Banks as dealers of credit money: Comparing the roles of banks and non-banks in the provision of liquidity

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Abstract

Using a general equilibrium monetary model for banks, we analyze the role of banks in providing liquidity to the financial market and the transmission of the financial liquidity channel of monetary policy. In the model, the roles of banks in the payment system give banks additional abilities to provide liquidity. Because bank deposits can be used as means of payment, banks can directly create and lend new deposits that are not backed by money collected from depositors. As a result, the private banking system has the ability to supply loans elastically to meet the stochastic liquidity needs of the economy with very little need to borrow from the central bank. We show that the existence of banks is important to non-banks. When aggregate liquidity is limited, the attempt of non-bank investment funds to provide more liquidity insurance to shareholders may lead to higher volatility in asset prices without actually giving more liquidity to shareholders. New inside money provided by banks can reduce the volatility of asset prices and help non-banks perform their risk-sharing functions more effectively. We also show how the interest rate policy can be transmitted to asset prices by affecting the liquidity constraint of banks.

Journal of Economic Literature Classification: E4, E5, G12, G21

Keywords: banking, inside money, payment system, liquidity, asset prices, monetary policy

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1 Introduction

With the development of the financial system, non-bank financial intermediaries are becoming increasingly important, many services previously performed by banks are now gradually taken away by non-banks. An interesting question is: will non-banks completely replace banks in the near future? Are there special roles played by banks? Previous answers to this question usually focus on the characteristics of bank loans. For example, banks still have some advantage in providing loans to small borrowers. In this paper, we address this question from a different angle. We argue that the liquidity provision functions of banks are different from those of non-banks. The roles of banks in the payment system give them additional abilities to provide liquidity quickly, and the liquidity provision functions of banks are essential to the smooth functioning of non-banks.

This paper has two main contributions. First, we build a general equilibrium monetary model for banks based on the "credit-creation" approach, and show that the private banking system has the ability to supply loans elastically to meet the liquidity needs of the economy with very little need to borrow from the central bank. As we will explain below, we think the standard view of banking as a mechanism for allocating depositors' money does not fully capture the functions of banks. We instead model banking as a *mechanism for providing liquidity through book-keeping.* We explicitly model the micro-structure of the payment process and analyze how banks optimally decide their lending when they face the liquidity constraint imposed by the settlement requirement. We show that banks can endogenously choose the optimal reserve level, and supply loans elastically to meet the stochastic liquidity needs of the economy.

Second, we analyze how the liquidity provided by banks can help non-banks perform their liquidity provision functions. In our model, non-bank investment funds provide liquidity-risk sharing to shareholders, but they can only collect and reallocate the existing liquidity in the economy. We find that the risk-sharing policy that is individually optimal to funds may lead to adverse aggregate outcomes. When an investment fund attempts to provide more risk-sharing, it will try to reduce the differences between the payments that will be made to different shareholders. But if aggregate liquidity is limited, the liquidity that the investment fund can raise by selling assets will be limited. As a result, the payments to those shareholders who have immediate liquidity needs will also be limited. Smaller differences between the payments to different shareholders will actually be achieved solely by lower future payments to the remaining shareholders who do not have immediate liquidity needs. The market price of

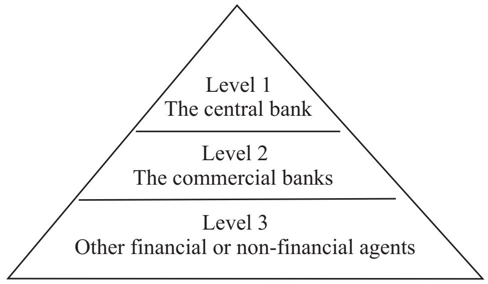


Figure 1: The hierarchy of the payment system

assets must decrease more before the optimal risk-sharing policy is satisfied and the investment fund stops selling assets. As a result, asset price will be more volatile. New liquidity created by banks can help investment funds provide more liquidity, and asset price will also be more stable.

The approach we used in this paper to model banks is somewhat different from the standard approach. The reason we adopt this different approach is as follows. A typical view of banking is that banks are intermediaries between depositors and borrowers and their function is to transfer the savings of depositors to borrowers. According to this view, there is no essential difference between banks and non-bank mutual funds. Both of them collect idle cash and then lend those cash to borrowers. Banks are merely middle-man between depositors and borrowers. The same view is also adopted by most models in the banking literature, where banking is usually modelled as a mechanism for optimally allocating the resources entrusted by depositors.

We think the above "middle-man" view of banking does not fully capture the functions of banks. An important difference between banks and non-banks is that they have different functions in the payment system. Because bank deposits are used as means of payment, banks can provide liquidity by directly creating and lending new deposits.

The basic intuition can be seen from Figure 1. The payment system has a hierarchical structure and the relationships between different levels are asymmetric. The payments of the lower levels of the payment system are settled with the debt issued by the higher levels of the payment system. For example, bank money is settled with central bank money, and payments among non-bank agents can be settled with bank money or central bank money. Since the debt of higher levels is used as means of payment by lower levels, the higher levels can provide liquidity to lower levels by expanding their debt. For example, the central bank can supply liquidity by creating and lending new central bank money. Similarly, banks can also provide liquidity by creating and lending new bank money.[1] As a result, the role of banks in the payment system gives banks some advantage in providing liquidity because the debt of banks is widely accepted as means of payment. As we will show in more detail in our model, in reality, the creation and circulation of bank money take the form of changes in the balance sheet of banks, and we can model banking as *a mechanism in which banks provide liquidity to non-bank agents through book-keeping*.

The basic structure of our model is an overlapping generations model with random relocation. There are two locations in which households live and financial intermediaries operate. In each period some of the households must move to the other location. Households can only invest in local banks and non-bank investment funds. Investment funds collect resources, make real investments, and provide risk-sharing to households. The shares of investment funds can not be used as means of payment and investment funds can only collect and reallocate the existing liquidity in the economy. Banks do not finance real investments in our model, as our focus is on banks' function in creating and lending inside money. We assume that movers must redeem their investment fund shares into money before they move to the other location. Investment funds may need to raise additional money to meet the redemption needs. If they raise money by selling assets, more selling can lead to lower prices because the cash available for purchasing assets is limited. We show that if aggregate liquidity is limited, the attempt of investment funds to provide more risk-sharing may cause higher volatility of asset prices without actually supplying more liquidity to movers. With new deposits provided by banks, investment funds will be able to provide more liquidity to movers. Since investment funds now sell less assets, asset price is more stable.

An important motivation for this paper is the recent studies in liquidity and asset prices, especially the works of Franklin Allen and Douglas Gale. They point out that when people sell assets, if the amount of cash that buyers can use to buy assets is limited, the market price of assets can deviate from the fundamental price decided by the primitive parameters such as asset returns and agents' risk aversion, which will lead to the "cash-in-the-market-pricing"(Allen and Gale(2005)). Most works in this area are still based on non-monetary models. For example, in Allen and Gale(1994, 2004b), people invest in short-term and long-term projects. If a large proportion of the agents turn out to be impatient and need to consume in the short-term, then long-term assets will be sold at low prices relative to short-

[1]Note that this relationship can not be reversed. For example, a non-bank financial intermediary such as a mutual fund can not issue debt and lend it to banks to be used to settle inter-bank balances. Banks must settle their balances with central bank money.

term consumption goods. Here, liquidity or "cash" is modelled as real consumption goods available for immediate delivery. Gale(2005) extends the framework into a monetary model with central bank money. He defines liquidity as means of payment. He shows that when both asset market and goods market are subject to the cash-in-advance constraint, liquidity shocks in the asset market can lead to relative price changes between assets and goods.

We follow Gale(2005) and define liquidity as means of payment. What is new in our paper is that we add sophisticated banking and payment system and show how the private banking system can also meet the liquidity needs of the economy. We focus on inside money instead of outside money, because most private agents can not directly borrow from the central bank, and, second, we wish to emphasize the fact that private institutions can also supply elastic aggregate liquidity by creating inside money without much need to borrow from the central bank.

Our paper can also be seen as an extension of Freeman(1996a,b). In Freeman's model, buyers issue personal debt to sellers when purchasing goods, and later both types of agents will travel to a central island where the debt will be settled with fiat money. But some sellers may have to leave before all buyers reach the central island. If those sellers sell their debt and if the cash available for purchasing debt is limited, the debt will be sold below its par value. Freeman then shows that welfare can be improved if the central bank or a clearinghouse bank can provide discount services. For example, the central bank or a clearinghouse bank can issue banknotes to buy those debts, and later collect those debts from buyers.

While our paper also adopts the basic mechanism that new money can be a source of liquidity, we extend Freeman's work in three important directions.

First, Freeman's model is more about the function of the central bank. The clearinghouse banks in his paper are similar to the central bank, as they provide liquidity by issuing banknotes and essentially face no liquidity constraints. In our paper, we focus on how the *private* commercial banking system can meet the liquidity needs of the economy without much need to borrow from the central bank. In particular, in our model, when commercial banks provide liquidity to non-bank agents, they themselves have liquidity concerns. We explicitly model the micro-structure of the payment process, and analyze how commercial banks optimally decide their liquidity provision when they face the liquidity constraint imposed by the settlement requirement.

Second, in Freeman's model, banks only issue banknotes, and do not issue deposits or collect money from depositors. Our paper analyzes the modern banking case where banknotes are not allowed. We show explicitly why the modern deposit banking system can be seen as a mechanism for providing liquidity through book-keeping.

Third, we also include non-bank financial intermediaries in the model and analyze how their liquidity-insurance policy can affect the volatility of asset prices.

We model how the interest rate policy can be transmitted to asset prices through the banking system. Although there is no reserve requirement for banks, banks still need central bank money to settle inter-bank balances. By reducing the lending rate, the central bank can partly relax the constraints faced by banks, and banks will supply more credit to non-banks at lower lending rate, which will in turn lead to more stable asset prices.

The basic environment(i.e, the overlapping generations model with random relocation) is taken from Champ, Smith and Williamson (1996). Their paper shows that the social welfare can be improved if banks can use private banknotes instead of central bank money to meet the random withdrawal needs of depositors. Our model focuses on how banks provide liquidity to borrowers instead of to depositors.

The rest of the paper is organized as follows. Section 2 describes the basic environment. Section 3 is the idealized case in which all assets can be used to make payments. Section 4 analyzes the case where payments must be made with money and investment funds can only raise money by selling assets. Section 5 looks at the case where investment funds can borrow from banks. Section 6 shows the numerical results for log utility function, and section 7 shows the results for more general utility functions. Section 8 provides some historical examples of the role of banks in providing emergency liquidity during financial market turmoils. Section 9 summarizes and concludes the paper.

2 The environment

In this part, we explain the basic environment of the model. Since there is no lending by commercial banks in section 3 and 4, the detailed environment for bank lending and the settlement process will be explained in section 5.

We consider an overlapping generations model with random relocation. Time is indexed by $t = 1, 2, ...$ There are two locations in the economy. In each period, a new generation is born at each of the two locations. In each generation, there are three types of agents: "households", "investment fund managers" and "bankers". We normalize the size of each type of agents to one. There is no population growth. Each generation lives for two periods. The first old generation of households in each location at time $t = 1$ is endowed with outside money M.

Agents only care about the consumption when they are old. There is a single good. Each household is endowed with e_h units of the good when young, and nothing when old. Young households save all their endowment. Households are risk-averse and they have a constant relative risk aversion(CRRA) utility function:

$$U(c) = \frac{c^{1-\sigma}}{1-\sigma} \qquad \sigma \geq 1 \tag{1}$$

When $\sigma = 1$, $U(c) = \ln c$.

Young investment fund managers can costlessly start new investment funds and young bankers can costlessly start new banks. There is free entry for investment funds and banks. Investment funds compete with each other by offering the best contract to shareholders and banks compete with each other by offering the best contract to depositors and borrowers.

Investment fund managers do not have endowments. And we assume away bankruptcy for banks using the following assumption: every banker has endowment e_b when old, which can be used to absorb the loss. Investment fund managers and bankers are risk neutral. Old fund managers and bankers consume all their wealth.

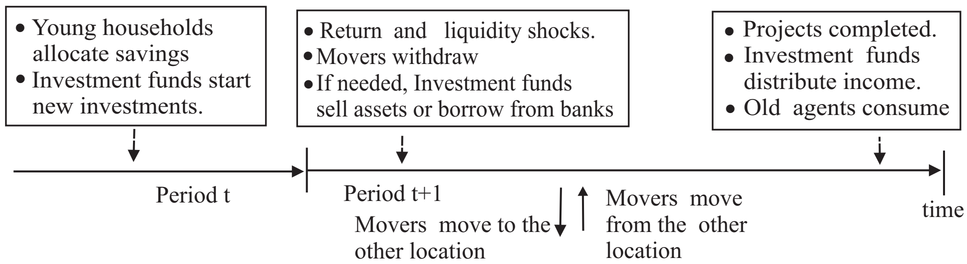
The good is non-storable but can be invested as capital to produce new goods in the next period. Real risky investments can only be made by investment funds. The gross return rate for the risky project is

$$R_k = A \tag{2}$$

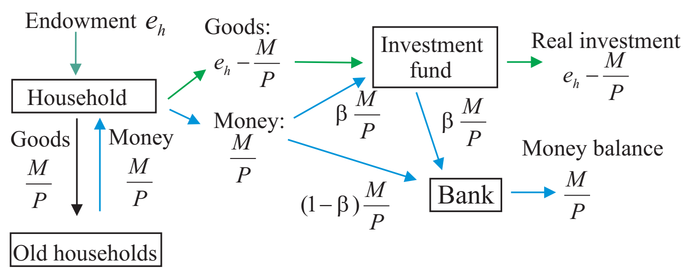
where A is the aggregate productivity. There are two values of A: A_H(high), A_L(low) with equal probability. A is i.i.d. in each period.

The main events and the initial portfolio allocation process are shown in Figure 2. We assume that young households can only invest in local investment funds and banks. For simplicity, we assume that each young household can at most invest in one investment fund and one bank. At the end of period t, young households sell part of their endowment to the old generation for money balance $\frac{M}{P}$. They invest the remaining endowment $e_h - \frac{M}{P}$ and part of the money $\beta\frac{M}{P}(\beta \in [0,1])$ into investment funds. They deposit the remaining money $(1-\beta)\frac{M}{P}$ into banks. Investment funds then make real investments and deposit the money $\beta\frac{M}{P}$ into banks. Banks will end up holding all the cash balance $\frac{M}{P}$. Note that investment funds can also choose to hold part of their assets in bank deposit, this happens when $\beta > 0$. If $\beta = 0$, investment funds only hold risky assets.

We focus on the symmetric case in which all investment funds and banks have the same size. We assume that the actual number of households is higher than the number of investment



(a) Events in the basic model



(b) Initial portfolio allocation

Figure 2:

funds and banks, so each investment fund has many shareholders and each bank has many depositors. We assume that young households are equally distributed among all investment funds and banks. We also assume that the actual number of investment funds is at least as high as the number of banks, and the deposit account of new investment funds are equally distributed among all banks. Deposit account holders are free to switch their account to other banks.

After real investments are made, we enter period $t+1$. At the beginning of period $t+1$, the productivity shock A is publicly observed. At the same time, the Nature decides the liquidity shock. A random fraction π of the old households(denoted as "movers") must move to the other location and consume there. π is distributed over $[0, \overline{\pi}]$, where $\overline{\pi} < 1$ is the upper bound of the distribution. The distribution function is $F(\pi)$. π is symmetric in the two locations. π is independently and identically distributed, so each old household has the same *ex ante* probability to be a mover.

Movers can not carry goods across locations. We assume there is a network between banks, and the value of bank deposits in everyone's banking account can be verified across locations, so movers can use bank deposits to make payments. More specifically, we assume that when movers move from location i to location j, they still keep their deposits in the banks in location i. And when they need to buy consumption goods in location j, they can pay using their deposits.

The value of other assets can not be verified across locations. In particular, we assume that people can not verify the value of investment fund shares, so investment fund shares are not accepted as means of payment across locations.(Except in section 3 where we consider the special case in which all assets can be used to make payments.) As a result, movers must use money to buy consumption goods. We assume that movers must have their deposits ready in their banking account when they move to the other location. As a result, movers must redeem their investment fund shares into bank deposits before move.

In order to meet the redemption needs, investment funds may need to raise additional money by selling assets or by borrowing from banks. When assets are sold, only the ownership is transferred to the buyer, the production process is not stopped. The investment funds will collect the return and pay it to the buyer at the end of the period.

The redemption process is as follows. After the shocks are realized, movers must send a withdrawal notice to the investment fund. Then the financial market opens. If the investment fund can not meet the withdrawal needs by its own riskless asset, then it can raise cash by selling assets to non-movers who have idle deposits. The fund can also choose to borrow from banks. The fund then pay movers by transferring bank deposits to them. For simplicity, we assume that movers only receive the payment after the transactions on financial market are completed, so the cash received from the fund can not be used to buy risky assets on the financial market.

After the redemption, movers move to the other location. At the end of $t+1$, risky projects are completed. Movers in each location use their bank deposits to buy consumption goods. Investment funds allocate returns to their shareholders and also repay the bank loan. Bankers consume the net income and old non-movers consume all their wealth.

For simplicity, we assume that non-bank agents always use bank deposits to make payments. Although people can choose to withdraw their deposits, in the equilibrium, people always keep their money in banks and there is no actual withdrawal of central bank currency.

We assume that banks must settle inter-bank balance with central bank money. Banks keep their reserves in the central bank deposit account, and the deposit rate paid by the central bank is normalized to zero. There is no official reserve requirement and banks can freely choose the reserve level.[2] We assume that non-bank agents can not directly borrow

[2]Countries such as Canada, New Zealand and UK have already eliminated the reserve requirement.

from the central bank, they can only borrow from banks, but banks can borrow from the central bank.

We will explain the details of bank lending and inter-bank payments in section 5. In the next two sections, we will first analyze two cases without bank lending. In section 3, we consider a special case in which all assets can be used to make payments. In this case, there is no liquidity risks. In section 4, movers must use money to make payments and the investment fund can only raise cash by selling assets on the financial market, which may lead to volatile asset prices.

3 The benchmark case: all assets can be used to make payments

In this part, we consider a special case in which there exists an imagined "Walrasian Auctioneer", who can verify the value of everything in the economy for free. As a result, movers can carry investment fund shares with them and use the shares to buy goods in the other location. The transactions are then cleared by the "Walrasian Auctioneer". Since π is symmetric in the two locations, investment funds and movers will be willing to accept this arrangement.

In this case, people only hold money as a store of value.(Money still has value because it can be used to buy goods from the young generation.) All assets are equally liquid.

3.1 The optimal choice

Let Z_p, Z_f and Z_k denote the portfolio, riskless assets and risky assets of a representative investment fund. And let $\alpha \geq 0$ denote the share of riskless assets, we have $Z_f = \alpha Z_p$ and $Z_k = (1-\alpha)Z_p$.

Let s denote the savings of each household. Households save all endowment and so $s = e_h$. Let $d = \omega s$ and $z_p = (1-\omega)s$ denote the bank deposit and investment fund shares held by each household. Let v denote the value of the household's portfolio(which is also the consumption level) in period $t+1$. Movers and non-movers have the same v:

$$v = s[\omega + (1-\omega)(\alpha + (1-\alpha)R_k)] = s[\kappa + (1-\kappa)R_k]$$

where

$$\kappa = \omega + (1-\omega)\alpha \tag{3}$$

is the total share of bank deposit in household's portfolio(i.e., both the deposit held directly by the household and the deposit held indirectly through the investment fund).

Because of competition, the contract of the investment fund should maximize the expected utility of households. Since the objective of the investment fund is the same as the objective of households, we can choose ω and α freely to maximize EU of households. ω and α will not be unique because the household has the same portfolio as long as κ is the same.

For simplicity, we use the example $[\omega = \kappa, \alpha = 0]$, that is, the investment fund only holds risky asset. We have $v = s(\omega + (1-\omega)R_k)$ and the expected utility of the household is

$$EU = \frac{1}{2}\frac{[s(\omega + (1-\omega)R_{k,H})]^{1-\sigma}}{1-\sigma} + \frac{1}{2}\frac{[s(\omega + (1-\omega)R_{k,L})]^{1-\sigma}}{1-\sigma} \tag{4}$$

where $R_{k,H} = A_h$ and $R_{k,L} = A_L$ are high and low risky asset returns. The first order condition is

$$\frac{\partial EU}{\partial \omega} = \frac{1}{2}\frac{s(1-R_{k,H})}{[s(\omega + (1-\omega)R_{k,H})]^{\sigma}} + \frac{1}{2}\frac{s(1-R_{k,L})}{[s(\omega + (1-\omega)R_{k,L})]^{\sigma}} = 0 \tag{5}$$

If $\frac{\partial EU}{\partial \omega} < 0$ at $\omega = 0$, then we have the corner solution $\omega = 0$. In this case, people do not hold any money balance and the outside money M is valueless in the equilibrium. If $\frac{\partial EU}{\partial \omega} > 0$ at $\kappa = 1$, then households only hold bank deposits in the equilibrium, no risky investments are made, and we have a pure-exchange overlapping generations economy. If we have $\frac{\partial EU}{\partial \omega} = 0$ for $\omega \in (0, 1)$, then money balance and real investments are both positive in the equilibrium.

