The Impact of Capital Gains Tax and Transaction Cost on Asset Bubbles

Yue Shen*†
Queen's University

July 6, 2015

Abstract

In this paper we investigate the effects of capital gains tax and transaction cost on asset bubbles. We construct a model of asset bubbles by incorporating purchases into the framework of Abreu and Brunnermeier (2003) so that capital gains can be evaluated. The capital gains tax helps deflate the bubble, but we find that the capital loss tax credit tends to offset this deflating effect. Under a perfect tax credit, the tax has no effect on the size of the bubble at all. Therefore dealing with bubbles with capital gains tax not only requires imposing the tax, but also tightening policies on tax credits. Besides that the transaction cost and the return from the outside option help reduce the bubble, we also find that a low transaction cost or a small outside option has a very large marginal effect on the bubble. This implies that when a central bank further lowers interest rates when they are already very low, this policy change can have a dramatic inflating effect on bubbles. To show that our results can be empirically tested, we compare several historical bubbles in different countries. We normalize the size of a bubble by its associated belief dispersion so that we can examine the effects of other factors such as taxes and transaction costs. We also propose a method to infer the belief dispersion from the price path of an actual bubble in the absence of explicit data on the belief.

1 Introduction

Since the 2008 US subprime mortgage crisis, housing prices in certain major cities outside the United States have been steadily increasing at speeds higher than their historical norms, including those in London (UK), Vancouver, Toronto, Beijing, Shanghai, Hong Kong[1], and most capital cities in Australia. For example, house prices in London rose by 18% in 2013 alone. The upsurging prices have raised serious concerns that bubbles are developing in these cities. Although the ongoing discussion on macroprudential policies might help regulate domestic financial practitioners, these policies seem to be ineffective when international "hot money" and private investors contribute greatly to the bubbles, and there is no consensus on the

*I am grateful to James Bergin and Frank Milne for numerous valuable comments. I thank Ruqu Wang, Marco Cozzi, Rui Gao, Steve Kivinen, Marie-Louise Vierø, Tianyi Wang, Jan Zabojnik, Chenggang Zhou, and participants in CEA and department seminars at Queen's. All errors are my own.

†Department of Economics, Queen's University, Kingston, Ontario, Canada. sheny@econ.queensu.ca

[1] Housing prices in hundreds of smaller cities in China are also inflating rapidly.

implementation of these policy tools. A different choice would be a tax. There is a large literature on the effects of financial transaction taxes on price volatilities, but to our knowledge the effectiveness of taxes on asset bubbles, especially capital gains tax, has not been studied in the existing literature.

With or without economic theories, governments and lawmakers are ready to intervene, or have already done so. Chancellor George Osborne of the British Parliament said that as of April 2015 he would introduce a capital gains tax on future gains made by non-residents who sell residential properties in the United Kingdom[2]. In 2013 the Chinese government introduced a 20% tax on capital gains from selling residential properties if a family owns multiple properties. There is an urgent need to assess the effectiveness of these tax policies on reducing bubbles.

We evaluate the effects of capital gains tax and transaction costs on asset bubbles. Our model incorporates purchases into the framework of Abreu and Brunnermeier (2003) (henceforth AB2003). When the asset fundamental rises, privately informed rational traders purchase the asset from behavioral agents, which continuously drives up the price. Each rational trader has incomplete knowledge about the new fundamental value and does not know how many others have a higher (or lower) belief. This belief dispersion is such that there is no common knowledge about the emergence of a bubble when the price rises above the fundamental value. As the price continues to rise, traders with the lowest belief stop purchasing and simply hold while others are buying. As the price is further driven up, low-belief traders start to sell, intermediate-belief traders stop buying and hold, and only high-belief traders are still buying. When traders with the highest belief stop buying, the bubble bursts.

In the unique equilibrium a bubble exists, and traders ride the bubble and try to make a profit from buying and selling. Upon the bursting, low-belief traders have already exited the market while others are caught in the crash. Each trader uses a trigger strategy that consists of two price thresholds: a selling price and a stop-buy price. The optimal selling strategy trades off the marginal price appreciation with the marginal risk of being caught in the crash. The stop-buy strategy dictates that a trader should not buy the asset if the expected after-tax profit is negative.

A trader with a profit is subject to a capital gains tax, while a trader with a loss is entitled to a tax credit.[3] The most favorable treatment of tax credits is refunding the trader an amount of the loss multiplied by the tax rate, which we call a perfect tax credit.[4] But in most countries the credits can only

[2]See Chancellor George Osborne's Autumn Statement 2013 speech on 5 December 2013 at the UK Parliament.

[3]The capital loss tax credit is also called capital loss deduction or tax carryover. It aims to reduce the tax burden when tax payers incur capital losses and help smooth fluctuations in their incomes so that the tax is levied on a long-term average base.

[4]This is also called a symmetric tax treatment or full loss refundability in the tax literature. Symmetry means an investor

be used to offset past or future gains, instead of an immediate refund, and there are restrictions on how gains can be offset, such as time restrictions, ceilings and inclusion rates.[5] We use a single parameter (the inclusion rate) to summarize these restrictions and allow it to vary from no credit at all to a perfect tax credit.

Our paper presents two main results. The first is that the tax credit can offset the deflating effect of capital gains tax on bubbles and when the tax credit is perfect, the tax has no effect on bubbles at all. The capital gains tax can reduce bubbles in our model because, intuitively, it widens the relative payoff difference between fleeing and being caught. This increased difference makes traders behave more cautiously by selling early to secure their gains. The lowered selling strategy then squeezes the stop-buy strategy downwards, which bursts the bubble early. The credit, on the other hand, serves as a compensation to a trader's loss such that the loss is also "taxed" and becomes smaller. It thus reduces the payoff difference and traders become less concerned about being caught. They behave aggressively by selling at high prices, which in turn encourages buying at higher prices and the bubble is inflated. Under a perfect tax credit, this compensatory effect completely neutralizes the effect of the tax, even when we raise the tax to 100%! This result suggests that to deflate a bubble with capital gains tax, a tax authority should not only impose or raise the tax, but also examine its tax credit policies and refrain from granting overly favorable tax credits on capital losses.

The second result is that, while it may not be surprising that a transaction cost or outside option (returns from alternative investment opportunities) can reduce bubbles, their marginal effects on bubbles are very large when they are small. When the cost or the outside option rises slightly from zero, a small gap between buying and selling is required (the profit margin). But since the bubble could burst in between this gap, a larger gap (profit margin) is needed to compensate the risk. A larger gap in turn means a larger probability of bursting in between, which in turn requires an even larger gap. Hence a small cost or outside option can significantly push down the stop-buy strategy and deflate bubbles. This result backs the practice in some countries to reserve financial transaction taxes at very low rates. If we interpret the outside option as the interest on treasure bonds, for instance, then conversely it implies that lowering interest rates when they are already low has a significant inflating effect on bubbles. This argument is

pays the tax whenever a profit occurs and receives a refunding whenever a loss occurs.

[5]In the United States, an individual who incurs capital losses can use the credit to offset any taxable income up to $3000 each year, and carry the unused credit forward indefinitely. A corporation can only offset capital gains (not other profit) with capital losses and carry it 3 years backward and 5 years forward, without a dollar limit. In Canada the tax code is similar, except that there is no $3000 limit on individuals, but only 50% of the capital gains/losses are included in the tax base for both individuals and corporations, and the tax credit can be carried 3 years backward and forward indefinitely.

supported by the timing of the Federal Reserve's low interest rate policies and the housing bubble in the United States between 2001 and 2008 and their potential causality. It also warns that the Bank of Canada's recent move of lowering the interest rate from 1% to 0.75% (a counter measure to the oil price plunge) will further inflate the Canadian housing market.

