use the interest rate policy to affect the equilibrium loan level and the asset prices(See Figure 6). For example, when the central bank reduces the interest rate, the lower expected borrowing cost for settlement balance will encourage banks to make more loans, which will lead to higher loan level, lower lending rate and higher asset price. The higher loan level will lead to higher demand for settlement balance. The central bank meets the demand for loans at the promised interest rate. So the supply of reserve is decided by the demand for reserve at the targeted interest rate. Thus, the supply of reserve is also endogenous.

Note that the central bank does not need to lend outside money to banks first and then banks lend those money to the public. Instead, the central bank can directly use the interest rate policy to encourage banks to expand their credit, which leads to an expansion of bank inside money.
6 Numerical Results: \( U = \ln c \)

This section shows the a numerical example for \( U = \ln c \). Note that this is not a calibration. The purpose of this example is to illustrate the intuition of the model.

6.1 Parameter values

Table 3: Values of the parameters

<table>
<thead>
<tr>
<th>( A_H )</th>
<th>( A_L )</th>
<th>( e_h )</th>
<th>( \delta )</th>
<th>( r^c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.21</td>
<td>0.85</td>
<td>1</td>
<td>0.03</td>
<td>0.03</td>
</tr>
</tbody>
</table>

Table 3 shows the parameter values. The return of the risky assets are \( R_{k,H} = A_H \) and \( R_{k,L} = A_L \). The expected return is \((A_H + A_L)/2 = 1.03\). \( A_H \) and \( A_L \) are chosen such that households will hold positive money balance even when all assets can be used to make payments. We want to show that even if there are no trading frictions, people can still hold money as a riskless asset. We set the household endowment \( e_h \) at 1, the loan management cost of banks \( \delta \) at 3% and the central bank lending rate \( r^c \) at 3%.

We assume that the liquidity shock is distributed according to

\[
\pi = 0.9\theta^a \tag{43}
\]

where \( \theta \) is uniform over \([0, 1]\). The highest liquidity shock is \( \pi = 0.9 \). \( a \) is used to adjust the density of \( \pi \). With higher \( a \), the density of \( \pi \) will be more concentrated on low values and households will hold lower monetary balance. We use \( a = 6 \). At this level, \( D_0 \) is low enough and we can see clearly the effects when banks borrow from the central bank. The steps for computing the equilibrium are explained in the Appendix.

6.2 Results

Table 4 shows \( \omega^* \) and the expected utility of households. Since we assume that the aggregate household wealth is 1, the aggregate real money balance \( \frac{M}{\pi} = D_0 \) is equal to \( \omega^* \).

Table 4 shows that households hold less money balance and more investment fund shares when liquidity supply is more elastic. The expected utility is the highest in case 1 when all assets can be used to make payments, and is the lowest in case 2 when money supply is fixed.
Table 4: Numerical example

<table>
<thead>
<tr>
<th></th>
<th>$\omega^*$</th>
<th>$E\ln c($households$)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) All assets can be used to make payments (section 3)</td>
<td>0.0476</td>
<td>0.0141</td>
</tr>
<tr>
<td>(2) Fixed money(section 4)</td>
<td>0.7013</td>
<td>0.0059</td>
</tr>
<tr>
<td>(3) With elastic inside money (section 5)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(3a) Central bank sets $r^c = 0.03$ for all levels of $\pi$</td>
<td>0.2460</td>
<td>0.0118989</td>
</tr>
<tr>
<td>(3b) CB sets $r^c = 0.02$ for $\pi \geq 0.5$. The policy is pre-announced</td>
<td>0.2438</td>
<td>0.0119093</td>
</tr>
</tbody>
</table>

$\omega$ is slightly lower in case 3b than in 3a. This is because in case 3b the central bank will set a lower interest rate for $\pi \geq 0.5$. The lower interest rate will lead to lower bank lending rate and higher asset prices. The difference in $\omega$ in these two cases is small because in our example, the density for $\pi \geq 0.5$ is small.

Figure 7 shows the market price of risky assets in case 2(equation 15).

Figure 8 shows the results for case 3a. $Q_k$ is decided according to equation (41). We can see that the aggregate money supply $D_0 + L(\pi)$ is stochastic. Higher liquidity shock will lead to higher money supply. The figure also shows that the changes in central bank loan $Eb$ is small compared to the changes in $L$. This implies that the private commercial banking system can meet the liquidity needs of the economy with only a very low need to borrow from the central bank.

Figure 9 compares $Q_k$ in case 1 and case 3a using the same scale. We can see that the asset price is more stable with elastic inside money.

Figure 10 compares cases 3a and 3b for $A = A_H$(the result for $A = A_L$ is similar). In case 3b, since the central bank lending rate is lower for $\pi > 0.5$, the equilibrium lending rate and
Figure 8: $Q_k$, $L$, $R$, $r^{d}$ and $Eb$ in case 3a (elastic money).
deposit rate are lower, and the asset prices are higher.

6.3 Implications

Our results have several implications.

Elasticity of liquidity: When liquidity needs take the form of demand for means of payment, it can partly be met with the creation of new inside money. Since the supply of new deposit is more elastic than the supply of real consumption goods, the liquidity supply in the real economy may be more elastic than those predicted by non-monetary models.

How to measure liquidity: Our results imply that the existing money aggregate may not be a good measure of the available “liquidity” (money) since new inside money can be elastically created to meet the demand for means of payment as long as people are willing to pay the borrowing cost. So if the banking system is working properly, then the interest rate may be a better measure of the availability of liquidity.

Aggregate money supply and the long-run price level: In our model, the aggregate money supply $D_0 + L(\pi)$ is stochastic. While the price level $P$ is constant in each period. So the ratio between the aggregate money supply and the price level is not stable. In the model, inside money is created when banks make loans. But the borrowers are required to pay back
those bank loans later. When bank loans are repaid, the deposits created during lending will be destroyed. So not all increases in aggregate money supply will lead to proportional changes in the long-run price level.

7 The relationship between liquidity-risk sharing through coalitions and bank money creation

Our model includes two types of liquidity provision mechanisms: liquidity risk-sharing through coalitions and liquidity provision through money creation. Liquidity risk-sharing means early withdrawers of liquidity will get a higher payment than if there were no coalitions. In the previous analysis, we use the log utility case to show the second mechanism. But the first mechanism is not actually used by households. Under the log utility function, the payment \( r_m \) is equal to the market value of the fund’s asset. If investment funds allocate all assets equally to shareholders and then movers sell assets directly to non-movers, then the payment to movers will be the same.

If \( \sigma > 1 \), then the investment fund would try to give movers a payment \( r_m \) that is higher than the market value of the fund’s asset. We ask two questions: Do investment funds actually help movers to achieve higher consumption? How the liquidity provision function of banks affect the liquidity-risk sharing function of investment funds?

We show below that if there is no banks, since the aggregate money is limited to the money held by people in their initial portfolio, investment funds will not actually help movers to achieve higher consumption. The incentive of investment funds to provide higher payment than the market value of the fund’s asset will only cause lower market prices. But with banks supplying elastic aggregate money, then investment funds can indeed help movers to achieve higher consumption.

7.1 No bank lending

We first consider the no-bank-lending case.

7.1.1 Some basic general equilibrium results

We first show two basic results.
1. If there is no bank lending, then once the total riskless asset $\kappa$ is decided, the consumption level of movers and non-movers are decided. The consumption level is not affected by risk-sharing provided by investment funds.

2. Once the initial distribution of money among investment funds and households are given, then the distribution of $r_m$ is decided. $r_m$ does not depend on the risk-sharing policy. But risk-sharing will cause higher volatility in asset prices.

These two results are proved in the following two propositions. We will then illustrate the results using numerical example.

**Proposition 6.** In the symmetric equilibrium, the distribution of $v_m$ and $v_n$ only depends on $\kappa$.

Proof: For notational convenience, we set $S = 1$. First, given $\kappa$, the total wealth of the economy $\kappa + (1 - \kappa)R_k$ is decided. In addition, $\pi_1$ only depends on $\kappa$. At $\pi_1$, $r_m = r_n = \alpha + (1 - \alpha)R_k$, and we also know that the payment to movers is equal to the cash of the investment fund $Z_f$ plus the cash collected from non-movers $(1 - \pi)D^h$, so we have

$$\frac{\pi_1Z_pr_m}{Z_f + (1 - \pi_1)D^h} = \frac{\omega + (1 - \omega)\alpha}{\omega + (1 - \omega)(\alpha + (1 - \alpha)R_k)} = \frac{\kappa}{\kappa + (1 - \kappa)R_k} \quad (44)$$

For $\pi \leq \pi_1$, $v_m = v_n = \kappa + (1 - \kappa)R_k$. For $\pi > \pi_1$, movers carry all the cash $\kappa$ with them, which means the risky assets will become the wealth of non-movers.

$$\pi v_m = \kappa \Rightarrow v_m = \frac{\kappa}{\pi} \quad (45)$$

$$(1 - \pi)v_n = (1 - \kappa)R_k \Rightarrow v_n = \frac{(1 - \kappa)R_k}{1 - \pi} \quad (46)$$

Thus, the distribution of $v_m$ and $v_n$ only depends on $\kappa$.■

The intuition for $\pi_1$ (44) is as follows. Once the cash constraint is binding, movers can only carry all the cash in the economy $\kappa$ with them to the other location. At $\pi_1$, each mover still receives the average wealth $\kappa + (1 - \kappa)R_k$, so $\pi_1$ is the ratio between the aggregate cash $\kappa$ and the aggregate wealth $\kappa + (1 - \kappa)R_k$.