Essentially, we have a standard portfolio choice problem with one risky and one riskless asset.

3.2 Equilibrium

We only focus on the stationary symmetric equilibrium. The equilibrium allocation is as follows. Households choose $\omega = \omega^*$ according to condition (5). Denote the aggregate saving as S(which is equals to e_h in the equilibrium because the total size of young households is normalized to 1). The young generation sells $\omega^* S$ of goods to the old generation for money, and the remaining goods $(1-\omega^*)S$ are invested in risky projects.

The consumption of each old household is $v = s(\omega^* + (1-\omega^*)R_k)$, where $s(1-\omega^*)R_k$ is the output from the risky projects and $s\omega^*$ is the goods purchased from the next young generation with money.

Since the aggregate value of the riskless asset chosen by the young generation is equal to the value of outside money spent by the old generation, we have

$$\omega^* S = \frac{M}{P} \tag{6}$$

Given M, this equation gives the equilibrium price P^*. In the stationary equilibrium, since $\omega^* S$ is the same in each period, and since M is fixed, P^* is the same in each period.

4 Liquidity provision through the financial market

In this section, we assume that there is no lending from the central bank or commercial banks. And we assume there is no cost for people to make payments using bank deposits. Since there is no bank loan and the central bank deposit rate is zero, the deposit rate for banks will also be zero. We assume investment funds can only raise cash by selling assets to non-movers. Since the amount of deposits is equal to the amount of reserve M, the money supply is essentially fixed at M.

We only care about the symmetric equilibrium, and we will use the following method in our analysis. We assume that all investment funds and households make symmetric choices, and we then check whether an individual household or investment fund wants to deviate from the symmetric equilibrium.

We assume that non-movers incur zero transaction costs when they buy assets on the financial market. Note that in this model, there is no need for any bank to collect the idle money from non-movers and then lend them to the investment fund because non-movers can directly use their money to buy assets from the investment fund. So in order for banks to be useful, they must do more than collecting and lending out the idle cash.

4.1 Optimal choices

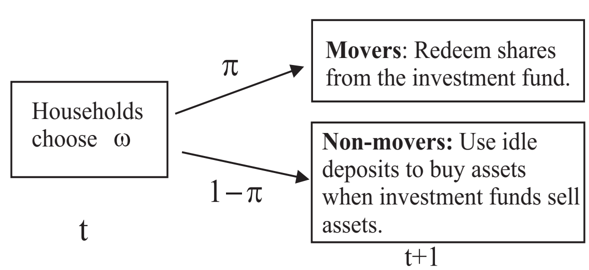


Figure 3: Household's choice

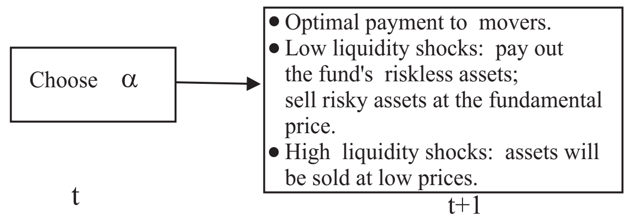


Figure 4: The choice of the investment fund

The choices of households and the investment fund are shown Figure 3 and 4. Households choose ω at the end of t. Given ω, the investment fund chooses α in t and the optimal payout policy in $t+1$ to maximize the expected utility of households.

Let r_m denote the return rate paid to movers at the beginning of $t+1$ and r_n the return rate paid to non-movers at the end of $t+1$. First, we need to find out the optimal r_m and r_n, then we need to find out the optimal ω and α in the initial portfolio.

Recall that R_k is the fundamental price of risky assets. And let Q_k denote the market price of risky assets when investment funds sell assets to non-movers. First, we need to decide the optimal r_m and r_n given Q_k, then we will use the result to derive the equilibrium distribution of Q_k. In the following analysis, we will ignore H and L from the notations whenever the analysis applies to both high and low productivity shocks.

Let v_m denote the value of movers' portfolio and v_n the value of non-movers' portfolio. We have

$$v_m = s\left[\omega + (1-\omega)r_m\right] \tag{7}$$

$$v_n = s\left[\omega\frac{R_k}{Q_k} + (1-\omega)r_n\right] \tag{8}$$

For movers, the value of the deposit is $s\omega$, and the payment from the fund is $s(1-\omega)r_m$. For non-movers, the payment from the investment fund is $s(1-\omega)r_n$. The gross return for the initial deposit balance $s\omega$ can be written as $\frac{R_k}{Q_k}$. There are three situations. If investment funds do not sell assets, then the return for the initial deposit is simply 1, which can be written as $\frac{R_k}{Q_k}$ because $R_k = Q_k$. Similarly, if investment funds sell some assets to non-movers but $R_k = Q_k$, then the return for the initial deposit is still 1. Finally, if $R_k > Q_k$, then non-movers will use all their initial deposit to buy assets, and the return is $\frac{R_k}{Q_k} > 1$.

The expected utility of the household is

$$\frac{1}{2}\int_0^1 \left[\pi\frac{(v_{m,H})^{1-\sigma}}{1-\sigma} + (1-\pi)\frac{(v_{n,H})^{1-\sigma}}{1-\sigma}\right] dF(\pi) + \frac{1}{2}\int_0^1 \left[\pi\frac{(v_{m,L})^{1-\sigma}}{1-\sigma} + (1-\pi)\frac{(v_{n,L})^{1-\sigma}}{1-\sigma}\right] dF(\pi) \tag{9}$$

where H and L are productivity shocks. Since r_m is chosen after the shocks are realized, the fund can choose the best r_m for each level of the shock. So the fund maximizes

$$\pi\frac{(v_m)^{1-\sigma}}{1-\sigma} + (1-\pi)\frac{(v_n)^{1-\sigma}}{1-\sigma} \tag{10}$$

subject to its budget constraints and the constraint $r_m \leq r_n$(payment to movers can not be higher than non-movers, otherwise non-movers will pretend to be movers and withdraw.) The details of the optimal payout policy is shown in the Appendix. We find that when $Q_k = R_k$, it is optimal to set $r_m = r_n$. When $Q_k < R_k$, since it is costly to raise cash by selling assets, the fund may not provide full insurance. If the constraint $r_m \leq r_n$ is not binding, then the fund tends to set a higher $\frac{r_m}{r_n}$ when σ is higher. That is, the fund will provide more risk sharing when people are more risk averse.

Since $\sigma = 1$ is the simplest case, for the remaining part of this section, we will show the result of $\sigma = 1$. The details of the general case ($\sigma \geq 1$) are shown in the Appendix.

Proposition 1. *For $U(c) = \ln c$, the optimal payout policy is*

$$r_m = \alpha + (1-\alpha)Q_k \tag{11}$$

$$r_n = \alpha\frac{R_k}{Q_k} + (1-\alpha)R_k = r_m\frac{R_k}{Q_k} \tag{12}$$

Proof: see the Appendix. ■

We can see that when $U = \ln c$, the optimal r_m is to pay the market value of the fund's asset. Given this payout policy, we have the following result:

Proposition 2. *When $\sigma = 1$, the equilibrium is defined by κ, and the solution for ω and α is not unique. As long as κ is the same, the equilibrium result is the same.*

Proof: We need to show that first, given Q_k, the value of the household's portfolio only depends on κ. Second, the same κ also gives the same distribution of Q_k. For the first part, substituting the optimal r_m and r_n(11 and 12) into (7) and (8)

$$\begin{aligned} v_m &= s\left[\omega + (1-\omega)(\alpha + (1-\alpha)Q_k)\right] = s\left[\kappa + (1-\kappa)Q_k\right] \\ v_n &= s\left[\omega\frac{R_k}{Q_k} + (1-\omega)(\alpha\frac{R_k}{Q_k} + (1-\alpha)R_k)\right] = s\left[\kappa\frac{R_k}{Q_k} + (1-\kappa)R_k\right] \end{aligned}$$

So given Q_k, the value of the portfolio only depends on κ. The second part of the proof is shown in the Appendix. $\blacksquare$.

In the remaining part of the section, we use the example ($\omega = \kappa$, $\alpha = 0$), that is, investment funds only hold risky assets. The payment to movers (11) becomes $r_m = Q_k$.

4.2 The distribution of $Q_k(\pi)$

Let π_1 denote the value of π above which the cash of non-movers is binding. We have the following result:

- If $\pi < \pi_1$, then after the investment funds raise enough cash by selling assets, the non-movers still have positive cash balance left, and we have $Q_k = R_k$.
- If $\pi \geq \pi_1$, non-movers will use all their deposits to buy assets. $Q_k = R_k$ at $\pi = \pi_1$, and $Q_k < R_k$ when $\pi > \pi_1$.

We first derive π_1, then we derive Q_k for $\pi > \pi_1$.

Recall that Z_k is the level of risky assets held by every investment fund. It is also the total investment made by the fund because we assume investment funds only hold risky assets. Let D_0 denote the value of deposits held by the households who belong to a representative investment fund. Since the choices of investment funds and households are symmetric, we can also use Z_k and D_0 to denote the level of aggregate risky assets and deposit balance.

At π_1, the total cash raised from non-movers is $(1 - \pi_1)D_0$, which is equal to the total redemption, and Q_k is still equal to R_k. Since r_m is equal to the market price of the fund's asset, we should have

$$\begin{aligned}
&\text{Total redemption by movers} = \pi * NV(\text{Total market value of the fund}'\text{s asset}) \\
&\Rightarrow \pi_1 = \frac{\text{Redemption}}{NV} = \frac{(1-\pi_1)D_0}{Z_k R_k}
\end{aligned}$$

Using the relationship $D_0 = \omega S$ and $Z_k = (1 - \omega)S$, we have

$$\pi_1 = \frac{\omega}{\omega + (1-\omega)R_k} \tag{13}$$

When $\pi > \pi_1$, $Q_k(\pi)$ can be decided according to

$$\begin{aligned}
\pi &= \frac{\text{Redemption}}{NV} = \frac{(1-\pi)D_0}{Z_k Q_k} \\
&\Longrightarrow Q_k(\pi) = \frac{\omega(1-\pi)}{\pi(1-\omega)}
\end{aligned} \tag{14}$$

So the distribution of $Q_k(\pi)$ is

$$Q_k(\pi) = \begin{cases} R_k & : \quad \pi \leq \pi_1 \quad (\mathrm{Non-movers'\ cash\ is\ not\ binding}) \\ \frac{\omega(1-\pi)}{\pi(1-\omega)} & : \quad \pi > \pi_1 \quad (\mathrm{Non-movers'\ cash\ is\ binding}) \end{cases} \tag{15}$$

From (14), we can see that when π is higher, the cash raised from non-movers, $(1-\pi)D_0$, will be lower, but the share of movers π is higher, so the asset price Q_k must decrease in order for (14) to hold.

4.3 Optimal choice of ω

Let ω_i denote the ω chosen by an individual household i. Household i chooses ω_i by taking ω_j chosen by all other agents ($j \neq i$) as given.

The value of household's portfolio is

$$\begin{aligned} v_m &= s\left[\omega_i + (1-\omega_i)Q_k\right] \\ v_n &= s\left[\omega_i \frac{R_k}{Q_k} + (1-\omega_i)R_k\right] = \frac{R_k}{Q_k} v_m \end{aligned}$$

And the expected utility of household i is

$$\begin{aligned} EU_i = \frac{1}{2}\int_0^1 & \left\{\pi \ln\left[s\left(\omega_i + (1-\omega_i)Q_{k,H}\right)\right] + (1-\pi)\ln\left[s\frac{R_{k,H}}{Q_{k,H}}(\omega_i + (1-\omega_i)Q_{k,H})\right]\right\} dF(\pi) \\ +\frac{1}{2}\int_0^1 & \left\{\pi \ln\left[s\left(\omega_i + (1-\omega_i)Q_{k,L}\right)\right] + (1-\pi)\ln\left[s\frac{R_{k,L}}{Q_{k,L}}(\omega_i + (1-\omega_i)Q_{k,L})\right]\right\} dF(\pi) \end{aligned}$$

$$\frac{\partial EU_i}{\partial \omega_i} = \frac{1}{2}\int_0^1 \frac{1-Q_{k,H}}{\omega_i + (1-\omega_i)Q_{k,H}} + \frac{1-Q_{k,L}}{\omega_i + (1-\omega_i)Q_{k,L}} dF(\pi) = 0 \tag{16}$$

In the symmetric equilibrium, given $\omega_j = \omega$ chosen by all other agents, household i should find it is optimal to set $\omega_i = \omega_j = \omega$.

4.4 General equilibrium

Similar to section 3.2, the equilibrium is still a repeated one-period portfolio choice problem.

The equilibrium is defined by $\omega = \omega^*$. Given the expected distribution of $Q_k(\pi)$, the equilibrium ω maximizes the expected utility of each household according to (16). Given the equilibrium ω, $Q_k(\pi)$ is decided according to equation (15).

The consumption of movers is v_m and the consumption of non-movers is v_n. The total real investment is $Z_k = (1-\omega^*)S$. The total goods sold by the young generation to the old generation is $D_0 = \omega^* S$. The nominal price level on the goods market is $P = \frac{M}{\omega^* S}$.

Bank's balance sheet				
	Asset		Liability	
	Reserve	Loan	Deposit (agent a)	Deposit (agent b)
Initial balance	0	0	0	0
1. Agent a borrows	0	x	x	0
2. Agent a pays agent b	0	x	0	x
3. Agent b pays agent a	0	x	x	0
4. Agent a repays the loan	0	0	0	0
Final balance	0	0	0	0

Table 1: An example for credit creation. In this special example, neither an initial injection of central bank money or resources collected from depositors are necessary for bank lending.

On the financial market, when $\pi < \pi_1$, the total cash used by non-movers to buy assets is equal to the redemption of movers $\pi Z_k R_k = \pi(1-\omega^*)SR_k$. When $\pi > \pi_1$, all cash of non-movers, which is $(1-\pi)D_0 = (1-\pi)\omega^* S$, is used to buy assets.

5 Elastic inside money

In this section, we analyze how banks provide liquidity through inside money creation. We also analyze the transmission mechanism of the central bank's interest rate policy.

5.1 Basics about credit money creation

We first use an idealized example to show the basic intuition of credit creation. We then show the main steps for bank lending in our formal model.

An example for credit creation

Suppose there is one bank in the economy and all agents make payments by transferring deposits between the deposit accounts of this bank. The transaction steps are shown in Table 1.

The initial asset and liability of the bank is set to zero, there is no outside money, nor there is any resources collected from depositors.

Suppose agent a needs to make payment but personal credit is not accepted. In step 1, agent a borrows from the bank. The bank lends by adding a number x into the borrower's account.(In reality, when banks make loans, what they actually do is to add a number into the borrower's deposit account.) As a result, both bank loan and bank deposit increase by x. The new deposit x is created. Note that the new deposit x is not collected from depositors, it is created by the bank, and the initial holder of the deposit is simply the borrower. There is no free lunch here because the newly created deposit x is bank's liability instead of asset, and the balance sheet is still balanced. Note that we can not have an increase in loan without an equivalent increase in deposit because otherwise the balance sheet will not be balanced.

The borrower a then uses the deposit to pay agent b. Suppose later agent b purchases goods from agent a by paying deposit x, then the deposit is transferred back to a. Finally, agent a uses the deposit to repay the bank loan. Here we assume the lending rate is zero. When the loan is repaid, both loan and deposit balance decrease by x. The deposit x is paid back to the bank and is destroyed by the bank, and the asset and liability of the bank are restored to zero.

This example shows how liquidity can be provided when we have a centralized book-keeping system. In this case, deposit is simply an entry in the bank's book. Since people accept bank deposit as means of payment, the bank can provide liquidity by creating and lending out new deposit. In this example, all changes in deposit are the result of the bank's lending activities. If centralized book-keeping is infeasible, then we will have multiple banks. Essentially, we will have a decentralized book-keeping system.

Our formal model will be more sophisticated than the above example with features such as multiple banks, money collected from depositors, and inter-bank payment flows. We show that since there are both payment inflows and outflows in the inter-bank payment process, the settlement balance will usually be smaller than the new loan level, so banks do not really need to collect an equivalent amount of money from depositors first and then transfer those money to borrowers. As long as banks can get enough reserve to settle the inter-bank balance, banks can create and lend new deposits that are not backed by money collected from depositors.

The key steps for bank lending

Let D_0 denote the initial deposit and reserve balance of a representative bank. The main steps for bank lending in our model are as follows.

First, when bank i makes loan L, it creates an equivalent amount of new deposit L.

Bank i makes new loan L	
Reserve: D_0	Initial deposit: D_0
Loan: $+L$	New deposit owned by the borrower: $+L$

Second, borrowers spend the money. If payments are made to depositors in other banks,

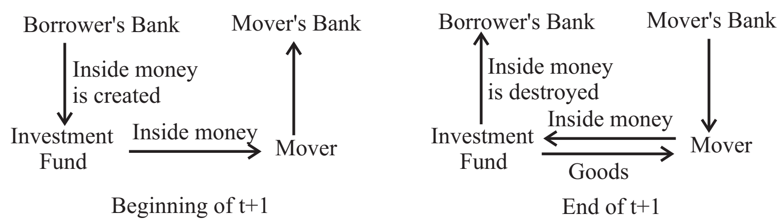


Figure 5: The flow of inside money.

then the inter-bank payments will create a liquidity constraint for the bank. The details will be analyzed later.

Third, when the borrower uses his deposit to repay the loan, we have

Loan L is paid back	
Loan: $-L$	Deposits owned by the borrower: $-L(1+r^l)$
	Bank Equity(interest income): $+Lr^l$

where r^l is the lending rate. The outstanding loan is reduced by L and the outstanding deposit is reduced by $L(1+r^l)$. The interest income is Lr^l.

In our model, the borrower are the investment funds. After the shocks are realized, if bank loan is needed, then banks create deposits and lend them to the investment funds, the investment funds then use the deposits to meet the redemption of movers. Movers then hold the deposits. At the end of period $t+1$, movers use their deposits to buy goods from the investment funds. Then the investment funds use the deposits to repay the bank loan, and the deposits are returned to banks and are destroyed. The main steps are shown in Figure 5.

5.2 The environment for bank lending and settlement

This part describes the environment for bank lending and inter-bank settlement. We focus on the case in which banks lend to the investment funds.

At the end of period t, depositors are equally distributed among the banks, and the deposit account of investment funds are also equally distributed among the banks. Initially, each investment fund only has deposit account in one of the banks. The shareholders of each investment fund are equally distributed among all banks. There is no interest payment to depositors for holding deposits between the end of t and the beginning of $t+1$.

At the beginning of period $t+1$, the productivity shock and the liquidity shock are realized. And then banks announce their deposit rate and lending rate. We assume that banks are competitive on the lending side and they take the market lending rate as given. And we assume that banks are also competitive on the deposit side and the resulting deposit contract maximizes the expected utility of depositors. In our model, there is no uncertainty in the payment of deposits, so the deposit rate that maximizes the expected utility of depositors is simply the highest deposit rate. We only focus on the symmetric equilibrium, and we assume that in the equilibrium, all banks simultaneously offer the same deposit rate r^d that gives zero expected profit to all banks.

Let π_2 denote the liquidity shock above which investment funds will borrow from banks. When $\pi \leq \pi_2$, investment funds will only raise money by selling assets. In this case, the deposit rate can only be zero because there is no income for banks.(We assume there is no cost for managing the deposits and allowing the depositors to use the payment facility). Since there is no bank loan, the level of deposits is the same as the level of reserves. This means all deposits are backed by reserves, so banks can never run out of reserves during the settlement process because the maximum outflow of payment is equal to the level of deposits.

Now suppose $\pi > \pi_2$ and bank loan is needed. We assume that investment funds can use the risky assets as collateral to borrow from banks. Each unit of bank loan incurs a management cost δ to the bank. Denote the net real lending rate as r^l, where $r^l \geq \delta$, and denote the gross lending rate as $R = 1 + r^l$.

The investment fund will borrow from the bank only when the borrowing cost is less than or equal to the cost for selling assets on the financial market. If the fund sells the asset, for each unit of asset with value R_k, the fund can get Q_k. If the fund borrows from the bank, for each unit of loan with future payment R_k, the fund can borrow $\frac{R_k}{1+r^l}$. In the equilibrium, we must have

$$Q_k = \frac{R_k}{1 + r^l} \tag{17}$$

that is, investment funds will only borrow from banks when the market price decreases to $\frac{R_k}{1+r^l}$. Since $r^l \geq \delta$, so $R \geq 1 + \delta$ and we must have $Q_k \leq \frac{R_k}{1+\delta}$ when bank loan is needed.

The main steps for lending and settlement are as follows(see Figure 6).

