Abstract

When does a specialized rather than a general education system result in superior labour market outcomes? This question is analyzed in an economy in which individuals differ in two ways: innate talent and how well informed they are about that talent. Some learn early on about which careers they are suited to while others become informed about their talents later in life. Firms value productivity, which is a function of raw talent and education. Well-designed education systems trade off the desire to capitalize on raw talent through specialization with the need to provide individuals with opportunities to learn about their career preferences through a broad education. Our main finding is that in a population where individuals have different talents and information regarding career suitability, education systems focused on providing a broad education can outperform those that specialize.

Keywords: Education systems, labour market screening
1 Introduction

Talent unfocused is potential wasted. Economists commonly ascribe “talents” or productivities to the various actors in their models but rarely consider the fact that nobody begins (or likely ends) life with perfect knowledge of their own abilities. Presumably education serves the role of enhancing that which nature has bestowed on us. It is therefore prudent to consider the structure of our education system not only in the training of our youth but in the discovery of their own unique abilities.

This paper analyzes the impact of educational institutions on labour market outcomes. Individuals make schooling decisions which have various consequences depending on their personal characteristics and the system in which they are educated. In particular we consider the relative merits of specialized and general school systems in a simple model where agents are heterogenous over both their talents and their knowledge of those talents. Individuals decisions are driven by the various employment prospects facing them when they enter the job market. As expected, we find that the relative value of each system depends on the underlying population parameters. We calculate welfare under each institution and characterize the conditions under which each is optimal. Particularly relevant is the degree to which education increases productivity rather than signalling talent. The more productive education, the more attractive a specialized education system relative to a general one. Also important is the extent to which individuals are informed about their talents, more informed individuals favours a specialized system.

Human capital is integral to the health of any society. This fact has spawned large literatures in both economics and sociology that address a broad array of issues. Typically, economists focus on the impacts that education may have on the growth and distribution of income. Some authors do consider the structure of the education system explicitly. For example Krueger and Kumar (2004) develop a model where there are varying degrees of what they refer to as vocational and general education. Their results suggest that Europe’s relatively poor performance vis a vis the U.S may be partially explained by an education system that favors the former. The analysis of education institutions themselves seems to be confined to this simple skilled versus general dichotomy. Along this vein is the literature on “streaming” or “tracking”. The value and fairness of such systems, which separate students based on some measure of innate ability, has sparked much debate in recent years. Some examples include Epple, Newlon and Romano (2002), Brunello and Gianinni (2001) and Bertocchi et al (2004).

Rather than imposing a mechanism that sorts individuals by talent, we consider a system that gives all individuals access to the same resources. A specialized system gives individuals complete flexibility in choosing the focus of their studies. For example, one may choose a course of study that specifically favors analytic over verbal acumen. On the other hand, a general system is a fixed curriculum that requires study of all subjects by all students (i.e no choices). In a sense our environment allows for a deeper analysis of what is considered “general education” in the existing literature. The resulting outcomes and disparities between individuals can be attributed to a different process than that found in existing models and

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1See for example Heckman and Krueger (2003) and Gradstein, Justman and Meier (2005).

2Perhaps the closest analog to a specialized system in our model is the secondary or post secondary system in North America.
has different implications for the size and distribution of the economic pie.

In a closely related paper, MacDonald (1980) considers the matching of firms and workers with different productivity characteristics. Taking the view that each group of individuals has some comparative advantage in a particular task, he considers the acquisition of education to increase information about type which leads to better labour market matches. His paper does not consider the impact of various institutions on these outcomes as is the focus here.

We find that a general system provides insurance against poor initial information and can be superior to a specialized system when there is considerable uncertainty about talent persists early in life. It is likely that the quality of information is not uniform across the income distribution which implies that the choice of education system has distributional consequences. In particular, the poor may have worse information and will likely benefit from general systems as they are less likely to fall behind.\footnote{As in MacDonald (1980).} The relevant factors in making a choice over institutions are the structure of information about talents, the productivity differences generated by various institutions and the how the inputs of individuals combine in production.