It is easy to see that given $\kappa$, if investment funds simply allocate all assets to shareholders and movers sell assets directly to non-movers before they move, then the consumption level of movers will be the same. When total money is not binding, movers get the average value of the total wealth. When total money is binding, every mover gets $\frac{\kappa}{\pi}$. 34
Proposition 7. In the symmetric case, if we fix the initial portfolio choice \( \omega \) and \( \alpha \), then the distribution of \( r_m \) will be the same for different \( \sigma \). But the distribution of \( Q_k \) will be different. Between \( \pi_1 \) and \( \pi_{\text{bind}} \) (the \( \pi \) above which the constraint \( r_m \leq r_n \) is binding), \( Q_k \) is lower for higher \( \sigma \).

Proof: Once \( \omega \) and \( \alpha \) are given, then \( \kappa \) is given, and \( \pi_1 \) is uniquely decided. For \( \pi \leq \pi_1 \), we have \( r_m = r_n = \alpha + (1 - \alpha)R_k \). For \( \pi > \pi_1 \), all the cash owned by investment funds and non-movers are used to pay movers, and we have

\[
\pi Z_p r_m = Z_f + (1 - \pi)D^h \Rightarrow r_m = \frac{Z_f + (1 - \pi)D^h}{\pi Z_p} = \frac{(1 - \omega)\alpha + (1 - \pi)\omega}{\pi(1 - \omega)} \tag{47}
\]

which is the same for different \( \sigma \). We show in the Appendix that between \( \pi_1 \) and \( \pi_{\text{bind}} \), we have

\[
\frac{Q_k}{R_k} = \left( \frac{\kappa(1 - \pi)}{R_k \pi(1 - \kappa)} \right)^\sigma \tag{48}
\]

This ratio is equal to 1 at \( \pi_1 \). For \( \pi > \pi_1 \), the right-hand-side is lower than 1. So given \( \kappa, R_k \) and \( \pi \), \( Q_k \) will be lower for higher \( \sigma \). ■

The intuition is as follows. Once the aggregate liquidity is decided by \( \omega \) and \( \alpha \), then the distribution of \( r_m \) will be uniquely decided. For \( \pi > \pi_1 \), when \( \sigma \) is higher, the investment fund would like to set a higher \( r_m \) given the market value of the fund’s asset. But since the payment to movers is limited by the liquidity in the economy, it is the price \( Q_k \) (and the market value of the fund’s asset) that must adjust in order for the optimal payout policy to be satisfied. If \( \sigma \) is higher, then \( Q_k \) must decrease more in order to satisfy the optimal payout policy. On the microeconomic level, each investment fund takes the price \( Q_k \) as given and tries to liquidate assets to raise cash in order to provide liquidity insurance to movers. But if the aggregate liquidity is limited, then the effort of investment funds is self-defeating, it will lead to more volatile asset prices, without actually providing more liquidity to movers.

7.1.2 Numerical examples

When \( \omega \) and \( \alpha \) are taken as given

We first fix the initial portfolio choice \( \omega \) and \( \alpha \) and see how changes in \( \sigma \) can affect the result. We use the example \([\omega = 0.4, \alpha = 0]\) (we set a low \( \omega \) so we can see clearly what will happen when \( Q_k \) goes to low levels.)

The results are shown in Figure 11. The main findings can be summarized as follows:
Liquidity shock \( \pi \) (non-movers) \( r \) (movers) \( r_m \) and \( r_n \): 

(a) \( r_m \) and \( r_n \): \( \sigma = 1 \)
(b) \( r_m \) and \( r_n \): \( \sigma = 1.2 \)
(c) \( r_m \) and \( r_n \): \( \sigma = 1.5 \)
(d) \( r_m \) and \( r_n \): \( \sigma = 2 \)
(e) \( r_m \) and \( r_n \)
(f) Asset price \( Q_k \)

Figure 11: Payout policy and \( Q_k \) when the initial portfolio is fixed at \([\omega = 0.4, \alpha = 0]\). The results for \( A = A_H \) are shown here.

1. The distribution of \( r_m \) is the same for different \( \sigma \).

2. When \( \sigma \) is higher, \( Q_k \) and \( r_n \) decrease more quickly between \([\pi, \pi_{bind}]\).

3. When the constraint \( r_m \leq r_n \) is binding, \( r_m \) is still the same, but \( Q_k \) decreases more slowly.

The reasons for result 1 and 2 are already explained above. The reason for result 3 is as follows. When \( r_m \leq r_n \) is binding, given \( Q_k \), the payment to movers will be lower than the optimal payment if \( r_m \) were allowed to be higher than \( r_n \). This will reduce the need for investment funds to liquidate assets. Thus, in the equilibrium, \( Q_k \) decreases more slowly. In this case, limiting the payment to movers does not really reduce the actual payment received by movers, but it helps stabilize the asset price.

Risk-sharing also means for higher \( \sigma \) the investment fund would set a smaller difference between \( r_n \) and \( r_m \). But in the above example, the smaller difference between \( r_n \) and \( r_m \) is actually achieved solely by a decrease in \( r_n \) without actually increasing the level of \( r_m \).
When ω and α are chosen freely

Now, we assume that people can optimally choose ω and α. In order to decide the equilibrium, we first need to analyze the response curves of the investment fund and the household. Let $R_{\text{household}}(\alpha)$ denote the response curve of the household and $R_{\text{fund}}(\omega)$ the response curve of the investment fund.

The main findings are as follows.

1. There exists a level of $\alpha = \alpha_{\text{bind}}$. When $\alpha < \alpha_{\text{bind}}$, the constraint $r_m \leq r_n$ is binding for positive probability. We find that it will cause $R_{\text{fund}}(\omega)$ to be slightly higher than $R_{\text{household}}(\alpha)$. So when $\alpha < \alpha_{\text{bind}}$, people will reduce $\omega$ and increase $\alpha$.

2. For $\alpha \geq \alpha_{\text{bind}}$, the constraint $r_m \leq r_n$ is not binding. The two response curves overlap with each other. In addition, all the optimal pairs of $[\omega, \alpha]$ have the same value of $\kappa$.

The detailed reason the above pattern is discussed in the Appendix.

Let $\omega_{\text{bind}}$ denote the $\omega$ when $\alpha = \alpha_{\text{bind}}$. The equilibriums are defined by the part of the response curves over $[\omega \leq \omega_{\text{bind}}, \alpha \geq \alpha_{\text{bind}}]$. For the log utility function, since $r_n = \frac{R_k}{Q_k} r_m$ and $\frac{R_k}{Q_k} \geq 1$, the constraint $r_m \leq r_n$ will never be binding, so the two response curves are the same.

The result

Here we only show the key result: the changes in asset price, in Figure 13. When the initial

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For example, if we start with $\alpha_0 = 0$, then $\omega_0 = \omega(\alpha_0)$ chosen by the household is defined by $R_{\text{household}}(\alpha = 0)$. Because $R_{\text{fund}}(\omega)$ is slightly higher than $R_{\text{household}}(\alpha)$, the fund will deviate by choosing an $\alpha$ higher than $\alpha_0$: $\alpha_1 = \alpha(\omega_0) > \alpha_0$. This will in turn cause households to deviate by choosing a lower $\omega$: $\omega(\alpha_1) < \omega_0$, and
portfolio is chosen freely, the main outcome is that $\kappa$ will be higher when people are more risk averse.\(^5\) As a result, for higher $\sigma$, the cash constraint is less likely to be binding. But once the cash constraint is binding, $Q_k$ decreases more quickly. The reason is the same as before, the incentive of investment funds to provide more risk-sharing only add more endogenous volatility in asset prices. Again, given $\kappa$, the consumption level of movers is not affected by the risk-sharing function of investment funds.

### 7.2 With bank lending

In this part, we show that with banks supplying elastic aggregate money, investment funds can indeed help movers to achieve a higher consumption level than if there is no risk-sharing. **When $\omega$ and $\alpha$ are taken as given**

We show the intuition by considering a case in which the initial portfolio is fixed at $[\omega = 0.2460, \alpha = 0]$ (This is the optimal equilibrium for $\sigma = 1$.) The results are shown in Figure 14.

The findings can be summarized as follows:\(^6\)

1. $r_m$, $r_n$ and the asset price $Q_k$ are more stable compared to the no-bank-lending case.

2. The levels of $r_m$ are no longer the same for different $\sigma$. When $\sigma$ is higher, investment funds can provide higher $r_m$ to movers.

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\(^5\)For $\sigma = 1$, we use $[\omega = \kappa = 0.7013, \alpha = 0]$. For $\sigma = 1.5$, we use $[\kappa = 0.7521, \omega_{\text{bind}} = 0.3731, \alpha_{\text{bind}} = 0.6046]$. And for $\sigma = 2$, we use $[\kappa = 0.7800, \omega_{\text{bind}} = 0.2130, \alpha_{\text{bind}} = 0.7205]$.

\(^6\)The differences in $Q_k$ for different $\sigma$ are very small so that we can only see one curve for $Q_k$ in the Figure.

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38
The key figure is 14(d). Recall that \( r_m \) under the log utility function is the market price of the fund’s asset. This is also the payment that movers would get if all investment funds allocate their assets to shareholders and shareholder trade assets by themselves.\(^7\) That is, if \( \sigma > 1 \) but there is no risk-sharing, then movers would get \( r_m(\sigma = 1) \) for the investments initially allocated into investment funds. But here, we can see that when \( \sigma > 1 \), the payment to movers is higher than \( r_m(\sigma = 1) \), which means movers achieve higher consumption than if there were no coalitions to provide risk-sharing.

