Leviathan governments and public debt

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Abstract

Casual observation suggests that governments are not as willing to save as they are to spend. The paper analyzes the dynamics of public spending, taxation and debt in a political agency model. Policy choices are made by short-lived politicians who can be only partially controlled through the electoral process. The main focus of the paper is to consider the impacts of binding limits on the public budget. A unique budget restriction that optimizes citizen welfare is shown to exist. The model characterizes the trade off between the benefits of better politician control and the cost that such restrictions will have in times of scarcity. The value of a limit depends both on the extent to which politicians goals deviate from their constituencies and how effectively the electoral process disciplines them when they misbehave. The results also suggest that the value of such a restriction depends on the fiscal position at the time in which it is imposed. In particular, the value of constraining politicians depends positively on the size of current public asset holdings.

Keywords: Public debt, political agency, debt controls.

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1 Introduction

The sizeable debts accumulated by most countries have likely not come about solely as a result of society’s needs\(^1\). Are governments as willing to save as they are to spend? Is it not strange to think of a government as a creditor? That politicians can rarely, if ever, be suitably labelled benevolent planners is the subject of a large literature on which this paper draws\(^2\). In particular we consider the behaviour of taxes and government debt within an electoral accountability model akin to those originally developed by Barro (1973) and Ferejohn (1986).

If incumbents are indeed inclined to spend too much, a natural question to ask is whether we should constrain them through some sort of constitutional restriction? This paper outlines a simple framework in which this question is addressed. The model characterizes the trade off between the benefits of better politician control and the cost that such restrictions will have in times of scarcity. The value of a limit depends both on the extent to which politicians goals deviate from their constituencies and how effectively the electoral process disciplines them for misbehaving. Further, the results suggest that such a constraint will have a positive/negative expected benefit depending on the fiscal position at the time in which it is imposed. In particular, the value of constraining politicians depends positively on the size of current public asset holdings.

In a related paper, Besley and Smart (2008) consider the implications that yardstick competition, transparency, tax limits and changes to the marginal cost of public funds have on the political equilibrium when there is both moral hazard and adverse selection. Bassetto and Sargent (2006) consider the value in constraining the government to issue debt only to finance capital items and not for ordinary budgets. The focus here is not only in capturing the deviation of government spending from some optimum (generally spending is too high in this framework) but also in the mix of financing between debt and taxation. In this regard we will employ a tax smoothing approach a la Barro (1979).

Battagliani and Coate (2008) propose a dynamic model with tax smoothing where public decisions are made via legislative bargaining. In their model, the value of balanced budget restrictions depends on the size of the tax base relative to desired public good spending. The larger this ratio, the more beneficial is such a restriction.

Alesina and Tabellini (2007) allow for debt in an agency model very similar to the one presented here. They use this theoretical framework to explain the existence of procyclical fiscal policy observed in many developing nations. Due to the nature of information in their model, incumbents borrow as much as possible every period so debt is not an effective instrument to smooth consumption (which of course doesn’t allow for an interesting discussion on debt controls).

The remainder of the paper is organized as follows. Section 2 outlines two versions of the basic finite period model, one with taxes and one without. Section 3 considers optimal debt limits. Section 4 extends the analysis to include and infinite horizon and section 5 concludes. All proofs

\(^1\)The U.S national “debt clock” in New York recently topped $10,000,000,000,000 [The Economist, Oct.18th, 2008].

\(^2\)For a survey see Besley (2006) or Persson Tabellini (2000).
can be found in the appendix.

# 2 The Model

Section 2 characterizes both the voter and politicians problem, the environment in which they interact and finally describes tax and spending behaviour.

## 2.1 Voters

As is the norm when using this framework, we assume society is represented by a single voter. Each period, the voter receives an endowment $y_t \in [\underline{y}, \overline{y}]$, which follows an i.i.d process characterized by distribution function $F(y)$.

Utility in period $t$ is derived from both private consumption $c_t$ and government services $g_t$ and defined as

$$U(c_t, g_t) = c_t + h(g_t)$$

(1)

Where $h()$ is concave and monotonically increasing. Private consumption and public services are defined by

$$c_t = (1 - \tau_t)y_t$$

$$g_t = \delta(\tau_t)y_t - (1 + r)b_{t-1} + b_t$$

Government borrowing at time $t$ is $b_t$, the tax rate is $\tau_t$ and the interest rate is fixed at $r$. The function $\delta()$ represents the distortionary costs associated with taxation (as in to Barro (1979)) that are not made explicit in the model and satisfies $1 > \delta'(\cdot) > 0$ and $\delta''(\cdot) < 0$. Debt taken in any period is to be repaid in full the following period.