To show that our results are empirically testable, we measure and compare four historical asset bubbles in different jurisdictions and fit them into the theoretical results. There is no consensus on how to compare different bubbles because there are numerous economic variables that could affect a bubble. We believe that the belief dispersion about the asset fundamental is the primary and intrinsic cause of the wildly distinctive sizes of these bubbles. Thus we normalize the size of a bubble by its associated belief dispersion so that we can compare the effects of other factors (taxes, transaction costs, etc.) on these bubbles. In the absence of explicit data on belief dispersion, we propose to use the largest price drop before the final crash as a proxy for the start of rational traders' attack on the bubble. In our model we can infer the belief dispersion from the price associated with this initial attack, since the latter is a function of the former. Given a sample size of four, however, our analyses are mainly descriptive and merely serve as a first step to testing the model. Besides the two main results, our model also generates other empirically testable predictions.

The framework of AB2003 is shown to be consistent with recent empirical studies of stock market data. Temin and Voth (2004) show that a major investor in the South Sea Bubble knew that a bubble was in progress and nonetheless invested in the stock and hence was profiting from riding the bubble. Brunnermeier and Nagel (2004) and Griffin et al. (2011) both study the tech bubble in the late 1990s. They show that instead of correcting the price bubble, hedge funds turned out to be the most aggressive investors. They profited during the upturn and unloaded their positions before the downturn.

Our model is also related to a large literature that studies the effects of taxes on asset prices. Constantinides (1983) shows that investors have incentives to sell assets with losses immediately and secure tax credits while deferring the selling of assets with gains to put off tax payment. His research focuses on trading under stochastic shocks whereas we focus on trading with private beliefs facing asset bubbles and crashes. In terms of the effects of the transaction tax on the volatility of asset prices, Westerhoff and Dieci (2006) show that the transaction tax can stabilize prices, whereas others suggest that the tax actually amplifies volatility (e.g. Lanne and Vesala (2010)). Empirical evidences show that the ability of the tax to reduce volatility is very limited (e.g. Umlauf (1993) and Hu (1998)).

The research on the effects of taxes on bubbles, on the other hand, is scarce. Scheinkman and Xiong

(2003) show that the financial transaction tax can substantially reduce speculative trading volume, but has only a limited impact on the size of the bubble. Our paper explicitly models an asset bubble and evaluates the effects of capital gains tax on trading behavior and the size of bubble, and is complementary to the above literature in understanding the effects of taxes on financial markets.

From a modeling perspective, our paper is related to a quickly growing literature on bubbles. For surveys, refer to Brunnermeier (2008), Brunnermeier (2001), Brunnermeier and Oehmke (2012) and Xiong (2013). AB2003 relies on the asynchronous timing of awareness to generate bubbles, whereas we transform the uncertainty from time to value/price and remove the sequential awareness assumption. Our model allows for any continuous and strictly increasing price path instead of a exogenous exponential price path. We also add purchases to their framework such that the price keeps increasing exactly because rational traders are buying. The bubble bursts in our model when no one wants to buy, which is in contrast to AB2003, where a threshold in accumulated sales triggers the bursting.

Doblas-Madrid (2012) removes behavioral agents from the framework of AB2003 and instead uses idiosyncratic liquidity shocks to force rational traders to sell to generate trades. The price is determined in equilibrium every period and agents also update their belief when facing the noisy price. This feature is difficult to apply to our setting because we do not have any noise in the price[6].

The remainder of this paper is organized as follows. Section 2 introduces the model. Section 3 shows that a trader's strategy space can be reduced, which gives us a simple game to solve. In Section 4 we solve a trader's problem, characterize the equilibrium and discuss several policy implications associated with the results. Section 5 studies several historical bubbles, measures and compares them and shows how to test the main results of the model. Section 6 concludes.

2 The model

There is one asset (henceforth the *asset*) with a total supply Q. Time t is continuous and at $t = 0$ the asset's fundamental value jumps up from p_0 to θ and does not change henceforth. θ is uniformly distributed on $[p_0, \infty)$[7] and is unobservable. Without loss of generality, we assume $p_0 = 0$. There are two types of agents: risk neutral rational traders (henceforth *traders*) and a large passive behavioral agent (or a pool of behavioral agents), though the latter is not our primary interest. All shares of the asset are held by the

[6]To conceal the very first strategic sale in our model, one must assume that there is an extra amount of money injected to balance the strategic sale at that moment, which can be difficult to justify.

[7]The improper uniform distribution on $[0, \infty)$ has a well defined posterior belief when we specify how signals are distributed. The uniform prior distribution gives tractable solutions and is adapted from Li and Milne (2014).

passive agent at the beginning. Rational traders have an outside investment option. The outside option provides a constant profit $R \geq 0$, which is common knowledge and uncorrelated with θ. A trader cannot hold the asset and the outside option at the same time and cannot switch to the other if she has bought one. Capital gains from both the asset and the outside option are subject to a capital gains tax, and a trader will choose the asset only when its expected profit is strictly higher than the outside option. At this moment we restrict that $R = 0$. In Section 4.6 we discuss the implication of strictly positive R. Each trader is infinitesimal and, without loss of generality, a trader's asset position is restricted and normalized to $[0, 1]$.[8]

The passive agent has an inverse asset supply function

$$p = \alpha(D_r) \tag{1}$$

where p is the price that is publicly observable, D_r is the total shares held by all traders (the aggregate position of all traders) and $\alpha(\cdot)$ is a continuous, strictly increasing function. When the fundamental value jumps up, traders start to buy the asset from the passive agent. This drives up the price continuously because, as shares are sold to traders, the passive agent keeps raising the price. This behavior can be interpreted as portfolio diversification requirements that make the risky asset more valuable to the passive agent when its weight decreases in her portfolio. Or it can be interpreted as an adverse selection problem where, when traders with private information keep buying the asset, it is natural for the uninformed passive agent to respond by raising the price, as in Kyle (1985). Similar behavioral asset supplies have also been adopted by De Long et al. (1990), where passive investors supply the asset at an increasing price when rational speculators are buying, and by Brunnermeier and Pedersen (2005) and Carlin et al. (2007), where long-term investors sell the asset when strategic traders' buy-back pushes up the price.

When the price rises above the fundamental value, we say a bubble emerges. When the price is driven so high that no one wants to buy any more, the price stops rising. Denote this random stopping price p_T and the time T. If $p_T > \theta$ at this moment, it will be clear shortly that the existence of the bubble becomes common knowledge and the bubble bursts at p_T *endogenously*.[9] We also assume that the price can be arbitrarily high when traders' aggregate position approaches Q, i.e. $\underset{D_r \longrightarrow Q}{lim} \alpha(D_r) = \infty$. This rules out the possibility that the asset price cannot catch up to its fundamental value simply because shares are running out.

[8] Limited short selling of the asset is allowed for traders.

[9] In a nutshell, given the strategy profile of all types of traders, p_T is a function of θ. Thus everyone can perfectly infer θ from the stopping price p_T.

After entering the market, a trader can buy and sell the asset at any time. A trader's purchases and sales cannot be observed by others. Since each trader is infinitesimal, her transaction is executed instantly at the spot price. After the bursting, the price is fixed at θ and the passive agent is willing to buy the asset only at this price. All traders who still hold the asset have to liquidate at this price. In the end, no trader holds the asset and all shares go back to the passive agent.