**When agents can choose \( \omega \) and \( \alpha \) freely**

We find that with bank lending, the shape of the response curves are similar to that in Figure 12. If there exists \( \alpha_{bind} > 0 \), then for \( \alpha < \alpha_{bind} \), \( R_{fund}(\omega) \) will be slightly higher than \( R_{household}(\alpha) \). For \( \alpha \geq \alpha_{bind} \), the two response curves overlap with each other. But in this case, different optimal pairs of \( \omega \) and \( \alpha \) will not give the same \( \kappa \). \( \kappa \) is higher for the pairs with lower \( \omega \)(higher \( \alpha \)). The reason is that higher \( \alpha \) increases the money that investment

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\(^7\)If the initial deposit of non-movers is binding, then we can assume that non-movers are allowed to borrow from banks to buy assets.
funds need to transfer to movers, this will cause higher interbank payment flows and higher borrowing of banks from the central bank. As a result, the lending rate tends to be higher and the asset price tends to be lower, which will cause people to hold more riskless assets. So the equilibrium with the highest \( \omega \) (lowest \( \alpha \)) is the most efficient equilibrium.

The results are shown in 15. The main finding is that for higher \( \sigma \), people will choose higher \( \kappa \), and the cash constraint and the bank reserve constraint is likely to be binding.\(^8\)

7.3 Summary

Our results shows that the ability of banks to relax the aggregate liquidity constraint is important for non-bank coalitions to provide insurance for aggregate liquidity risks. Without banks, coalitions can not actually provide more liquidity to people who need liquidity early, and risk-sharing will only make asset prices more volatile. But with banks supplying elastic aggregate money, people who need liquidity early can indeed achieve higher consumption

\(^8\)The constraint \( r_m \leq r_n \) is never binding for log utility functions. The constraint is not binding either for \( \sigma = 1.5 \) given the parameter values. For \( \sigma = 1 \), the optimal equilibrium is \([\omega = 0.2460, \alpha = 0]\). For \( \sigma = 1.5 \), it is \([\omega = 0.4394, \alpha = 0]\). And for \( \sigma = 2.0 \), it is \([\kappa = 0.5629, \omega_{bind} = 0.5350, \alpha_{bind} = 0.0600]\).
through the risk-sharing function provided by coalitions.9

Note that we get the above result because we use a general equilibrium model to endogenously decide the asset price. In partial equilibrium models, we usually exogenously assume that the liquidation price is a constant proportion of the fundamental value, that is, 
\[ Q_k = \lambda_q R_k, \] 
where \( \lambda_q < 1 \) is a constant. With this type of assumption, coalitions will be able to provide higher consumption to people who need liquidity early. The reason is that we essentially assume that coalitions can sell unlimited amount of asset at the same price \( \lambda_q R_k \). In our model, higher sales will cause lower \( Q_k \), if there no bank lending, then the market liquidity is limited to money already existed in the economy. As a result, by using general equilibrium model, we get new insights about the relationship between liquidity-risk sharing and the ability of banks to relax the aggregate liquidity constraint.

8 Capital constraint

8.1 The assumptions

In the previous analysis, we assume that banks are only subject to the liquidity constraint. In reality, the ability of banks to provide liquidity through deposit creation will be limited by the capital level of banks. Here we show some basic results when the capital constraint is binding, more detailed modelling will be carried out in another paper.

We will not try to endogenize the optimal bank capital level and the optimal banking regulation. We directly make the following two assumptions:

- Bank capital is exogenously given by \( e_b R_k \).
- Banks are only allowed to lend up to \( L = \rho e_b R_k \), where \( \rho \) is exogenously given by regulation.

We will show two basic results

1. When the capital constraint is binding, bank lending is limited, the asset price will be more volatile.

9If we allow banks to perform both the function of inside money creation and liquidity risk-sharing, we will get a type of mechanism which is different from the traditional mechanism of liquidity-risk sharing through coalitions. The reason is that people can directly use bank deposit to make payments without actually withdrawing currency from banks, so the need for bank deposit holders to make payments will not create an equivalent amount of liquidity shortage of banks. We will analyze this in a different paper.
2. If the regulator requires banks to evaluate their capital using the market price, then it will add more volatility to asset prices.

The intuition for the second result is as follows. Suppose all assets are subject to the same discount when evaluated at the market price, then the market value of bank’s capital is $e_bQ_k$. When the market price $Q_k$ is lower, bank capital will be lower, so bank lending will be lower, which will in turn cause further decrease in the market price.

When loan supply is limited by the capital constraint, the market interest rate must rise so that the loan demand will reduce to the level of loan supply. We use the log utility function as an example. When bank loan is binding, the equilibrium condition is

$$\pi Z_k Q_k = (1 - \pi)D_0 + L = (1 - \pi)D_0 + \rho e_b R_k$$

(49)

We have

$$Q_k = \frac{(1 - \pi)D_0 + \rho e_b R_k}{\pi Z_k}$$

(50)

and the equilibrium lending rate is $\frac{R_k}{Q_k}$. We assume banks still set the highest $r^d$ which gives them zero expected profit, then $r^d$ is still decided according to (40).

If banks are required to evaluate bank capital using market price (mark-to-market (MTM)), then when the constraint is binding, we have $L = \rho e_b Q_k$ and

$$\pi Z_k Q_k = (1 - \pi)D_0 + \rho e_b Q_k$$

$$\Rightarrow Q_k = \frac{(1 - \pi)D_0}{\pi Z_k - \rho e_b}$$

(51)

8.2 Numerical Results

We assume $e_b = 0.04$ and $\rho = 12.5$. We select the level of $e_b$ such that the capital constraint is only binding for high levels of $\pi$. All other variables are the same as before.

Figure 16 shows the result when $L = \rho e_b R_k$. The result for $A = A_L$ is shown here, $A_H$ is similar. We can see that when the constraint is binding, $Q_k$ will be decreasing more quickly in $\pi$.

Given the same portfolio, the result for mark-to-market is shown in Figure 17. When the constraint is binding, lower market price causes $L$ to decrease, and $Q_k$ is more volatile.

Figure 18 shows the intuition of how mark-to-market can cause higher volatility. We use $\pi = 0.85$ in the example. If bank capital is evaluated at $R_k$, then the loan level is $L$ at $a'$,
which we can denote as $L(a')$. Given $L(a')$, the market price is $Q(a'')$, this is the market price if there is no MTM. But if there is MTM, then given $Q(a'')$, the loan level will decrease to $L(a''')$. This process will continue until we reach the equilibrium.

The above results imply that when the capital constraint is binding, policies other than the
interest rate policy will be needed. For example, regulators can temporarily allow banks not to test their capital value using the market price (for example, in reality, banks can evaluate the asset value using historical costs). Policies such as temporary suspension of the capital adequacy requirement will also be helpful.

9 Summary and conclusion

This paper studies the role of banks in providing liquidity through inside money creation. We build a general equilibrium model for banks based on the monetary loanable funds theory, we analyze how banks supply loans when they face liquidity and capital constraints. We explain why the ability of banks to relax the aggregate liquidity constraint through credit money creation is important to financial stability. The main results are as follows:

1. The role of banks in the payment system gives banks additional ability to provide liquidity. Since bank deposit is used as means of payment, banks can expand credit by creating new deposit. Deposit created by banks can help meet liquidity needs and stabilize asset prices.

2. When people use non-bank coalitions to share aggregate liquidity risks, the function of banks is important. Without elastic aggregate liquidity, coalitions can not actually provide higher consumption to shareholders who need liquidity early, and risk-sharing will only add more volatility to asset prices. But with banks supplying elastic aggregate liquidity, risk-sharing coalitions can help people to achieve less risky consumptions.

3. Under the interest rate policy, the money supply is decided endogenously and is not directly controlled by the central bank, but the central bank can still use the interest rate policy to indirectly affect the money supply by affecting the incentive of banks to provide loans. But if the capital constraint is binding, then other policies, such as temporarily suspending the capital adequacy requirement, will be needed.

Instead of modelling banking as a mechanism for transferring savings of depositors to borrowers, we instead focus on the function of banks in providing liquidity through money creation. A unique feature of banks is that they can directly lend out their own debt (i.e., deposit) without first collecting equivalent amount of money from depositors, this makes it easier for banks to provide liquidity quickly, which is also why banks act as important backup
source of liquidity to the financial system. Banks in our model are very similar to market makers. When they provide liquidity, they use liquid assets (the newly created bank deposit) to buy illiquid assets (the loan of borrowers). The result is that borrowers become more liquid while banks bear more liquidity risks.

In this paper, we didn’t consider credit risks faced by banks. We will extend the model to include default risks. We will then analyze how limited lending abilities of banks will interact with liquidity shocks and asset prices on the financial market.

A

A.1 Main Notations

A: the productivity factor  
H, L: aggregate return shocks  
π: liquidity shocks  
$R_k$: return (fundamental value) of risky assets  
$Q_k$: market price of risky assets  
$Z_p, Z_f, Z_k$: total portfolio, riskless assets and risky assets held by the investment fund  
s(S): individual (aggregate) savings of households  
d: deposit of each household  
ω: riskless assets in household’s portfolio  
α: riskless assets in the investment fund’s portfolio  
κ: riskless assets in the aggregate portfolio  
r^d(r^l): bank deposit (lending) rate  
r^c: central bank lending rate  
δ: management cost for bank loan  
b: central bank loan  
$L$: equilibrium loan level  
$π_1, π_2, π_3$: when $π > π_1$, $Q_k < R_k$; when $π > π_2$, bank loan $L > 0$; when $π > π_3$, central bank loan $Eb > 0$.  
X: payment outflow of the bank when investment funds pay movers  
$M$: nominal level of outside money  
P: nominal price level  
$D_0$: initial real deposit balance  
$NV$: market value of the investment fund  
$R$: gross lending rate  
N: the number of subperiods in the settlement process  
n: the period in which bank $i$ is chosen to make the payment

A.2 The detailed transaction steps

In this part, we list and explain the transactions and the resulted changes on the balance sheet of the bank. In the model, we assume that the inter-bank settlement at the end of period $t + 1$ is carried out based on net balance. But for illustrative purposes, we show the transactions one by one. In the balance sheet, we normalize the initial bank equity to zero. We put the interest costs and interest income of the bank under the entry “Bank Equity”.