It is generally agreed upon that optimal government policy should be countercyclical and in fact this is what we see in the majority of the developed world. This is justified through a variety of arguments including tax-smoothing or simply smoothing in that discretionary spending should remain constant in the face of temporary fluctuations. Given the framework here, countercyclical policy is optimal as voters wish to smooth public spending in the face of uncertainty both due to inefficiencies in the tax system and concavity in the utility of public goods. The extent to which this can be achieved will however be constrained by the political environment as discussed below.

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3 A persistent process is more realistic but it is of little consequence to the analysis here.

4 Utility is linear in private consumption to avoid the question of private saving which adds complication and little else.

5 Assuming a small open economy.

6 For discussions of these and related issues see Lane (2003), Talvi and Vegh (2005) or Alesina et. al (2007)
2.1.1 Government

The government consists of an elected party whose actions each period are taken by individual politicians. Politicians themselves are transitory and only last one period, but are not completely irresponsible and care about the party and its re-election⁷. There is no shortage of explanations as to why governments may overspend⁸. Here we will abstract away from these and assume simply that governments gain utility from spending your money. There are however no “rents”, as in many political agency models, and funds are never completely wasted. This is an attempt to capture the pressures on governments from bureaucracies, interest groups etc. as well as the benefits politicians receive personally from handing out money. Aside from spending, the politician’s utility depends on a sense of party loyalty, responsibility and/or legacy concerns. Formally, politician utility will be a function of current spending $g_t$ and a constant term $L$, which captures loyalty or legacy concerns. Let the function be separable and linear so that politician utility is given by

$$G(g_t, L) = g_t + L = \delta(\tau_t)y_t - (1 + r)b_{t-1} + b_t + L$$  \hspace{1cm} (2)

Where the politician does not receive $L$ if the party is not re-elected. Party loyalty $L$ is an attempt to capture the idea that the incumbent is a member of a party whose horizon is longer term. This can also or partly be viewed as a legacy or ego term in that a politician may get utility from being viewed as responsible and competent, not to mention ethical obligations and employment prospects after politics. Thus not only do they owe the party and care about re-election but they wished to be viewed in a good light once they are out of power.

2.1.2 Politics

The approach to politics is adapted from Barro (1973), Ferejohn (1986) and Persson, Roland and Tabellini (1997). In this set-up, elections are held at the end of each period, after government policy is chosen. Promises are meaningless and politicians can only be held accountable for misbehaving after the fact. This is achieved simply by assuming that in such a case, the politician is ousted and an identical challenger is brought in. This amounts to a simple agency problem in which voters are assumed to coordinate on an incentive compatible voting rule, which is observed by the politician before policy is undertaken. Alesina and Tabellini (2007) extend this approach to allow for public debt and use this theoretical framework to explain the existence of procyclical fiscal policy observed in many developing nations.

Generally, the government is assumed to have some informational advantage that is exploited to obtain rents. However, due to the nature of the relationship between the government and its public, this informational asymmetry is not necessary to obtain a suboptimal outcome.⁹ We will

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⁷If politicians last longer than a single period then there is the question of re-electing an incumbent vs. a challenger. There are many papers that consider this issue but it adds little to analysis here so we avoid the complication.

⁸See Persson and Tabellini (2000) or Besley (2006) for a description of the literature

⁹Persson, Roland and Tabellini (1997) make this distinction explicitly when the distinguish between what they
assume here that voters are fully informed but can still only punish politicians in the next election. The very fact that they are in power allows incumbents to overspend regardless of whether voters observe it. This must be accounted for in the voters calculus and thus saving will not be optimal in general. Allowing for an informational asymmetry here would add some realism and exacerbate any inefficiencies but is cumbersome and is not necessary for the analysis.

At the beginning of period $t$, income is realized and the state is represented $(b_{t-1}, y_t)$. A voting rule is a tax rate $\tau$ and saving/borrowing level $b$ which implies spending $g$. Given the rule and the state, an incumbent chooses taxes and public spending. If the government follows the rule the party is re-elected and if not they are ousted\textsuperscript{10}. The timing is as follows

1. The period begins with debt/saving level $(1+r)b_{t-1}$
2. Voters observe current income $y_t$ and set a voting rule, which is a tax $\tau_t$ and borrowing $b_t$ (implying a current spending level $g_t$)
3. The government chooses $\tau_t, b_t$ and thus $g_t$.