Recall that D_0 is the deposit and reserve balance for each bank at the beginning of $t + 1$. After the shocks, the financial market opens, and the investment funds sell assets to non-movers. When bank loan is needed, $\frac{R_k}{Q_k}$ is equal to the lending rate $1 + r^l$, which is higher than the deposit rate, so non-movers will use all their deposits to buy assets. So after the transactions, non-movers transfer all their deposits to the investment funds. (This is "settlement 1" in Figure 6). Because all bank deposits are still backed by reserves, banks will not face a liquidity constraint no matter how the inter-bank settlement process is carried out.

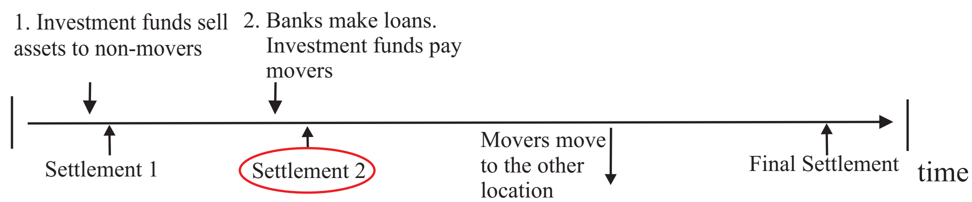


Figure 6: The lending and settlement process

In the symmetric case, each fund sells the same amount of asset, and after the payment, the deposit balance in each bank is still D_0.

After all payments by non-movers are completed, the financial market closes. We then enter step 2 in which banks make loans to the investment funds. If an investment fund has its deposit account in bank i, then it will first choose to borrow from bank i. The investment fund can also borrow from other banks if their lending rate is lower. In the symmetric case, every investment fund borrows from its own bank. If an investment fund borrows from bank i, it also keeps the newly borrowed money with bank i before making payments to movers.

Investment funds then use all their deposits to pay movers. The process is "settlement 2" in Figure 6. Since banks have created new deposits during lending, the payment may not be fully covered by initial reserves and banks may face binding liquidity constraint during the settlement process. The details will be explained later.

We assume that after each mover receives all of his money, he can also switch his deposit account to a new bank if that bank offers a higher deposit rate. We use this assumption to force banks to be competitive on the deposit side. If the deposit rate offered by a bank were lower than the market rate, then all its depositors would switch to other banks and this bank would run out of reserves in the settlement process. In the symmetric case, all banks offer the same deposit rate and no mover would switch banks. As a result, in "settlement 2", we only need to consider the inter-bank payments caused by investment funds paying their movers.

After the redemption process is completed, movers move to the other location.

For simplicity, we ignore the liquidity constraint for banks during the transactions at the end of $t+1$. We assume that after the transactions are completed, there is a settlement between banks based on net balance. As we will show later, after all transactions are completed, the deposits in each bank will be fully backed by reserves. So the liquidity constraint for banks will not be binding during the "final settlement" because in the worst case, all deposits in a bank are paid to other banks and the reserve level is reduced to zero.

As a result, we only need to consider the liquidity constraint in "settlement 2".

The settlement method

Since our paper is not about the optimal design of the settlement process, we will use a simple model to capture the liquidity constraint generated by the settlement requirement.

We assume that inter-bank payments are settled according to "Real-Time Gross Settlement" method.[3] More specifically, we assume that there are N banks in the economy, with N being a very large number. And the redemption process will be separated in to N subperiods. We normalize the total time length of the redemption process to 1. The time length of each subperiod is $\frac{1}{N}$. In each subperiod, a bank is randomly chosen by the Nature to make payments to other banks, and the inter-bank balance is settled right away(i.e, the transfer of reserve happens right away). In the next subperiod, another bank is randomly chosen. During this process, any negative balance of reserve must be met by borrowing from the central bank.

For simplicity, we assume there is no inter-bank loan market, and banks can use the collateral collected from investment funds as collateral when borrowing from the central bank. Also, we assume that the central bank consumes the interest by purchasing consumption goods, so the outside money is always restored to M at the end of each period.

5.3 The portfolio choice

The value of household's portfolio are

$$v_m = s\left[\omega + (1-\omega)r_m\right](1+r^d) \tag{18}$$

$$v_n = s\left[\omega \frac{R_k}{Q_k} + (1-\omega)r_n\right] \tag{19}$$

In this case, r^d can be positive. The deposit interest is paid at the end of the period. The portfolio of non-movers is not affected by the deposit rate. As we will show below, when r^d is positive, $\frac{R_k}{Q_k}$ is equal to $1+r^l$, which is higher than $1+r^d$. So non-movers will use all initial deposits to buy assets and earn the return $\frac{R_k}{Q_k}$.

We derive the optimal payout policy in the Appendix. We find that it is similar to the no-bank-loan case. When $Q_k = R_k$, we have $r_m = r_n$. When $Q_k < R_k$, the investment fund tends to set higher $\frac{r_m}{r_n}$ when people are more risk averse.

In the case of $U = \ln c$, we find that r_m is still equal to the market price of the fund. Also, the best equilibrium is for households to hold all the riskless assets ($\alpha = 0$). The basic

[3] This method is used in many large value inter-bank payment system. Note that the gross settlement method is not actually the best settlement method for the environment given in this paper, we use it only because it provides a better way to capture the liquidity constraint generated by the settlement process. For example, it is easy to show that settlement based on net inter-bank balance imposes lower liquidity constraint.

reason is that investment funds need to transfer the riskless assets back to movers during the redemption process. Lower α can reduce the payment and so banks are less likely to borrow from the central bank. In the following analysis, we will show the result for $[U = \ln c, \alpha = 0]$. The results for the general case are shown in the Appendix. We will also discuss some results of the general case in section 7.

5.4 Bank's problem

5.4.1 The expected borrowing from the central bank

In this part, we compute the expected central bank loan during the settlement process.

Let L_i denote the loan made by bank i and $L_j = L$ denote the loan made by all other banks $j \neq i$. Right after the loan is made, the deposit balance for bank i is $D_0 + L_i$ and the deposit balance for all other banks $j \neq i$ is $D_0 + L_j$, where L_i and L_j are new deposits created during lending.

Then investment funds use all their deposits to pay the movers. We use X to denote the payment made by each bank, then

$$X_i = (1-\pi)D_0 + L_i \tag{20}$$

$$X_j = (1-\pi)D_0 + L_j \tag{21}$$

where $(1-\pi)D_0$ is the deposits raised by selling assets to non-movers.

Recall that the settlement process is divided into N subperiods and in each subperiod a bank is chosen to make the payment to other banks. Since we assume the shareholders of each fund are evenly distributed among all banks, when bank i is chosen to make the payment, the payment made to the shareholders in the same bank is $\frac{1}{N}X_i$, and the payment made to each of the other $N-1$ banks is also $\frac{1}{N}X_i$, and the total payment outflow is $\frac{N-1}{N}X_i$. The patter is symmetric for all other banks $j \neq i$.

Let n denote the subperiod in which bank i is chosen to make the payment. And let $k = 1, 2, ..., N$ denote the subperiod. The patter of payment flow is shown in Table 2.

Table 3 shows the accumulated flow of payments. For example, column 1 shows the result if bank i is chosen to make the payment in subperiod 1. In subperiod 1(row 1), the outflow of payment is $\frac{(N-1)X_i}{N}$. In each subperiod $k > 1$, bank i receives $\frac{X_j}{N}$. Similarly, in column 2, bank i receives $\frac{X_j}{N}$ in $k = 1$, makes the payment $\frac{(N-1)X_i}{N}$ in $k = 2$, and receives $\frac{X_j}{N}$ in each of the subperiods $k > 2$.

$$\begin{pmatrix} \text{Out} & \text{In} & \text{In} & \cdots \\ \text{In} & \text{Out} & \text{In} & \cdots \\ \text{In} & \text{In} & \text{Out} & \cdots \\ \text{In} & \text{In} & \text{In} & \cdots \\ \vdots & \vdots & \vdots & \ddots \\ \textit{Bank } 1 & \textit{Bank } 2 & \textit{Bank } 3 & \cdots \end{pmatrix}$$

Table 2: "Out" is payment outflow and "in" is payment inflow. Column n shows the result if a bank makes its payment in subperiod n. The bank receives payments from other banks for subperiods $k < n$ and $k > n$.

$$\begin{pmatrix} \frac{(N-1)X_i}{N} & -\frac{X_j}{N} & -\frac{X_j}{N} & \cdots & -\frac{X_j}{N} \\ \frac{(N-1)X_i}{N}-\frac{X_j}{N} & \frac{(N-1)X_i}{N}-\frac{X_j}{N} & -\frac{2X_j}{N} & \cdots & -\frac{2X_j}{N} \\ \frac{(N-1)X_i}{N}-\frac{2X_j}{N} & \frac{(N-1)X_i}{N}-\frac{2X_j}{N} & \frac{(N-1)X_i}{N}-\frac{2X_j}{N} & \cdots & -\frac{3X_j}{N} \\ \frac{(N-1)X_i}{N}-\frac{3X_j}{N} & \frac{(N-1)X_i}{N}-\frac{3X_j}{N} & \frac{(N-1)X_i}{N}-\frac{3X_j}{N} & \cdots & -\frac{4X_j}{N} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{(N-1)X_i}{N}-\frac{(N-2)X_j}{N} & \frac{(N-1)X_i}{N}-\frac{(N-2)X_j}{N} & \frac{(N-1)X_i}{N}-\frac{(N-2)X_j}{N} & \cdots & -\frac{(N-1)X_j}{N} \\ \frac{(N-1)X_i}{N}-\frac{(N-1)X_j}{N} & \frac{(N-1)X_i}{N}-\frac{(N-1)X_j}{N} & \frac{(N-1)X_i}{N}-\frac{(N-1)X_j}{N} & \cdots & \frac{(N-1)X_i}{N}-\frac{(N-1)X_j}{N} \end{pmatrix}$$

Table 3: Accumulated flow of payments. The accumulated flow in subperiod k(row k) is the total outflow minus the total inflow up to that subperiod. Column n shows the accumulated flow if bank i makes the payment in subperiod n.

Let $FL(k, n)$ denote the accumulated flow in row k column n. Banks are required to borrow from the central bank as long as $FL(k, n) > D_0$. Let $b(k, n)$ denote the central bank loan.

$$b(k, n) = \max(0, FL(k, n) - D_0) \tag{22}$$

The value of $FL(k, n)$ depends on X_i. Since $X_i = (1-\pi)D_0 + L_i$, higher L_i will lead to higher payment X_i. Given L_j, higher X_i has two effects: it will increase the level of $b(k, n)$ when $b(k, n)$ is positive, and it will also increase the probability for $b(k, n)$ to be positive.

Suppose bank i makes the payment in period n, we define the accumulated borrowing as

$$\bar{b}_n = \frac{1}{N}\Sigma_{k=1}^{N} b(k, n)$$

And the interest cost for central bank loan is $r^c \bar{b}_n$, where r^c is the central bank lending rate.

Before the settlement process starts, the expected future borrowing is defined as

$$Eb = \frac{1}{N}\Sigma_{n=1}^{N}\bar{b}_n$$

When N is large, we can get a closed-form solution for Eb, the result is as follows:

Proposition 3. *Suppose N is very large. Given L_j, when $Eb > 0$, it can be written as*

$$Eb(L_i) = \frac{1}{6}\frac{(X_i - X_j - D_0)^3}{X_j^2} + \frac{1}{2}\frac{(X_i - X_j - D_0)^2}{X_j} + \frac{1}{2}(X_i - X_j - D_0) + \frac{1}{6}X_j \tag{23}$$

In the symmetric case($L_i = L_j = L$), $Eb(L) > 0$ when $L > \pi D_0$(when $X > D_0$).

Proof: See the Appendix ■

In the symmetric case, the payment X is $(1-\pi)D_0 + L$. If $L > \pi D_0$, then $X > D_0$ and the payments will not be fully covered by reserves.

5.4.2 The loan supply curve

In this part, we derive the bank loan supply curve.

After all inter-bank payments in "settlement 2" are completed, bank i's deposit balance is

$$\begin{aligned} & \text{deposit after loan making} - \text{payment outflow } + \text{payment inflow} \\ = \quad & (D_0 + L_i) - \frac{N-1}{N}X_i + \frac{N-1}{N}X_j \approx (D_0 + L_i) - X_i + X_j = D_0 + L_j \end{aligned} \tag{24}$$

And the reserve balance is

$$\begin{aligned} & \text{initial reserve } - \text{payment outflow } + \text{payment inflow} \\ & = D_0 - \frac{N-1}{N}X_i + \frac{N-1}{N}X_j \approx D_0 - (X_i - X_j) = D_0 - (L_i - L_j) \end{aligned}$$

We assume that banks pay the interest of central bank loan $r^c\bar{b}_n$ at the end of "settlement 2". For simplicity, we focus on the case in which the reserve balance after paying the interest, $D_0 - (L_i - L_j) - r^c\bar{b}_n$, is positive. That is, D_0 is high enough to cover marginal increases in L_i and the interest cost $r^c\bar{b}_n$.(Since we focus on the symmetric case, $L_i - L_j$ means small marginal deviations of L_i from L_j.) So banks do not need to borrow any central bank loan after the settlement process is completed.

The profit of the bank is

$$\Pi = [D_0 - (L_i - L_j) - r^c\bar{b}_n] + L_i(R - \delta) - (1 + r^d)(D_0 + L_j) \tag{25}$$

The first term is the remaining reserves, the second term is the value of bank loan, and the third term is the gross payment to deposits.

The expected profit is

$$E\Pi = [D_0 - (L_i - L_j)] - r^c Eb + L_i(R - \delta) - (1 + r^d)(D_0 + L_j) \tag{26}$$

If no central bank loan is needed and $Eb = 0$, then (26) becomes

$$E\Pi = [D_0 - (L_i - L_j)] + L_i(R - \delta) - (1 + r^d)(D_0 + L_j) \tag{27}$$

In the equilibrium, bank i should not be able to increase the profit by changing L_i. The first order condition is

$$\frac{\partial E\Pi}{\partial L_i} = -1 + R - \delta = 0 \tag{28}$$

which gives

$$R = 1 + \delta \qquad (\textbf{ loan supply curve when Eb=0}) \tag{29}$$

This is the bank loan supply curve when liquidity shock is low and $Eb = 0$. In the symmetric case we have $L_i = L_j = L$. The deposit rate can be computed by applying the zero expected profit condition ($E\Pi = 0$) to equation (27), the result is $r^d = 0$.

If Eb is positive, then the first order condition is

$$\frac{\partial E\Pi}{\partial L_i} = -1 - r^c \frac{\partial Eb(L_i)}{\partial L_i} + (R - \delta) = 0 \tag{30}$$

Using (20) and (23), we have

$$\frac{\partial Eb(L_i)}{\partial L_i} = \frac{1}{2}\left(1 + \frac{X_i - X_j - D_0}{X_j}\right)^2 \tag{31}$$

and (30) becomes

$$R = 1 + \delta + r^c \frac{1}{2}\left(1 + \frac{X_i - X_j - D_0}{X_j}\right)^2$$

This is the loan supply curve of bank i given L_j. In the symmetric case($L_i = L_j = L$), the supply curve becomes:

$$\begin{aligned} R &= 1 + \delta + r^c \frac{1}{2}\left(1 - \frac{D_0}{X}\right)^2 \\ &= 1 + \delta + r^c \frac{1}{2}\left(1 - \frac{D_0}{(1-\pi)D_0 + L}\right)^2 \quad (\textbf{loan supply curve when Eb>0}) \end{aligned} \tag{32}$$

R is increasing in L. From (32), we can see that when $L > \pi D_0$, $\frac{D_0}{(1-\pi)D_0+L} < 1$ and so $R > 1 + \delta$. When L is very large, R approaches $1 + \delta + \frac{1}{2} r^c$.

5.5 The bank loan equilibrium

In this part, we derive the loan demand curve and then solve the bank loan equilibrium.

The demand curve for bank loan can be derived from the payout policy of the investment fund, which is

$$\pi = \frac{\text{Redemption}}{NV} = \frac{(1-\pi)D_0 + L(\pi)}{Z_k Q_k(\pi)} \tag{33}$$

And the market price for asset is

$$Q_k(\pi) = \frac{R_k}{R(\pi)} \tag{34}$$

where $R(\pi)$ is the bank lending rate when the liquidity shock is π. Substitute $Q_k(\pi)$ into equation (33) and we have the demand curve for bank loan

$$R(\pi) = \frac{\pi Z_k R_k}{(1-\pi)D_0 + L(\pi)} = \frac{\pi Z_k R_k}{X} \qquad \textbf{(loan demand curve)} \tag{35}$$

R is decreasing in L, so when the lending rate is lower, the borrowing is higher.

Bank loan equilibrium

When $Eb = 0$, the loan supply curve is $R = 1 + \delta$, and $Q_k = \frac{R_k}{1+\delta}$. Using (33), the equilibrium loan level is

$$\begin{aligned} L &= \text{Total redemption} - \text{cash raised from nonmovers} \\ &= \pi Z_k Q_k - (1-\pi)D_0 = \pi Z_k \frac{R_k}{1+\delta} - (1-\pi)D_0 = S\left[\pi(1-\omega)\frac{R_k}{1+\delta} - (1-\pi)\omega\right] \end{aligned} \tag{36}$$

When $Eb > 0$, (32) and (35) give the following result:

Proposition 4. *The equilibrium $L(\pi)$ and $R(\pi)$ are*

$$L^*(\pi) = S\left(\frac{r^c\omega + \pi(1-\omega)R_k + \sqrt{(r^c\omega + \pi(1-\omega)R_k)^2 - 2(1+\delta+\frac{r^c}{2})r^c\omega^2}}{2(1+\delta+\frac{r^c}{2})} - (1-\pi)\omega\right) \tag{37}$$

$$R^*(\pi) = \frac{2(1+\delta+\frac{r^c}{2})\pi(1-\omega)R_k}{r^c\omega + \pi(1-\omega)R_k + \sqrt{(r^c\omega + \pi(1-\omega)R_k)^2 - 2(1+\delta+\frac{r^c}{2})r^c\omega^2}} \tag{38}$$

Proof: Eliminating R from (32) and (35), we get a quadratic equation for X.

$$X^2\left(1+\delta+\frac{r^c}{2}\right) - X\left(\frac{r^c}{2}D_0 + \pi Z_k R_k\right) + \frac{r^c}{2}D_0^2 = 0$$

After we solve for X, we have $L^*(\pi) = X - (1-\pi)D_0$. $R^*(\pi)$ is decided according to (35). Finally, we can simplify the results using the relationship $D_0 = \omega S$ and $Z_k = (1-\omega)S$. ■

Given the equilibrium $L(\pi)$ and $R(\pi)$, we can solve for $Eb(\pi)$ from equation (23) by setting $L_i = L_j = L^*(\pi)$, and then solve for the equilibrium deposit rate $r^d(\pi)$ from equation (26) by setting $E\Pi = 0$. The result is as follows:

Proposition 5. *When $Eb \geq 0$, the expected central bank loan is*

$$Eb = \frac{-D_0^3}{6X^2} + \frac{D_0^2}{2X} - \frac{D_0}{2} + \frac{X}{6} \tag{39}$$

and the equilibrium deposit rate is[4]

$$r^d(\pi) = \frac{\frac{Lr^c}{2}(1-\frac{D_0}{X})^2 - r^c Eb}{D_0 + L} = \frac{\frac{Lr^c}{2}(1-\frac{D_0}{X})^2 - r^c(\frac{-D_0^3}{6X^2} + \frac{D_0^2}{2X} - \frac{D_0}{2} + \frac{X}{6})}{D_0 + L} \tag{40}$$

where $X = (1-\pi)D_0 + L$, and $L = L^*(\pi)$ is the equilibrium loan level.

We can see that if L is very high compared to D_0, then $X \approx L$ and $\frac{D_0}{X} \approx 0$, and we have $Eb \approx \frac{L}{6}$, and $r^d \approx \frac{r^c}{3}$.

Once the equilibrium loan level is decided, then the aggregate deposit level in the economy is also decided. The aggregate deposit(i.e, money supply) is $D_0 + L(\pi)$, where D_0 is the initial deposit balance, and $L(\pi)$ is the deposits that are created by banks during lending. Note that since banks can lend by creating new credit money, the aggregate bank loan level is elastic and is not directly limited by the monetary funds saved by depositors.(For example, L can certainly be higher than D_0.) Instead, it is the lending activities of banks that decide the aggregate deposits in the economy. By creating and lending out deposits $L(\pi)$, banks increase the aggregate money that people can use to make payments, and thus help to relax the aggregate financial constraint imposed by the existing money.

In most standard banking models, depositors are the people who provide liquidity to banks. It is usually assumed that the depositors have some idle cash that they will not use in the near future, and they lend those money to banks, and banks then lend those money to borrowers. In our model, the movers will end up holding all the deposits $D_0 + L$. So the final deposit holders are actually the people who need liquidity. It would be inappropriate to say that the movers provide liquidity to banks. In our model, the liquidity provider is the bank, it creates and lends out new deposit, which is then used by people to make payments.

A summary of the steps for solving the loan equilibrium

The steps for solving the loan equilibrium can be summarized as follows.