The initial deposit and reserve balance is $D_0$, $πD_0$ is held by movers and $(1 − π)D_0$ by non-movers. After the shocks, the investment fund sells assets to non-movers and raises cash $(1 − π)D_0$. The investment fund also borrows $L$ from the bank. After the loan is made, ‘loan’ and “deposit” on the balance sheet increase by the same amount $L$, and the new deposit $L$ is created. The fund then uses all its deposits $L + (1 − π)D_0$ to meet the redemption needs of movers, and movers end up holding all the deposits $D_0 + L$. After the redemption process is
completed, the bank pays the interest cost $r^d b_n$ of the central bank loan.

At the end of the period, the central bank consumes the interest income by paying central bank money to the investment fund. The reserve balance of the bank and the deposit of the investment fund will increase by the same amount. Banks then pay deposit interest to movers. Movers then use their deposits to buy goods. When movers from location $i$ buy goods in location $j$, their deposit balance decreases by $(D_0 + L)(1 + r^d)$. And when movers from location $j$ buy goods from the investment fund in location $i$, the investment fund’s deposit balance increases by $(D_0 + L)(1 + r^d)$.

The investment fund then repays the bank loan. The outstanding loan is reduced by $L$, and the outstanding deposit is reduced by $L(1 + r^d)$, and the interest income of the bank is $L r^d$. The bank then spends the income $[L r^d - r^d b_n - r^d(D_0 + L)]$ to buy goods from the investment fund. Note that the income is usually positive because the lending rate $r^d$ includes the management cost for bank loans. If $\delta$ is not too small, then $L r^d$ should be higher than the interest costs of the bank. In case where the income is negative, then the bank can sell its endowment to absorb the loss. When the bank spends the income, the bank makes the payment by increasing the deposit balance of the investment fund by the same amount. The increase in deposit (bank’s liability) by $[L r^d - r^d b_n - r^d(D_0 + L)]$ reduces bank’s equity by the

<table>
<thead>
<tr>
<th>Asset Side</th>
<th>Reserve (Fund)</th>
<th>Loan (movers)</th>
<th>Deposit (non-movers)</th>
<th>Deposit (Fund)</th>
<th>Bank Equity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Balance</td>
<td>$D_0$</td>
<td>$\pi D_0$</td>
<td>$(1 - \pi) D_0$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Balance before the liquidity shock</td>
<td>$D_0$</td>
<td>$\pi D_0$</td>
<td>$(1 - \pi) D_0$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Changes in each account after the liquidity shock</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Investment fund sells assets to non-movers</td>
<td>+$L$</td>
<td></td>
<td>$-(1 - \pi) D_0$</td>
<td>$(1 - \pi) D_0$</td>
<td></td>
</tr>
<tr>
<td>Bank makes loans to the investment fund</td>
<td></td>
<td></td>
<td></td>
<td>+$L$</td>
<td></td>
</tr>
<tr>
<td>Movers redeem fund shares</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bank pays the borrowing cost</td>
<td>$-r^d b_n$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Changes in each account at the end of the Period</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>The central bank consumes the interest</td>
<td>$+r^d b_n$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bank pays the deposit interest</td>
<td></td>
<td></td>
<td></td>
<td>$+r^d(D_0 + L)$</td>
<td></td>
</tr>
<tr>
<td>Movers($i$) buy goods in location $j$</td>
<td></td>
<td></td>
<td>$-(L + D_0)(1 + r^d)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Movers($j$) buy goods in location $i$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Investment fund repays the bank loan</td>
<td>$-L$</td>
<td></td>
<td>$-L(1 + r^d)$</td>
<td>$+L r^d$</td>
<td></td>
</tr>
<tr>
<td>Bank spends the interest income</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Final Balance</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Balance</td>
<td>$D_0$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$D_0$</td>
</tr>
</tbody>
</table>
same amount.

After all the above steps are completed, the investment fund will then transfer the deposit balance $D_0$ to non-movers, who will then use the deposit to purchase goods from the young generation.

B No bank lending: the general case $\sigma \geq 1$

B.1 The optimal payout policy of the investment fund

We first derive the optimal payout policy. We also prove proposition 1 for the log-utility function when we prove the following results for the general case.

**Proposition 8.** If $Q_k = R_k$, then the optimal policy is to set $v_m = v_n$, and $r_m = r_n = \alpha + (1 - \alpha)R_k$. When $Q_k < R_k$, if the constraint $r_m \leq r_n$ is not binding, then the optimal policy is to set

$$\frac{v_m}{v_n} = \frac{\omega + (1 - \omega)r_m}{\omega R_k + (1 - \omega)r_n} = \left(\frac{Q_k}{R_k}\right)^{\frac{1}{\sigma}}$$

(52)

Given $\omega$ and $Q_k$, $\frac{r_m}{r_n}$ is increasing in $\sigma$. If the constraint $r_m \leq r_n$ is binding, then the optimal policy is $r_m = r_n$. For the log utility function ($\sigma = 1$), we have

$$r_m = \alpha + (1 - \alpha)Q_k$$

$$r_n = \alpha \frac{R_k}{Q_k} + (1 - \alpha)R_k = r_m \frac{R_k}{Q_k}$$

The meaning of the proposition is as follows. First, as long as $Q_k = R_k$, there is no cost to raise cash by selling assets, and it is optimal to fully insure the liquidity risk and give movers and non-movers the same return. Second, if $Q_k < R_k$, since it is costly to raise cash, the investment fund may not provide full insurance. When people are more risk averse (higher $\sigma$), it is optimal to set a higher $\frac{r_m}{r_n}$, which means to give a higher payment to movers. When the constraint $r_m \leq r_n$ is binding, it is optimal to set $r_m = r_n$. For the log utility function, the optimal $r_m$ is simply to pay the market value of the fund’s asset.

**Proof:** We first analyze the case when the fund does not need to sell assets. The budget constraints are

$$\pi r_m = \phi \alpha$$

$$(1 - \pi)r_n = (1 - \phi)\alpha + (1 - \alpha)R_k$$

where $\phi \alpha$ is the riskless asset used to pay movers, $(1 - \phi)\alpha$ is the unused riskless asset and $(1 - \alpha)R_k$ is the value of the risky assets. Using the budget constraints to replace $r_m$ and $r_n$ in $v_m$ and $v_n$ (equation (7) and (8)), the fund’s problem (10) can be written as (we eliminate the common “$s$” from $v_m$ and $v_n$)

$$\frac{\pi (\omega + (1 - \omega)\frac{\phi \alpha}{\pi})^{1 - \sigma}}{1 - \sigma} + (1 - \pi) \frac{\left(\omega + (1 - \omega)\frac{(1 - \phi)\alpha + (1 - \alpha)R_k}{1 - \pi R_k}\right)^{1 - \sigma}}{1 - \sigma}$$
Here, we use \( R_k = 1 \) since there is no liquidation of assets. Taking the derivative with respect to \( \phi \) and simplifying the terms, we get

\[
\frac{1}{(\omega + (1 - \omega)\frac{2\alpha}{\pi})^\sigma} \times \frac{1}{(\omega + (1 - \omega)\frac{(1-\phi)\alpha + (1-\alpha)R_k}{1-\pi})^\sigma} = 0 \tag{53}
\]

that is, \( \frac{1}{v_m} - \frac{1}{v_n} = 0 \), which means \( v_m = v_n \) and \( r_m = r_n = \alpha + (1 - \alpha)R_k \).

Next, suppose the fund needs to sell assets. Let \( \eta \) denote the share of risky assets that is liquidated. The budget constraints are

\[
\pi r_m = \alpha + (1 - \alpha)\eta Q_k \tag{54}
\]

\[
(1 - \pi) r_n = (1 - \alpha)(1 - \eta)R_k \tag{55}
\]

The fund maximizes

\[
\pi \left( \omega + (1 - \omega)\frac{\alpha + (1-\alpha)\eta Q_k}{\pi} \right)^{1-\sigma} + (1 - \pi) \left( \omega \frac{R_k}{Q_k} + (1 - \omega)\frac{(1-\alpha)(1-\eta)R_k}{1-\pi} \right)^{1-\sigma} \tag{56}
\]

Taking the derivative with respect to \( \eta \) and simplifying the terms, we get

\[
\frac{Q_k}{(\omega + (1 - \omega)\frac{\alpha + (1-\alpha)\eta Q_k}{\pi})^\sigma} - \frac{R_k}{(\omega \frac{R_k}{Q_k} + (1 - \omega)\frac{(1-\alpha)(1-\eta)R_k}{1-\pi})^\sigma} = 0
\]

which can be written as

\[
\frac{\omega + (1 - \omega)r_m}{\omega \frac{R_k}{Q_k} + (1 - \omega)r_n} = \left( \frac{Q_k}{R_k} \right)^{\frac{1}{\sigma}} \tag{57}
\]

When \( R_k = Q_k \), we have \( r_m = r_n \).

Now suppose \( \frac{Q_k}{R_k} < 1 \). When \( \sigma > 1 \), given \( \frac{Q_k}{R_k} \), \( \left( \frac{Q_k}{R_k} \right)^{\frac{1}{\sigma}} \) is increasing in \( \sigma \). So higher \( \sigma \) will increase the level of \( r_m \) relative to \( r_n \). That is, people share more liquidity risks when they are more risk averse. When \( \sigma \to \infty \), \( \left( \frac{Q_k}{R_k} \right)^{\frac{1}{\sigma}} \to 1 \), and equation (57) would imply that \( r_m > r_n \). Once the constraint \( r_m \leq r_n \) is binding, the fund sets \( r_m = r_n \).