Voters wish to set a re-election rule that optimizes both private consumption and public spending to the extent possible (given the behaviour of the incumbent government). Before deriving an incentive compatible voting rule, we must consider the behaviour of an incumbent politician. When not seeking re-election, utility is given by the highest possible value of current spending given as

$$G(g_t, 0) = \overline{G_t} = \delta(\tau) y_t - (1 + r)b_{t-1} + \overline{b}$$

Where $\overline{\tau}$ is the highest taxes can be set (exogenous) and $\overline{b}$ is the limit on borrowing\textsuperscript{11}. The debt limit could be a rule (as will be discussed later) but for now can be interpreted as the most that can be repaid with certainty. Politicians will abide by the voting rule if the following inequality holds

$$\delta(\tau_t)y_t + b_t \geq \delta(\overline{\tau})y_t + \overline{b} - L$$

Where $\tau_t, b_t$ are the tax rate and borrowing/saving level under the rule. The next section considers the behaviour of taxes and debt in a two-period setting. Once a framework is established, we can consider the possible value in constraining the government.

\textsuperscript{10}The government would always tax and spend as much as possible so there is no concern of too little public spending
\textsuperscript{11}The maximum tax rate could be considered the highest taxes can be set without revolt. This is likely related to current public opinion (i.e $\overline{\tau} = f(\tau_t)$) but for simplicity is fixed here
2.2 Optimal spending/voting

There are two periods; 1 and 2. In the second period, the voter chooses an optimal tax rate given income $y_2$, the current debt $(1 + r)b_1$ and the incentives of the politician (there is no borrowing in the last period). The problem is solved recursively so that in period 1 the voter chooses both taxes and borrowing/saving given optimal behaviour in period 2, which is determined by

$$\max_{\tau_2} \quad (1 - \tau_2)y_2 + h(\delta(\tau_2)y_2 - (1 + r)b_1)$$

s.t

$$\delta(\tau_2)y_2 \geq \delta(\bar{\tau})y_2 - L$$

Let $\tau^*_2$ be the solution to this problem which will either equalize the marginal value of public and private spending (when unconstrained) or satisfy the political constraint with equality (when constrained). Further denote period 2 utility at the optimum $\tau^*_2$ as the value function $V_2(b_1)$. If initial debt is $d$ (exogenous) and $\beta$ is the discount factor, the period 1 problem is

$$\max_{\tau_1, b_1} \quad (1 - \tau_1)y_1 + h(\delta(\tau_1)y_1 + b_1 - d) + \beta V_2(b_1)$$

s.t

$$\delta(\tau_1)y_1 + b_1 \geq \delta(\bar{\tau})y_1 + \bar{b} - L$$

$$b_1 \leq \bar{b}$$

The corresponding Lagrangian expression is

$$\mathcal{L} = (1 - \tau_1)y_1 + h(\delta(\tau_1)y_1 + b_1 - d) + \beta V_2(b_1) + \lambda_1(\delta(\tau_1)y_1 + b_1 - \delta(\bar{\tau})y_1 - \bar{b} + L) + \lambda_2(\bar{b} - b_1)$$

The first order conditions are,

$$[h'(\delta(\tau_1)y_1 + b_1 - d) + \lambda_1]\delta'(\tau_1) = 1$$

$$h'(\delta(\tau_1)y_1 + b_1 - d) + \beta(1 + r)h'(\delta(\tau_2)y_2 - (1 + r)b_1) + \lambda_1 - \lambda_2 = 0$$

Equation (9) is the equalization of marginal returns between private and public consumption. Condition (10) is a familiar euler equation that relates the value of public consumption between the two periods. The optimal voting rule obviously depends on the importance of the two constraints, which create 4 possible outcomes considered in turn.

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12 There is no uncertainty here. We could allow for a random draw in period two and thus maximize expected utility but this adds little.
Case 1: $\lambda_1 = \lambda_2 = 0$

In the unconstrained case, the voter would like to use public debt to smooth the distortions brought about through tax finance (as well as smooth public consumption over periods). If income is low and/or debt is high today relative to tomorrow, then it is optimal to borrow to cover shortfalls rather than use inefficiently high taxes. Denoting optimal policies with a *, we have

**Lemma 1** Borrowing is increasing in initial debt $d$ and decreasing in $y_1$. Taxes are increasing in both $y_1$ and $d$ so that

$$\frac{\partial b^*_1}{\partial d} > 0, \quad \frac{\partial b^*_1}{\partial y_1} < 0, \quad \frac{\partial \tau^*_1}{\partial d} > 0, \quad \frac{\partial \tau^*_1}{\partial y_1} > 0$$

We see that tax and borrowing behaviour are quite simple but may be distorted by either politics or borrowing constraints. With the former, there will be too much overall spending. With the latter, there will be too much tax financing. If both constraints bind, there will be too much spending and taxes will be too high relative to debt.