The paper is organized as follows. Section 2 describes the basic model and section 3 introduces general and specialized educational institutions. Section 4 analyzes welfare and efficiency implications. Section 5 discusses extensions to the basic framework, section 6 outlines further avenues for research and section 7 concludes. All proofs are found in the appendix.

## 2 The Model

We develop a two-period labour market model in which individuals invest in education to enhance productivity. In the first period, individuals may be ignorant about their true talents when choosing education. In the second period, uncertainty about talent is resolved and individuals further invest in education prior to entering the job market.

Typically, “talent” is viewed as an endowment that bestows some general level of advantage on an individual. However, this ignores the reality that an individual may be talented in one field while lacking in another. We consider individuals that differ in their talent across two fields labelled $A$ and $B$. In addition, we take the distribution of talents across the population to be heterogenous.

Specifically, we define affinity or talent of an agent in field $f \in \{A, B\}$ as $\theta_f^i \in \{\theta_f^L, \theta_f^H\}$\footnote{We assume $\theta_f^L = \theta_f^P, \theta_f^A = \theta_f^P$.} where $\theta_f^L$ represents low ability and $\theta_f^H$ represents high ability (i.e. $\theta_f^H > \theta_f^L$), in field $f$. By an individual of type $\theta_{ij}$, where $i, j \in \{H, L\}$, we mean an individual with innate talents $\theta_{ij} = (\theta_A^i, \theta_B^j)$. For simplicity, we ignore people who are talented in both sectors so that effectively the population is composed of three types: $\theta_{HL}$’s (individuals suited to field $A$), $\theta_{LH}$’s (individuals suited to field $B$) and $\theta_{LL}$’s (individuals that are not particularly suited to either field). The population proportions of $\theta_{HL}, \theta_{LH}$ and $\theta_{LL}$ types are $p_{HL}, p_{LH}$ and $1 - p_{HL} - p_{LH}$, respectively.

Individuals do not know their type initially, but receive a signal at the beginning of
period 1. Specifically, in period 1, a fraction $\alpha (< p_{HL})$ find out that they are type HL, a fraction $\beta (< p_{LH})$ find out that are type LH, a fraction $\gamma (< 1 - p_{HL} - p_{LH})$ discover they are type LL while a fraction $(1 - \alpha - \beta - \gamma)$ remain uninformed about their type. The uncertainty regarding career suitability is completely resolved for all individuals at the beginning of period 2. Throughout, we assume that type information remains private and thereby non-contractible.

To enhance their productivity individuals may obtain education in each period. In period 1, everyone receives a fixed amount of education through either a general or specialized system. A general education system provides access to a broad based education without the ability for individuals to specialize in a particular field of study. On the other hand, a specialized education system provides access to a narrow education in field A or B.

In period 2, individuals have a choice in the type and amount of education they can acquire (i.e. a specialized system prevails) before entering the labour market. However, the costs of acquiring education in period 2 are dependent on first period education choices and systems. Consider a type $\theta_{ij}$ who chose field $f'$ in period 1 under a specialized system in period 1. The cost of acquiring $e$ units of education in field $f$ is $c^f(e, \theta_{fi}, f')$ where $c_{ef} > 0, c_{\theta f} < 0, c_{ef} > 0$ and $c_{\theta f} < 0$ for all $f$. Moreover, switching fields across periods is costly: $c^f(e, \theta_{fi}, f') > c^f(e, \theta_{fi}, f)$ for $f \neq f'$. Under a general system, there is no switching fields so that the corresponding cost function is simply given as $c^f_g(e, \theta_{fi})$. The general education system itself imposes a direct cost on acquiring second period education but one that is less severe than the cost of switching fields in a specialized system. That is, $c^f(e, \theta_{fi}, f') > c^g_f(e, \theta_{fi})$ for $f' \neq f$.

Two types of firms operate in the economy, type A and B. Type f firm’s output can be produced solely by individuals educated in field $f$. Moreover, an individual’s productivity is a function of both an individual’s innate talent and education. Firms screen out individuals with low productivity from individuals with high productivity by offering separating wage contracts contingent on education amounts. We employ the informational equilibrium concept of Riley (1979). Two firms of are assumed to operate in each sector so that competition among firms, within each sector, requires that firms pay individuals their marginal product.