When \( \sigma = 1 \), (57) becomes

\[
\frac{\omega + (1 - \omega)r_m}{\omega \frac{R_k}{Q_k} + (1 - \omega)r_n} = \frac{Q_k}{R_k}
\]

which gives \( r_n = r_m \frac{R_k}{Q_k} \). Substitute this into (54) and (55) and we get \( \eta = \pi - \frac{\alpha(1-\pi)}{(1-\alpha)Q_k} \), which gives \( r_m = \alpha + (1 - \alpha)Q_k \) and \( r_n = \alpha \frac{R_k}{Q_k} + (1 - \alpha)R_k \). ■

### B.2 The values of \( r_m \), \( r_n \) and \( Q_k \) in the symmetric equilibrium

In this part, we take the initial portfolio choice \( \alpha \) and \( \omega \) as given and solve for \( r_m \), \( r_n \) and \( Q_k \) in the symmetric equilibrium.
We can separate $\pi$ into three ranges: $[0, \pi_1]$, $[\pi_1, \pi_{bind}]$ and $[\pi_{bind}, \pi]$. Non-movers’ cash is binding for $\pi \geq \pi_1$. And for $\pi > \pi_{bind}$, the constraint $r_m \leq r_n$ is binding.

Below $\pi_1$, we have $Q_k = R_k$ and $r_m = r_n = \alpha + (1 - \alpha) R_k$. At $\pi_1$, the payment to movers is equal to the cash collected from non-movers plus the cash held by the fund, and we have

$$\pi_1 Z_p r_m = Z_f + (1 - \pi_1) D^h$$

$$\Rightarrow \pi_1 = \frac{D^h + Z_f}{D^h + Z_p r_m} = \frac{\omega + (1 - \omega) \alpha}{\omega + (1 - \omega)(\alpha + (1 - \alpha) R_k)} = \frac{\kappa}{\kappa + (1 - \kappa) R_k}$$

(58)

(59)

So $\pi_1$ only depends on $\kappa$.

For $\pi > \pi_1$, in the symmetric equilibrium, we have

$$\pi Z_p r_m = Z_f + (1 - \pi) D^h \Rightarrow r_m = \frac{Z_f + (1 - \pi) D^h}{\pi Z_p} = \frac{(1 - \omega) \alpha + (1 - \pi) \omega}{\pi (1 - \omega)}$$

(60)

Having solved $r_m$, we can use (54), (55) and (57) to solve for equilibrium $Q_k$ and $r_n$.

Between $[\pi_1, \pi_{bind}]$, the constraint $r_m \leq r_n$ is not binding. Using (54) and (55), we can write $r_n$ as a function of $r_m$ and $Q_k$

$$r_n = \frac{(1 - \alpha) R_k}{1 - \pi} - \frac{R_k (\pi r_m - \alpha)}{Q_k (1 - \pi)} = \frac{(1 - \alpha) R_k}{1 - \pi} - \frac{R_k \omega}{Q_k (1 - \omega)}$$

(61)

Then substitute $r_m$ (60) and $r_n$ (61) into (57), we have

$$\frac{\omega}{\pi} = \left( \frac{Q_k}{R_k} \right)^{\frac{1}{\sigma}}$$

(62)

Arranging terms, we get

$$\frac{\omega + (1 - \omega) \alpha}{\pi} = \left( \frac{Q_k}{R_k} \right)^{\frac{1}{\sigma}} (1 - \omega) (1 - \alpha) R_k \Rightarrow Q_k = R_k^{1 - \sigma} \left( \frac{\kappa (1 - \pi)}{\pi (1 - \kappa)} \right)^{\sigma}$$

(63)

Substitute $Q_k$ back to (61) and we get the solution for $r_n$

$$r_n = \frac{(1 - \alpha) R_k}{1 - \pi} - \left( \frac{R_k \pi (1 - \kappa)}{\kappa (1 - \pi)} \right)^{\sigma} \frac{\omega}{1 - \omega}$$

(64)

Also, from (63), we have

$$\frac{Q_k}{R_k} = \left( \frac{\kappa (1 - \pi)}{R_k \pi (1 - \kappa)} \right)^{\sigma}$$

(65)

This ratio is equal to 1 at $\pi_1$. For $\pi > \pi_1$, the RHS is lower than 1. So given $\kappa$, $R_k$ and $\pi$, $Q_k$ will be lower for higher $\sigma$.

At $\pi_{bind}$, we have $r_m = r_n$. Using (60) and (64), we have

$$r_m = r_n \Rightarrow (1 - \omega) \alpha + (1 - \pi) \omega = \frac{(1 - \alpha) R_k}{1 - \pi} - \left( \frac{R_k \pi (1 - \kappa)}{\kappa (1 - \pi)} \right)^{\sigma} \frac{\omega}{1 - \omega}$$

(66)
This equation implicitly defines $\pi_{\text{bind}}$.

Above $\pi_{\text{bind}}$, $r_m$ is still (60), and we have

$$r_m = r_n = \frac{(1 - \omega)\alpha + (1 - \pi)\omega}{\pi(1 - \omega)}$$  \hspace{1cm} (67)

And using the budget constraints (54) and (55), we get

$$Q_k = \frac{(1 - \pi)\omega}{(1 - \omega)(1 - \alpha) - \frac{(1 - \pi)((1 - \omega)\alpha + (1 - \pi)\omega)}{\pi R_k}}$$  \hspace{1cm} (68)

So the distribution for $Q_k$ is

$$Q_k(\pi) = \begin{cases} 
R_k & : \pi \leq \pi_1 = \frac{\kappa}{\kappa + (1 - \kappa)R_k} \\
R_k^{1 - \sigma} \left(\frac{\kappa(1 - \pi)}{\pi(1 - \kappa)}\right)^{\sigma} & : \pi_1 < \pi \leq \pi_{\text{bind}} \\
equation 68 & : \pi_1 > \pi_{\text{bind}}
\end{cases}$$  \hspace{1cm} (69)

We can see that if the constraint $r_m \leq r_n$ is not binding, then the distribution of $Q_k$ only depends on $\kappa$. This can be seen from (68) where $Q_k$ for $\pi < \pi_{\text{bind}}$ only depends on $\kappa$ (remember that $\pi_1(59)$ only depends on $\kappa$).

For the log utility function, $r_n = \frac{R_k}{Q_k} r_m$, so the constraint $r_m \leq r_n$ is never binding. As a result, under the log utility function, the distribution of $Q_k$ only depends on $\kappa$.

**B.3 The first order conditions for $\omega$ and $\alpha$**

This part derives the first order conditions for the representative household and the investment fund. When deciding the optimal choice, the representative household and the investment fund will take the choices of other agents and the distribution of $Q_k$ as given.

For notational convenience, we set $s = 1$, so $v_m = \omega + (1 - \omega) r_m$ and $v_n = \omega \frac{R_k}{Q_k} + (1 - \omega) r_n$. The expected utility is

$$EU = \frac{1}{2} \int_0^1 [\pi \frac{(v_{m,H})^{1 - \sigma}}{1 - \sigma} + (1 - \pi) \frac{(v_{n,H})^{1 - \sigma}}{1 - \sigma} + \pi \frac{(v_{m,L})^{1 - \sigma}}{1 - \sigma} + (1 - \pi) \frac{(v_{n,L})^{1 - \sigma}}{1 - \sigma}] dF(\pi)$$  \hspace{1cm} (70)

And the first order condition for $\omega$ is

$$\frac{\partial EU}{\partial \omega} = \frac{1}{2} \int_0^1 \frac{\pi (1 - r_{m,H})}{(\omega + (1 - \omega)r_{m,H})^{\sigma}} + \frac{(1 - \pi)(\frac{R_k}{Q_k} - r_{n,H})}{(\omega \frac{R_k}{Q_k} + (1 - \omega)r_{n,H})^{\sigma}} dF(\pi)$$

$$+ \frac{1}{2} \int_0^1 \frac{\pi (1 - r_{m,L})}{(\omega + (1 - \omega)r_{m,L})^{\sigma}} + \frac{(1 - \pi)(\frac{R_k}{Q_k} - r_{n,L})}{(\omega \frac{R_k}{Q_k} + (1 - \omega)r_{n,L})^{\sigma}} dF(\pi)$$  \hspace{1cm} (71)

The first order condition for $\alpha$ is

$$\frac{\partial EU}{\partial \alpha} = \frac{(1 - \omega)}{2} \int_0^1 \frac{\partial r_{m,H}}{\partial \alpha} \frac{v_m}{v_{m,H}} + (1 - \pi) \frac{\partial r_{n,H}}{\partial \alpha} \frac{v_n}{v_{n,H}} + \pi \frac{\partial r_{m,L}}{\partial \alpha} \frac{v_m}{v_{m,L}} + (1 - \pi) \frac{\partial r_{n,L}}{\partial \alpha} \frac{v_n}{v_{n,L}} dF(\pi)$$  \hspace{1cm} (72)
We still need to decide $\frac{\partial r_m}{\partial \alpha}$ and $\frac{\partial r_n}{\partial \alpha}$. When $\pi \leq \pi_1$, since $r_m = r_n = \alpha + (1 - \alpha)R_k$, we have

$$\frac{\partial r_m}{\partial \alpha} = \frac{\partial r_n}{\partial \alpha} = 1 - R_k$$  \hspace{1cm} (73)

For $\pi > \pi_1$, we first need to solve for $r_m$ and $r_n$ by taken $Q_k$ as given.