Case 2: $\lambda_1 > 0$, $\lambda_2 = 0$

When the political constraint binds then there is too much public spending. As $\lambda_2 = 0$ however, the mix of financing between debt and taxes given that level of spending remains optimal.

Case 3: $\lambda_1 = 0$, $\lambda_2 > 0$

Here the ability to use debt financing is hampered by the constraint and thus public spending will be lower than optimal as it is more costly.

Case 4: $\lambda_1 > 0$, $\lambda_2 > 0$

When both constraints bind, there is no choices in that debt is at the limit and taxes are defined by the participation constraint. It is unclear whether there is too much or too little spending in this case.

With this characterization of optimal debt and tax behaviour, we can consider the value of limiting governments through $\bar{b}$, which up to this point has been considered given. Denote total utility by $W(\bar{b})$ which is utility under optimal tax and spending behaviour characterized by (5). Changes to $\bar{b}$ would only have an impact in cases 2-4. In particular, the envelope theorem implies

$$\frac{\partial W}{\partial \bar{b}} = -\lambda_1 + \lambda_2$$ (11)

Of course these multipliers are both non-negative so the impact is generally ambiguous. If we are in case 2, the debt limit is not binding but the political constraint is, then tightening it will increase utility as it reduces the politicians leverage. In case 3, the opposite is true and reducing $\bar{b}$ limits the ability to borrow and has no benefit. In case 4 it is not clear whether changes in the debt limit will be harmful or beneficial. A tighter debt limit would reduce the politicians leverage but if politics just lead to higher tax financing it may do more harm than good. As both constraints bind in this case we can characterize the impact of changes in $\bar{b}$ quite simply as
\[ \frac{\partial W}{\partial b} = h'(\delta(\tau)y_1 - \frac{L}{y_1} + \bar{b} - d) + \beta(1 + r)h'(\delta(\tau_2)y_2 - (1 + r)\bar{b}) \] (12)

Define \( \bar{b}^* \) as the debt limit which satisfies \( \frac{\partial W}{\partial b}(\bar{b}^*) = 0 \). For a debt limit greater than this, further increases have a negative impact on utility. For a debt limit less than this, increases have a positive effect\(^\text{13}\). Assume that \( \beta(1 + r) = 1\)\(^\text{14}\), then \( \bar{b}^* \) is defined by

\[ \bar{b}^* = \frac{1}{2 + r} \left( \delta(\tau_2)y_2 - \delta(\bar{\tau}) + \frac{L}{y_1} + d \right) \] (13)

We could do some comparative statics calculations to further our intuition on such changes, and while this may shed some light on the issue, the important question that remains is when we expect these constraints to bind? Is it worth the possible inefficiency of being up against a debt limit if it helps control politician overspending? This problem is quite complex, so we shall simplify. Consider the following which is assumed to hold throughout

**Assumption 1** \( L > [\delta(\tau) - \delta(\tau_1)]y_1 \)

Effectively, this says that an incumbent is limited in the amount it can overspend using taxes. This will hold if \( \tau \) is not too large, if there is sufficient curvature in \( \delta() \) or the politician is “benevolent enough”. For instance, we could define \( \tau = \tau + \epsilon \) where \( \epsilon \) is small so that governments can only raise taxes by some “reasonable” amount above the optimum without being thrown out of office or not being able to pass the legislation\(^\text{15}\). This seems reasonable and there are a variety of papers that make similar assumptions. For instance that taxes are generally more easily observed than debt financing when there is a lack of transparency over government actions (due perhaps to creative accounting practices)\(^\text{16}\). Rewriting the political constraint and note that if assumption 1 holds, both constraints cannot bind at the same time.

\[ b_1 \geq (\bar{b}) - L + [\delta(\tau) - \delta(\tau_1)]y_1 \] (14)

We are now concerned only with cases 2 and 3. Either the political constraint is binding or the debt constraint is binding. Changes to \( \bar{b} \) have opposite impacts on both of these constraints so the question of optimal debt limits becomes is one of when we expect these constraints to bind.

The value in government debt in this model is two-fold; to smooth tax distortions and optimize the timing of public consumption under a varying income process. In fact, if we fix the tax rate, the timing of funds is still an issue and can be looked as a simple one-dimensional constrained spending/savings problem. When assumption 1 holds, the two problems are essentially equivalent

\(^{13}\)We ensure that this is indeed the case in the next section. In particular, the proof for proposition 1 shows that there is a unique debt limit that optimizes indirect utility

\(^{14}\)This assumption is made purely for simplicity and can be relaxed without difficulty.

\(^{15}\)Of course then we would have to bound the choice set of \( \tau \) appropriately.