The profits to a firm operating in sector $f$ from employing a worker of type $\theta_{ij}$ with $e$ units of education in field $f'$ at wage $w$ is given by:

$$
\Pi = \begin{cases} 
  y(e, \theta_{fi}) - we & \text{if } f = f' \\
  0 & \text{otherwise}
\end{cases}
$$

where $y(e, \theta_{fi})$ is the output produced by the worker. Individual utility from a wage contract $(w, e)$ is:

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5One can view the signal as the outcome of a search process in period 1 in which agents differ in their search costs.

6For simplicity we assume that signals are uncorrelated with type and that the aggregate signal process is common knowledge.

7For example, one can view this a general system equipping an individual with a half unit of education in each of A and B while a specialized system provides an individuals with one unit of education in either A or B.

8One can think of these as two different industries.
\[ U(e|\theta_{ij}, f) = \begin{cases} 
  w - c^f(e, \theta^f_i, f') & \text{under the specific education system} \\
  w - c^g(e, \theta^f_i) & \text{under the general education system.} 
\end{cases} \]

The timing is summarized in the following figure:

Figure 1: Timing

<table>
<thead>
<tr>
<th>Time 1</th>
<th>Time 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Signals observed</td>
<td>Choose education type a or b</td>
</tr>
</tbody>
</table>

3 General Model

3.1 Full Information Benchmark

As a benchmark we consider a world in which individuals have perfect information about their own type.\(^9\) As signals are completely informative, nobody makes mistakes in period 1. From a firms perspective this collapses to the basic screening problem with two types (either high or low). As this model is widely used we shall not bother to discuss it here but will use the classic results as a benchmark in what follows. Below we analyze the case of imperfect information over type and use this to contrast the value of two different education systems. In the following sections, we relax the assumption that individuals are perfectly informed about their job characteristics by their private signals early in life. As noted earlier, to capture the reality that some individuals are better informed about their preferences than others, we allow for asymmetry in the quality of information signals.

3.2 Specific Education

As in the previous section, we model a specific education system as one that provides individuals in period 1 with a choice: they may choose to specialize in A or B. As noted earlier, the choice in period 1 affects the cost of acquiring education in period 2.

We begin by looking at the second period problem in which individuals seek employment. Employers are unable to observe innate ability nor can they ascertain directly if an individual has received better career suitability signals early in life. That is, firms are unable to observe first period choices. Nevertheless, employers can observe the level of education an individual acquires. As a result, given that they care only about total productivity, employers will wish to offer separating contracts to distinguish the various types. In the analysis that follows

\(^9\)We are not interested in full information of the firms and only consider a second best world in this sense.
we shall consider the behaviour of a firm in a single sector as the problem of firms in either sector is identical.

To determine exactly how many types will prevail within each sector, we need to examine the behaviour of those HL and LH types that make mistakes in period 1 (for those who don’t make mistakes and those who are untaledent, behaviour will be obvious). These individuals face a cost function given by \( c^f(e, \theta_f, f') = c^f(e, \theta_f, f) / \phi \) for acquiring second period education where \( 0 < \phi < 1 \). For instance, for any given amount of education \( e \), HL types that chose \( B \) in period 1 are better off switching to sector A than staying in sector B as

\[
\begin{align*}
\text{w} - c^A(e, \theta_H, B) &= y'(e, \theta_H) - c^A(e, \theta_H, A) / \phi \\
&> y'(e, \theta_L) - c^A(e, \theta_H, A) / \phi \\
&> y'(e, \theta_L) - c^B(e, \theta_L, B)
\end{align*}
\]

where \( w = y'(e, \theta_f) \) due to competition among firms. As a result there will be three types in each sector in period 2.