Each trader receives a private signal v that is uniformly distributed on $[\theta - \frac{\eta}{2}, \theta + \frac{\eta}{2}]$. We call a trader with signal v a type v trader or trader v. There is enough mass of traders (potentially infinite) for each type v.[10] Let $\Phi(\theta|v)$ denote the cumulative distribution function of the belief of a trader v conditioned on v, and $\phi(\theta|v)$ the corresponding probability density function. Given a signal v, the posterior belief of a trader is that θ is uniformly distributed on $[v - \frac{\eta}{2}, v + \frac{\eta}{2}]$. Therefore $\phi(\theta|v) = \frac{1}{\eta}$ and $\Phi(\theta|v) = \frac{\theta-(v-\frac{\eta}{2})}{\eta}$.[11] Figure 1 depicts the posterior belief about θ for traders v, v' and v''. These different posterior beliefs reflect

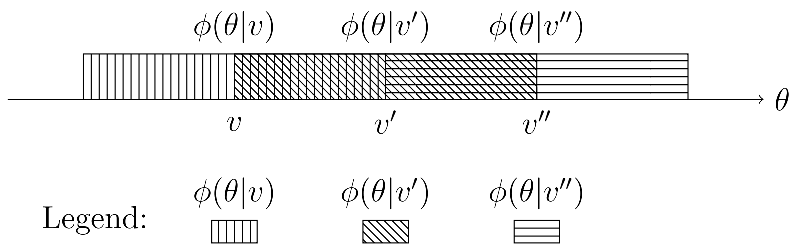


Figure 1: Posterior beliefs

different opinions about the asset fundamental value. A trader is not sure about her position among the population $[\theta - \frac{\eta}{2}, \theta + \frac{\eta}{2}]$, i.e., a trader does not know how many others' signals are lower or higher than hers. This is an important element in the model because the lack of common knowledge about "what all others are thinking about" prevents traders from perfectly coordinating with each other. In contrast, in the standard literature with a common posterior belief, perfect coordination rules out the existence of the bubble by backward induction.

Traders enter the market gradually and steadily due to an exogenous friction that is not modeled here. This friction is simply in order to achieve a gradual upward price path before the crash. Without this friction the bubble still exists, but the run-up becomes instantaneous: all traders rush into the market all at once at the beginning and push the price up to the peak infinitely fast, the bubble bursts and the price

[10]This is to rule out the possibility that the price cannot catch up to its fundamental value simply because we are running out of buyers.

[11]When $\theta < \eta$, traders with $v < \frac{\eta}{2}$ have a truncated belief support because θ cannot be below zero. This causes these traders to have different strategies. As explained in Section 4 these traders are not important and thus we ignore these traders. When $\theta < \frac{\eta}{2}$, some traders will receive negative signals, but this is perfectly compatible with the assumption that $\theta \geq 0$.

then plummets back. Such a friction could arise because traders need time to sell their other assets first or to wait for the maturity of other investments to buy this one, or simply because they did not notice this asset earlier. With this friction, traders enter the market in a continuous stream. We assume that the newcomers are always confident enough about the asset value and in particular the highest type, $\theta + \frac{\eta}{2}$, is entering at every instant.[12] If we let $u(t, v)$ denote the mass of traders entering the market at time t with signal v, then $u(t, \theta + \frac{\eta}{2}) > 0$, $\forall t > 0$. Thus the price is smoothly increasing[13] and there will be no ambiguity when the price stops rising at the peak when all traders cease buying. We can thus simplify the bursting of the bubble as a vertical drop right at the peak.[14] But beyond that $u(t, \theta + \frac{\eta}{2}) > 0$, traders (and the behavioral agent) have no knowledge about $u(t, v)$. Thus traders anticipate a smooth and strictly increasing price path before the crash, and are unaware when some of them start to sell strategically since they do not know the shape of the path.[15] This unawareness parallels and simplifies traders' uncertainty in a real market about other traders' behavior when facing a noisy price, and complex belief updating (if traders have some partial knowledge) is avoided, so that we can focus on our main targets.

A trader incurs a fixed transaction cost c each time she changes her position, irrespective of the price or the volume of the transaction.A trader's profit before tax is thus determined by her purchase prices and sale prices, minus the transaction costs. There is a capital gains tax of rate τ, which is levied when a trader has a *realized* profit (after deducting the fixed transaction cost). If the trader incurs a loss, then she is entitled to a tax credit. We summarize complex tax credit policies (such as ceilings and expirations) into a single parameter τ_c, which can vary between 0 and τ. When a trader has a realized capital loss L, she gets a refund $\tau_c L$.[16] A more lenient tax credit policy, such as a longer period within which the credits can be used, corresponds to a higher τ_c. The capital loss tax credit is said to be perfect when $\tau_c = \tau$.

To rule out the possibility that some types of traders never stop buying so that the bubble grows forever, we assume that the size of the bubble has an upper bound $\overline{B}$. Once $p - \theta > \overline{B}$, the bubble bursts exogenously. We are only interested in the endogenous bursting, so $\overline{B}$ is large enough.[17] Lastly, a

[12]This guarantees that the bubble will not burst accidentally and prematurely. It is actually simpler to imagine that the newcomers always have a full support $[\theta - \frac{\eta}{2}, \theta + \frac{\eta}{2}]$, though this is not necessary.

[13]Our model allows an arbitrary price path, as long as it is continuous and strictly increasing.

[14]As we can observe in real markets in Section 5.1, there are usually gradual but short downturns after the peaks but before the largest crashes. Facing noisy prices, some traders are uncertain about the coming of the final downturns within this period.

[15]We assume that $u(t, v)$ is large enough so that it outweighs the strategic exiting. Alternatively we can assume that there is no market entry friction but all traders finance their purchase by borrowing and there is friction on the availability of loans. The loans are available in a stream. By allowing transferring of rational sellers' repayment directly to rational buyers who are borrowing, the stream of loans can also achieve a smoothly increasing price path that is irresponsive to strategic sales.

[16]Constantinides (1983) models the tax credit in a similar way.

[17]We will show that in the equilibrium with endogenous bursting the size of the bubble size is finite and constant so that $\overline{B}$ is never binding.

technical assumption is that all traders selling exactly at the instant when the bubble bursts receive the full pre-crash price.

Figure 2 shows a simple example of the dynamics. In each panel, traders' types (signals) are continuously distributed between $\theta - \frac{\eta}{2}$ and $\theta + \frac{\eta}{2}$ along the horizontal axis, given θ, while the vertical axis is the mass density for each type of traders. The shaded area is the mass of traders who hold the asset. If we assume that every buyer holds the same amount of shares (which will turn out to be the case in equilibrium), the shaded area is proportional to D_r. In panel (a) at $t = 0$, no one has entered the market yet. In panel (b), some traders have entered and bought. In panel (c), the price is high enough such that low types are no longer buying but simply hold (the shaded area of "hold" freezes and does not rise any more), while the rest are entering and buying. In panel (d), the price is even higher such that not only does "no-buy" spread to higher types of traders, low types now start to sell. The highest types, however, are still buying. In panel (f), the "no-buy" finally reaches the highest type, $\theta + \frac{\eta}{2}$, who stops buying. As a result, no one buys any more and the price stops rising and the bubble bursts. Note that in Figure 2, we have assumed that all traders use a trigger strategy, where a trader will not restart buying once she has stopped buying and she will never re-enter the market once she has sold. This strategy will turn out to be the equilibrium strategy.

3 Preliminary analysis

In this section, we define the equilibrium, impose two technical assumptions on traders' strategies and show that the dynamic game can be simplified to a static-like game by reducing traders' strategy space. We will establish that in equilibrium traders use trigger strategies (Proposition 3.1) and their decisions are strictly increasing (Lemma 3.1). Readers who are not interested in these details can jump to Section 4.

Definition 3.1. *A trading equilibrium is a Perfect Bayesian Nash equilibrium in which traders hold the (correct) belief: whenever a trader v is not buying (temporarily or permanently), she (correctly) believes that all traders with signal equal to or smaller than v are not buying.*

This definition imposes a natural assumption on traders' equilibrium beliefs, without which it will be difficult to characterize an equilibrium.

With the positive fixed transaction cost c, a trader will trade only a finite number of times. Due to the linearity of the problem and the cost c, it is optimal for traders to either hold the maximum long position or not hold any asset at all. At any given price, a trader's asset value is linear in her position.