\(^{16}\)For example Alesina and Tabellini (2007) make an informational assumption of this type
when the focus is on debt controls (although the interpretation is slightly different). The impacts are
the same in that a debt control can be; welfare enhancing (through better behaved governments), or
welfare reducing (if the constraint binds and more borrowing is constrained). This simpler version
is outlined below and will be used to consider optimal restrictions on $\bar{b}$. A note before moving on
is that if assumption 1 fails to hold then the two problems are generally not equivalent and the
question of constraining governments must consider limiting tax behaviour as well. This is left to
further work.

2.2.1 Fixed taxes

This section outlines the two-period problem with a fixed tax rate. Voter utility is simply given by
public good consumption $h(g)$ which is ideally smoothed over each period. Public good spending
in period $t$ is now simply

$$g_t = y_t - (1 + r)b_{t-1} + b_t$$

Where $y_t$ is now considered government revenue in period $t$ (as with a fixed tax rate and random
income above). The voter gives the politician a voting rule which governs spending in that period.
A politician either follows the rule and gets the party re-elected or spends as much as possible and
is ousted. The political constraint is

$$b_t \geq \bar{b} - L$$

The voters problem is the same as above; given an income draw and a current debt, they choose
a level of borrowing/saving that is optimal given that the political constraint is not violated. As
above, there are two periods and the problem will be solved recursively. In the last period there is
no uncertainty and no borrowing so that all current wealth is just consumed. In period 1, borrow-
ing is chosen to optimize consumption between the last two periods to the extent possible given
politician incentives.

Period 2:

Simply consume all wealth, utility is given by

$$h(y_2 - (1 + r)b_1)$$

Period 1:

The voter sets the rule to optimize borrowing subject to the constraints, which amounts to solving

$$\max_{b_1} h(y_1 - (1 + r)b_0 + b_1) + \beta h(y_2 - (1 + r)b_1)$$
Again assuming $\beta(1 + r) = 1$, in the interior, optimal borrowing is given by

$$b^*_1(y_1, b_0) = \frac{y_2 - (y_1 - (1 + r)b_0)}{2 + r}$$

We can see that borrowing when unconstrained is strictly increasing (decreasing) in future (current) wealth. For the problem to be interesting there must be some period 1 wealth levels at which the constraints will bind. Let $w$ be that at which borrowing is constrained from above (would like to borrow more) and $\overline{w}$ be the wealth level such that they would like to save more, but can’t due to politics. These cut-offs are defined formally by

$$b^*_1(w) = \overline{b} \Rightarrow w = y_2 - \overline{b}(2 + r)$$

and

$$b^*_1(\overline{w}) = \overline{b} - L \Rightarrow \overline{w} = y_2 - (\overline{b} - L)(2 + r)$$

Figure 1 depicts optimal borrowing behaviour in period 1

### 3 Optimal debt limits

It is useful to consider another period before discussing optimal $\overline{b}$. Period 1 behaviour is described completely by $b^*_1(y_1, b_0)$ as described in (15) and the wealth cutoffs $\overline{w}$ and $w$. Consider the period preceding period 1 which we label period 0. Assume the voter is unconstrained in this initial period (otherwise there is no decision to make). The value function of the entire problem from a period 0 perspective is then (where $w_0$ is initial wealth)

$$V(w_0) = \max_{b_0} h(w_0 + b_0) + \beta \int_{y}^{\overline{w} + (1+r)b_0} \left[ h(y_1 - (1 + r)b_0 + \overline{b}) + \beta h(y_2 - (1 + r)\overline{b}) \right] f(y_1) dy_1 +$$

$$\beta \int_{\overline{w} + (1+r)b_0}^{\overline{y}} \left[ h(y_1 - (1 + r)b_0 + b^*_1) + \beta h(y_2 - (1 + r)b^*_1) \right] f(y_1) dy_1 +$$

$$\beta \int_{\overline{y}}^{\overline{w} + (1+r)b_0} \left[ h(y_1 - (1 + r)b_0 + \overline{b} - L) + \beta h(y_2 - (1 + r)(\overline{b} - L)) \right] f(y_1) dy_1$$

Let $b^*_0 = b^*_0(w_0)$ be optimal borrowing in period 0 which satisfies the expression above.

Borrowing increases consumption of public goods today and increases debt tomorrow. In the two period case, we are concerned with the chance of ending up constrained which is exogenous.
Optimal period 1 borrowing/saving

\[ w_1 = y_1 - (1 + r)b_0 \]
Considering the period 0 problem highlights the fact that a longer time horizon means more sophisticated voting behaviour. From the period 0 perspective the optimal choice now influences the chance of ending up constrained in period 1.