[4] We replace R in (26) using the loan supply curve (32).

First, we build a general equilibrium model for inter-bank settlement process, we use it to decide Eb and then derive the loan supply curve. Then we derive the bank loan demand curve using the optimal payout policy of the investment fund. Then we solve the equilibrium loan level L and the lending rate R.

Once L is solved, the aggregate deposit is also decided because banks create equivalent amount of deposit during lending, and so the total deposit is $D_0 + L$. Finally, we use the zero expected profit condition to solve for the deposit rate.

The distribution of Q_k: The distribution of Q_k is

$$Q_k(\pi) = \begin{cases} R_k & : \ \pi \leq \pi_1 = \frac{\omega}{\omega+(1-\omega)R_k} & (\text{Nonmovers' cash is not binding}) \\ \frac{\omega(1-\pi)}{\pi(1-\omega)} & : \ \pi_1 < \pi < \pi_2 = \frac{\omega}{\omega+(1-\omega)\frac{R_k}{1+\delta}} & (\text{Nonmovers' cash is binding}) \\ \frac{R_k}{1+\delta} & : \ \pi_2 \leq \pi \leq \pi_3 = \frac{\omega}{(1-\omega)\frac{R_k}{1+\delta}} & (\text{Bank loan L} > 0, \text{Eb} = 0) \\ \frac{R_k}{R(\pi)} & : \ \pi > \pi_3 & (\text{Bank loan L} > 0, \text{Eb} > 0) \end{cases} \tag{41}$$

The definition of π_1 is still the same as in the no-bank-loan case. Non-movers use all their deposits to buy assets if $\pi \geq \pi_1$. π_2 is the π above which the investment funds borrow positive bank loans. Below π_2, the investment funds only raise cash by selling assets on the financial market. The distribution of Q_k for $\pi < \pi_2$ is the same as when there is no bank lending(equation 15). For example, below π_1, the asset is sold for its fundamental value. Between π_1 and π_2, Q_k is lower than R_k, but since $Q_k > \frac{R_k}{1+\delta}$, it is not worthwhile for the investment funds to borrow from banks. At π_2, Q_k decreases to $\frac{R_k}{1+\delta}$, and the funds start to borrow from banks. π_3 is the level of π above which the expected central bank loan Eb is positive. So between π_2 and π_3, the lending rate is $R = 1 + \delta$(equation 29), and $Q_k = \frac{R_k}{1+\delta}$. Above π_3, $Eb > 0$, and $Q_k = \frac{R_k}{R(\pi)}$, where $R(\pi)$ is defined in (38).

π_2 and π_3 can be decided as follows. At π_2, $Q_k = \frac{R_k}{1+\delta}$, non-movers use all their deposits to buy assets and the bank loan is still zero, so we have

$$\pi_2 = \frac{\text{Redemption}}{Z_k Q_k} = \frac{(1-\pi_2)D_0}{Z_k \frac{R_k}{1+\delta}} \Rightarrow \pi_2 = \frac{\omega}{\omega + (1-\omega)\frac{R_k}{1+\delta}}$$

Since $Eb > 0$ when $L > \pi D_0$, at π_3, we have $L(\pi_3) = \pi_3 D_0$. Since $Q_k(\pi_3)$ is still $\frac{R_k}{1+\delta}$, we have

$$\pi_3 = \frac{\text{Redemption}}{Z_k Q_k} = \frac{(1-\pi_3)D_0 + L(\pi_3)}{Z_k \frac{R_k}{1+\delta}} = \frac{D_0}{Z_k \frac{R_k}{1+\delta}} = \frac{\omega}{(1-\omega)\frac{R_k}{1+\delta}} \tag{42}$$

The effects of the central bank lending rate r^c

Please note that r^c does not need to be constant, the central bank can actively change

r^c contingent on π. When r^c is a function of π, the solutions for $R(\pi)$ and $L(\pi)$ are still the same. r^c affects the equilibrium by affecting the loan supply curve. When $Eb = 0$, r^c does not affect the equilibrium. When $Eb > 0$, lower r^c reduces the slope of the supply curve (32). Since the demand curve is downward sloping and the supply curve is upward sloping, a lower slope of the supply curve will lead to higher $L^*(\pi)$ and lower $R^*(\pi)$.

5.6 The general equilibrium

Using v_m and v_n from (18) and (19), the expected utility for a representative household i is

$$\begin{aligned}
&EU_i = \\
&\frac{1}{2}\int_0^1 \left\{\pi \ln\left[(1+r^d)s\left(\omega_i + (1-\omega_i)Q_{k,H}\right)\right] + (1-\pi)\ln\left[s\frac{R_{k,H}}{Q_{k,H}}(\omega_i + (1-\omega_i)Q_{k,H})\right]\right\} dF(\pi) \\
&+\frac{1}{2}\int_0^1 \left\{\pi \ln\left[(1+r^d)s\left(\omega_i + (1-\omega_i)Q_{k,L}\right)\right] + (1-\pi)\ln\left[s\frac{R_{k,L}}{Q_{k,L}}(\omega_i + (1-\omega_i)Q_{k,L})\right]\right\} dF(\pi)
\end{aligned}$$

Because of the log utility function, $(1 + r^d)$ does not affect the first order condition for ω_i. So the first order condition is the same as equation (16).

The symmetric equilibrium can be defined as follows.

At the beginning of $t+1$, given ω, the return shock A and liquidity shock π determine Q_k according to (41). When $\pi < \pi_1$, the total cash used by non-movers to buy assets is equal to the redemption $\pi Z_k R_k$. For $\pi \geq \pi_1$, non-movers use all their initial deposit $(1 - \pi)D_0$ to buy assets. Bank loan is zero below π_2. Between $[\pi_2, \pi_3]$, L is (36), $R = 1 + \delta$ and $r^d = 0$. For $\pi > \pi_3$, L, R and r^d are decided according to (37), (38) and (40). Every bank makes the same amount of loan $L(\pi)$, charges the same R and offers the same r^d.

The consumption by movers is v_m, and the consumption by non-movers is v_n. Since Eb is also the average borrowing for all banks, the aggregate cost for central bank loan is $r^c Eb$.

At the end of t, given the expected distribution of $Q_k(\pi)$ and the interest rates in $t + 1$, households choose ω to maximize their expected utility. In the symmetric equilibrium, all households choose the same ω. Given ω, the aggregate investment is $(1 - \omega)S$. The goods sold to the old generation is ωS, and the price level is given by $\omega S = \frac{M}{P} = D_0$.

5.7 Endogenous money supply and the transmission mechanism of the central bank interest rate policy

In our model, the central bank can not directly control the money supply, the money supply is not decided exogenously according to the money multiplier, it is endogenously decided by the

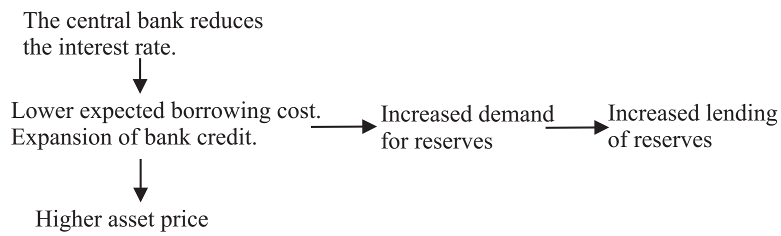


Figure 7: The transmission mechanism of the central bank interest rate policy

credit transactions of private agents. The loan supply and loan demand are decided together in the general equilibrium. The money supply is "endogenous" in the sense that it is greatly affected by the money demand. For example, when the demand for bank loan is low, the supply will also be low because banks can not lend when people do not want to borrow. And when the demand is high, the equilibrium supply will also tend to be high.

Although money supply is not directly controlled by the central bank, the central bank can still use the interest rate policy to affect the equilibrium loan level and the asset prices(See Figure 7). For example, when the central bank reduces the interest rate, the lower expected borrowing cost for settlement balance will encourage banks to make more loans, which will lead to higher loan level, lower lending rate and higher asset price. The higher loan level will lead to higher demand for settlement balance. The central bank meets the demand for loans at the promised interest rate. So the supply of reserve is decided by the demand for reserve at the targeted interest rate. Thus, the supply of reserve is also endogenous.

Note that the central bank does not need to lend outside money to banks first and then banks lend those money to the public. Instead, the central bank can directly use the interest rate policy to encourage banks to expand their credit, which leads to an expansion of bank inside money.

5.8 The detailed transaction steps

In order to provide a clear illustration of the model, in Appendix A.3, we list and explain the detailed steps for transactions and the resulted changes on the balance sheet of the bank. We track the details of the monetary flows among households, the investment fund, the commercial bank and the central bank.

5.9 Two related cases

In the above section, we analyze the case when people can use bank deposits to make payments. Here we compare it to two other cases. In the first case, banks can issue private banknotes and in the second case people can only use central bank currency to make payments.

5.9.1 Free private banknotes

In this part, we show that if banks can freely issue banknotes, then they will have higher ability in providing liquidity.

We assume that bank deposits can not be used across locations but private banknotes can be used. Banks are free to issue private banknotes. Each note can be redeemed into one unit of central bank money. We assume that people trust the banks and they are willing to accept banknotes as payments. More specifically, when an investment fund pays the movers, it simply withdraw banknotes from its bank and pay the movers, movers accept the banknotes. Movers also withdraw their initial deposits in the form of banknotes. Movers than carry those banknotes with them to the other location. After investment funds sell their goods, they use the received banknotes to repay the bank loan. The final settlement at the end of the period is the same as in the previous case(Figure 6), it is based on net balance and thus will not impose liquidity constraint to banks. All outstanding banknotes are redeemed into central bank money at the end of the period.

Compared to the previous case, the difference here is that when investment funds pay movers, movers accept those notes and carry them to the other location. No inter-bank payment flows happen in this step. As a result, banks have lower liquidity constraint.

The profit for an individual bank i is

$$D_0 + L_i(R - \delta) - (L_i + D_0) \tag{43}$$

The first D_0 is the reserve kept in the central bank. The first L_i is the loan level, and the last term L_i+D_0 is the total outstanding banknotes.(The investment fund withdraws $L_i+(1-\pi)D_0$, movers withdraw πD_0.) Here we assume no interest is paid for holding banknotes.

The first order condition for L_i gives

$$R = 1 + \delta \tag{44}$$

which is the same as (29). So when banknotes are allowed, banks are subject to lower liquidity constraint, and the credit supply will be more elastic.

5.9.2 Payment only with central bank money

In this part, we assume that people can only use central bank money to make payments across locations. Since movers will withdraw all their deposits $D_0 + L$, and the total initial central bank money is D_0, banks must borrow L from the central bank. Note that in this case, banks can only repay the central bank loan at the *end* of the period, while in Figure 6, the central bank loan is repaid at the end of "settlement 2". So the lending rate should be at least as high as r^c. We denote it as r_2^c. The lending rate for banks will be

$$R = 1 + \delta + r_2^c \tag{45}$$

We can see that bank credit will be more costly than if people can directly use bank deposits to make payments.

6 Numerical Results: $U = \ln c$

This section shows the a numerical example for $U = \ln c$. Note that this is not a calibration. The purpose of this example is to illustrate the intuition of the model.

6.1 Parameter values

Table 4: Values of the parameters

A_H	A_L	e_h	δ	r^c
1.21	0.85	1	0.03	0.03

Table 4 shows the parameter values. The return of the risky assets are $R_{k,H} = A_H$ and $R_{k,L} = A_L$. The expected return is $(A_H + A_L)/2 = 1.03$. A_H and A_L are chosen such that households will hold positive money balance even when all assets can be used to make payments. We want to show that even if there are no trading frictions, people can still hold money as a riskless asset. We set the household endowment e_h at 1, the loan management cost of banks δ at 3% and the central bank lending rate r^c at 3%.

We assume that the liquidity shock is distributed according to

$$\pi = 0.9\theta^a \tag{46}$$

where θ is uniform over $[0, 1]$. The highest liquidity shock is $\overline{\pi} = 0.9$. a is used to adjust the density of π. With higher a, the density of π will be more concentrated on low values and households will hold lower monetary balance. We use $a = 6$. At this level, D_0 is low enough and we can see clearly the effects when banks borrow from the central bank. The steps for computing the equilibrium are explained in the Appendix.

6.2 Results

Table 5 shows ω^* and the expected utility of households. Since we assume that the aggregate household wealth is 1, the aggregate real money balance $\frac{M}{P} = D_0$ is equal to ω^*.

Table 5: Numerical example

	ω^*	$E \ln c$(households)
(1) All assets can be used to make payments (section 3)	0.0476	0.0141
(2) Fixed money(section 4)	0.7013	0.0059
(3) With elastic inside money (section 5)		
(3a) Cental bank sets $r^c = 0.03$ for all levels of π	0.2460	0.0118989
(3b) CB sets $r^c = 0.02$ for $\pi \geq 0.5$. The policy is pre-announced	0.2438	0.0119093

Table 5 shows that households hold less money balance and more investment fund shares when liquidity supply is more elastic. The expected utility is the highest in case 1 when all assets can be used to make payments, and is the lowest in case 2 when money supply is fixed.[5]

ω is slightly lower in case 3b than in 3a. This is because in case 3b the central bank will set a lower interest rate for $\pi \geq 0.5$. The lower interest rate will lead to lower bank lending rate and higher asset prices. The difference in ω in these two cases is small because in our example, the density for $\pi \geq 0.5$ is small.

Figure 8 shows the market price of risky assets in case 2(equation 15).

Figure 9 shows the results for case 3a. Q_k is decided according to equation (41). We can see that the aggregate money supply $D_0 + L(\pi)$ is stochastic. Higher liquidity shock will lead to higher money supply. The figure also shows that the changes in central bank loan Eb is small compared to the changes in L. This implies that the private commercial banking system can meet the liquidity needs of the economy with only a very low need to borrow from the central bank.

Figure 10 compares Q_k in case 1 and case 3a using the same scale. We can see that the asset price is more stable with elastic inside money.

[5] The numerical results for cases in section 5.9 are not shown here. They are similar to case 3 with a different interest rate. For example, when banks can freely issue banknotes, the interest rate tends to be lower and the loan supply will be more elastic.

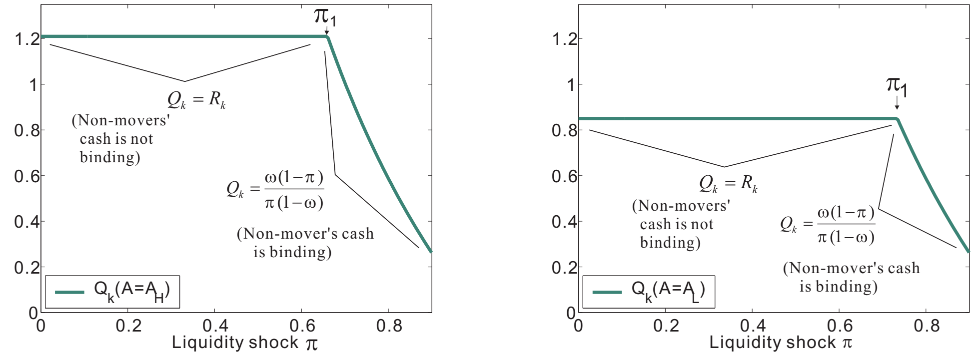


Figure 8: Q_k in case 2: no bank loan. Note that π_1 is different for A_H and A_L.

Figure 11 compares cases 3a and 3b for $A = A_H$(the result for $A = A_L$ is similar). In case 3b, since the central bank lending rate is lower for $\pi > 0.5$, the equilibrium lending rate and deposit rate are lower, and the asset prices are higher.

7 Results: the general case $\sigma \geq 1$

In this part, we show the results of the general case. We find that if aggregate liquidity is limited, the attempt of investment funds to provide more risk-sharing to shareholders may lead to higher volatility of asset prices without actually supplying more liquidity to shareholders. Inside money supplied by banks can help investment funds provide more liquidity to shareholders and perform their risk-sharing functions more effectively.

We will discuss some analytical results, and then show the numerical examples. We first consider the no-bank-lending case.

7.1 No bank lending

7.1.1 Some basic general equilibrium results

We first discuss some basic analytical results.

Proposition 6. *In the symmetric equilibrium, the distribution of v_m and v_n only depends on κ.*

Proof: For notational convenience, we set $S = 1$. First, given κ, the total wealth of the economy $\kappa + (1-\kappa)R_k$ is decided. In addition, π_1 only depends on κ. At π_1, $r_m = r_n = \alpha + (1-\alpha)R_k$, and we also know that the payment to movers is equal to the cash of the

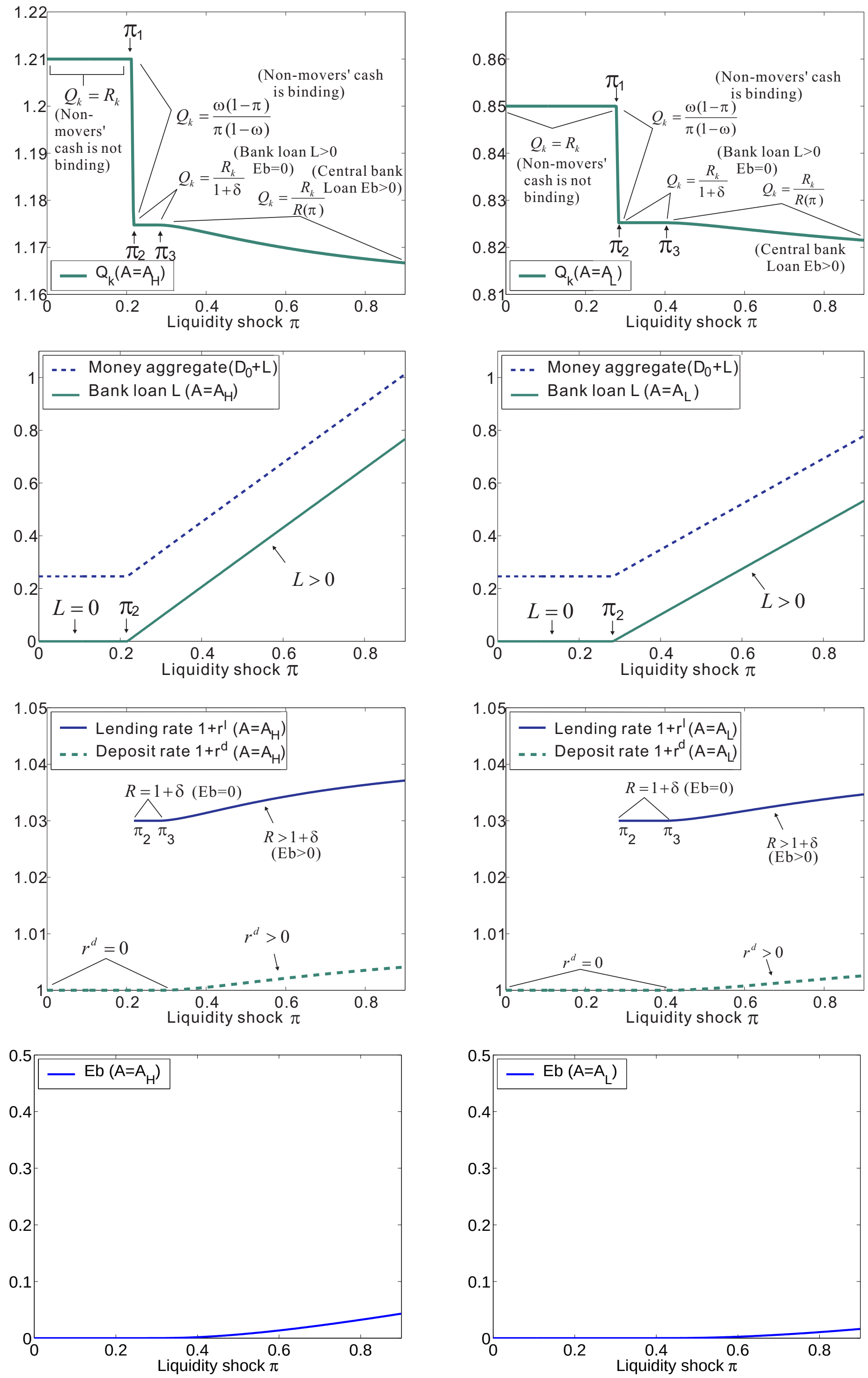


Figure 9: Q_k, L, R, r^d and Eb in case 3a(elastic money).