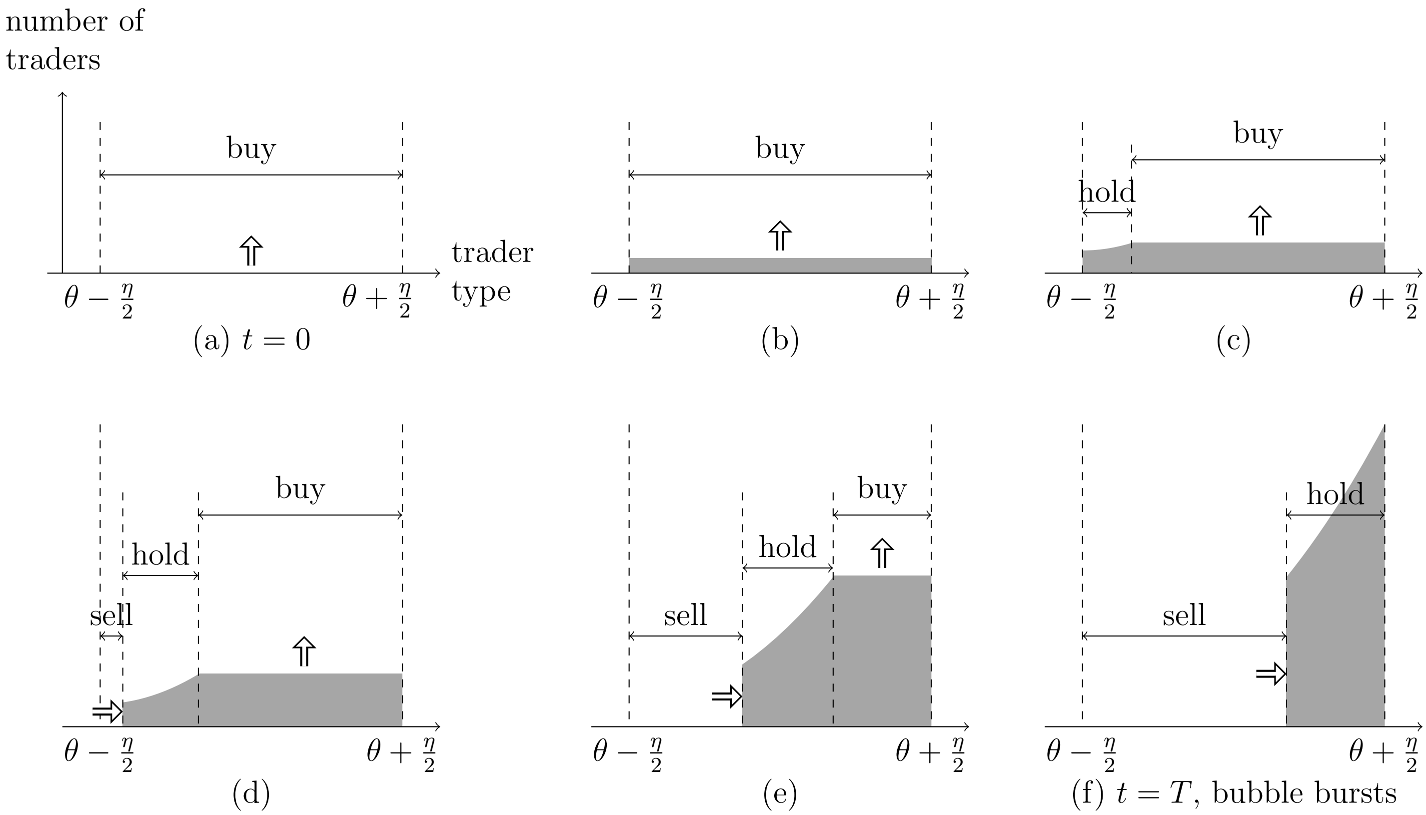


Figure 2: Dynamics of the mass of shareholders

With the transaction cost, if buying is profitable, then it is optimal to buy to the maximum long position; conversely, if selling is profitable, then it is optimal to sell all shares[18]. Hence, the space of a trader's asset position in equilibrium reduces to $\{0, 1\}$.

Because we assumed a simple linear tax rate, a trader's profit or losses realized after sales (whether in the current tax year or previous years) and realized tax payments and benefits are all sunk and will not affect her future decisions. The trading history affects her decisions only when she is currently holding the asset and whether she can sell before the crash is uncertain, i.e. her most recent purchase price may affect her selling decision. At any given price, a trader who has entered the market has three options: buy to the maximum long position (*buy*), not change her current position (*hold*), and sell all her shares (*sell*). Let $A(p, v, h, P_p)$ denote the strategy of an in-market trader v at price p with position $h \in \{0, 1\}$ and the most recent purchase price P_p. P_p is relevant only when $h = 1$. Then A is defined on $[0, \infty) \times [-\frac{\eta}{2}, \infty) \times \{0, 1\} \times [0, \infty) \to \{buy,\ hold,\ sell\}$.

Definition 3.1 immediately implies the following corollary, which states that when trader v is not buying, then all types weakly lower than v are not buying, and when she is buying, all types weakly higher than

[18] See Appendix A and proof of Lemma 1 in AB2003 for details.

her are still buying, irrespective of trading histories or current positions.[19] This is because the belief in Definition 3.1 must be correct for all types of traders.

Corollary 3.1. *$A(p, v, h, P_p) \neq$ buy $\Longrightarrow A(p, v', h', P'_p) \neq$ buy, $\forall v' \leq v$ and $\forall p, h, P_p, h', P'_p$;*
$A(p, v, h, P_p) =$ buy $\Longrightarrow A(p, v', h', P'_p) =$ buy, $\forall v' \geq v$ and $\forall p, h, P_p, h', P'_p$.

Corollary 3.1 implies that traders' strategies are symmetric in the sense that traders with the same signal are either all buying or all not buying. Now we can formally define the bursting price p_T:

$$p_T = inf\{p | A(v, p, h, P_p) \neq buy, \forall v \in [\theta - \frac{\eta}{2}, \theta + \frac{\eta}{2}], \forall h \text{ and } \forall P_p\}$$

That is, the bubble bursts when no one wants to buy.

Let $P^*_b(v)$ denote the price at which the action of trader v is not *buy* for the first time. Let $P^*_s(v, P_p)$ denote the price at which trader v sells her shares for the first time, given her purchase price P_p (since her position is either 0 or 1 and this is the first-time sale, P_p is well defined.). By definition, $P_p < P^*_b(v) \leq P^*_s(v, P_p), \forall v$ and $\forall P_p$.

To derive an equilibrium, we need two technical assumptions:

Assumption 3.1. *$P^*_b(v)$ is continuous in v, $\forall v$ and differentiable $\forall v > \frac{\eta}{2}$, and $P^*_s(v, P_p)$ is continuous in both v and P_p.*

Lemma 3.1. *$P^*_b(v)$ is strictly increasing in v, $\forall v \geq \frac{\eta}{2}$.*

Then the inverse function $P^{*-1}_b(v)$, $\forall v \geq \frac{\eta}{2}$, is well defined.

Assumption 3.2. *A trader cannot switch actions more than once in one instant.*

Then we have the following proposition.

Proposition 3.1. *(Trigger-strategy): In equilibrium, if an in-market trader's action is not "buy", she will never restart buying again. If a trader has sold her shares, she will never return to the market.*

Proposition 3.1 implies that when the highest type $\theta + \frac{\eta}{2}$ decides to stop buying for the first time at $P^*_b(\theta + \frac{\eta}{2})$, all other types have already done so. Since no one is buying at this moment, the bubble bursts at $p_T = P^*_b(\theta + \frac{\eta}{2})$. Let $\Theta(p_T) \equiv P^{*-1}_b(p_T) - \frac{\eta}{2}$. Then if the bubble bursts at p_T, the realized θ must be $\Theta(p_T)$. This is how everyone can perfectly infer θ from the bursting price p_T.