Employers are aware that there are three types (high ability types that have chosen the correct field in period 1, high ability types that would switch if the incentives were right and low ability types), offer separating contracts contingent on education amounts. That is, within each industry, firms offer contracts of the form \((w_L, e_L), (w_{HS}, e_{HS}), (w_H, e_H)\) where \( w_H > w_{HS} > w_L \) and \( e_H > e_{HS} > e_L \). We have effectively reduced the second period problem to a screening problem with three types. In the informational equilibrium there are no pooling equilibria and there is at exactly one separating equilibrium.

Intuitively, the separating equilibrium provides the lowest types (types LL here) with their optimal bundle. It also requires that second lowest types (types HL that have switched) acquire enough education to distinguish themselves from the lowest types. Finally, the highest types (types HL that have chosen the correct field in period 1) are required to acquire the highest amount of education to distinguish themselves from the other two types. Formally, the equilibrium in the second period is characterized as follows:

**Lemma 1** In the unique separating equilibrium of the second period screening game between firms and employees, the optimal contracts are:

- \( e^*_L \) solves the low type’s problem and \( w^*_L = y'(e^*_L, \theta_L) \),

- \( w_{HS}^*, e_{HS}^* \) solve

\[
\begin{align*}
&w_L^* - c(e^*_L, \theta_L) = w_{HS} - c(e_{HS}, \theta_L) \\
&w_{HS} = y'(e_{HS}, \theta_H)
\end{align*}
\]

- \( w^*_H, e^*_H \) solve

\[
\begin{align*}
&w_{HS}^* - c(e^*_H, \theta_H) / \phi = w_H - c(e_H, \theta_H) / \phi \\
&w_H = y'(e_H, \theta_H)
\end{align*}
\]
where we have omitted the sector superscripts as the contracts are identical in each sector.

Now, given that the above separating contracts are offered in period 2, individuals choose to make their first period decision to maximize their lifetime welfare. Clearly, this implies that all informed HL types choose to enter sector $A$ while all informed LH types $B$ choose to enter sector $B$. Moreover, all informed LL types will be indifferent between choosing $A$ or $B$. Without loss of generality, we will assume that LL types choose $A$ or $B$, each with probability $1/2$.

To ensure that the problem is interesting, we require that uninformed types be indifferent between choosing $A$ or $B$ in the second period. Given that the wages prevailing in each sector will be identical, to ensure this requires

$$p_{HL} - \alpha = p_{LH} - \beta$$

which will be assumed throughout. Hence, an equal number of uninformed types will enter each sector in equilibrium.

### 3.3 General Education

We model a general education system as one that prevents individuals from choosing a particular field in period 1. One can view it as requiring individuals to acquire a fixed amount of education in both fields.

Clearly, the general education system provides hedging for those that are uninformed about their true types in period 1. This is because under the general education system uninformed LH and HL types will receive some education in the field to which they are suited. The hedging, while beneficial to uninformed types, is costly to informed LH and HL types because it forces these types to acquire education in an area to which they are not suited. This is in sharp contrast to the specialized education system in which uninformed types that make a mistake in their first period choice incur a penalty relative to informed types.

Since agents are not faced with a choice in period 1, they only need to decide how much education to acquire in the second period. The impact of the general education is modeled via an additional cost of acquiring education of type $A$ ($B$) in period 2 for types HL (LH).

All LH and HL agents face a second period cost function $c_f(e, \theta^f) = c_f(e, \theta^f, f)/\rho$ where $0 < \phi < \rho < 1$ for all $f$ and all $i$.

In period 2, firms in each sector face a screening problem with 2 productivity types as there is no switching at the beginning of period 2. Formally, the equilibrium in the second period is characterized as follows:

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10 The idea is that there may be knowledge that is required for second period education that these types were not able to acquire in period 1 and so they need to catch up in period 2.
Figure 2: Separating Equilibrium under Specialized Education

\[ U_L \]

\[ U_H \]

\[ U_{HS} \]

\[ \pi_H = 0 \]

\[ \pi_{HS} = 0 \]

\[ \pi_L = 0 \]

\( (w_L, e_L) \)

\( (w_{HS}, e_{HS}) \)

\( (w_H, e_H) \)
Lemma 2 In the unique separating equilibrium of the second period screening game between firms and employees, given a general education system in period 1, the contracts are:

\- \( e^*_L \) solves the low type’s problem and \( w^*_L = y'(e, \theta_L) \),

\- \( w^*_H, e^*_H \) solve

\[
\begin{align*}
  w^*_L - c_y(e^*_L, \theta_L) &= w_H - c_y(e_H, \theta_H)/\rho \\
  w_H &= y'(e_H, \theta_H)
\end{align*}
\]

Under a general education system, better information in period 1 conveys no advantage in period 2, so that period 1 signals are irrelevant.