With this in mind, we return to our focus of characterizing optimal debt limits. To gain some intuition, consider that changes in the debt limit have the following effects; when the constraints are binding from above and below (which relate directly to cases 2 and 3 above)\textsuperscript{17}.

\[
\frac{\partial V(w_0)}{\partial b} = \beta \int_{y}^{w+(1+r)b_0} \left[ h'(y_1 - (1 + r)b_0 + \bar{b}) - h'(y_2 - (1 + r)\bar{b}) \right] f(y_1)dy_1 +
\]

> 0, effect of easing debt constraint when borrowing is constrained

\[
\beta \int_{y}^{\bar{b}} \left[ h'(y_1 - (1 + r)b_0^* + \bar{b} - L) - h'(y_2 - (1 + r)(\bar{b} - L)) \right] f(y_1)dy_1
\]

< 0, increasing the borrowing constraint gives the politician more leverage

**Proposition 1** There exists a unique optimal debt limit \(\bar{b}^*\) which is increasing in \(w_0\)

The optimal \(\bar{b}\) depends on initial wealth. With higher initial wealth (and thus less borrowing/more saving in period 0), there is less chance of being constrained by the debt limit and more incentive to save in period 1. In fact if initial wealth is high enough there may be no chance of hitting the upper bound. To gain some intuition, consider the following simple example where we can solve explicitly for the optimal debt limit.

### 3.1 Numerical Example

Let the function \(h()\) be given by

\[ h(g) = g - \frac{g^2}{2M} \]

Where \(M\) is large enough to ensure that \(h'(\) is always non-negative. Furthermore, let government resources or wealth take on three values \(y_L \leq w, y_M \in (w, \bar{w})\) and \(y_H \geq \bar{w}\) which occur with probabilities \(p_L, p_M\) and \(p_H\) respectively. Let \(y_2 = y_M\) and \(b^*_1\) be the optimal borrowing when unconstrained\textsuperscript{18}. From above we have expected welfare over the two periods as

\[
W = p_L \left[ y_L + \bar{b} - \frac{(y_L + \bar{b})^2}{2M} + \beta \left( y_M - (1 + r)\bar{b} - \frac{(y_M - (1 + r)\bar{b})^2}{2M} \right) \right] +
\]

\textsuperscript{17}Leibniz integral rule and the envelope theorem are both used to attain the derivative.

\textsuperscript{18}There is no need to solve this explicitly as this term will drop out due to the envelope theorem.
\[ p_M \left[ y_M + b^*_1 - \frac{(y_M + b^*_1)^2}{2M} + \beta \left( y_M - (1+r)b^*_1 - \frac{(y_M - (1+r)b^*_1)^2}{2M} \right) \right] + \]
\[ p_H \left[ y_H + \bar{b} - L - \frac{(y_H + \bar{b} - L)^2}{2M} + \beta \left( y_M - (1+r)(\bar{b} - L) - \frac{(y_M - (1+r)(\bar{b} - L))^2}{2M} \right) \right] \]

Optimizing with respect to \( \bar{b} \) yields the following optimal debt limit

\[ \bar{b}^* = \frac{p_L(y_M - y_L) - p_H(y_H - y_M) + p_H(2+r)L}{(2+r)(p_L + p_H)} \]

We can see that the optimal debt limit is

- increasing in \( L \)
- increasing in the probability and “severity” of the low state
- decreasing in the probability and “size” of the good state
- positive or negative

The optimal rule depends on the chances of ending up debt or savings constrained as expected. The more well behaved politicians are (the larger is \( L \)), the larger the optimal debt limit to allow for debt financing should it be necessary.

4 Infinite horizon

This section considers the issue in an infinite horizon setting. We’ll analyze the simpler case where tax rates are fixed for continuity\(^{19}\). As above, private consumption is fixed (and thus ignored) and government revenues are an i.i.d process. Denote current debt by \( d \), current borrowing \( b \) and let a \( ' \) represent next period values. The voter is now optimizing over

\[ E_0 \sum_{t=0}^{\infty} \beta^t h(g_t) \]

Government incentives and political structure are the same as above. The problem is characterized by the following Bellman equation.

\[ P(d, y) = \max_b \left[ h(y - (1+r)d + b) + \beta E P(b, y') \right] \quad (16) \]

\(^{19}\)We can easily extend this to the more general case with tax smoothing as this section does not attempt to derive any analytic results.
Where the maximization is again subject to

\[ b \geq \bar{b} - L \]

(17)

\[ b \leq \bar{b} \]

(18)

The corresponding Lagrangian expression is

\[ \mathcal{L} = h(g) + \beta EP(b, y') + \lambda_1 [b - (\bar{b} + L)] + \lambda_2(\bar{b} - b) \]

yielding the first order condition

\[ h'(g) = \beta E P(b, y') + \lambda_2 - \lambda_1 \]