[19] For the ease of description, we allow a trader with asset position 1 to use action *buy*, though she is not able to further increase her holding due to the limit.

Proposition 3.1 further reduces a trader's strategy space to $\{P_b^*(v), P_s^*(v, P_p)\}$. After $t = 0$, traders keep entering the market and buying the asset. If a trader v has bought the asset at price P_p ($< P_b^*(v)$), she holds and waits until the price rises to $P_s^*(v, P_p)$ then she sells all her shares and will never restart buying again. If she has not entered the market, then at price $P_b^*(v)$ she will no longer try to enter the market. Readers can review the price dynamics under this strategy profile in Figure 2 and a sketch of traders' strategies and where they end up in Figure 3.[20]

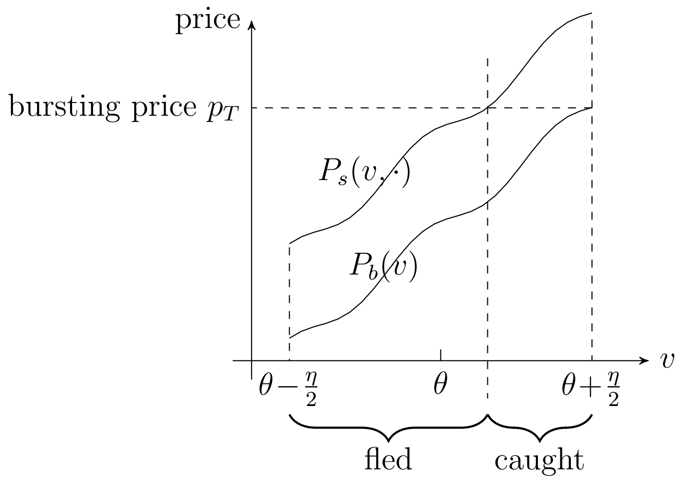


Figure 3: Traders' strategies and outcomes

4 Reduced-form game

From Lemma 3.1 and Proposition 3.1 we know that for an arbitrary θ and the signal profile generated by θ, the bursting price is fully determined by the stop-buy strategy $P_b^*(\cdot)$, if all other traders follow this strategy.[21] Therefore, given $P_b^*(\cdot)$, a trader needs to best respond to it. This is the reduced-form game we discuss in this section.

After time T if a trader v sells before the crash, she gets the selling price. Otherwise, she gets the post-crash price θ. Now we solve the individual trader's problem backwards, starting with her sale decision. Consider a trader v who has bought at price P_p and plans to sell at P_s. Let $\omega(P_p, P_s)$ denote her expected profit. Then, given that all other traders follow a strategy $P_b^*(\cdot)$, trader v would like to maximize $\omega(P_p, P_s)$ by choosing an optimal selling price P_s,

$$\underset{P_s}{max} \quad \omega(P_p, P_s) \tag{2}$$

[20]As this is a sketch, the effect of P_p on P_s is not depicted here.
[21]Since $p_T = P_b^*(\theta + \frac{\eta}{2})$

This gives her optimal selling strategy $P_s^*(v, P_p)$.

Knowing her selling plan, trader v should buy the asset until the price reaches a level of P_b such that

$$\omega(P_b, P_s^*(v, P_b)) = (1-\tau)R = 0 \tag{3}$$

i.e. she should stop buying at price P_b.[22] Then as long as $\omega(P_p, P_s^*(v, P_p)) > 0$ under the current price P_p, trader v should keep buying. In an equilibrium it must be that $P_b(\cdot) = P_b^*(\cdot)$.

All traders are ready to stop buying at $P_b^*(\cdot)$ if they have not bought yet. But once a trader has bought (of course below $P_b^*(\cdot)$), then she is no longer a concern to others because all that matters is the stop-buy decisions of those who have not bought yet (and are entering or waiting to enter the market). Recall that the crash is triggered by stop-buy decisions (of the highest type of traders). Call a trader v a *break-even trader* if she has not bought yet and finds that buying at the current price P_p and selling optimally at $P_s^*(v, P_p)$ gives a zero expected profit so that she decides to stop buying at P_p. Only break-even traders are important to identify the bursting price and the size of the bubble. So the equilibrium is defined by equations (2) and (3). Now we focus on the strategies of break-even traders. The price at which a break-even trader stops, P_b, is just her stop-buy strategy $P_b^*(v)$. We can write a break-even trader's (planned) selling strategy as $P_s^*(v, P_b^*(v))$, though she never actually buys (so her selling strategy never gets implemented).

To better understand $\omega(P_p, P_s)$, we write it semantically as follows (not considering any tax yet).

$$\omega(P_p, P_s) = \int_{v-\frac{\eta}{2}}^{\Theta(P_s)} \Big[\text{post-crash profit}\Big] \phi(\theta|v)d\theta + \Big[\text{pre-crash profit}\Big] \times Prob\Big[\text{sell before the crash}\Big]$$

The first term is trader v's *expected* post-crash profit, in which case the bubble bursts before she sells. The probability of selling before the crash is the subjective probability that $\theta > \Theta(P_s)$.[23] Since a trader can only sell at price θ after the crash, her post-crash profit in the brackets depends on θ. The second term is her expected pre-crash profit, in which case θ is higher than $\Theta(P_s)$ so that trader v will be able to sell before the crash.

To write $\omega(P_p, P_s)$ formally, let $G_{pre} = P_s - (P_p + 2c)$ denote a trader's pre-crash sale profit (before tax), and $G_{post} = \theta - (P_p + 2c)$ denote the trader's post-crash sale profit (before tax, negative in case of

[22] The bubble will not burst below $P_b(v)$ for trader v according to Corollary 3.1. P_b is unique because in equilibrium $\omega(\cdot, \cdot)$ decreases in its first variable, and the second variable, P_s^* at its optimum, does not affect ω due to the envelope theorem.

[23] The upper bound of the integral is $\Theta(P_s)$ because the trader will be caught in the crash if $P_s > p_T = P_b^*(\theta + \frac{\eta}{2})$. If we inverse $P_b^*(\cdot)$ and rearrange, then equivalently, she will be caught in the crash if $\Theta(P_s) > \theta$. Intuitively, if the realized θ is smaller than $\Theta(P_s)$, then the majority of the population have signals lower than v and v is too high.

loss). A trader is taxed only if $G_{pre} > 0$[24] or $G_{post} > 0$ and not taxed otherwise. She also receives a tax benefit $-\tau_c G_{post}$ if $G_{post} < 0$.

Since the purchase price P_p affects the taxability of G_{post}, $\omega(P_p, P_s)$ has two different possible forms depending on P_p. When P_p is high enough such that $\Theta(P_s) - (P_p + 2c) < 0$, G_{post} is always negative and is non-taxable.[25] In this case, given that all other traders follow strategy $P_b^*(\cdot)$, trader v's expected profit is

$$NT \text{ trader: } \omega(P_p, P_s) = (1-\tau_c) \int_{v-\frac{\eta}{2}}^{\Theta(P_s)} G_{post}\phi(\theta|v)d\theta + (1-\tau)G_{pre}\left[1 - \Phi(\Theta(P_s)|v)\right] \tag{4}$$

We call a trader who faces such an expected profit an NT trader. When a trader's purchase price P_p is low enough such that $P_p + 2c < \Theta(P_s)$, it is possible that the post-crash sale is profitable and taxable.[26] In this case the expected payoff is[27]

$$T \text{ trader: } \omega(P_p, P_s) = (1-\tau_c) \int_{v-\frac{\eta}{2}}^{P_p+2c} G_{post}\phi(\theta|v)d\theta + (1-\tau)\left[\int_{\theta=P_p+2c}^{\Theta(P_s)} G_{post}\phi(\theta|v)d\theta + G_{pre}\left[1 - \Phi(\Theta(P_s)|v)\right]\right] \tag{5}$$

We call a trader who faces such an expected profit a T trader. A trader must be either a T trader or an NT trader.