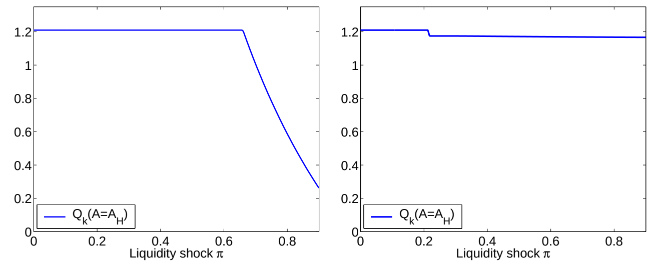


Figure 10: Compare Q_k in cases 1(no inside money) and 3a(with inside money). $A = A_H$

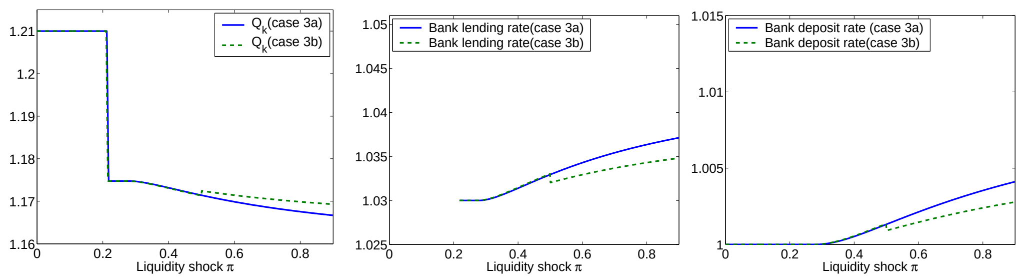


Figure 11: Compare cases 3a and 3b. $A = A_H$.

investment fund Z_f plus the cash collected from non-movers $(1-\pi)D^h$, so we have

$$\begin{aligned} & \pi_1 Z_p r_m = Z_f + (1-\pi_1)D^h \\ \Rightarrow \quad & \pi_1 = \frac{D^h + Z_f}{D^h + Z_p r_m} = \frac{\omega + (1-\omega)\alpha}{\omega + (1-\omega)(\alpha + (1-\alpha)R_k)} = \frac{\kappa}{\kappa + (1-\kappa)R_k} \end{aligned} \tag{47}$$

For $\pi \le \pi_1$, $v_m = v_n = \kappa + (1-\kappa)R_k$. For $\pi > \pi_1$, movers carry all the cash κ with them, which means the risky assets will become the wealth of non-movers.

$$\pi v_m = \kappa \quad \Rightarrow \quad v_m = \frac{\kappa}{\pi} \tag{48}$$

$$(1-\pi)v_n = (1-\kappa)R_k \quad \Rightarrow \quad v_n = \frac{(1-\kappa)R_k}{1-\pi} \tag{49}$$

Thus, the distribution of v_m and v_n only depends on κ.■

The intuition for (47) is as follows. Once the cash constraint is binding, movers can only carry all the cash in the economy κ with them to the other location. At π_1, each mover still receives the average wealth $\kappa + (1-\kappa)R_k$, so π_1 is the ratio between the aggregate cash κ and the aggregate wealth $\kappa + (1-\kappa)R_k$.

(48) and (49) imply that, when liquidations of assets lead to lower Q_k and r_n, the lower payment $(1-\omega)r_n$ will be exactly offset by the income earned by non-movers from purchasing assets on the financial market, and v_n will not be affected by Q_k and r_n. Note that this result

only applies to symmetric equilibrium. If different non-movers hold different levels of cash and investment fund shares, then there could be a wealth redistribution among non-movers.

Proposition 7. *In the symmetric case, if we fix the initial portfolio choice* ω *and* α*, then the distribution of* r_m *will be the same for different* σ*. But the distribution of* Q_k *will be different. Between* π_1 *and* π_{bind}*(the* π *above which the constraint* $r_m \leq r_n$ *is binding),* Q_k *is lower for higher* σ*.*

Proof: Once ω and α are given, then κ is given, and π_1 is uniquely decided. For $\pi \leq \pi_1$, we have $r_m = r_n = \alpha + (1-\alpha)R_k$. For $\pi > \pi_1$, all the cash owned by investment funds and non-movers are used to pay movers, and we have

$$\pi Z_p r_m = Z_f + (1-\pi)D^h \Rightarrow r_m = \frac{Z_f + (1-\pi)D^h}{\pi Z_p} = \frac{(1-\omega)\alpha + (1-\pi)\omega}{\pi(1-\omega)} \tag{50}$$

which is the same for different σ. We show in the Appendix that between π_1 and π_{bind}, we have

$$\frac{Q_k}{R_k} = \left(\frac{\kappa(1-\pi)}{R_k \pi(1-\kappa)}\right)^{\sigma} \tag{51}$$

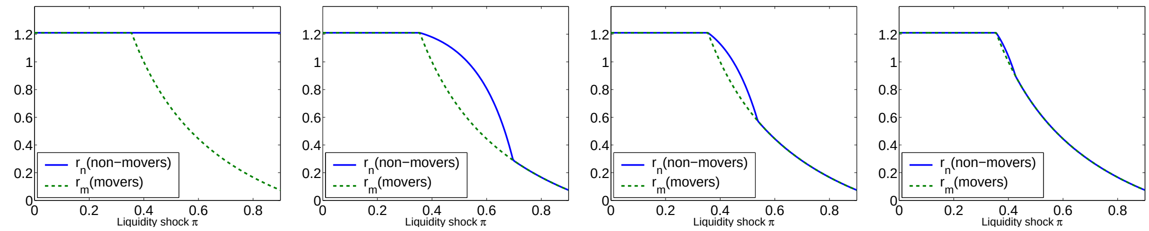
This ratio is equal to 1 at π_1. For $\pi > \pi_1$, the right-hand-side is lower than 1. So given κ, R_k and π, Q_k will be lower for higher σ.■

The intuition is as follows. Once aggregate liquidity is decided by ω and α, then the distribution of r_m will be uniquely decided. For $\pi > \pi_1$, given the same level of Q_k, the investment fund tends to set a higher r_m when σ is higher. But since the payment to movers is limited by the liquidity in the economy, it is the price Q_k that must be adjusted in order for the optimal payout policy to be satisfied. If σ is higher, then Q_k must decrease more in order to satisfy the optimal payout policy. On the microeconomic level, each investment fund takes the price Q_k as given and tries to liquidate assets to raise cash in order to provide liquidity insurance to movers. But if aggregate liquidity is limited, then the effort of investment funds is self-defeating, it will lead to more volatile asset prices, without actually providing more liquidity to movers. Numerical examples are shown below.

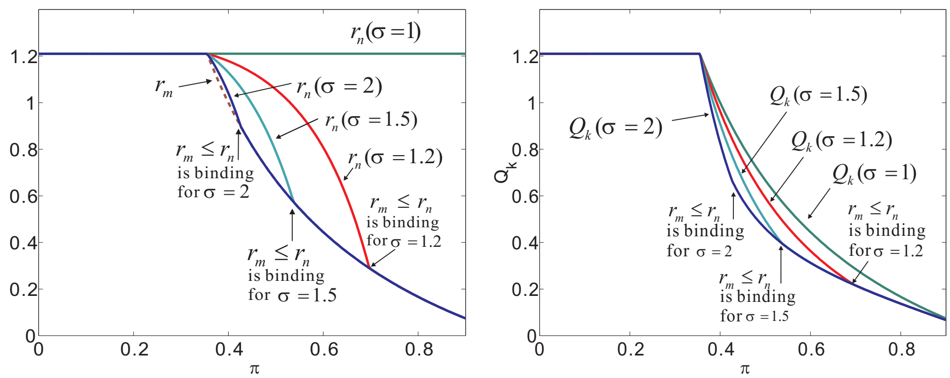
7.1.2 Numerical examples

When ω and α are taken as given

We first fix the initial portfolio choice ω and α and see how changes in σ can affect the result. We use the example $[\omega = 0.4, \alpha = 0]$(we set a low ω so we can see clearly what will happen when Q_k goes to low levels.)



(a) r_m and r_n: $\sigma = 1$ (b) r_m and r_n: $\sigma = 1.2$ (c) r_m and r_n: $\sigma = 1.5$ (d) r_m and r_n: $\sigma = 2$



(e) r_m and r_n (f) Asset price Q_k

Figure 12: Payout policy and Q_k when the initial portfolio is fixed at $[\omega = 0.4, \alpha = 0]$. The results for $A = A_H$ are shown here.

The results are shown in Figure 12. The main findings can be summarized as follows:

1. The distribution of r_m is the same for different σ.

2. When σ is higher, Q_k and r_n decrease more quickly between $[\pi, \pi_{bind}]$.

3. When the constraint $r_m \leq r_n$ is binding, r_m is still the same, but Q_k decreases more slowly.

The reasons for result 1 and 2 are already explained above. The reason for result 3 is as follows. When $r_m \leq r_n$ is binding, given Q_k, the payment to movers will be lower than the optimal payment if r_m were allowed to be higher than r_n. This will reduce the need for investment funds to liquidate assets. Thus, in the equilibrium, Q_k decreases more slowly. In this case, limiting the payment to movers does not really reduce the actual payment received by movers, but it helps stabilize the asset price.

When σ is higher, the investment fund will provide more risk-sharing among shareholders. This means there will be a smaller difference between r_n and r_m. When an individual fund

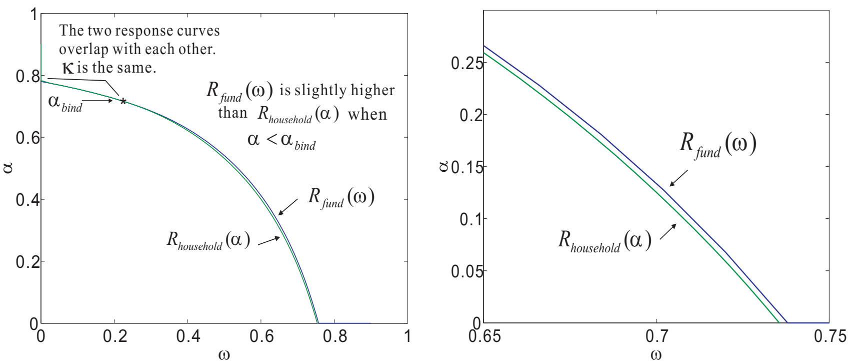


Figure 13: Response curves when $\sigma = 2$

attempts to provide more risk-sharing, it will take the market price as given and try to increase the level of r_m relative to r_n. But in the above example, the smaller difference between r_n and r_m is actually achieved solely by a *decrease* in r_n without actually increasing the level of r_m. We can see that when aggregate liquidity is limited, investment funds may not be able to perform the risk-sharing function effectively. The effort to provide more risk-sharing can lead to more volatile asset prices.

Note that the action of each investment fund is still optimal on the individual level. But their actions may lead to un-intended macroeconomic outcome and every fund may end up worse off. This is because every fund does not take into account the externality that its action may impose on the market price.

The shape of the response curves

Now, we assume that people can optimally choose ω and α. In order to decide the equilibrium, we first need to analyze the response curves of the investment fund and the household. Let $R_{household}(\alpha)$ denote the response curve of the household and $R_{fund}(\omega)$ the response curve of the investment fund.

The example for $\sigma = 2$ is shown in Figure 13. The main findings are as follows.

1. There exists a level of $\alpha = \alpha_{bind}$. When $\alpha < \alpha_{bind}$, the constraint $r_m \leq r_n$ is binding for positive probability.[6] We find that it will cause $R_{fund}(\omega)$ to be slightly higher than

[6] The result means that the constraint $r_m \leq r_n$ is less likely to be binding when α is higher. The intuition is as follows: For $\pi > \pi_1$, we have $v_n = \frac{(1-\kappa)R_k}{1-\pi}$. Also, $v_n = \omega\frac{R_k}{Q_k} + (1-\omega)r_n$. Between $[\pi_1, \pi_{bind}]$, v_n and Q_k only depend on κ. Given κ, if we have $\frac{R_k}{Q_k} > v_n$, then when ω is lower(α is higher), r_n needs to be higher in order to give the same v_n, and the constraint $r_m \leq r_n$ will less likely to be binding.

$R_{household}(\alpha)$. So when $\alpha < \alpha_{bind}$, people will reduce ω and increase α.[7]

2. For $\alpha \geq \alpha_{bind}$, the constraint $r_m \leq r_n$ is not binding. The two response curves overlap with each other. In addition, all the optimal pairs of $[\omega, \alpha]$ have the same value of κ.

Let ω_{bind} denote the ω when $\alpha = \alpha_{bind}$. The equilibriums are defined by the response curve for $[\omega \leq \omega_{bind}, \alpha \geq \alpha_{bind}]$. In the Appendix, we explain why the two response curves overlap with each other when $\alpha \geq \alpha_{bind}$, and why those equilibriums have the same κ. The basic reason is as follows: We know that both the investment fund and the household try to maximize the household's utility by taking the distribution of Q_k as given. We show that when Q_k is taken as given, the value of the portfolio v_m and v_n can be written as functions of κ.(See equations 82 and 83) Essentially, both the investment fund and the household try to choose the best κ. So given α, households will choose ω to get the best κ, and given ω, investment funds will not want to deviate from the original α. The reason that different equilibriums have the same κ is that as long as $\alpha \geq \alpha_{bind}$, the distribution of Q_k only depends on κ.(See equation 69)

For the log utility function, since $r_n = \frac{R_k}{Q_k} r_m$ and $\frac{R_k}{Q_k} \geq 1$, the constraint $r_m \leq r_n$ will never be binding, so the two response curves are the same.

The results when ω and α are chosen freely

Figure 14 shows the numerical result. For $\sigma = 1$, we use $[\omega = \kappa = 0.7013, \alpha = 0]$. For $\sigma = 1.5$, we use $[\kappa = 0.7521, \omega_{bind} = 0.3731, \alpha_{bind} = 0.6046]$. And for $\sigma = 2$, we use $[\kappa = 0.7800, \omega_{bind} = 0.2130, \alpha_{bind} = 0.7205]$. The findings can be summarized as follows:

1. For higher σ, people will choose higher κ and lower real investments.

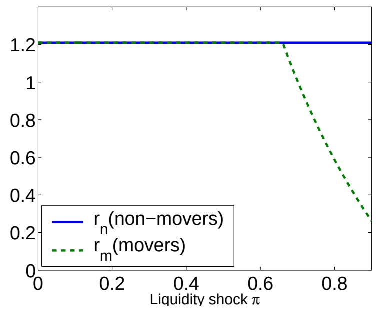
2. Higher κ makes it less likely for the cash constraint to be binding. But once the cash constraint is binding, then Q_k decreases more quickly for higher σ.

7.2 With bank lending

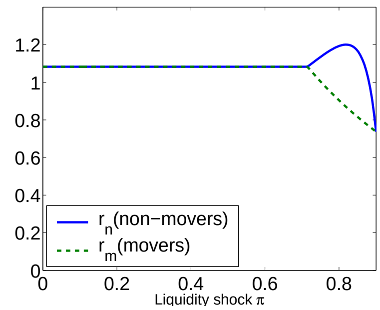
When ω and α are taken as given

We first consider a case in which we fix the initial portfolio at $[\omega = 0.2460, \alpha = 0]$(This is

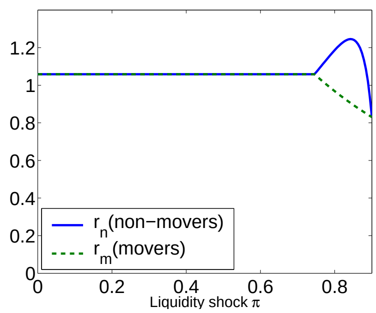
[7] For example, if we start with $\alpha_0 = 0$, then $\omega_0 = \omega(\alpha_0)$ chosen by the household is defined by $R_{household}(\alpha = 0)$. Because $R_{fund}(\omega)$ is slightly higher than $R_{household}(\alpha)$, the fund will deviate by choosing an α higher than α_0: $\alpha_1 = \alpha(\omega_0) > \alpha_0$. This will in turn cause households to to deviate by choosing a lower ω: $\omega(\alpha_1) < \omega_0$, and so on.



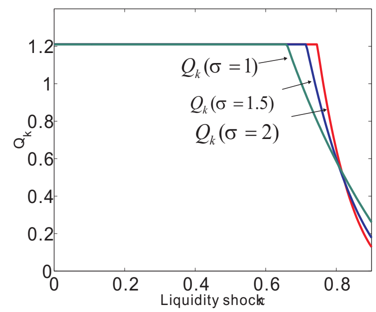
(a) r_m and r_n: $\sigma = 1$



(b) r_m and r_n: $\sigma = 1.5$



(c) r_m and r_n: $\sigma = 2$



(d) Q_k

Figure 14: Payout policy and Q_k when agents choose the optimal portfolio. The results for $A = A_H$ are shown here.

the optimal equilibrium for $\sigma = 1$.) The results are shown Figure 15. The findings can be summarized as follows:

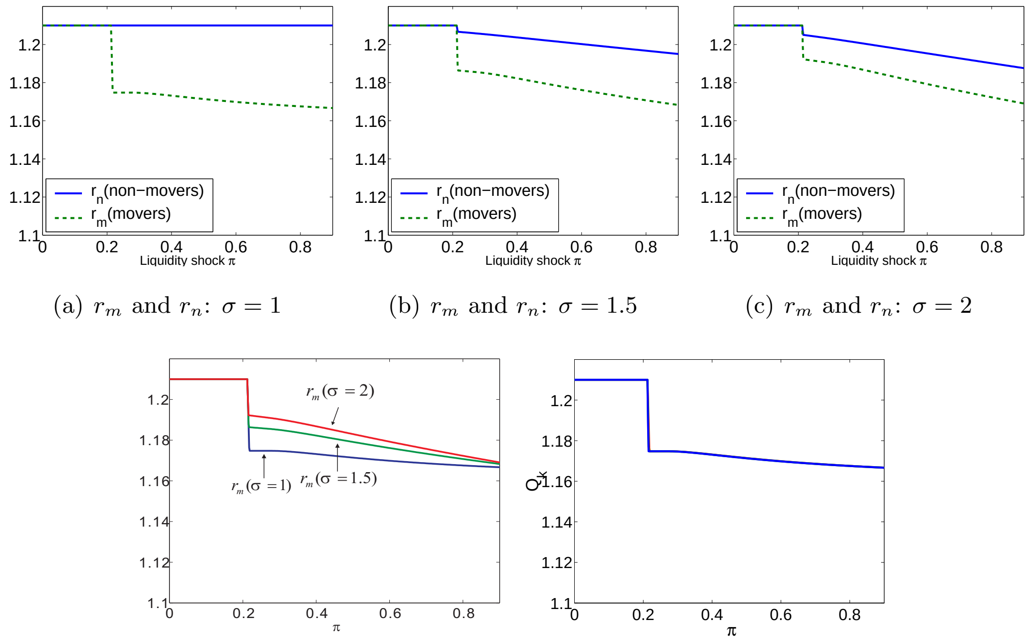
1. Different from Figure 12, in this case, the levels of r_m are no longer the same for different σ. With the help of banks, when σ is higher, investment funds can provide higher r_m to movers. Thus, investment funds can perform their function of risk-sharing more effectively.

2. r_m, r_n and the asset price Q_k are more stable compared to the no-bank-lending case.

The differences in Q_k for different σ are very small so that we can only see one curve for Q_k in the Figure. When σ is higher, because investment funds set a higher r_m and borrow more loans, the interest rate will be slightly higher and Q_k will be slightly lower.

When agents can choose ω and α freely

We find that with bank lending, the shape of the response curves are similar to that in Figure 13. If there exists $\alpha_{bind} > 0$, then for $\alpha < \alpha_{bind}$, $R_{fund}(\omega)$ will be slightly higher than



(a) r_m and r_n: $\sigma = 1$ (b) r_m and r_n: $\sigma = 1.5$ (c) r_m and r_n: $\sigma = 2$

(d) r_m (e) Q_k for σ = 1, 1.5 and 2.0

Figure 15: Payout policy and Q_k when the initial portfolio is fixed at $[\omega = 0.2460, \alpha = 0]$. $A = A_H$.

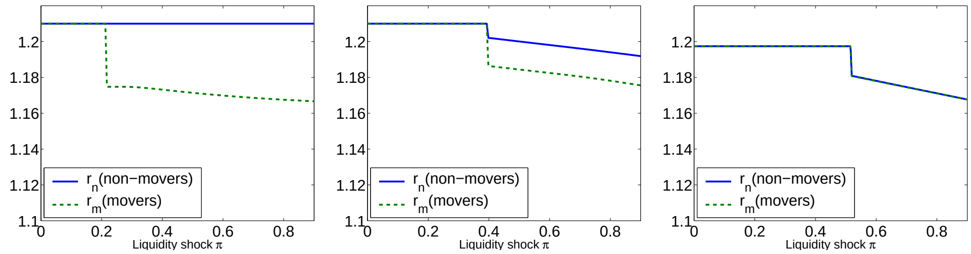
$R_{household}(\alpha)$. For $\alpha \geq \alpha_{bind}$, the two response curves overlap with each other. But in this case, different optimal pairs of ω and α will not give the same κ. We find that people choose higher κ when ω is lower (α is higher). The reason is that higher α increases the money that investment funds need to transfer to movers, this will cause higher interbank payment flows and higher borrowing of banks from the central bank. As a result, the lending rate tends to be higher and the asset price tends to be lower, which will cause people to hold more riskless assets. So the equilibrium with the highest ω(lowest α) is the most efficient equilibrium.