First, for $\pi \in [\pi_1, \pi_{bind}]$, using (54) and (55), we can write $r_m$ as $\frac{\alpha + (1 - \alpha)Q_k}{\pi}$ and $r_n$ as $\frac{(1 - \alpha)(1 - \eta)R_k}{1 - \pi}$. Substituting them into (57) and arranging terms, we get

$$\eta = \frac{-\omega - (1 - \omega)\frac{\alpha}{\pi} + (\frac{Q_k}{R_k})^{\frac{1}{\alpha}} \left[ \omega \frac{R_k}{Q_k} + \frac{(1 - \omega)(1 - \alpha)R_k}{1 - \pi} \right]}{(1 - \omega)(1 - \alpha)Q_k + (\frac{Q_k}{R_k})^{\frac{1}{\alpha}} \left[ 1 - \omega \right] (1 - \alpha)R_k}$$  \hspace{1cm} (74)

Substitute $\eta$ into (54) and (55) and we get

$$r_m = \frac{1}{\pi}(\alpha + Q_k(1 - \alpha)\eta) = \frac{1}{\pi} \left( \alpha + Q_k \left[ -\omega - (1 - \omega)\frac{\alpha}{\pi} + (\frac{Q_k}{R_k})^{\frac{1}{\alpha}} \left[ \omega \frac{R_k}{Q_k} + \frac{(1 - \omega)(1 - \alpha)R_k}{1 - \pi} \right] \right] \right)$$  \hspace{1cm} (75)

$$r_n = \frac{R_k(1 - \alpha)(1 - \eta)}{1 - \pi} = \frac{R_k}{1 - \pi} \left( 1 - \alpha \right) - \frac{-\omega - (1 - \omega)\frac{\alpha}{\pi} + (\frac{Q_k}{R_k})^{\frac{1}{\alpha}} \left[ \omega \frac{R_k}{Q_k} + \frac{(1 - \omega)(1 - \alpha)R_k}{1 - \pi} \right]}{(1 - \omega)(1 - \alpha)Q_k + (\frac{Q_k}{R_k})^{\frac{1}{\alpha}} \left[ 1 - \omega \right] (1 - \alpha)R_k}$$  \hspace{1cm} (76)

And so

$$\frac{\partial r_m}{\partial \alpha} = \frac{1}{\pi} \left( 1 + Q_k \left[ -(1 - \pi) - \pi(\frac{Q_k}{R_k})^{\frac{1}{\alpha}} \frac{1}{\pi} R_k \right] \right)$$  \hspace{1cm} (77)

$$\frac{\partial r_n}{\partial \alpha} = \frac{R_k}{1 - \pi} \left[ -1 - \left( -(1 - \pi) - \pi(\frac{Q_k}{R_k})^{\frac{1}{\alpha}} \frac{1}{\pi} R_k \right) \right]$$  \hspace{1cm} (78)

In the symmetric equilibrium, $(\frac{Q_k}{R_k})^{\frac{1}{\alpha}} = \frac{\kappa(1 - \pi)}{\pi \kappa (1 - \pi)}$ (equation 65). Rearranging terms, we get

$$\frac{\partial r_m}{\partial \alpha} = \frac{1}{\pi} \frac{\kappa(1 - Q_k)}{\kappa + (1 - \kappa)Q_k}$$

$$\frac{\partial r_n}{\partial \alpha} = \frac{R_k}{1 - \pi} \frac{1 - \kappa(1 - Q_k)}{\kappa + (1 - \kappa)Q_k}$$

For $\pi > \pi_{bind}$, since $r_m = r_n$, using (54) and (55) and taking $Q_k$ as given, we get

$$r_m = r_n = \frac{R_k(\alpha + (1 - \alpha)Q_k)}{(1 - \pi)Q_k + \pi R_k}$$  \hspace{1cm} (79)

and we have

$$\frac{\partial r_m}{\partial \alpha} = \frac{\partial r_n}{\partial \alpha} = \frac{R_k(1 - Q_k)}{(1 - \pi)Q_k + \pi R_k}$$  \hspace{1cm} (80)
B.4 The equilibrium when \( r_m \leq r_n \) is not binding

This part considers the features of the equilibriums when the constraint \( r_m \leq r_n \) is not binding. We have the following result: First, the response curves of the household and the investment fund overlap with each other. Second, the equilibrium is defined by \( \kappa \). As long as \( \kappa \) is equal to the equilibrium \( \kappa \), then people can choose different combinations of \([\omega, \alpha]\).

The response curves
First, we explain why the response curves overlap with each other when the constraint \( r_m \leq r_n \) is not binding. The response curves are simply the first order conditions of \( \omega \) and \( \alpha \) (equation 70 and 72). Denote the response curve of the household and the investment fund as \( R_{\text{household}}(\alpha) \) and \( R_{\text{fund}}(\omega) \). Let \( \omega(\alpha) \) denote the optimal choice of the household by taken \( \alpha \) as given and \( \alpha(\omega) \) the optimal choice of the investment fund by taking \( \omega \) as given. Then on the response curves, we have \( \alpha(\omega(\alpha_0)) = \alpha_0 \) and \( \omega(\alpha(\omega_0)) = \omega_0 \). The reason is that given the distribution of \( Q_k \), the portfolio of movers and non-movers can be written as functions of \( \kappa \). So when the investment fund chooses the best \( \alpha \) given \( \omega \), or when the household chooses \( \omega \) given \( \alpha \), they essentially choose the best \( \kappa \).

The portfolio of movers is \( v_m = \omega + (1 - \omega)r_m \) and the portfolio of non-movers is \( v_n = \omega \frac{R_k}{Q_k} + (1 - \omega)r_m \). For \( \pi \leq \pi_1 \), \( r_m = r_n = \alpha + (1 - \alpha)R_k \), and so
\[
v_m = v_n = \omega + (1 - \omega)(\alpha + (1 - \alpha)R_k) = \kappa + (1 - \kappa)R_k \quad (81)
\]
For \( \pi > \pi_1 \), when \( Q_k \) is given, the solutions for \( r_m \) and \( r_n \) are (75) and (76). After some arrangement of equations, we get
\[
v_m = \omega + (1 - \omega)r_m = \left(\frac{Q_k}{R_k}\right)^{\frac{1}{2}} \frac{R_k}{Q_k} (\kappa + (1 - \kappa)Q_k) \quad (82)
\]
\[
v_n = \omega \frac{R_k}{Q_k} + (1 - \omega)r_m = \left(\frac{Q_k}{R_k}\right)^{\frac{1}{2}} \frac{R_k}{Q_k} (\kappa + (1 - \kappa)Q_k) \quad (83)
\]
So given \( Q_k \), \( v_m \) and \( v_n \) can be written as functions of \( \kappa \).

The equilibrium \( \kappa \)
Given the equilibrium level of \( \kappa \), different combinations of \([\omega, \alpha]\) which give the same \( \kappa \) will also be the equilibrium.

We’ve already shown that in the symmetric equilibrium, when the constraint \( r_m \leq r_n \) is not binding, \( \pi_1 \) and \( Q_k \) only depend on \( \kappa \). From the previous analysis, we know that households and investment funds try to maximize \( EU \) given \( v_m \) and \( v_n \) specified in (82) and (83). We can see that \( v_m \) and \( v_n \) only depend on \( \pi_1 \) and \( Q_k \). In the equilibrium, given \( Q_k \), households and investment funds would find that the equilibrium \( \kappa \) is optimal. And so if we keep the same \( \kappa \) but change the combination of \( \omega \) and \( \alpha \), then since \( Q_k \) does not change, households and investment funds will still choose the same \( \kappa \) because they still face the same portfolio choice problem.
C With bank lending: the general case $\sigma \geq 1$

C.1 The optimal payout policy when bank loan is allowed

The optimal policy of the investment fund is as follows:

**Proposition 9.** If $Q_k = R_k$, then it is optimal to set $v_m = v_n$ and $r_m = r_n = \alpha + (1 - \alpha)R_k$. When $Q_k < R_k$, if the constraint $r_m \leq r_n$ is not binding, then the optimal policy is to set

$$\frac{v_m}{v_n} = \frac{(\omega + (1 - \omega)r_m)(1 + r^d)}{\omega \frac{R_k}{Q_k} + (1 - \omega)r_n} = \left(\frac{Q_k(1 + r^d)}{R_k}\right)^{\frac{1}{\sigma}} \quad (84)$$

Given $\omega$, $Q_k$ and $r^d$, $\frac{r_m}{r_n}$ is increasing in $\sigma$. If the constraint $r_m \leq r_n$ is binding, then the optimal policy is $r_m = r_n$. For the log utility function ($\sigma = 1$), the optimal policy is still $r_m = \alpha + (1 - \alpha)Q_k$ and $r_n = \alpha \frac{R_k}{Q_k} + (1 - \alpha)R_k$.

**Proof:** When $Q_k = R_k$, there is no bank borrowing and $r^d = 0$. The problem is the same as in Proposition 1.

Let $\eta_1$ denote the share of assets sold on the financial market and let $\eta_2$ denote the share of assets used as collateral to borrow from banks. Define $\eta = \eta_1 + \eta_2$. When $Q_k < R_k$, the budget constraint is

$$\pi r_m = \alpha + (1 - \alpha)\eta_1 Q_k + (1 - \alpha)\eta_2 Q_k = \alpha + (1 - \alpha)\eta Q_k \quad (85)$$

$$\quad (1 - \pi) r_n = (1 - \alpha)(1 - \eta)R_k \quad (86)$$

Set $s = 1$. We have

$$v_m = [\omega + (1 - \omega)r_m](1 + r^d) \quad (87)$$

$$v_n = \omega \frac{R_k}{Q_k} + (1 - \omega)r_n \quad (88)$$

and the fund’s problem (10) becomes

$$\pi \left[ \frac{(\omega + (1 - \omega)\frac{\alpha + (1 - \alpha)\eta Q_k}{\pi})(1 + r^d)}{1 - \sigma} \right]^{1 - \sigma} + (1 - \pi) \frac{(\omega \frac{R_k}{Q_k} + (1 - \omega)\frac{(1 - \alpha)(1 - \eta)R_k}{1 - \pi})^{1 - \sigma}}{1 - \sigma}$$

Taking the derivative with respect to $\eta$ and simplifying the terms, we get

$$\frac{Q_k(1 + r^d)}{[(\omega + (1 - \omega)\frac{\alpha + (1 - \alpha)Q_k}{\pi})(1 + r^d)]^{\sigma}} - \frac{R_k}{(\omega \frac{R_k}{Q_k} + (1 - \omega)\frac{(1 - \alpha)(1 - \eta)R_k}{1 - \pi})^{\sigma}} = 0$$

which can be written as (84). We can also write it as

$$\frac{\omega + (1 - \omega)r_m}{\omega \frac{R_k}{Q_k} + (1 - \omega)r_n} = \left(\frac{Q_k}{R_k}\right)^{\frac{1}{\sigma}} (1 + r^d)^{\frac{1}{\sigma} - 1} \quad (89)$$

When $\frac{Q_k}{R_k} < 1$, if $r^d = 0$, it is clear that the RHS of (89) is increasing in $\sigma$. $r^d$ is positive only when investment funds borrow positive loans from banks. In this case, $\frac{Q_k}{R_k} = \frac{1}{1 + r^d}$, and the RHS of (89) can be written as $\frac{Q_k}{R_k} \left(\frac{1 + r^d}{1 + r^d}\right)^{\frac{1}{\sigma}}$, which is increasing in $\sigma$ since $r^d > r^d$.