Because the bursting is triggered by break-even traders, we say that the bubble is an NT (T) bubble if break-even traders are NT (T) traders.[28] It turns out that both types of bubbles exist but each belongs to a different zone in the parameter space (see Figure 8). Since all historical bubbles we explore are NT bubbles, the parameters (τ and c) associated with the T bubble are less likely to be observed in practice and the characterization of the two types of bubbles are similar, we relegate the T bubble to Appendix D.

By conjecturing that $P_s^*(v, P_p)$ and $P_b^*(v)$ are linear in v, we can solve for the unique equilibrium strategies of break-even traders in an NT bubble from equations (2), (3) and (4), and have the following proposition:[29]

Proposition 4.1. ***(Equilibrium with NT bubble)*** *When* $\frac{c}{\eta} < \frac{3-2\sqrt{2}}{2}$ *and* $\begin{cases} \tau < \tau^{TNT1} \text{ and } 0 \le \tau_c \le \tau \text{ or} \\ \tau > \tau^{TNT1} \text{ and } \tau_c^1 \le \tau_c \le \tau \end{cases}$, *there exists a unique trading equilibrium in which the bubble size is* $B = \frac{\eta}{2} + D_{NT} > 0$, *the bubble bursts at*

[24] In equilibrium, pre-crash sales always give positive profits. A pre-crash sale is a scheduled sale. If a trader buys at P_p and sells at planned P_s and incurs a loss, then he will not buy at P_p.

[25] Given $P_b^*(\cdot)$, the highest G_{post} a trader can have is when $\theta = \Theta(P_s)$, i.e. the bubble bursts right before she sells. If this G_{post} is still negative, G_{post} is always negative.

[26] This happens when $\theta > P_p + 2c$.

[27] In the first integral the post-crash sale incurs a loss and is not taxable, and there is a tax benefit; in the second integral the post-crash sale is profitable and taxable. The third term is the expected payoff from selling before the crash, which is also taxable.

[28] In equilibrium all break-even traders are either all NT or all T traders.

[29] This corresponds to an interior solution of equation (2).

$\theta + B$ *and all the break-even traders are NT traders with*

$$\begin{cases} P_b^*(v) = v + D_{NT} \\ P_s^*(v, P_b^*(v)) = v + \frac{\eta}{1-\tau_c}\left[h_{NT} + \frac{2(\tau-\tau_c)}{1-2\tau+\tau_c} d_{NT}\right] \end{cases}$$

where $D_{NT} \equiv \frac{\eta}{1-\tau_c}(h_{NT} - d_{NT})$, $h_{NT} \equiv \frac{1}{2}[1 - 2\tau + \tau_c] - (\tau - \tau_c)\frac{2c}{\eta}$, $d_{NT} \equiv \sqrt{\frac{4c}{\eta}(1 - 2\tau + \tau_c)(1 - \tau)}$, $\tau^{TNT1} \equiv \frac{1}{2} - \frac{\frac{4c}{\eta}}{(1-\frac{2c}{\eta})^2}$ *and* $\tau_c^1 \equiv 2\tau - 1 + 8(1 - \tau)\frac{\frac{2c}{\eta}}{(1+\frac{2c}{\eta})^2}$.

Non-break-even traders' strategies will be characterized in Section 4.4. For traders with $v \leq \frac{\eta}{2}$, their strategies do not affect, but are rather determined by, strategies of those with $v > \frac{\eta}{2}$ and are more complicated while less important, so we omit the characterization.

The equilibrium strategies in an NT bubble are depicted in Figure 4. The two solid lines, $P_s^*(v, P_b^*(v))$ and $P_b^*(v)$, are break-even traders' strategies,[30] which is a special case of Figure 3. The selling and stop-buy strategies of all traders (break-even and non-break-even) always increase in her signal. Low-belief traders (signals in the lower range of $[\theta - \frac{\eta}{2}, \theta + \frac{\eta}{2}]$) always flee the market before the crash, while the rest are caught.

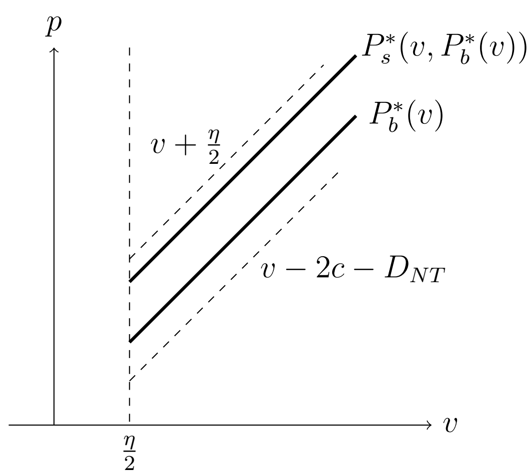


Figure 4: Equilibrium strategies with an NT bubble

The bubble size is $B = \frac{\eta}{2} + D_{NT}$, which is a constant and does not depend on the realization of θ. It can be verified that B increases in the belief dispersion η and the tax credit rate τ_c and decreases in the tax rate τ and the fixed transaction cost c. The first relationship, $\frac{\partial B}{\partial \eta} > 0$, is consistent with AB2003. Now we depict the latter three relationships: $\frac{\partial B}{\partial \tau_c} > 0$, $\frac{\partial B}{\partial \tau} < 0$ and $\frac{\partial B}{\partial \frac{c}{\eta}} < 0$ in Figure 5 and 6 (measured in normalized bubble size) and discuss their implications. In both figures, the dark (red) surface in the upper area represents NT bubbles and the light (blue) surface in the lower area represents T bubbles. In particular the two main results of our paper are related to the third and the fourth relationship and are

[30]Dashed lines are about non-break-even traders' strategies. See Section 4.4.

explained in Section 4.2 to 4.3.

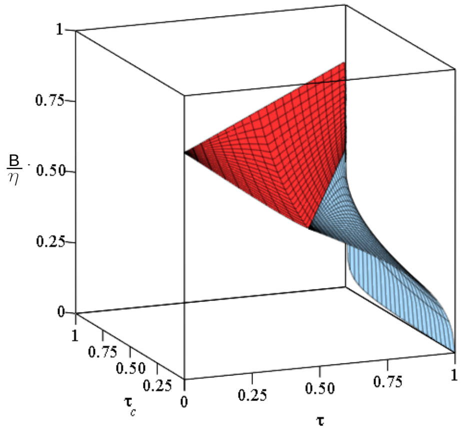
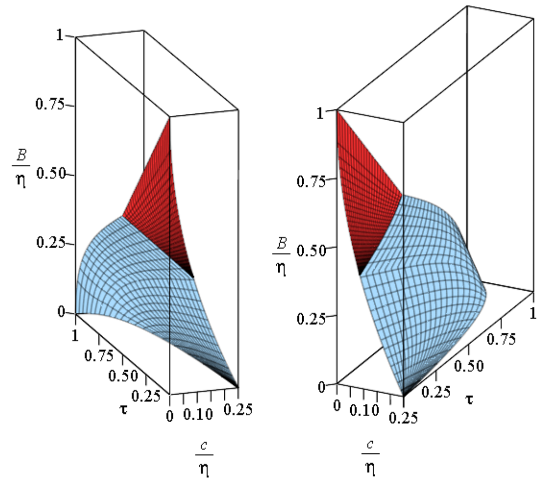


Figure 5: Normalized bubble size ($c = 0.01$, $R = 0$)



View 1 View 2

Figure 6: Normalized bubble size ($\tau_c = 0$, $R = 0$)