For $\sigma = 1$, the constraint $r_m \leq r_n$ is never binding. The constraint is not binding either for $\sigma = 1.5$ in this example. Figure 16 shows the result. For $\sigma = 1$, the optimal equilibrium is $[\omega = 0.2460, \alpha = 0]$. For $\sigma = 1.5$, it is $[\omega = 0.4394, \alpha = 0]$. And for $\sigma = 2.0$, it is $[\kappa = 0.5629, \omega_{bind} = 0.5350, \alpha_{bind} = 0.0600]$. The main result is that higher σ will lead to higher κ, so the cash constraint is less likely to be binding.

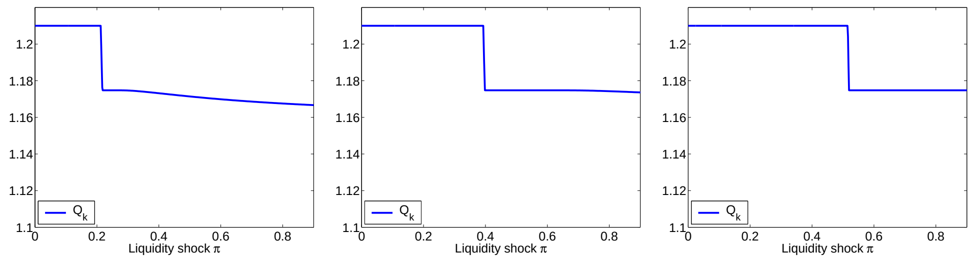
7.3 Implications

Our results have several implications.

Elasticity of liquidity: When liquidity needs take the form of demand for means of payment,



(a) r_m and r_n:$\sigma = 1$ (b) r_m and r_n:$\sigma = 1.5$ (c) r_m and r_n:$\sigma = 2$



(d) Q_k: $\sigma = 1$ (e) Q_k: $\sigma = 1.5$ (f) Q_k: $\sigma = 2$

Figure 16: Payout policy and Q_k in the optimal portfolio. For $\sigma = 2$, r_m and r_n are almost identical so we can only see one curve in the Figure. The results for $A = A_H$ are shown here.

it can partly be met with the creation of new inside money. Since the supply of new deposit is more elastic than the supply of real consumption goods, the liquidity supply in the real economy may be more elastic than those predicted by non-monetary models.

How to measure liquidity: Our results imply that the existing money aggregate may not be a good measure of the available "liquidity"(money) since new inside money can be elastically created to meet the demand for means of payment as long as people are willing to pay the borrowing cost. So if the banking system is working properly, then the interest rate may be a better measure of the availability of liquidity.

Aggregate money supply and the long-run price level: In our model, the aggregate money supply $D_0 + L(\pi)$ is stochastic. While the price level P is constant in each period. So the ratio between the aggregate money supply and the price level is not stable. In the model, inside money is created when banks make loans. But the borrowers are required to pay back those bank loans later. When bank loans are repaid, the deposits created during lending will be destroyed. So not all increases in aggregate money supply will lead to proportional changes in the long-run price level.

8 Some historical examples

In this section, we show some historical examples for the role played by banks in providing emergency liquidity. We will discuss three cases: the 1987 stock market crash, the financial market turmoil in fall 1998, and the crisis after Sept 11, 2001. The purpose is show that: First, banks performed important roles in providing emergency liquidity during those financial market turmoils; Second, bank loan supply is endogenous, lending is higher when there is a higher demand for liquidity; Third, expansion of inside money helps to meet liquidity needs.

8.1 1987 stock market crash

From the close of trading Tuesday, October 13, 1987 to the close of trading Money October 19, the Dow Jones Industrial Average declined by almost one third. Especially, on October 19, "Black Monday", the Dow Jones Industrial Average plunged 508 points, the largest one-day drop in history.

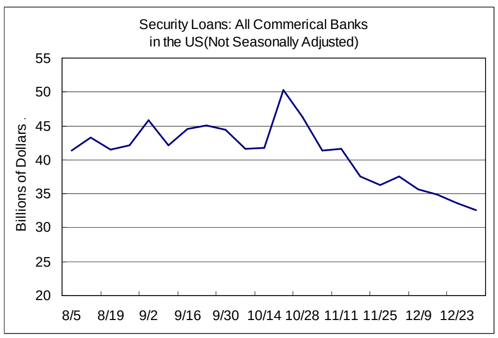


Figure 17: Security loans during the 1987 stock market crash.

Figure 17 shows the security loans made by commercial banks during the stock market crash. Security loans are loans to brokers and dealers and other loans for the purpose of purchasing and carrying securities. We can see that there was a large increase in leading during the crisis.[8]

8.2 LTCM crisis in 1998

In Autumn 1998, the default of Russian on its government bond and the near-collapse of the hedge fund "long-term capital management" led to volatile asset prices on the financial

[8]The data used in this section is collected from the Statistical Releases of the Board of Governors of the Federal Reserve System. Weekly data is used. See the Appendix for more details.

market.[9]

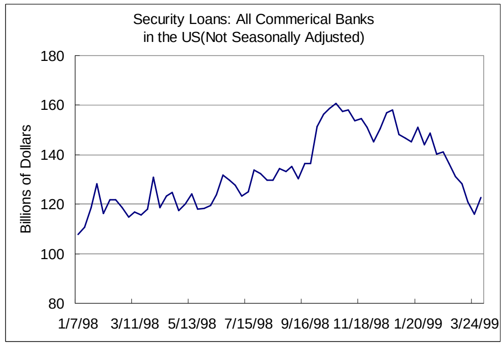


Figure 18: Security loans in 1998.

Figure 18 shows that during the financial market turmoils, there was a large increase in security loans. The loan level only started to decrease in early 1999.

The crisis on the financial market also made it more difficult for non-financial firms to raise liquidity. Figure 19 shows the yield spread between the three-month commercial paper and three-month Treasury Bills. We can see that the spread was clearly higher during that period, especially in October. We can use the spread to indirectly measure the difficulty for firms to raise liquidity through the financial market. Figure 19 also shows that there was a big increase in the level of commercial and industry loans when the spread was high. This is consistent with our model in the sense that banks supply more liquidity to firms when it is more difficult to raise liquidity from the financial market.

Figure 20 shows the total deposit and the sum of cash and government securities held by banks. We use the changes in "cash+government securities" to measure how much increase in the bank deposit was caused by flows of new funds into the banking system.[10]

The result implies that some of the increase in bank deposit was caused by flows of new funds into the banking system. But a large part of the increase was not caused by new funds.

[9]On August 17 1998, Russian devalued the rouble and defaulted on its government bond. The following "flight to liquidity" in the global financial market caused the yield spread of liquid and illiquid asset to increase. LTCM suffered huge losses due to high positions in illiquid assets. If the fund were allowed to fail and creditors started to liquidate the fund's asset, it may lead to lower market prices. In order to avoid a systemic crisis, the Fed helped organize a meeting of major banks on 23, September. The result was a $3.65 billion rescue package from leading U.S. investment and commercial banks. In exchange the participants received 90% of LTCM's equity. See Lowenstein(2000) for details.

[10]We add government security because some of the funds that flow into banks may be used by banks to buy government bonds from the central bank, so changes in cash may not fully reflect the flow of new funds into the bank system.

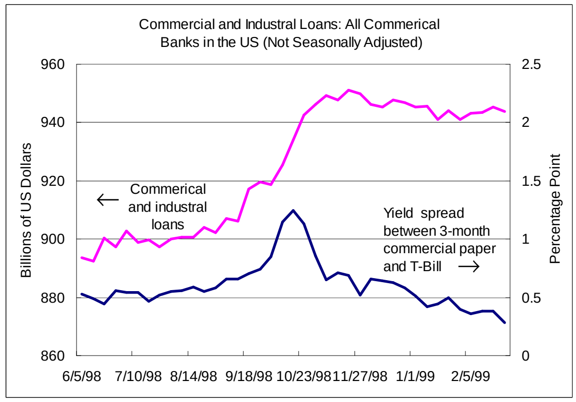


Figure 19: Yield spread and commercial and industrial loans. (Yield spread= yield for 3-month AA-rated non-financial-firm commercial paper − yield for 3-month Treasury Bills.)

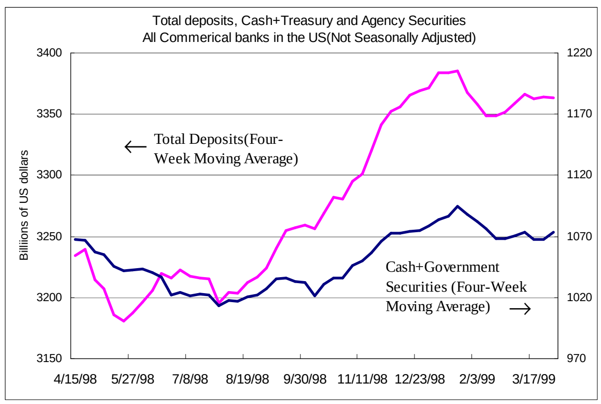


Figure 20: Total deposits and "cash+government security" held by banks.(Four-Week moving average in period t is the average of the variable from period $t-3$ to t.)

The previous results for bank loan imply that the remaining increase in bank deposit was largely the result of the increase in bank lending.[11] This is consistent with the argument of our model that the creation of new inside money can help to meet people's liquidity needs.

8.3 September 2001

From Figure 21, we can see that right after Sept 11th, 2001, banks provided emergency liquidity to the security industry.

[11] From July to December, security loans increased by about 40 billion dollars, and commercial and industrial loans increased by about 50 billion dollars. While the increase in deposit not explained by new funds was about 105 billion dollars.

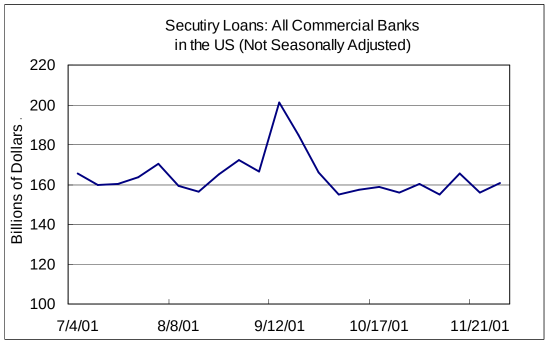


Figure 21: Security loans in September 2001.

9 Summary and conclusion

This paper analyzes the role of banks in providing liquidity to the financial market and the transmission of the financial liquidity channel of monetary policy. We explain why the roles of banks in the payment system give banks additional abilities to provide liquidity, and why the liquidity provision functions of banks are important to non-banks. The results can be summarized as follows:

1. We show that the private banking system has the ability to endogenously choose the optimal reserve level and supply loans elastically to meet the stochastic liquidity needs of the economy.

2. We compare banks and non-banks. The crucial difference between banks and non-banks is that the debt of banks is used as means of payment. Non-bank investment funds can only reallocate the existing liquidity, while banks can change the aggregate liquidity by creating and lending their own debt. Bank lending is not limited by the savings of depositors. As long as banks can meet the settlement requirement, they can create and lend new deposits which are not backed by money collected from depositors. So banks are not simply middle-man who transfers resources from depositors to borrowers. They can actively add more liquidity by creating and lending out new deposits. Since banks can lend their own debt, banks have some advantage in providing liquidity quickly in case of emergency. This is why banks are an important backup source of liquidity to the economy.

3. We show that banks are important to non-banks. We find that both the liquidity provision functions of banks and non-banks can help to meet people's liquidity needs.

But the ability of investment funds to provide liquidity is limited by the aggregate liquidity in the economy. The attempt of investment funds to provide more risk-sharing to shareholders may lead to higher volatility of asset prices without actually supplying more liquidity to shareholders. Inside money provided by banks can help investment funds to perform risk-sharing more effectively. With elastic bank money, people will also invest more in non-banks.

4. We also model the transmission of the central bank's policy. By cutting interest rates, the central bank can partly relax the constraints faced by banks, and banks will lend at lower lending rate, which will in turn help to support asset prices on the financial market. We show that under the interest rate policy, both money supply and reserve supply are endogenously decided.

Our results imply that banks will not be completely replaced by non-bank institutions. As long as payments need to be made with money, the market will create bank-like intermediaries to provide the service of elastic means of payment.

This paper is also related to an important question about liquidity provision: how the liquidity provision structure of the economy is determined? The answer is that it is largely determined by the structure of the payment system. Once the hierarchy of the payment system is decided, a corresponding liquidity provision structure emerges. Since agents at the lower levels of the payment system settle their payments with the debt issued by the institutions at the higher levels, the latter can provide liquidity to the former by creating and lending its debt. In reality, this liquidity provision will take the form of book-keeping.

Note that this paper is not about banks and non-banks *per se*. More generally, we can think of "banks" as institutions at the higher levels in the payment system and "non-banks" as those at the lower levels. The key idea is that the role played by institutions in the payment system can have important effects on the abilities of those institutions to provide liquidity. For example, we can extend the model to include multiple levels of banks. Some big banks directly settle on the book of the central bank while the remaining smaller banks settle on the books of the big banks. In this case, the big banks can provide liquidity to the small banks through book-keeping. Similarly, we can also extend the model to include different levels of non-banks. The different acceptability of their debt as means of payment may give them different abilities to provide liquidity.

Our model can be extended to analyze other issues in banking. In particular, many previous studies about the liquidity provision functions of banks and the design of the financial system are based on non-monetary models. We think it is important to re-evaluate some of the previous results using monetary models which explicitly model the payment system. It would be interesting to see whether the results are different under the new framework.

In this paper, we build a general equilibrium monetary model for banks, and apply the model to analyze the difference between banks and non-banks. We can also apply this model to other issues such as how inside money can affect consumption and real investments. In particular, in this paper, we analyze how elastic inside money can be good for the economy when we have liquidity shocks. It will be interesting to analyze how elastic inside money can also be bad for the economy by causing volatile output and inflations. We are currently working on this topic.

A

A.1 Main Notations

A: the productivity factor
H, L: aggregate return shocks
π: liquidity shocks
R_k: return(fundamental value) of risky assets
Q_k: market price of risky assets
Z_p, Z_f, Z_k: total portfolio, riskless assets and risky assets held by the investment fund
$s(S)$: individual(aggregate)savings of households
d: deposit of each household
ω: riskless assets in household's portfolio
α: riskless assets in the investment fund's portfolio
κ: riskless assets in the aggregate portfolio
$r^d(r^l)$: bank deposit(lending) rate
r^c: central bank lending rate
δ: management cost for bank loan
b: central bank loan
L: equilibrium loan level
π_1, π_2, π_3: when $\pi > \pi_1$, $Q_k < R_k$; when $\pi > \pi_2$, bank loan $L > 0$; when $\pi > \pi_3$, central bank loan $Eb > 0$.
X: payment outflow of the bank when investment funds pay movers
M: nominal level of outside money
P: nominal price level
D_0: initial real deposit balance
NV: market value of the investment fund
R: gross lending rate
N: the number of subperiods in the settlement process
n: the period in which bank i is chosen to make the payment

A.2 Data

The data used in section 8 is collected from the Statistical Releases of the Board of Governors of the Federal Reserve System. Weekly data is used. The data is available at: http://www.federalreserve.gov/releases. Assets and Liabilities of Commercial Banks are listed under "H.8". Selected interest rates are listed under "H.15".

Treasury and Agency Securities: b1003b; Commercial and Industrial Loans: b1021b; Security loans: b1030b; Cash Assets: b1048b; Total Deposits: b1058b; Yield for Three-Month

Table 6: Lending and settlement for a representative bank

Balance sheet of a representative commercial bank in location i (when $L > 0$)						
	Asset Side		Liability side			
	Reserve	Loan (Fund)	Deposit (movers)	Deposit (non-movers)	Deposit (Fund)	Bank Equity
Balance before the liquidity shock						
Balance	D_0	0	πD_0	$(1-\pi)D_0$	0	0
Changes in each account after the liquidity shock						
Investment fund sells assets to non-movers				$-(1-\pi)D_0$	$+(1-\pi)D_0$	
Bank makes loans to the investment fund		$+L$			$+L$	
Movers redeem fund shares			$+L+$ $(1-\pi)D_0$		$-L-$ $(1-\pi)D_0$	
Bank pays the borrowing cost	$-r^c\bar{b}_n$					$-r^c\bar{b}_n$
Changes in each account at the end of the Period						
The central bank consumes the interest	$+r^c\bar{b}_n$				$+r^c\bar{b}_n$	
Bank pays the deposit interest			$+r^d(D_0+L)$			$-r^d(D_0+L)$
Movers(i) buy goods in location j			$-(L+D_0)\times$ $(1+r^d)$			
Movers(j) buy goods in location i					$+(L+D_0)\times$ $(1+r^d)$	
Investment fund repays the bank loan		$-L$			$-L(1+r^l)$	$+Lr^l$
Bank spends the interest income					$+[Lr^l - r^c\bar{b}_n$ $-r^d(D_0+L)]$	$-[Lr^l - r^c\bar{b}_n$ $-r^d(D_0+L)]$
Final Balance						
Balance	D_0	0	0	0	D_0	0

Nonfinancial Firm Commercial Paper: H15_NFCP_M3; Yield for Three-Month Treasury Bills: H15_TB_M3.

A.3 The detailed transaction steps

In this part, we list and explain the transactions and the resulted changes on the balance sheet of the bank. In the model, we assume that the inter-bank settlement at the end of period $t+1$ is carried out based on net balance. But for illustrative purposes, we show the transactions one by one. In the balance sheet, we normalize the initial bank equity to zero. We put the interest costs and interest income of the bank under the entry "Bank Equity".

The initial deposit and reserve balance is D_0, πD_0 is held by movers and $(1-\pi)D_0$ by non-movers. After the shocks, the investment fund sells assets to non-movers and raises cash $(1-\pi)D_0$. The investment fund also borrows L from the bank. After the loan is made , 'loan" and "deposit" on the balance sheet increase by the same amount L, and the new deposit L is created. The fund then uses all its deposits $L + (1-\pi)D_0$ to meet the redemption needs of movers, and movers end up holding all the deposits $D_0 + L$. After the redemption process is completed, the bank pays the interest cost $r^c\bar{b}_n$ of the central bank loan.

At the end of the period, the central bank consumes the interest income by paying central bank money to the investment fund. The reserve balance of the bank and the deposit of the investment fund will increase by the same amount. Banks then pay deposit interest to movers. Movers then use their deposits to buy goods. When movers from location i buy goods in location j, their deposit balance decreases by $(D_0 + L)(1 + r^d)$. And when movers from location j buy goods from the investment fund in location i, the investment fund's deposit balance increases by $(D_0 + L)(1 + r^d)$.

The investment fund then repays the bank loan. The outstanding loan is reduced by L, and the outstanding deposit is reduced by $L(1 + r^l)$, and the interest income of the bank is Lr^l. The bank then spends the income $[Lr^l - r^c\bar{b}_n - r^d(D_0 + L)]$ to buy goods from the investment fund. Note that the income is usually positive because the lending rate r^l includes the management cost for bank loans. If δ is not too small, then Lr^l should be higher than the interest costs of the bank. In case where the income is negative, then the bank can sell its endowment to absorb the loss. When the bank spends the income, the bank makes the payment by increasing the deposit balance of the investment fund by the same amount. The increase in deposit (bank's liability) by $[Lr^l - r^c\bar{b}_n - r^d(D_0 + L)]$ reduces bank's equity by the same amount.

After all the above steps are completed, the investment fund will then transfer the deposit balance D_0 to non-movers, who will then use the deposit to purchase goods from the young generation.

B No bank lending: the general case $\sigma \geq 1$

B.1 The optimal payout policy of the investment fund

We first derive the optimal payout policy. We also prove proposition 1 for the log-utility function when we prove the following results for the general case.

Proposition 8. *If $Q_k = R_k$, then the optimal policy is to set $v_m = v_n$, and $r_m = r_n = \alpha + (1-\alpha)R_k$. When $Q_k < R_k$, if the constraint $r_m \leq r_n$ is not binding, then the optimal policy is to set*

$$\frac{v_m}{v_n} = \frac{\omega + (1-\omega)r_m}{\omega\frac{R_k}{Q_k} + (1-\omega)r_n} = \left(\frac{Q_k}{R_k}\right)^{\frac{1}{\sigma}} \tag{52}$$

Given ω and Q_k, $\frac{r_m}{r_n}$ is increasing in σ. If the constraint $r_m \leq r_n$ is binding, then the optimal policy is $r_m = r_n$. For the log utility function($\sigma = 1$), we have

$$\begin{aligned} r_m &= \alpha + (1-\alpha)Q_k \\ r_n &= \alpha\frac{R_k}{Q_k} + (1-\alpha)R_k = r_m\frac{R_k}{Q_k} \end{aligned}$$

■

The meaning of the proposition is as follows. First, as long as $Q_k = R_k$, there is no cost to raise cash by selling assets, and it is optimal to fully insure the liquidity risk and give movers and non-movers the same return. Second, if $Q_k < R_k$, since it is costly to raise cash, the investment fund may not provide full insurance. When people are more risk averse(higher σ), it is optimal to set a higher $\frac{r_m}{r_n}$, which means to give a higher payment to movers. When the constraint $r_m \leq r_n$ is binding, it is optimal to set $r_m = r_n$. For the log utility function, the optimal r_m is simply to pay the market value of the fund's asset.