This and the envelope condition gives the following stochastic Euler equation

\[ h'(g) = \beta (1 + r) E (h'(g')) + \lambda_2 - \lambda_1 \]

inter-temporal wedge

A social planner would smooth consumption and set government spending such that there is a random walk of marginal utility. The political environment generally prevents this and creates an “inter-temporal wedge”, which as in the two-period case depends on which constraint binds. There is an important difference in the infinite horizon problem however. The initial wealth which was fixed in the finite period problem, now evolves due in part to any debt limit. Thus we can capture the behaviour of public monies over the short and long run. We expect that the imposition of a debt control will have short run costs that will be higher if the initial wealth is low. These costs are offset by the ability to control politicians in the long run. For future work, as a complement to the two period model, it would be interesting to characterize the effect of debt limits on the stationary distribution of wealth at least for some specified function and parameter values.

5 Conclusion

Politicians hinder the ability of governments to accumulate wealth. Incumbents are less likely to behave during good economic times as they can’t resist spending money when it’s around. The process of elections mitigates the problem as it imposes some degree of discipline on incumbents. That this is insufficient is the reason for this paper and many like it in the vast literature on political economy.

\[ \text{In the unconstrained case the planner will in fact save an infinite amount to finance an infinite consumption as the problem is not stationary. This can be avoided by assuming that at some very large level of consumption, marginal utility falls to zero. Otherwise we could consider a model where the politician gets some extra utility from spending when not seeking re-election and that could serve to limit asset accumulation.} \]
Imposing restrictions on borrowing/saving seems a natural solution to the problem but the impacts of such a restriction are generally ambiguous. Allowing governments flexibility has value and the gains of a tighter limit due to control of politicians is offset by a lessened ability to smooth consumption (and tax distortions in the more general case). The value of imposing borrowing limits depends on how irresponsible governments are, which is captured simply in the model by the parameter $L$. The larger is $L$, the more likely a limit will do more harm than good.

The results also suggest that the current fiscal position is important when considering a debt constraint. If one can be imposed during good economic times it is more likely to be beneficial. If governments spend and borrow too much however, we have a chicken and egg problem as imposing debt controls may not be ideal if the government is always in debt. To address this we must be able to describe the evolution of public debt and specifically the steady state distribution of wealth. Doing so requires a fully dynamic framework as is laid out in section 4 and is left for future work.

6 Appendix

Proof Lemma 1.

Consider first $\frac{\partial b^*_1}{\partial d} > 0$, $\frac{\partial \tau^*_1}{\partial d} > 0$. The first order conditions for the unconstrained problem are

$$h'(\delta(\tau_1)y_1 + b_1 - d)\delta'(\tau_1) = 1$$

(19)

$$h'(\delta(\tau_1)y_1 + b_1 - d) + \beta(1+r)h'(\delta(\tau_2)y_2 - (1+r)b_1) = 0$$

(20)

The implicit function theorem assures us of the existence of the functions $\tau^*_1(d)$ and $b^*_1(d)$. Differentiating the F.O.C’s fully and separating the relevant partials provides the results. These two derivatives are given by

$$\frac{\partial \tau_1}{\partial d} (h''(\delta(\tau_1)y_1 + b_1 - d)[\delta'(\tau_1)]^2y_1 + h'(\delta(\tau_1)y_1 + b_1 - d)\delta''(\tau_1)) + \frac{\partial b_1}{\partial d} h''(\delta(\tau_1)y_1 + b_1 - d)\delta'(\tau_1)$$

(21)

$$= h''(\delta(\tau_1)y_1 + b_1 - d)\delta'(\tau_1)$$

(22)

$$\frac{\partial \tau_1}{\partial d} (\delta'(\tau_1)y_1) + \frac{\partial b_1}{\partial d} (2 + r) = 1$$

(23)

We have two equations and two unknowns. Using the labels in the underbraces and rewriting in matrix form we have the following.

$$\begin{bmatrix} a & b \\ d & e \end{bmatrix} \begin{bmatrix} \frac{\partial \tau_1}{\partial \delta_1} \\ \frac{\partial b_1}{\partial \delta_1} \end{bmatrix} = \begin{bmatrix} c \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} \frac{\partial \tau_1}{\partial \delta_1} \\ \frac{\partial b_1}{\partial \delta_1} \end{bmatrix} = \frac{1}{ae-bd} \begin{bmatrix} ec-b \\ -dc+a \end{bmatrix}$$
To show the result, first calculate the following

\[ ae - bd = (1 + r)(h''(\delta(\tau_1)y_1 + b_1 - d)\delta'(\tau_1))^2y_1 + (2 + r)(h'(\delta(\tau_1)y_1 + b_1 - d)\delta''(\tau_1) < 0 \]  \hspace{1cm} (24)

\[ ec - b = (1 + r)(h''(\delta(\tau_1)y_1 + b_1 - d)\delta'(\tau_1) < 0 \]  \hspace{1cm} (25)

\[ -dc + a = h'(\delta(\tau_1)y_1 + b_1 - d)\delta''(\tau_1) < 0 \]  \hspace{1cm} (26)