4.1 $\frac{\partial B}{\partial \tau} < 0$

The intuition of how tax affects rational traders' decisions is as follows. The capital gains tax is distortionary in that the tax is effectively imposed only on pre-crash transactions for NT traders and selling strategies are distorted downward. This can be seen from the first-order condition of (4) w.r.t. P_s,

$$1 - \tau = \left[(1-\tau)G_{pre} - (1-\tau_c)G^{\Theta}_{post}\right] \frac{1}{P^{*\prime}_b(P^{*-1}_b(P_s))} \frac{\phi(\Theta(P_s)|v)}{1 - \Phi(\Theta(P_s)|v)} \tag{6}$$

where $G^{\Theta}_{post} \equiv \Theta(P_s) - P_p - 2c$. The first-order condition can be interpreted in terms of marginal benefit and cost. Consider a trader who wants to sell at $P_s + \delta$ instead of P_s. On the left-hand side is the marginal benefit $(1-\tau)\delta$ (the after-tax price appreciation). On the right-hand side is the marginal cost of being caught in the crash, which equals the profit difference[31] multiplied by the probability or hazard rate of bursting between P_s and $P_s + \delta$. Dividing both sides by δ and letting $\delta \to 0$, we have equation (6). If we normalize this first-order condition by dividing $1-\tau$ on both sides, we have $1 = \left[G_{pre} - \frac{1-\tau_c}{1-\tau}G^{\Theta}_{post}\right] \frac{1}{P^{*\prime}_b} \frac{\phi}{1-\Phi}$. We call

$$G_{pre} - \frac{1-\tau_c}{1-\tau}G^{\Theta}_{post} \tag{7}$$

the normalized payoff difference between pre- and post-crash. See Figure 7 for the equilibrium selling strategy P^*_{s1} determined by hazard rate h_1 and normalized payoff difference npd_1 under tax rate τ_1.[32] If

[31] The first term in the brackets is the pre-crash profit, which is always positive and taxable, and the second term is post-crash payoff, which is negative for an NT trader.

[32] Our equilibrium is different than the endogenous equilibrium in AB2003 in that the hazard rate in AB2003 is a constant because their bubble is triggered by a fixed fraction of sale, while in our model the fraction of traders caught in the crash varies with the gap between P^*_s and P^*_b so that the hazard rate is not a constant.

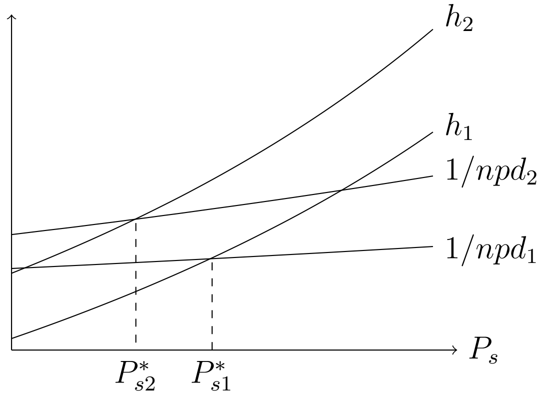


Figure 7: Equilibrium P_s^* determined by the payoff difference and hazard rate under different tax rates

τ increases from τ_1 to τ_2, the normalized payoff difference tends to increase. Then a break-even trader has to lower her P_s ($G_{pre} = P_s - P_p - 2c$ falls) so that the first-order condition still holds and her P_s is optimal.[33] Intuitively, facing a larger payoff difference between fleeing and being caught due to a rise in the tax, break-even traders behave more cautiously and conservatively by selling early and securing their gains sooner.[34] See Figure 7 for the equilibrium selling strategy P_{s2}^* under a higher tax rate τ_2.

The change in a break-even trader's selling decision then transmits to her stop-buy decision. Since a break-even trader's expected profit (4) is zero, a rise in tax, combined with a fall in P_s, will force P_p to decrease.[35] Intuitively, a rise in tax distorts a break-even trader's selling decision P_s downwards, and a lower P_s reduces the profit margin and in turn squeezes the stop-buy decision P_b downwards, which bursts the bubble early.

When evaluating the effects of taxes on asset prices empirically, usually two forces that move the prices in opposite directions are considered: the lock-in effect (a rise in tax causes traders to defer selling and thus tends to reduce supply and raise the price) and the capitalization effect (a rise in tax discourages buying and thus tends to reduce demand and lower the price). In our model the behavior agent is part of the supply side and does not respond to the tax incidence and thus there is no lock-in effect on them. However, the absence of lock-in effect from the behavioral agent is not why the tax can deflate the bubble in our model. The driving force of asset bubbles is rational traders in our model as well as in the real world (as demonstrated by Griffin et al. (2011) and Brunnermeier and Nagel (2004)). It is their responses to the tax incidence that deflate bubbles. In particular, that they stop buying early when facing a higher

[33]If we do not take into account the equilibrium response of $P_b^*(\cdot)$, then clearly a rise in τ increases the normalized payoff difference. Recall that $G_{post}^{\Theta} < 0$. In equilibrium, while a break-even trader's P_s indeed decreases, the final effects of a rise in τ are that the hazard rate rises while the normalized payoff difference decreases and their product remains unity.

[34]This does not conflict with the lock-in effect in that the lock-in effect applies to traders who have already bought with a given purchase cost while break-even traders are those who have not bought whose stop-buy strategies vary with the tax.

[35]In addition, the probability of being caught is also higher in equilibrium.

tax bursts the bubble prematurely.[36]

4.2 $\frac{\partial B}{\partial \tau_c} > 0$ and the ineffectiveness of the tax under perfect tax credit

The tax credit rate, τ_c, works in the opposite direction to the tax rate τ and serves as a compensation to the loss from being caught in the crash. In particular, the post-crash loss G_{post} in the first-order condition (6) is scaled by a factor $1 - \tau_c$. So the tax credit is a "tax" on G_{post}, which reduces the loss. If τ_c increases, the normalized payoff difference decreases. With a smaller payoff difference, traders care less about being caught and behave more aggressively by selling at higher prices. A higher P_s then enlarges the profit margin and encourages buying at higher prices, which allows the bubble to grow larger. This suggests that weakening tax credits has an effect similar to raising the tax rate in deflating bubbles. This result can be useful when raising the tax is difficult or infeasible for economic or political reasons.

A special case is that the tax credit is perfect, i.e. $\tau_c = \tau$. In this case the equilibrium is simplified to $P_b^*(v) = v + \dfrac{\eta}{2} - \sqrt{4c\eta}$ and $P_s^*(v) = v + \dfrac{\eta}{2}$, with a bubble $B = \eta - \sqrt{4c\eta}$.[37] Notice that now the bubble stays at its tax-free level and is unaffected by the capital gains tax, even if we impose a 100% capital gains tax! This is because the first-order condition becomes $1 - \tau = \Big[(1-\tau)G_{pre} - (1-\tau)G_{post}\Big] \frac{1}{P_b^{*\prime}} \frac{\phi}{1-\Phi}$, which, after normalization, is exactly the same as without tax. Hence the perfect tax credit provides a perfect compensation to the loss in crash, reduces the payoff difference between pre- and post-crash and restores it to its tax-free level. It induces high selling prices and encourages aggressively high purchase prices. Traders behave as if there is no tax and the tax credit entirely neutralizes the deflating effect of the tax. In Figure 5 this corresponds to the horizontal top edge of the dark (red) surface, where $\tau_c = \tau$ and the bubble is not responsive to the increasing tax rate.