When $\sigma = 1$, it is easy to see that the result is the same as in Proposition 1. ■
C.2 The bank loan supply curve

We first prove Proposition 3.

**Proof:** In Table 2, in each column $n$, the maximum accumulated payment is $(\frac{(N-1)X_i}{N} - \frac{(n-1)X_j}{N})$, which happens in period $k = n$(the diagonal of the matrix) when banks are chosen to make the payment. And for $k > n$, the accumulated payment is $(\frac{(N-1)X_i}{N} - \frac{(k-1)X_i}{N})$. If $N$ is very large, then $(\frac{(N-1)X_i}{N} - \frac{(k-1)X_i}{N}) \approx X_i$. We set $1 - \frac{n-1}{N}$ as $\lambda_{max}$ and $1 - \frac{k-1}{N}$ as $\lambda$, then for $k \geq n$, we can write

$$FL_{max} = X_i + (\lambda_{max} - 1)X_j$$

$$FL(k) = X_i + (\lambda - 1)X_j$$

And the central bank loan is

$$b(k) = \max(FL(k) - D_0, 0)$$

Note that in Table 2, in each column, the accumulated flow $FL(k)$ for $k \geq n$ is the same as the $FL(k)$ in the previous column. Let $\lambda$ denote the level of $\lambda$ at which $b(k) = 0$. Using (91) and (92), we get

$$\lambda = \frac{D_0 + X_j - X_i}{X_j} \quad \lambda \in [0, 1]$$

$b(k) > 0$ if $\lambda > \lambda$.

When $N$ is large, we can take $\lambda$ as continuous, and the expected loan can be written as

$$Eb(L_i) = \int_{\lambda}^{1} \int_{\lambda}^{\lambda_{max}} b(k) d\lambda d\lambda_{max} = \int_{\lambda}^{1} \int_{\lambda}^{\lambda_{max}} ([X_i + (\lambda - 1)X_j] - D_0) d\lambda d\lambda_{max}$$

The integral of $b(k)$ over $[\lambda, \lambda_{max}]$ is the borrowing for each realized $n$(i.e., each column of the matrix). The integral over $[\lambda, 1]$ denotes the changes in $\lambda_{max}$ caused by the changes in $n$(i.e., different columns of the matrix). $b(k)$ is positive only when $\lambda$ and $\lambda_{max}$ are $> \lambda$.

$$Eb(L_i) = \int_{\lambda}^{1} \int_{\lambda}^{\lambda_{max}} [X_i - X_j - D_0 + \lambda X_j] d\lambda d\lambda_{max}$$

$$= \int_{\lambda}^{1} \left[ (\lambda_{max} - \lambda)(X_i - X_j - D_0) + \frac{\lambda_{max}^2 - \lambda^2}{2} X_j \right] d\lambda_{max}$$

$$= \left( \frac{\lambda_{max}^2}{2} - \lambda_{max} |\lambda| \lambda \right) (X_i - X_j - D_0) + \frac{1}{2} \left( \frac{\lambda_{max}^3}{3} - \lambda_{max} |\lambda| \lambda^2 \right) X_j$$

$$= \left( \frac{1 - \lambda^2}{2} - (1 - \lambda) \lambda \right) (X_i - X_j - D_0) + \frac{1}{2} \left( \frac{1 - \lambda^3}{3} - (1 - \lambda) \lambda^2 \right) X_j$$

Replacing $\lambda$ with (93) and arranging terms, we get

$$Eb(L_i) = \frac{1}{6} \frac{(X_i - X_j - D_0)^3}{X_j^2} + \frac{1}{2} \frac{(X_i - X_j - D_0)^2}{X_j} + \frac{1}{2} (X_i - X_j - D_0) + \frac{1}{6} X_j$$
In the general case, we have

\[ X_i = Z_f + (1 - \pi)D^h + L_i \]  
\[ X_j = Z_f + (1 - \pi)D^h + L_j \]  

where \( Z_f \) is the riskless asset of the investment fund, \((1 - \pi)D^h\) is the money collected from non-movers. The method is the same and it can be shown that in the symmetric case we still have

\[ R = 1 + \delta + \frac{r^c}{2} \left( 1 - \frac{D_0}{X} \right)^2 \]  
\[ r^d = \frac{Lc^c(1 - \frac{D_0}{X})^2 - r^c E_b}{D_0 + L} = \frac{Lc^c(1 - \frac{D_0}{X})^2 - r^c(\frac{-D_0^2}{6X^2} + \frac{D_0^2}{2X} - \frac{D_0}{2} + \frac{X}{6})}{D_0 + L} \]

where \( D_0 \) is \( D^h + Z_f \) and \( X = Z_f + (1 - \pi)D^h + L \).

C.2.1 The Equilibrium solutions for \( r_m \), \( r_n \), \( Q_k \), \( L \), \( R \) and \( r^d \).

This part derives the equilibrium solutions in period \( t + 1 \) by taken \( \omega \) and \( \alpha \) as given. We first consider the case in which the constraint \( r_m \leq r_n \) is not binding.

Recall that at \( \pi_2 \), investment funds start to borrow from banks. And at \( \pi_3 \), banks start to borrow from the central bank. Everything for \( \pi < \pi_2 \) is the same as in the non-bank lending case. In order to get the solution for \( \pi \geq \pi_2 \), we first decide \( \pi_2 \) and \( \pi_3 \).

**Derive \( \pi_2 \) and \( \pi_3 \)**

\( \pi_2 \) can be decided as follows. At \( \pi_2 \), \( R = 1 + \delta \), \( Q_k = \frac{R}{1 + \delta} \) and \( L = 0 \). At the same time, \( \frac{Q_k}{R_k} \) should satisfy (65), and so we have

\[ \frac{Q_k}{R_k} = \frac{1}{1 + \delta} = \frac{\kappa(1 - \pi)}{R_k \pi_2(1 - \kappa)} \]  
\[ \Rightarrow \pi_2 = \frac{\kappa}{R_k(1 - \kappa)\left(\frac{1}{1 + \delta}\right)^\frac{1}{\sigma} + \kappa} \]  

\( \pi_3 \) can be decided as follows. At \( \pi_3 \), \( R = 1 + \delta \), \( r^d = 0 \) and \( Q_k = \frac{R_k}{1 + \delta} \). At \( \pi_3 \), \( X = D_0 \). Since \( X = Z_f + (1 - \pi)D^h + L \) and \( D_0 = Z_f + D^h \), so \( L = \pi D^h \). Thus, we have

\[ \pi Z_f r_m = Z_f + (1 - \pi)D^h + \pi D^h \]  
\[ \Rightarrow \pi r_m = \alpha + (1 - \pi)\frac{\omega}{1 - \omega} + \pi \frac{\omega}{1 - \omega} = \alpha + \frac{\omega}{1 - \omega} \]  
\[ \Rightarrow r_m = \frac{1}{\pi}(\alpha + \frac{\omega}{1 - \omega}) \]

Also, comparing (85) and (100), we have

\[ (1 - \alpha)\eta Q_k = \frac{\omega}{1 - \omega} \Rightarrow \eta = \frac{\omega}{(1 - \omega)(1 - \alpha)Q_k} \]
Substituting $\eta$ into (86) and we have
\[
r_n = \frac{1}{1 - \pi} \left( (1 - \alpha) - \frac{\omega}{(1 - \omega)Q_k} \right) R_k \tag{103}\]

Then substitute (101) and (103) into the optimal payout policy (89) and we have
\[
\frac{\omega + (1 - \omega) \frac{1}{\pi} (\alpha + \frac{\omega}{1 - \omega})}{\omega(1 + \delta) + (1 - \omega) \frac{1}{1 - \pi} \left( (1 - \alpha) - \frac{\omega}{(1 - \omega)Q_k} \right) R_k} = \left( \frac{1}{1 + \delta} \right)^{\frac{1}{2}} \tag{104}\]

Arranging terms, we get
\[
\omega \left[ (1 + \delta)^{1-\frac{1}{\pi}} - 1 \right] \pi^2 - \pi \left[ \kappa - \omega + (1 + \delta)^{-\frac{1}{\pi}} (1 - \kappa) R_k \right] + \kappa = 0 \tag{105}\]

When $\sigma = 1$, the solution is $\frac{\kappa}{\kappa - \omega + (1 - \kappa) \frac{R_k}{1 + \delta}}$. When $\sigma > 1$, the smaller one of the two solutions is $\pi_3$. Note that given $\kappa$, $\pi_3$ is affected by $\omega$. For example, if $\omega = 0$(all riskless assets are held by the investment fund), then $\pi_3 = \pi_2$.