Proof: We first analyze the case when the fund does not need to sell assets. The budget constraints are

$$\begin{aligned} \pi r_m &= \phi\alpha \\ (1-\pi)r_n &= (1-\phi)\alpha + (1-\alpha)R_k \end{aligned}$$

where $\phi\alpha$ is the riskless asset used to pay movers, $(1-\phi)\alpha$ is the unused riskless asset and $(1-\alpha)R_k$ is the value of the risky assets. Using the budget constraints to replace r_m and r_n in v_m and v_n (equation (7) and (8)), the fund's problem (10) can be written as (we eliminate the common "s" from v_m and v_n)

$$\pi\frac{(\omega + (1-\omega)\frac{\phi\alpha}{\pi})^{1-\sigma}}{1-\sigma} + (1-\pi)\frac{(\omega + (1-\omega)\frac{(1-\phi)\alpha+(1-\alpha)R_k}{1-\pi})^{1-\sigma}}{1-\sigma}$$

Here, we use $\frac{R_k}{Q_k} = 1$ since there is no liquidation of assets. Taking the derivative with respect to ϕ and simplifying the terms, we get

$$\frac{1}{(\omega + (1-\omega)\frac{\phi\alpha}{\pi})^{\sigma}} - \frac{1}{(\omega + (1-\omega)\frac{(1-\phi)\alpha+(1-\alpha)R_k}{1-\pi})^{\sigma}} = 0 \tag{53}$$

that is, $\frac{1}{v_m^\sigma} - \frac{1}{v_n^\sigma} = 0$, which means $v_m = v_n$ and $r_m = r_n = \alpha + (1-\alpha)R_k$.

Next, suppose the fund needs to sell assets. Let η denote the share of risky assets that is liquidated. The budget constraints are

$$\pi r_m = \alpha + (1-\alpha)\eta Q_k \tag{54}$$

$$(1-\pi)r_n = (1-\alpha)(1-\eta)R_k \tag{55}$$

The fund maximizes

$$\pi\frac{(\omega + (1-\omega)\frac{\alpha+(1-\alpha)\eta Q_k}{\pi})^{1-\sigma}}{1-\sigma} + (1-\pi)\frac{(\omega\frac{R_k}{Q_k} + (1-\omega)\frac{(1-\alpha)(1-\eta)R_k}{1-\pi})^{1-\sigma}}{1-\sigma} \tag{56}$$

Taking the derivative with respect to η and simplifying the terms, we get

$$\frac{Q_k}{(\omega + (1-\omega)\frac{\alpha+(1-\alpha)\eta Q_k}{\pi})^{\sigma}} - \frac{R_k}{(\omega\frac{R_k}{Q_k} + (1-\omega)\frac{(1-\alpha)(1-\eta)R_k}{1-\pi})^{\sigma}} = 0$$

which can be written as

$$\frac{\omega + (1-\omega)r_m}{\omega\frac{R_k}{Q_k} + (1-\omega)r_n} = \left(\frac{Q_k}{R_k}\right)^{\frac{1}{\sigma}} \tag{57}$$

When $R_k = Q_k$, we have $r_m = r_n$.

Now suppose $\frac{Q_k}{R_k} < 1$. When $\sigma > 1$, given $\frac{Q_k}{R_k}$, $\left(\frac{Q_k}{R_k}\right)^{\frac{1}{\sigma}}$ is increasing in σ. So higher σ will increase the level of r_m relative to r_n. That is, people share more liquidity risks when they are more risk averse. When $\sigma \to \infty$, $\left(\frac{Q_k}{R_k}\right)^{\frac{1}{\sigma}} \to 1$, and equation (57) would imply that $r_m > r_n$. Once the constraint $r_m \leq r_n$ is binding, the fund sets $r_m = r_n$.

When $\sigma = 1$, (57) becomes

$$\frac{\omega + (1-\omega) r_m}{\omega \frac{R_k}{Q_k} + (1-\omega) r_n} = \frac{Q_k}{R_k}$$

which gives $r_n = r_m \frac{R_k}{Q_k}$. Substitute this into (54) and (55) and we get $\eta = \pi - \frac{\alpha(1-\pi)}{(1-\alpha)Q_k}$, which gives $r_m = \alpha + (1-\alpha)Q_k$ and $r_n = \alpha \frac{R_k}{Q_k} + (1-\alpha)R_k$. ■

B.2 The values of r_m, r_n and Q_k in the symmetric equilibrium

In this part, we take the initial portfolio choice α and ω as given and solve for r_m, r_n and Q_k in the symmetric equilibrium.

We can separate π into three ranges: $[0, \pi_1]$, $[\pi_1, \pi_{bind}]$ and $[\pi_{bind}, \overline{\pi}]$. Non-movers' cash is binding for $\pi \geq \pi_1$. And for $\pi > \pi_{bind}$, the constraint $r_m \leq r_n$ is binding.

Below π_1, we have $Q_k = R_k$ and $r_m = r_n = \alpha + (1-\alpha)R_k$. At π_1, the payment to movers is equal to the cash collected from non-movers plus the cash held by the fund, and we have

$$\pi_1 Z_p r_m = Z_f + (1-\pi_1) D^h \tag{58}$$

$$\Rightarrow \quad \pi_1 = \frac{D^h + Z_f}{D^h + Z_p r_m} = \frac{\omega + (1-\omega)\alpha}{\omega + (1-\omega)(\alpha + (1-\alpha)R_k)} = \frac{\kappa}{\kappa + (1-\kappa)R_k} \tag{59}$$

So π_1 only depends on κ.

For $\pi > \pi_1$, in the symmetric equilibrium, we have

$$\pi Z_p r_m = Z_f + (1-\pi) D^h \Rightarrow r_m = \frac{Z_f + (1-\pi)D^h}{\pi Z_p} = \frac{(1-\omega)\alpha + (1-\pi)\omega}{\pi(1-\omega)} \tag{60}$$

Having' solved r_m, we can use (54), (55) and (57) to solve for equilibrium Q_k and r_n.

Between $[\pi_1, \pi_{bind}]$, the constraint $r_m \leq r_n$ is not binding. Using (54) and (55), we can write r_n as a function of r_m and Q_k

$$r_n = \frac{(1-\alpha)R_k}{1-\pi} - \frac{R_k(\pi r_m - \alpha)}{Q_k(1-\pi)} = \frac{(1-\alpha)R_k}{1-\pi} - \frac{R_k}{Q_k}\frac{\omega}{1-\omega} \tag{61}$$

Then substitute r_m(60) and r_n(61) into (57), we have

$$\frac{\omega + (1-\omega)\frac{(1-\omega)\alpha + (1-\pi)\omega}{\pi(1-\omega)}}{\omega \frac{R_k}{Q_k} + (1-\omega)\left(\frac{(1-\alpha)R_k}{1-\pi} - \frac{R_k}{Q_k}\frac{\omega}{1-\omega}\right)} = \left(\frac{Q_k}{R_k}\right)^{\frac{1}{\sigma}} \tag{62}$$

Arranging terms, we get

$$\frac{\omega + (1-\omega)\alpha}{\pi} = \left(\frac{Q_k}{R_k}\right)^{\frac{1}{\sigma}} \frac{(1-\omega)(1-\alpha)R_k}{1-\pi} \Longrightarrow Q_k = R_k^{1-\sigma}\left(\frac{\kappa(1-\pi)}{\pi(1-\kappa)}\right)^{\sigma} \tag{63}$$

Substitute Q_k back to (61) and we get the solution for r_n

$$r_n = \frac{(1-\alpha)R_k}{1-\pi} - \left(\frac{R_k\pi(1-\kappa)}{\kappa(1-\pi)}\right)^{\sigma} \frac{\omega}{1-\omega} \tag{64}$$

Also, from (63), we have

$$\frac{Q_k}{R_k} = \left(\frac{\kappa(1-\pi)}{R_k\pi(1-\kappa)}\right)^{\sigma} \tag{65}$$

This ratio is equal to 1 at π_1. For $\pi > \pi_1$, the RHS is lower than 1. So given κ, R_k and π, Q_k will be lower for higher σ.

At π_{bind}, we have $r_m = r_n$. Using (60) and (64), we have

$$r_m = r_n \Longrightarrow \frac{(1-\omega)\alpha + (1-\pi)\omega}{\pi(1-\omega)} = \frac{(1-\alpha)R_k}{1-\pi} - \left(\frac{R_k\pi(1-\kappa)}{\kappa(1-\pi)}\right)^{\sigma} \frac{\omega}{1-\omega} \tag{66}$$

This equation implicitly defines π_{bind}.

Above π_{bind}, r_m is still (60), and we have

$$r_m = r_n = \frac{(1-\omega)\alpha + (1-\pi)\omega}{\pi(1-\omega)} \tag{67}$$

And using the budget constraints (54) and (55), we get

$$Q_k = \frac{(1-\pi)\omega}{(1-\omega)(1-\alpha) - \frac{(1-\pi)((1-\omega)\alpha + (1-\pi)\omega)}{\pi R_k}} \tag{68}$$

So the distribution for Q_k is

$$Q_k(\pi) = \begin{cases} R_k & : \quad \pi \leq \pi_1 = \frac{\kappa}{\kappa + (1-\kappa)R_k} \\ R_k^{1-\sigma}\left(\frac{\kappa(1-\pi)}{\pi(1-\kappa)}\right)^{\sigma} & : \quad \pi_1 < \pi \leq \pi_{bind} \\ \text{equation 68} & : \quad \pi_1 > \pi_{bind} \end{cases} \tag{69}$$

We can see that if the constraint $r_m \leq r_n$ is not binding, then the distribution of Q_k only depends on κ. This can be seen from (68) where Q_k for $\pi < \pi_{bind}$ only depends on κ(remember that π_1(59) only depends on κ).

For the log utility function, $r_n = \frac{R_k}{Q_k} r_m$, so the constraint $r_m \leq r_n$ is never binding. As a result, under the log utility function, the distribution of Q_k only depends on κ.

B.3 The first order conditions for ω and α

This part derives the first order conditions for the representative household and the investment fund. When deciding the optimal choice, the representative household and the investment fund will take the choices of other agents and the distribution of Q_k as given.

For notational convenience, we set $s = 1$, so $v_m = \omega + (1-\omega)r_m$ and $v_n = \omega\frac{R_k}{Q_k} + (1-\omega)r_n$. The expected utility is

$$EU = \frac{1}{2}\int_0^1 \left[\pi\frac{(v_{m,H})^{1-\sigma}}{1-\sigma} + (1-\pi)\frac{(v_{n,H})^{1-\sigma}}{1-\sigma} + \pi\frac{(v_{m,L})^{1-\sigma}}{1-\sigma} + (1-\pi)\frac{(v_{n,L})^{1-\sigma}}{1-\sigma}\right] dF(\pi) \quad (70)$$

And the first order condition for ω is

$$\begin{aligned}\frac{\partial EU}{\partial \omega} &= \frac{1}{2}\int_0^1 \frac{\pi(1-r_{m,H})}{(\omega+(1-\omega)r_{m,H})^\sigma} + \frac{(1-\pi)(\frac{R_{k,H}}{Q_{k,H}} - r_{n,H})}{(\omega\frac{R_{k,H}}{Q_{k,H}} + (1-\omega)r_{n,H})^\sigma} dF(\pi) \\ &+ \frac{1}{2}\int_0^1 \frac{\pi(1-r_{m,L})}{(\omega+(1-\omega)r_{m,L})^\sigma} + \frac{(1-\pi)(\frac{R_{k,L}}{Q_{k,L}} - r_{n,L})}{(\omega\frac{R_{k,L}}{Q_{k,L}} + (1-\omega)r_{n,L})^\sigma} dF(\pi) \quad (71)\end{aligned}$$

The first order condition for α is

$$\frac{\partial EU}{\partial \alpha} = \frac{(1-\omega)}{2}\int_0^1 \pi\frac{\frac{\partial r_{m,H}}{\partial\alpha}}{v_{m,H}^\sigma} + (1-\pi)\frac{\frac{\partial r_{n,H}}{\partial\alpha}}{v_{n,H}^\sigma} + \pi\frac{\frac{\partial r_{m,L}}{\partial\alpha}}{v_{m,L}^\sigma} + (1-\pi)\frac{\frac{\partial r_{n,L}}{\partial\alpha}}{v_{n,L}^\sigma} dF(\pi) \quad (72)$$

We still need to decide $\frac{\partial r_m}{\partial \alpha}$ and $\frac{\partial r_n}{\partial \alpha}$. When $\pi \leq \pi_1$, since $r_m = r_n = \alpha + (1-\alpha)R_k$, we have

$$\frac{\partial r_m}{\partial \alpha} = \frac{\partial r_n}{\partial \alpha} = 1 - R_k \quad (73)$$

For $\pi > \pi_1$, we first need to solve for r_m and r_n by taken Q_k as given.

First, for $\pi \in [\pi_1, \pi_{bind}]$, using (54) and (55), we can write r_m as $\frac{\alpha+(1-\alpha)\eta Q_k}{\pi}$ and r_n as $\frac{(1-\alpha)(1-\eta)R_k}{1-\pi}$. Substituting them into (57) and arranging terms, we get

$$\eta = \frac{-\omega - (1-\omega)\frac{\alpha}{\pi} + (\frac{Q_k}{R_k})^{\frac{1}{\sigma}}\left[\omega\frac{R_k}{Q_k} + \frac{(1-\omega)(1-\alpha)R_k}{1-\pi}\right]}{\frac{(1-\omega)(1-\alpha)Q_k}{\pi} + (\frac{Q_k}{R_k})^{\frac{1}{\sigma}}\frac{(1-\omega)(1-\alpha)R_k}{1-\pi}} \quad (74)$$

Substitute η into (54) and (55) and we get

$$r_m = \frac{1}{\pi}(\alpha + Q_k(1-\alpha)\eta) = \frac{1}{\pi}\left(\alpha + Q_k\frac{-\omega-(1-\omega)\frac{\alpha}{\pi} + (\frac{Q_k}{R_k})^{\frac{1}{\sigma}}\left[\omega\frac{R_k}{Q_k} + \frac{(1-\omega)(1-\alpha)R_k}{1-\pi}\right]}{\frac{(1-\omega)Q_k}{\pi} + (\frac{Q_k}{R_k})^{\frac{1}{\sigma}}\frac{(1-\omega)R_k}{1-\pi}}\right) \quad (75)$$

$$\begin{aligned} r_n &= \frac{R_k(1-\alpha)(1-\eta)}{1-\pi} \\ &= \frac{R_k}{1-\pi}\left((1-\alpha) - \frac{-\omega-(1-\omega)\frac{\alpha}{\pi} + (\frac{Q_k}{R_k})^{\frac{1}{\sigma}}\left[\omega\frac{R_k}{Q_k} + \frac{(1-\omega)(1-\alpha)R_k}{1-\pi}\right]}{\frac{(1-\omega)Q_k}{\pi} + (\frac{Q_k}{R_k})^{\frac{1}{\sigma}}\frac{(1-\omega)R_k}{1-\pi}}\right) \quad (76)\end{aligned}$$

And so

$$\frac{\partial r_m}{\partial \alpha} = \frac{1}{\pi}\left(1 + Q_k\left(\frac{-(1-\pi) - \pi(\frac{Q_k}{R_k})^{\frac{1}{\sigma}}R_k}{(1-\pi)Q_k + \pi(\frac{Q_k}{R_k})^{\frac{1}{\sigma}}R_k}\right)\right) \tag{77}$$

$$\frac{\partial r_n}{\partial \alpha} = \frac{R_k}{1-\pi}\left(-1 - \left(\frac{-(1-\pi) - \pi(\frac{Q_k}{R_k})^{\frac{1}{\sigma}}R_k}{(1-\pi)Q_k + \pi(\frac{Q_k}{R_k})^{\frac{1}{\sigma}}R_k}\right)\right) \tag{78}$$

In the symmetric equilibrium, $(\frac{Q_k}{R_k})^{\frac{1}{\sigma}} = \frac{\kappa(1-\pi)}{R_k\pi(1-\kappa)}$(equation 65). Rearranging terms, we get

$$\begin{aligned}\frac{\partial r_m}{\partial \alpha} &= \frac{1}{\pi}\frac{\kappa(1-Q_k)}{\kappa + (1-\kappa)Q_k} \\ \frac{\partial r_n}{\partial \alpha} &= \frac{R_k}{1-\pi}\frac{(1-\kappa)(1-Q_k)}{\kappa + (1-\kappa)Q_k}\end{aligned}$$

For $\pi > \pi_{bind}$, since $r_m = r_n$, using (54) and (55) and taking Q_k as given, we get

$$r_m = r_n = \frac{R_k(\alpha + (1-\alpha)Q_k)}{(1-\pi)Q_k + \pi R_k} \tag{79}$$

and we have

$$\frac{\partial r_m}{\partial \alpha} = \frac{\partial r_n}{\partial \alpha} = \frac{R_k(1-Q_k)}{(1-\pi)Q_k + \pi R_k} \tag{80}$$

B.4 The equilibrium when $r_m \leq r_n$ is not binding

This part considers the features of the equilibriums when the constraint $r_m \leq r_n$ is not binding. We have the following result: First, the response curves of the household and the investment fund overlap with each other. Second, the equilibrium is defined by κ. As long as κ is equal to the equilibrium κ, then people can choose different combinations of $[\omega, \alpha]$.

The response curves

First, we explain why the response curves overlap with each other when the constraint $r_m \leq r_n$ is not binding. The response curves are simply the first order conditions of ω and α(equation 70 and 72). Denote the response curve of the household and the investment fund as $R_{household}(\alpha)$ and $R_{fund}(\omega)$. Let $\omega(\alpha)$ denote the optimal choice of the household by taken α as given and $\alpha(\omega)$ the optimal choice of the investment fund by taking ω as given. Then on the response curves, we have $\alpha(\omega(\alpha_0)) = \alpha_0$ and $\omega(\alpha(\omega_0)) = \omega_0$. The reason is that given the distribution of Q_k, the portfolio of movers and non-movers can be written as functions of κ. So when the investment fund chooses the best α given ω, or when the household chooses ω given α, they essentially choose the best κ.

The portfolio of movers is $v_m = \omega + (1-\omega)r_m$ and the portfolio of non-movers is $v_n = \omega\frac{R_k}{Q_k} + (1-\omega)r_m$. For $\pi \leq \pi_1$, $r_m = r_n = \alpha + (1-\alpha)R_k$, and so

$$v_m = v_n = \omega + (1-\omega)(\alpha + (1-\alpha)R_k) = \kappa + (1-\kappa)R_k \tag{81}$$

For $\pi > \pi_1$, when Q_k is given, the solutions for r_m and r_n are (75) and (76). After some arrangement of equations, we get

$$v_m = \omega + (1-\omega) r_m = \frac{(\frac{Q_k}{R_k})^{\frac{1}{\sigma}} \frac{R_k}{Q_k}(\kappa + (1-\kappa)Q_k)}{(1-\pi) + \pi(\frac{Q_k}{R_k})^{\frac{1}{\sigma}} \frac{R_k}{Q_k}} \tag{82}$$

$$v_n = \omega \frac{R_k}{Q_k} + (1-\omega) r_m = \frac{\frac{R_k}{Q_k}(\kappa + (1-\kappa)Q_k)}{(1-\pi) + \pi(\frac{Q_k}{R_k})^{\frac{1}{\sigma}} \frac{R_k}{Q_k}} \tag{83}$$

So given Q_k, v_m and v_n can be written as functions of κ.

The equilibrium κ
Given the equilibrium level of κ, different combinations of $[\omega, \alpha]$ which give the same κ will also be the equilibrium.

We've already shown that in the symmetric equilibrium, when the constraint $r_m \leq r_n$ is not binding, π_1 and Q_k only depend on κ. From the previous analysis, we know that households and investment funds try to maximize EU given v_m and v_n specified in (82) and (83). We can see that v_m and v_n only depend on π_1 and Q_k. In the equilibrium, given Q_k, households and investment funds would find that the equilibrium κ is optimal. And so if we keep the same κ but change the combination of ω and α, then since Q_k does not change, households and investment funds will still choose the same κ because they still face the same portfolio choice problem.

C With bank lending: the general case $\sigma \geq 1$

C.1 The optimal payout policy when bank loan is allowed

The optimal policy of the investment fund is as follows:

Proposition 9. *If $Q_k = R_k$, then it is optimal to set $v_m = v_n$ and $r_m = r_n = \alpha + (1-\alpha)R_k$. When $Q_k < R_k$, if the constraint $r_m \leq r_n$ is not binding, then the optimal policy is to set*

$$\frac{v_m}{v_n} = \frac{(\omega + (1-\omega)r_m)(1+r^d)}{\omega \frac{R_k}{Q_k} + (1-\omega) r_n} = \left(\frac{Q_k(1+r^d)}{R_k}\right)^{\frac{1}{\sigma}} \tag{84}$$

Given ω, Q_k and r^d, $\frac{r_m}{r_n}$ is increasing in σ. If the constraint $r_m \leq r_n$ is binding, then the optimal policy is $r_m = r_n$. For the log utility function($\sigma = 1$), the optimal policy is still $r_m = \alpha + (1-\alpha)Q_k$ and $r_n = \alpha \frac{R_k}{Q_k} + (1-\alpha)R_k$.