4 Welfare

We define the lifetime ex-ante welfare of an individual as the sum of his expected welfare in period 1 and 2 before the realization of his private signal in period 1. Therefore, under a general education system, the ex-ante expected welfare of an individual, \( EU^g \), is given by the weighted average utilities of the high types (\( U^g_H \)) and low types (\( U^g_L \)) in period 2

\[
EU^g = (p_{HL} + p_{LH})U^g_H + (1 - p_{HL} - p_{LH})U^g_L
\]

where we have normalized the first period utilities to 0.

Under a specific education system, the ex-ante welfare calculation is more difficult. Let the second period utility of HL’s and LH’s that have learned their type in period 1 be denoted by \( U^H_{HS} \). Let the corresponding utility for types HL and LH that have switched (i.e. made mistakes in period 1) be denoted by \( U^H_{HS} \) and finally let the utility attained by low types be denoted by \( U^L \). Then, the ex-ante expected welfare, \( EU^s \), is the weighted average of these types:

\[
EU^S = [(\alpha + \beta) + \frac{1}{2}((p_{HL} - \alpha) + (p_{LH} - \beta))]U_H + \frac{1}{2}[(p_{HL} - \alpha) + (p_{LH} - \beta)]U_{HS} \\
+ (1 - p_{HL} - p_{LH})U_L
\]

The following result emphasizes the importance of education being productive:

Theorem 1 When education only serves as a means to signal talent (i.e. does not enhance productivity), a general education system always results in higher ex-ante welfare. More generally, the value of specialization is increasing in the extent to which education enhances productivity.

When education is productive, we can summarize the comparisons of ex-ante welfare in the following two results:
Theorem 2 For any given $\alpha$, $\beta$, $\rho$ and $\phi$

1. the ex-ante welfare under a general system relative to specialized one is increasing in $\rho$,

2. there exists a $\rho^*$ such that for all $\rho > \rho^*$, the ex-ante welfare under the generalized system is higher than under the specialized one.

This result essentially says that as long as the cost of taking a general education in period 1 is sufficiently low relative to the perfect information benchmark, imposing a general education system increases ex-ante welfare. The result below looks at how the a general education provides insurance for talent risk in period 1.

Theorem 3 Welfare under a general education system relative to a specialized one is decreasing in the number of informed high types in period 1:

$$\frac{\partial \Delta U}{\partial (\alpha + \beta)} < 0$$

5 Numerical Example

This section illustrates the results of section 4 through a numerical example. We use the following functional form for the cost functions:

$$c^A(e, \theta_i^A, A) = c^B(e, \theta_i^B, B) = \frac{e^2}{\theta_i}, i = H, L$$
$$c^A(e, \theta_i^A, B) = c^B(e, \theta_i^B, A) = \frac{e^2}{\theta_i \phi}, i = H, L$$
$$c^g_A(e, \theta_i^A) = c^g_B(e, \theta_i^B) = \frac{e^2}{\theta_i \rho}, i = H, L$$

Profits are given by the following:

$$\Pi = \theta_i^f e - we$$

Under a general education system in the first period, the low types maximize over education amount.

$$\max_{e_L} \theta_L e - \frac{e^2}{\theta_L \rho}$$

where we have omitted the sector superscripts as the problem is the same in both sectors. The first order condition yields the following optimal bundle

$$\left(e_L^*, w_L^*\right) = \left(\frac{\rho \theta_L^g}{2}, \frac{\rho \theta_L^g}{2}\right)$$

where we have used the zero profit condition to determine the wage. Using the incentive constraint for the low types, the optimal education amount for the high is implicitly
given by the following quadratic equation:
\[
\frac{\rho \theta^3}{4} = \theta H e^*_H - \frac{(e^*_H)^2}{\rho \theta L}
\]