The distribution for $Q_k$ takes the following form:
\[
Q_k(\pi) = \begin{cases} 
R_k & : \pi_1 \leq \pi \leq \pi_1 \frac{\kappa}{\kappa + (1 - \kappa) R_k} \\
R_k^{1 - \sigma} \left( \frac{\kappa (1 - \pi)}{\pi (1 - \pi)} \right)^{\sigma} & : \pi_1 < \pi < \pi_2 = \frac{\kappa}{R_k (1 - \kappa) (1 + \delta)^{1 - \frac{1}{\pi}} + \kappa} \\
\frac{R_k^{1 - \delta}}{R_k(\pi)} & : \pi_2 \leq \pi \leq \pi_3 \\
\frac{R_k}{R_k(\pi)} & : \pi > \pi_3 
\end{cases} \tag{106}\]

**Equilibrium solutions over $\pi_2$ and $\pi_3$**

Over $[\pi_2, \pi_3]$, $R = 1 + \delta$, $r^d = 0$ and $Q_k = \frac{R_k}{1 + \delta}$. We still need to decide $r_m$, $r_n$ and $L$. Since $r^d = 0$, (89) is the same as (57), and the solution for $\eta$, $r_m$ and $r_n$ are simply (74), (75) and (76) with $Q_k = \frac{R_k}{1 + \delta}$. Knowing $\eta$, we can decide $L$ from the budget constraint (85). Since $L$ is equal to the total external cash minus the cash from non-movers, so
\[
L = Z_p(1 - \alpha)\eta Q_k - (1 - \pi)D^h = S[(1 - \omega)(1 - \alpha)\eta Q_k - (1 - \pi)\omega] \tag{107}\]

**Equilibrium solutions for $\pi > \pi_3$**

We will set $L$ as the variable that we try to solve, and we express all other variables as a function of $L$. The equilibrium is defined by the following conditions. 1. The budget constraints (85) and (86); 2. The optimal payout policy (89); 3. Asset Price on the financial market: $Q_k = \frac{R_k}{R}$; 4. The cash paid to movers is equal to the fund’s own money plus the money raised from the financial market and the bank.
\[
\pi Z_p r_m = Z_f + (1 - \pi)D^h + L \tag{108}\]

5. The loan supply curve (97) which defines the relationship between $L$ and $R$; 6. $r^d$(equation 98) derived from the zero expected profit condition. We can write (97) as $R(L)$ and (98) as $r^d(L)$. Then $Q_k(L) = \frac{R_k}{R(L)}$. We can also write (108) as
\[
r_m = \frac{1}{\pi Z_p} (Z_f + (1 - \pi)D^h + L) = \frac{1}{\pi} \left( \alpha + (1 - \pi) \frac{\omega}{1 - \omega} + \frac{L}{S(1 - \omega)} \right) \tag{109}\]
which we define as \( r_m(L) \). Then using the two budget constraints (85) and (86), we have

\[
r_n = \frac{R_k(1 - \alpha)}{1 - \pi} - \frac{R(L)(\pi r_m(L) - \alpha)}{1 - \pi}
\]

which we define as \( r_n(L) \). Substitute \( r_m(L) \), \( r_n(L) \), \( Q_k(L) \), \( R(L) \), and \( r^d(L) \) into the optimal payout policy (89), and we can get an equation in which the only unknown is \( L \):

\[
\frac{\omega + (1 - \omega)r_m(L)}{\omega R(L) + (1 - \omega)r_n(L)} = \left( \frac{1}{R(L)} \right)^{\frac{1}{\delta}} (1 + r^d(L))^{\frac{1}{\delta} - 1}
\]

where \( R(L) \), \( r^d(L) \), \( r_m(L) \) and \( r_n(L) \) are (97), (98), (109), and (110). This equation implicitly defines the equilibrium \( L \). After deciding \( L \), all other variables can then be decided.

**When \( r_m \leq r_n \) is binding**

Let \( \pi_{bind} \) denote the \( \pi \) above which the constraint \( r_m \leq r_n \) is binding. We first consider the case when \( \pi_1 < \pi_{bind} < \pi_2 \). At \( \pi_2 \), \( r_m \) is still (60), and \( r_n \) is (79) with \( Q_k = \frac{R_k}{1 + \sigma} \), equating \( r_m \) and \( r_n \) gives the value of \( \pi_2 \). At \( \pi_3 \), \( Q_k = \frac{R_k}{1 + \sigma} \), \( r_m \) is (101) and \( r_n \) is (103). Equating \( r_m \) and \( r_n \) gives \( \pi_3 \).

For equilibrium values of variables. For \( \pi \leq \pi_{bind} \), everything is the same as in the non-binding case. For \( [\pi_{bind}, \pi_2] \), we use (67) and (68). Over \( [\pi_2, \pi_3] \), \( r_m \) and \( r_n \) are (79) with \( Q = \frac{R_k}{1 + \sigma} \). For \( \pi > \pi_3 \), we can solve the equilibrium using the same method as in the non-binding case, the only difference is that instead of using condition (111), we use the condition \( r_m(L) = r_n(L) \).

If \( \pi_{bind} \in [\pi_2, \pi_3] \) or \( \pi_{bind} > \pi_3 \), then we can decide \( \pi_{bind} \) using simulation methods. The method for deciding the equilibrium values is the same as explained above.

**C.2.2 The first order conditions for \( \omega \) and \( \alpha \)**

This part derives the first order conditions for representative household and investment fund.

Using \( EU(70), v_m(87) \) and \( v_n(88) \), we get

\[
\frac{\partial EU}{\partial \omega} = \frac{1}{2} \int_0^1 \frac{\pi(1 - r_m(H))(1 + r^d_H)}{[(\omega + (1 - \omega)r_m,H)(1 + r^d_H)]^\sigma} + \frac{(1 - \pi)(R_k,H - r_n,H)}{(\omega Q_k,H + (1 - \omega)r_n,H)^\sigma} dF(\pi) \\
+ \frac{1}{2} \int_0^1 \frac{\pi(1 - r_m,L)(1 + r^d_L)}{[(\omega + (1 - \omega)r_m,L)(1 + r^d_L)]^\sigma} + \frac{(1 - \pi)(R_k,L - r_n,L)}{(\omega Q_k,L + (1 - \omega)r_n,L)^\sigma} dF(\pi)
\]

The first order condition for \( \alpha \) is

\[
\frac{\partial EU}{\partial \alpha} = \frac{1 - \omega}{2} \int_0^1 \left( \pi \frac{\partial r_m,H}{\partial \alpha} (1 + r^d_H) + (1 - \pi) \frac{\partial r_n,H}{\partial \alpha} v^\sigma \right) dF(\pi) \\
+ \pi \frac{\partial r_m,L}{\partial \alpha} (1 + r^d_L) + (1 - \pi) \frac{\partial r_n,L}{\partial \alpha} v^\sigma
\]
We need to decide \( \frac{\partial r_m}{\partial \alpha} \) and \( \frac{\partial r_n}{\partial \alpha} \). When \( \pi \leq \pi_1 \), since \( r_m = r_n = \alpha + (1 - \alpha) R_k \), we get \( \frac{\partial r_m}{\partial \alpha} = \frac{\partial r_n}{\partial \alpha} = 1 - R_k \). For \( \pi > \pi_1 \), we first need to solve for \( r_m \) and \( r_n \) by taken \( Q_k \) and \( r^d \) as given. Note that equations (85) and (86) are the same as (54) and (55), and the only difference between (57) and (89) is that the RHS is changed from \( (Q_k R_k)^{\frac{1}{\sigma}} \) into \( (Q_k R_k)^{\frac{1}{\sigma}} (1 + r^d)^{\frac{1}{\sigma} - 1} \). It turns out that we only need to modify the solutions of \( \eta, \ r_m, r_n \) in the no-lending case (74, 75 and 76) by changing \( (Q_k R_k)^{\frac{1}{\sigma}} \) into \( (Q_k R_k)^{\frac{1}{\sigma}} (1 + r^d)^{\frac{1}{\sigma} - 1} \). And so \( \frac{\partial r_m}{\partial \alpha} \) and \( \frac{\partial r_n}{\partial \alpha} \) are equations (77) and (78) with the term \( (Q_k R_k)^{\frac{1}{\sigma}} \) replaced by \( (Q_k R_k)^{\frac{1}{\sigma}} (1 + r^d)^{\frac{1}{\sigma} - 1} \).

If the constraint \( r_m \leq r_n \) is binding, then for \( \pi > \pi_{bind} \), \( \frac{\partial r_m}{\partial \alpha} \) and \( \frac{\partial r_n}{\partial \alpha} \) are the same as (80).

The response curves

We can still show that as long as \( r_m \leq r_n \) is not binding, then the response curve of the household and the investment fund will overlap with each other. The proof is omitted because it is essentially the same as in the no-bank-lending case. But in this case, different combinations of \( [\omega, \alpha] \) will given different values of \( \kappa \). This is because increasing the level of \( \alpha \) will increase the monetary payment from investment funds to movers during the redemption process, so banks will be more likely to borrow from the central bank. Thus, equilibriums with high \( \omega \) and low \( \alpha \) will be more efficient.

D Steps for computing the numerical example

What follows are the steps for computing the equilibrium \( \sigma = 1 \) when there are bank lending. The method is similar for other cases.

1. Start with an initial value of \( \omega \), the aggregate risky investment is \( (1 - \omega)e_h \). Given \( A_H \) and \( A_L \), the asset returns are \( R_{k,H} = A_H \) and \( R_{k,L} = A_L \).

2. Select a large number of \( \pi \) according to \( F(\pi) \). For each \( \pi \), compute the equilibrium values of \( Q_k, L, R \) and \( r^d \). Then compute the first order condition for \( \omega \). Iterate on \( \omega \) until the first order condition converges to zero.

3. Given the equilibrium \( \omega \), we compute the equilibrium \( Q_k, L \) and \( R \) and \( r^d \).

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