Given these we have

\[ \frac{\partial\tau_1}{\partial d} = \left(\frac{1}{ae - bd}\right)(ec - b) > 0 \]  \hspace{1cm} (27)

\[ \frac{\partial b_1}{\partial d} = \left(\frac{1}{ae - bd}\right)(-dc + a) > 0 \]  \hspace{1cm} (28)

The other two results are obtained in the same manner. □

**Proof Proposition 1.**

It is tedious but straightforward to show that \( \frac{\partial^2 V(w_0)}{\partial b^2} < 0 \) so that a stationary point is a unique maximum. Replacing \( w = y_2 - (\bar{b} - L)(2 + r) \) and \( \bar{w} = y_2 - \bar{b}(2 + r) \), we can express the impact on welfare of a change in \( \bar{b} \) as follows (using both the envelope theorem and Leibniz integral rule).

\[
\frac{\partial V}{\partial \bar{b}} = \beta \int_{y_2}^{y_2+1+r(\bar{b}^* - \bar{b})(2+r)(y_1)} [h'(y_1 - (1 + r)\bar{b}^* + \bar{b} - (1 + r)\bar{b})] f(y_1)dy_1 \geq 0, \text{ effect of easing debt constraint when borrowing is constrained}
\]

\[
\beta \int_{y_2+1+r(\bar{b}^* - \bar{b})}^{y_2+1+r(\bar{b}^* - \bar{b})} [h'(y_1 - (1 + r)\bar{b}^* + \bar{b} - (1 + r)\bar{b})] f(y_1)dy_1 \leq 0, \text{ increases in the borrowing constraint give the politician more leverage}
\]

As the debt limit becomes larger (smaller), the positive (negative) terms shrink and eventually approach zero as there is no positive probability of hitting the debt (political) constraint. Specifically,

\[
\lim_{\bar{b} \to y_2} \int_{y_2}^{y_2+1+r(\bar{b}^* - \bar{b})(2+r)} f(y_1)dy_1 = 0 \Rightarrow \frac{\partial V(w_0)}{\partial \bar{b}} < 0
\]
Similarly,

\[
\lim_{b \to y_2 + L - \overline{y}} y_2 + (1 + r)b_0^* - (\overline{b} - L)(2 + r) \leq \lim_{b \to y_2 + L - \overline{y}} y_2 + L - \overline{b} = \overline{y}
\]

\[
\Rightarrow \lim_{b \to y_2 + L - \overline{y}} \int_{y_2 + (1 + r)b_0^* - (\overline{b} - L)(2 + r)}^{\overline{y}} f(y_1) dy_1 = 0 \Rightarrow \frac{\partial V(w_0)}{\partial \overline{b}} > 0
\]

Where the last inequalities in each come from the fact that only the relevant term in (29) is zero at these limits (which is easy to show). Since \(V(\overline{b})\) is continuous, there is some point in where the derivative is zero.

To show that the optimal debt limit is decreasing in initial wealth is suffices to show that \(\frac{\partial^2 V}{\partial \overline{b} \partial w_0} < 0\). Again using Leibniz rule we have the following.

\[
\frac{\partial^2 V}{\partial \overline{b} \partial w_0} = \beta \int_{y_2}^{y_2 + (1 + r)b_0^* - (\overline{b} - L)(2 + r)} h''(y_1 - (1 + r)b_0^* + \overline{b}) \left[ - (1 + r) \frac{\partial b_0}{\partial w_0} \right] f(y_1) dy_1 + (30)
\]

Which is always less than zero when period 0 borrowing is not increasing in wealth or \(\frac{\partial b_0}{\partial w_0} \leq 0\).

The impact of \(L\), or the degree of politician benevolence on the optimal debt limit is ambiguous. In fact, if \(\frac{\partial b_0}{\partial L} \geq \beta\) then the cross partial derivative is actually negative. This is somewhat surprising as one would imagine that the larger \(L\), the more value in relaxing the borrowing constraint. This is something I would like to consider further.

References


Besley, T., (2006), “Principled Agents? The political economy of good government, the Lindahl lectures”, *Oxford University Press*