(21)

Solving this equation and keeping the root that yields \( e^*_H > e^*_L \), we have the following optimal bundle for the high types:

\[
(e^*_H, w^*_H) = \left( \frac{\rho \theta L(\theta_H + \sqrt{\theta_H^2 - \theta_L^2})}{2}, \frac{\rho \theta H \theta L(\theta_H + \sqrt{\theta_H^2 - \theta_L^2})}{2} \right)
\]

(22)

Similarly, under a specific education system in the first period, the optimal problem for the low types is simply

\[
\max_{e_L} \theta L e - \frac{e^2}{\theta L}
\]

(23)

The first order condition yields the following optimal bundle

\[
(e^*_L, w^*_L) = \left( \frac{\theta^2 L}{2}, \frac{\theta^3 L}{2} \right)
\]

(24)

Again, using the incentive constraint for the low types, we can compute the minimum education amount that high types that have made a mistake in period 1 would need to undertake to differentiate themselves from low types. Therefore, the optimal contract high types that switch is as follows:

\[
(e^*_HS, w^*_HS) = \left( \frac{\theta_H \theta L(1 + \sqrt{1 - \frac{\theta_H^2}{\theta^2 L}})}{2}, \frac{\theta_H^2 L(1 + \sqrt{1 - \frac{\theta_H^2}{\theta^2 L}})}{2} \right)
\]

(25)

Now, the incentive constraint for the high types that switch ensures that they (weakly) prefer their own contract rather than the high types’ contract:

\[
w^*_HS - \frac{(e^*_HS)^2}{\theta_H \phi} = w_H - \frac{(e_H)^2}{\theta_H \phi}
\]

(26)

Then, noting that \( w_H = e_H \theta_H \), the above incentive constraint is again a quadratic equation in \( e_H \). Solving for the optimal contract for the high types we have:

\[
(e^*_H, w^*_H) = \left( \frac{\theta_H \theta L[(\frac{\theta_H \phi}{\theta^2 L} - 1) + (\frac{\theta_H \phi}{\theta^2 L} - \sqrt{1 - \frac{\theta_H^2}{\theta^2 L}})]}{2}, \frac{\theta_H^2 L[(\frac{\theta_H \phi}{\theta^2 L} - 1) + (\frac{\theta_H \phi}{\theta^2 L} - \sqrt{1 - \frac{\theta_H^2}{\theta^2 L}})]}{2} \right)
\]

(27)

Now, using the optimal contracts, it is clear that both Theorem 1 and 2 hold. [still to be shown formally]
6 Extensions/Further work

In modelling the first period problem, we have not explicitly incorporated the search costs that give rise to the differences in acquiring education in the second period. From a theoretical perspective, modelling the search costs directly would shed some light on how a search friction affects the information-rent in the principal-agent model.

We would like to consider how varying the value of education in production (i.e. making it more or less productive) influences the results. Moreover, we could also consider the effects of varying the production functions of firms such that firms across sectors would compete for the same workers. For instance, this would arise if the labour inputs of the various types were compliments in production.

Finally, we would like to investigate what implications this model has for issues of redistribution. Perhaps, we could compare the effects of redistribution via a tax and transfer scheme in period 2 with the redistribution achieved by making everyone choose a general education in period 1. The thinking is that it might be possible for the government to achieve some of its redistributional objectives by altering the scope and nature of the education system, while reducing the distortionary impact of taxation in later periods.

7 Conclusions

Using a simple model, we have characterized the idea that choice allows some to advance their talents but forces others to narrow their focus prematurely. In our framework, firms screen for talent using productive education. The environment in which educational investments is made is defined by the particular system in which individuals find themselves. We find that a general curriculum may provide a superior outcome to a specialized one when there is sufficient uncertainty over individuals types. Further, the degree to which this is true also depends on how well a general system prepares students for further education relative to specialization.