Proof: When $Q_k = R_k$, there is no bank borrowing and $r^d = 0$. The problem is the same as in Proposition 1.

Let η_1 denote the share of assets sold on the financial market and let η_2 denote the share of assets used as collateral to borrow from banks. Define $\eta = \eta_1 + \eta_2$. When $Q_k < R_k$, the budget constraint is

$$\pi r_m = \alpha + (1-\alpha)\eta_1 Q_k + (1-\alpha)\eta_2 Q_k = \alpha + (1-\alpha)\eta Q_k \tag{85}$$

$$(1-\pi) r_n = (1-\alpha)(1-\eta)R_k \tag{86}$$

Set $s = 1$. We have

$$v_m = [\omega + (1-\omega) r_m] (1 + r^d) \tag{87}$$
$$v_n = \omega \frac{R_k}{Q_k} + (1-\omega) r_n \tag{88}$$

and the fund's problem (10) becomes

$$\pi \frac{\left[(\omega + (1-\omega) \frac{\alpha + (1-\alpha)\eta Q_k}{\pi})(1 + r^d) \right]^{1-\sigma}}{1-\sigma} + (1-\pi) \frac{(\omega \frac{R_k}{Q_k} + (1-\omega) \frac{(1-\alpha)(1-\eta) R_k}{1-\pi})^{1-\sigma}}{1-\sigma}$$

Taking the derivative with respect to η and simplifying the terms, we get

$$\frac{Q_k (1 + r^d)}{\left[(\omega + (1-\omega) \frac{\alpha + (1-\alpha)\eta Q_k}{\pi})(1 + r^d) \right]^{\sigma}} - \frac{R_k}{(\omega \frac{R_k}{Q_k} + (1-\omega) \frac{(1-\alpha)(1-\eta) R_k}{1-\pi})^{\sigma}} = 0$$

which can be written as (84). We can also write it as

$$\frac{\omega + (1-\omega) r_m}{\omega \frac{R_k}{Q_k} + (1-\omega) r_n} = \left(\frac{Q_k}{R_k} \right)^{\frac{1}{\sigma}} (1 + r^d)^{\frac{1}{\sigma} - 1} \tag{89}$$

When $\frac{Q_k}{R_k} < 1$, if $r^d = 0$, it is clear that the RHS of (89) is increasing in σ. r^d is positive only when investment funds borrow positive loans from banks. In this case, $\frac{Q_k}{R_k} = \frac{1}{1+r^l}$, and the RHS of (89) can be written as $\frac{Q_k}{R_k} \left(\frac{1+r^l}{1+r^d} \right)^{1-\frac{1}{\sigma}}$, which is increasing in σ since $r^l > r^d$.

When $\sigma = 1$, it is easy to see that the result is the same as in Proposition 1. ■

C.2 The bank loan supply curve

We first prove Proposition 3.

Proof: In Table 3, in each column n, the maximum accumulated payment is $\frac{(N-1)X_i}{N} - \frac{(n-1)X_j}{N}$, which happens in period $k = n$(the diagonal of the matrix) when banks are chosen to make the payment. And for $k > n$, the accumulated payment is $\frac{(N-1)X_i}{N} - \frac{(k-1)X_j}{N}$. If N is very large, then $\frac{(N-1)X_i}{N} \approx X_i$. We set $1 - \frac{n-1}{N}$ as λ_{max} and $1 - \frac{k-1}{N}$ as λ, then for $k \geq n$, we can write

$$FL_{\max} = X_i + (\lambda_{\max} - 1) X_j \tag{90}$$
$$FL(k) = X_i + (\lambda - 1) X_j \tag{91}$$

And the central bank loan is

$$b(k) = \max(FL(k) - D_0, 0) \tag{92}$$

Note that in Table 3, in each column, the accumulated flow $FL(k)$ for $k \geq n$ is the same as the $FL(k)$ in the previous column. Let $\underline{\lambda}$ denote the level of λ at which $b(k) = 0$. Using (91) and (92), we get

$$\underline{\lambda} = \frac{D_0 + X_j - X_i}{X_j} \qquad \underline{\lambda} \in [0, 1] \tag{93}$$

$b(k) > 0$ if $\lambda > \underline{\lambda}$.

When N is large, we can take λ as continuous, and the expected loan can be written as

$$Eb(L_i) = \int_{\underline{\lambda}}^{1} \int_{\underline{\lambda}}^{\lambda_{\max}} b(k) d\lambda d\lambda_{\max} = \int_{\underline{\lambda}}^{1} \int_{\underline{\lambda}}^{\lambda_{\max}} \left([X_i + (\lambda - 1) X_j] - D_0 \right) d\lambda d\lambda_{\max}$$

The integral of $b(k)$ over $[\underline{\lambda}, \lambda_{\max}]$ is the borrowing for each realized n(i.e., each column of the matrix). The integral over $[\underline{\lambda}, 1]$ denotes the changes in $\lambda_{\max}$ caused by the changes in n(i.e., different columns of the matrix). $b(k)$ is positive only when λ and $\lambda_{\max}$ are $> \underline{\lambda}$.

$$\begin{aligned}
Eb(L_i) &= \int_{\underline{\lambda}}^{1} \int_{\underline{\lambda}}^{\lambda_{\max}} [X_i - X_j - D_0 + \lambda X_j] \, d\lambda d\lambda_{\max} \\
&= \int_{\underline{\lambda}}^{1} \left[(\lambda_{\max} - \underline{\lambda})(X_i - X_j - D_0) + \frac{\lambda_{\max}^2 - \underline{\lambda}^2}{2} X_j \right] d\lambda_{\max} \\
&= \left(\frac{\lambda_{\max}^2}{2} |_{\underline{\lambda}}^{1} - \lambda_{\max} |_{\underline{\lambda}}^{1} \underline{\lambda} \right) (X_i - X_j - D_0) + \frac{1}{2} \left(\frac{\lambda_{\max}^3}{3} |_{\underline{\lambda}}^{1} - \lambda_{\max} |_{\underline{\lambda}}^{1} \underline{\lambda}^2 \right) X_j \\
&= \left(\frac{1 - \underline{\lambda}^2}{2} - (1 - \underline{\lambda}) \underline{\lambda} \right) (X_i - X_j - D_0) + \frac{1}{2} \left(\frac{1 - \underline{\lambda}^3}{3} - (1 - \underline{\lambda}) \underline{\lambda}^2 \right) X_j
\end{aligned}$$

Replacing $\underline{\lambda}$ with (93) and arranging terms, we get

$$Eb(L_i) = \frac{1}{6} \frac{(X_i - X_j - D_0)^3}{X_j^2} + \frac{1}{2} \frac{(X_i - X_j - D_0)^2}{X_j} + \frac{1}{2}(X_i - X_j - D_0) + \frac{1}{6} X_j \tag{94}$$

■

In the general case, we have

$$X_i = Z_f + (1 - \pi) D^h + L_i \tag{95}$$

$$X_j = Z_f + (1 - \pi) D^h + L_j \tag{96}$$

where Z_f is the riskless asset of the investment fund, $(1 - \pi) D^h$ is the money collected from non-movers. The method is the same and it can be shown that in the symmetric case we still have

$$R = 1 + \delta + r^c \frac{1}{2} \left(1 - \frac{D_0}{X} \right)^2 \tag{97}$$

$$r^d = \frac{\frac{Lr^c}{2}(1 - \frac{D_0}{X})^2 - r^c Eb}{D_0 + L} = \frac{\frac{Lr^c}{2}(1 - \frac{D_0}{X})^2 - r^c(\frac{-D_0^3}{6X^2} + \frac{D_0^2}{2X} - \frac{D_0}{2} + \frac{X}{6})}{D_0 + L} \tag{98}$$

where D_0 is $D^h + Z_f$ and $X = Z_f + (1 - \pi) D^h + L$.

C.2.1 The Equilibrium solutions for r_m, r_n, Q_k, L, R and r^d.

This part derives the equilibrium solutions in period $t+1$ by taken ω and α as given. We first consider the case in which the constraint $r_m \leq r_n$ is not binding.

Recall that at π_2, investment funds start to borrow from banks. And at π_3, banks start to borrow from the central bank. Everything for $\pi < \pi_2$ is the same as in the non-bank lending case. In order to get the solution for $\pi \geq \pi_2$, we first decide π_2 and π_3.

Derive π_2 and π_3

π_2 can be decided as follows. At π_2, $R = 1+\delta$, $Q_k = \frac{R}{1+\delta}$ and $L = 0$. At the same time, $\frac{Q_k}{R_k}$ should satisfy (65), and so we have

$$\begin{aligned} & \frac{Q_k}{R_k} = \frac{1}{1+\delta} = \left(\frac{\kappa(1-\pi)}{R_k\pi_2(1-\kappa)}\right)^{\sigma} \\ \Rightarrow \qquad & \pi_2 = \frac{\kappa}{R_k(1-\kappa)\left(\frac{1}{1+\delta}\right)^{\frac{1}{\sigma}} + \kappa} \end{aligned} \tag{99}$$

π_3 can be decided as follows. At π_3, $R = 1+\delta$, $r^d = 0$ and $Q_k = \frac{R_k}{1+\delta}$. At π_3, $X = D_0$. Since $X = Z_f + (1-\pi)D^h + L$ and $D_0 = Z_f + D^h$, so $L = \pi D^h$. Thus, we have

$$\begin{aligned} \pi Z_p r_m &= Z_f + (1-\pi)D^h + \pi D^h \\ \Rightarrow \pi r_m &= \alpha + (1-\pi)\frac{\omega}{1-\omega} + \pi\frac{\omega}{1-\omega} = \alpha + \frac{\omega}{1-\omega} & (100) \\ \Rightarrow r_m &= \frac{1}{\pi}(\alpha + \frac{\omega}{1-\omega}) & (101) \end{aligned}$$

Also, comparing (85) and (100), we have

$$(1-\alpha)\eta Q_k = \frac{\omega}{1-\omega} \Rightarrow \eta = \frac{\omega}{(1-\omega)(1-\alpha)Q_k} \tag{102}$$

Substituting η into (86) and we have

$$r_n = \frac{1}{1-\pi}\left((1-\alpha) - \frac{\omega}{(1-\omega)Q_k}\right)R_k \tag{103}$$

Then substitute (101) and (103) into the optimal payout policy (89) and we have

$$\frac{\omega + (1-\omega)\frac{1}{\pi}(\alpha + \frac{\omega}{1-\omega})}{\omega(1+\delta) + (1-\omega)\frac{1}{1-\pi}\left((1-\alpha) - \frac{\omega}{(1-\omega)Q_k}\right)R_k} = \left(\frac{1}{1+\delta}\right)^{\frac{1}{\sigma}} \tag{104}$$

Arranging terms, we get

$$\omega\left[(1+\delta)^{1-\frac{1}{\sigma}} - 1\right]\pi^2 - \pi\left[\kappa - \omega + (1+\delta)^{-\frac{1}{\sigma}}(1-\kappa)R_k\right] + \kappa = 0 \tag{105}$$

When $\sigma = 1$, the solution is $\frac{\kappa}{\kappa - \omega + (1-\kappa)\frac{R_k}{1+\delta}}$. When $\sigma > 1$, the smaller one of the two solutions is π_3. Note that given κ, π_3 is affected by ω. For example, if $\omega = 0$(all riskless assets are held by the investment fund), then $\pi_3 = \pi_2$.

The distribution for Q_k takes the following form:

$$Q_k(\pi) = \begin{cases} R_k & : \ \pi \leq \pi_1 = \pi_1 = \frac{\kappa}{\kappa+(1-\kappa)R_k} \\ R_k^{1-\sigma}\left(\frac{\kappa(1-\pi)}{\pi(1-\kappa)}\right)^{\sigma} & : \ \pi_1 < \pi < \pi_2 = \frac{\kappa}{R_k(1-\kappa)\left(\frac{1}{1+\delta}\right)^{\frac{1}{\sigma}}+\kappa} \\ \frac{R_k}{1+\delta} & : \ \pi_2 \leq \pi \leq \pi_3 \\ \frac{R_k}{R(\pi)} & : \ \pi > \pi_3 \end{cases} \tag{106}$$

Equilibrium solutions over π_2 and π_3
Over $[\pi_2, \pi_3]$, $R = 1+\delta$, $r^d = 0$ and $Q_k = \frac{R_k}{1+\delta}$. We still need to decide r_m, r_n and L. Since $r^d = 0$, (89) is the same as (57) , and the solution for η, r_m and r_n are simply (74), (75) and (76) with $Q_k = \frac{R_k}{1+\delta}$. Knowing η, we can decide L from the budget constraint (85). Since L is equal to the total external cash minus the cash from non-movers, so

$$L = Z_p(1-\alpha)\eta Q_k - (1-\pi)D^h = S[(1-\omega)(1-\alpha)\eta Q_k - (1-\pi)\omega] \tag{107}$$

Equilibrium solutions for $\pi > \pi_3$
We will set L as the variable that we try to solve, and we express all other variables as a function of L. The equilibrium is defined by the following conditions. 1.The budget constraints (85) and (86); 2.The optimal payout policy (89); 3.Asset Price on the financial market: $Q_k = \frac{R_k}{R}$; 4.The cash paid to movers is equal to the fund's own money plus the money raised from the financial market and the bank.

$$\pi Z_p r_m = Z_f + (1-\pi)D^h + L \tag{108}$$

5.The loan supply curve (97) which defines the relationship between L and R; 6.r^d(equation 98) derived from the zero expected profit condition. We can write (97) as $R(L)$ and (98) as $r^d(L)$. Then $Q_k(L) = \frac{R_k}{R(L)}$. We can also write (108) as

$$r_m = \frac{1}{\pi Z_p}(Z_f + (1-\pi)D^h + L) = \frac{1}{\pi}\left(\alpha + (1-\pi)\frac{\omega}{1-\omega} + \frac{L}{S(1-\omega)}\right) \tag{109}$$

which we define as $r_m(L)$. Then using the two budget constraints (85) and (86), we have

$$r_n = \frac{R_k(1-\alpha)}{1-\pi} - \frac{R(L)(\pi r_m(L) - \alpha)}{1-\pi} \tag{110}$$

which we define as $r_n(L)$. Substitute $r_m(L)$, $r_n(L)$, $Q_k(L)$, $R(L)$, and $r^d(L)$ into the optimal payout policy (89), and we can get an equation in which the *only* unknown is L:

$$\frac{\omega + (1-\omega)r_m(L)}{\omega R(L) + (1-\omega)r_n(L)} = \left(\frac{1}{R(L)}\right)^{\frac{1}{\sigma}} (1 + r^d(L))^{\frac{1}{\sigma}-1} \tag{111}$$

where $R(L)$, $r_d(L)$, $r_m(L)$ and $r_n(L)$ are (97), (98), (109), and (110). This equation implicitly defines the equilibrium L. After deciding L, all other variables can then be decided.
When $r_m \leq r_n$ is binding
Let π_{bind} denote the π above which the constraint $r_m \leq r_n$ is binding. We first consider the

case when $\pi_1 < \pi_{bind} < \pi_2$. At π_2, r_m is still (60), and r_n is (79) with $Q_k = \frac{R_k}{1+\delta}$, equating r_m and r_n gives the value of π_2. At π_3, $Q_k = \frac{R_k}{1+\delta}$. r_m is (101) and r_n is (103). Equating r_m and r_n gives π_3.

For equilibrium values of variables. For $\pi \leq \pi_{bind}$, everything is the same as in the non-binding case. For $[\pi_{bind}, \pi_2]$, we use (67) and (68). Over $[\pi_2, \pi_3]$, r_m and r_n are (79) with $Q = \frac{R_k}{1+\delta}$. For $\pi > \pi_3$, we can solve the equilibrium using the same method as in the non-binding case, the only difference is that instead of using condition (111), we use the condition $r_m(L) = r_n(L)$.

If $\pi_{bind} \in [\pi_2, \pi_3]$ or $\pi_{bind} > \pi_3$, then we can decide π_{bind} using simulation methods. The method for deciding the equilibrium values is the same as explained above.

C.2.2 The first order conditions for ω and α

This part derives the first order conditions for representative household and investment fund.

Using EU(70), v_m(87) and v_n(88), we get

$$\begin{aligned}\frac{\partial EU}{\partial \omega} &= \frac{1}{2}\int_0^1 \frac{\pi(1-r_{m,H})(1+r_H^d)}{[(\omega+(1-\omega)r_{m,H})(1+r_H^d)]^\sigma} + \frac{(1-\pi)(\frac{R_{k,H}}{Q_{k,H}} - r_{n,H})}{(\omega\frac{R_{k,H}}{Q_{k,H}} + (1-\omega)r_{n,H})^\sigma} dF(\pi) \\ &+ \frac{1}{2}\int_0^1 \frac{\pi(1-r_{m,L})(1+r_L^d)}{[(\omega+(1-\omega)r_{m,L})(1+r_L^d)]^\sigma} + \frac{(1-\pi)(\frac{R_{k,L}}{Q_{k,L}} - r_{n,L})}{(\omega\frac{R_{k,L}}{Q_{k,L}} + (1-\omega)r_{n,L})^\sigma} dF(\pi) \quad (112)\end{aligned}$$

The first order condition for α is

$$\begin{aligned}\frac{\partial EU}{\partial \alpha} &= \frac{(1-\omega)}{2}\int_0^1 \left(\pi\frac{\frac{\partial r_{m,H}}{\partial \alpha}(1+r_H^d)}{v_{m,H}^\sigma} + (1-\pi)\frac{\frac{\partial r_{n,H}}{\partial \alpha}}{v_{n,H}^\sigma}\right. \\ &\left. + \pi\frac{\frac{\partial r_{m,L}}{\partial \alpha}(1+r_L^d)}{v_{m,L}^\sigma} + (1-\pi)\frac{\frac{\partial r_{n,L}}{\partial \alpha}}{v_{n,L}^\sigma}\right) dF(\pi) \quad (113)\end{aligned}$$

We need to decide $\frac{\partial r_m}{\partial \alpha}$ and $\frac{\partial r_n}{\partial \alpha}$. When $\pi \leq \pi_1$, since $r_m = r_n = \alpha + (1-\alpha)R_k$, we get $\frac{\partial r_m}{\partial \alpha} = \frac{\partial r_n}{\partial \alpha} = 1 - R_k$. For $\pi > \pi_1$, we first need to solve for r_m and r_n by taken Q_k and r^d as given. Note that equations (85) and (86) are the same as (54) and (55), and the only difference between (57) and (89) is that the RHS is changed from $\left(\frac{Q_k}{R_k}\right)^{\frac{1}{\sigma}}$ into $\left(\frac{Q_k}{R_k}\right)^{\frac{1}{\sigma}}(1+r^d)^{\frac{1}{\sigma}-1}$. It turns out that we only need to modify the solutions of η, r_m, r_n in the no-lending case(74, 75 and 76) by changing $\left(\frac{Q_k}{R_k}\right)^{\frac{1}{\sigma}}$ into $\left(\frac{Q_k}{R_k}\right)^{\frac{1}{\sigma}}(1+r^d)^{\frac{1}{\sigma}-1}$. And so $\frac{\partial r_m}{\partial \alpha}$ and $\frac{\partial r_n}{\partial \alpha}$ are equations (77) and (78) with the term $\left(\frac{Q_k}{R_k}\right)^{\frac{1}{\sigma}}$ replaced by $\left(\frac{Q_k}{R_k}\right)^{\frac{1}{\sigma}}(1+r^d)^{\frac{1}{\sigma}-1}$.

If the constraint $r_m \leq r_n$ is binding, then for $\pi > \pi_{bind}$, $\frac{\partial r_m}{\partial \alpha}$ and $\frac{\partial r_n}{\partial \alpha}$ are the same as (80).

The response curves

We can still show that as long as $r_m \leq r_n$ is not binding, then the response curve of the household and the investment fund will overlap with each other. The proof is omitted because it is essentially the same as in the no-bank-lending case. But in this case, different combinations of $[\omega, \alpha]$ will given different values of κ. This is because increasing the level of α will increase the monetary payment from investment funds to movers during the redemption process, so banks will be more likely to borrow from the central bank. Thus, equilibriums with high ω and low α will be more efficient.

D Steps for computing the numerical example

What follows are the steps for computing the equilibrium $\sigma = 1$ when there are bank lending. The method is similar for other cases.

1. Start with an initial value of ω, the aggregate risky investment is $(1-\omega)e_h$. Given A_H and A_L, the asset returns are $R_{k,H} = A_H$ and $R_{k,L} = A_L$.

2. Select a large number of π according to $F(\pi)$. For each π, compute the equilibrium values of Q_k, L, R and r^d. Then compute the first order condition for ω. Iterate on ω until the first order condition converges to zero.

3. Given the equilibrium ω, we compute the equilibrium Q_k, L and R and r^d.

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