Although not explicit in the analysis, it is likely that information is not uniform across socioeconomic groups. Thus changes towards or away from specialization could have interesting redistributive consequences\textsuperscript{11}.

Finally, we note that a general education may have benefits we don’t consider here. A general education may provide many but important intangibles that are not be easily modelled. For instance, advocates of a broad based liberal arts education often emphasize the importance to society of having well rounded citizens. Although this type of reasoning does not naturally lend itself to economic analysis, it is certainly plausible and important nonetheless.

8 Appendix B: Proofs

Proof Lemma 1. As noted in the last footnote we adopt the equilibrium concept advanced \textsuperscript{11}This was initially a motivation for the paper. In the future, we would like to consider the signal component more deeply and include this type of reasoning.
by Riley (1979). This implies a unique separating equilibrium exists that given the lowest
types his optimal bundle and requires that the screen be sufficiently high to deter him from
wanting to take the next highest type’s bundle. An analogous argument holds for separating
out the high types that switch from the high types that do not.

**Proof Lemma 2.** See Riley 1979. This is the basic R-S screening game cast in the Riley
settings with types given by $\theta_L$ and $\theta_H$ and education is productive.

**Proof Theorem 1.** Still to come. 

**Proof Theorem 2.**

1. Define $\Delta U = EU^g - EU^s$. Then, using (11) and (12) we have

$$
\Delta U = (pHL + pLH)(U^g_H - \frac{U_H + U_H}{2}) - (\alpha + \beta)(\frac{U_H}{2} - \frac{U_H}{2}) +
(1 - pHL - pLH)(U^g_L - U_L)
$$

(28)

Now, $\frac{\partial(\Delta U)}{\partial \rho} = (pHL + pLH)(\frac{\partial U^g_H}{\partial \rho}) + (1 - pHL - pLH)(\frac{\partial U^g_L}{\partial \rho})$ as all other terms in (13)
are independent of $\rho$. Moreover, given that the low types receive their optimal bundle
in the separating equilibrium, by the envelope theorem $\frac{\partial U^g_L}{\partial \rho} > 0$. Under appropriate
conditions on the profit function we can show that $\frac{\partial U^g_H}{\partial \rho} > 0$. Hence, $\frac{\partial(\Delta U)}{\partial \rho} > 0$.

2. It suffices to show that $\lim_{\rho \to 1} \Delta U > 0$. Now, as $\rho \to 1$, $U^L_g$ approaches $U_L$ so that the
last term in (13) goes to zero. Then,

$$(pHL + pLH)(U^g_H - \frac{U_H + U_H}{2}) - (\alpha + \beta)(\frac{U_H}{2} - \frac{U_H}{2}) = (pHL + pLH)U^g_H - (\frac{pHL + pLH + \alpha + \beta}{2})U_H - (\frac{pHL + pLH - \alpha - \beta}{2})U_H S$$

$$> (pHL + pLH)(U^g_H - U_H)$$

as $U_H > U_H$. Now, as $\rho \to 1$, $U^g_H$ approaches the utility for the high type under the
typical problem with only two types which dominates the corresponding utility of the
high types when there are three types. Hence, $\lim_{\rho \to 1} \Delta U > 0$. Now, for $\rho = \phi$ we can
show that $\Delta U < 0$. Again, from (13) the last term will be negative as $U^g_L = U_L S < U_L$. Also,
for $\rho = \phi$ we have $U^g_H = U_H S$ so that combining this with $U_H > U_H$, implies that
$\Delta U < 0$. Now, noting that $\Delta U$ is a differentiable function of $\rho$ implies the existence
of $\rho^*$ via the intermediate value theorem.
**Proof Theorem 3.** From (13) we have \[ \frac{\partial (\Delta U)}{\partial (\alpha + \beta)} = \frac{-(U_H - U_{HS})}{2}. \] Now, it is easy to see that \( U_H > U_{HS} \) as acquiring education is more costly for high types that switch than for high types that do not. Hence, \( \frac{-(U_H - U_{HS})}{2} < 0. \)

**References**