A 28% capital gains tax was introduced in the United Kingdom on April 6, 2015, on gains from residential property by overseas investors that, most analysts believe, targets the potential housing bubble in London that has been growing since 2009. To take the advantage of this policy change to its full extent, our paper suggests that a tax authority should examine its policies on tax credit before imposing or increasing the tax rate, because an overly favorable tax credit can entirely offset the deflating effects of the tax and leave the bubble unaffected, no matter how much the tax rate is raised.[38] Measures to limit

[36] If we allow the behavioral agent to respond to a tax rise by deferring or even suspending selling and if we allow the price to rise, the price will still be driven to the same level at the peak of the bubble and then crashes, because the crash is triggered by rational traders and their stop-buy strategies are not affected by the behavioral agent.

[37] This equilibrium requires that $\frac{2c}{\eta} < \frac{1}{2}$. The difference between NT and T traders disappears, since equation (4) is equivalent to (5) when $\tau_c = \tau$. Break-even and non-break-even traders both sell at $P_s^*(v) = v + \frac{\eta}{2}$.

[38] One clarification is that this suggestion should be viewed as an ad hoc tax policy instead of a long-run, regular optimal tax policy. For those who believe that capital gains taxes should be calculated on the basis of long-run average (net) capital

the tax credits include reducing the tax credit rate τ_c, setting or tightening the dollar limit on the tax credit and shortening the duration within which tax credits can be carried forward or backward.

4.3 $\frac{\partial B}{\partial c} < 0$ and the large marginal effect of a small c

An increase in c directly erodes the profit margin between P_s and P_b and a break-even trader has to lower P_b to make a profit.[39] This lowered stop-buy strategy therefore deflates the bubble.

One implication arising from our model is that the transaction cost can have an arbitrarily large marginal effect on the bubble when the cost approaches zero: $lim_{c \to 0} \frac{\partial B}{\partial c} = -\infty$ (see Figure 6 View 2). Notice that this result does not depend on τ or τ_c. In addition, it does not even depend on the assumptions about belief and signal distributions in our model. The reason is as follows. When $c = 0$, the selling price and stop-buy price of a break-even trader coincide, i.e. $P_b^*(v) = P_s^*(v, P_b^*(v))$. In this case a break-even trader does not need a margin at all between her P_s and P_b.[40] When c increases slightly from zero, $2c$ is the minimal gap between purchase and selling prices for a break-even trader, if no risk of crash is considered. But since it is always possible that the bubble bursts in between this gap, a trader needs a larger profit margin/gap to compensate for the risk. This leads to a lower P_b. A larger gap means a higher probability of bursting in between, which requires an even larger gap. Hence the stop-buy strategy is lowered dramatically by a small transaction cost, which deflates the bubble significantly.[41]

The large marginal effect of the transaction cost on bubbles justifies the practice of implementing financial transaction taxes at very low rates. Although we use a fixed transaction cost instead of a proportional tax, every trader holds the same amount of the asset in our model and the effect of a fixed cost is quite close to that of a proportional transaction tax. Since the 1970s many countries have experimented with financial transaction taxes on trades of shares, bonds, currencies and derivatives. As mentioned earlier, empirical evidences show that these taxes tend to reduce trading volumes but not price volatilities, and some countries have decided to abolish the tax. Other countries[42] nevertheless still maintain these

gains, weakening the tax credit may seem unfair because, without the tax credit, investors are taxed whenever they have gains and are left alone when they incur losses, and hence the tax is not based on long-run average gains. So our suggestion is to apply this policy only when there is strong concern that a bubble may be in progress in a specific market. In this paper we do not intend to find an optimal tax policy that can automatically deflate bubbles when they arise and otherwise does not interfere with the market.

[39] Under an imperfect tax credit, selling price P_s actually rises in response to a rise in c. This can be seen from the fact that the change in normalized payoff difference (7) will be widened since $\frac{1-\tau_c}{1-\tau} > 1$. The net effect of c on P_b is nevertheless negative. Under a perfect tax credit P_s is invariant in c.

[40] With a zero profit margin, the tax has no effect.

[41] But P_b will not be shifted downwards infinitely, since the bubble size is also decreasing as $P_b^*(\cdot)$ decreases and the loss of being caught in the crash diminishes. There is a stop-buy price where the expected profit equals zero. So this marginal effect decreases in c.

[42] There were about 40 countries in 2011 that imposed financial transaction taxes.

taxes at low rates. For example, the United States currently levies a 0.0034% tax on stock transactions, the United Kingdom levies a 0.5% tax on share purchases and Japan has a 0.1−0.3% tax on stock transactions, and these countries are among those which have experienced the most notorious bubbles in history.

4.4 Non-break-even traders

In fact, all traders who have already bought the asset are non-break-even traders. An NT bubble actually includes both NT and T traders: those whose purchase prices are not too low are NT traders, and those who entered the market very early and bought the asset at very low prices (so it would be possible to make a profit even if caught in the crash) are T traders. $v - 2c - D_{NT}$ is the dividing line between the purchase prices of T and NT traders. Non-break-even traders' selling strategies are

$$P_s^*(v, P_p) = \begin{cases} \frac{1}{1-2\tau+\tau_c}\left[(1-\tau)v-(\tau-\tau_c)P_p+\frac{\eta}{1-\tau_c}\left[(1-\tau)h_{NT}+(\tau-\tau_c)d_{NT}\right]\right], \text{ if } P_p > v-2c-D_{NT} \text{ } (NT \text{ trader}) \\ v+\frac{\eta}{2}, \text{ if } P_p < v-2c-D_{NT} \text{ } (T \text{ trader}) \end{cases} \tag{8}$$

Non-break-even NT traders' selling prices span between $v + \frac{\eta}{2}$ and $P_s^*(v, P_b^*(v))$, while all T traders sell exactly at $v + \frac{\eta}{2}$. See Figure 4.

4.5 NT bubble, T bubble and no bubble

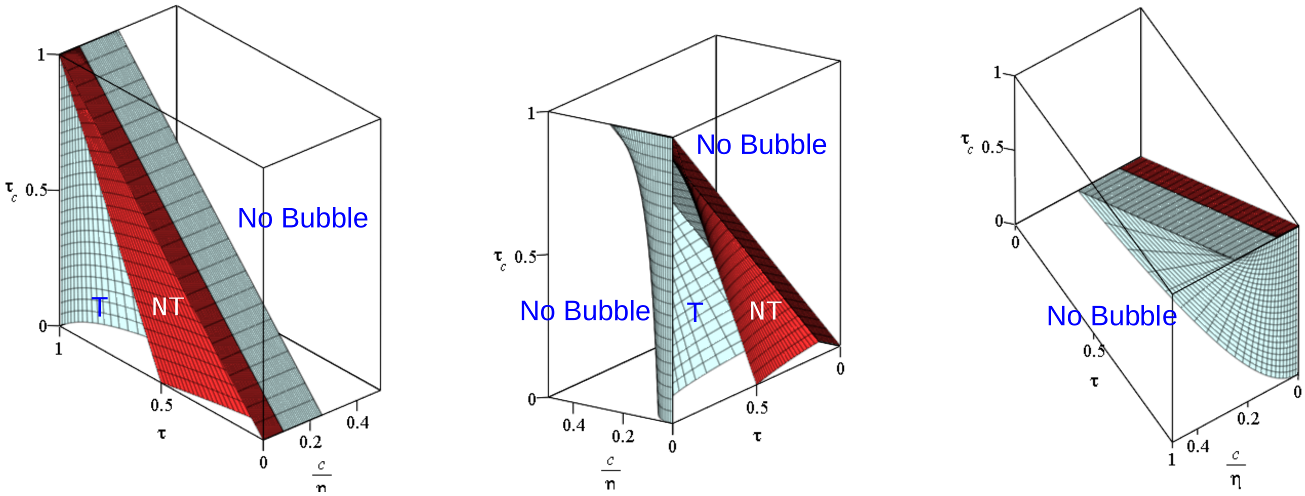
The NT bubble emerges when τ_c is high while τ and $\frac{c}{\eta}$ are low. When τ_c, τ and $\frac{c}{\eta}$ are moderate, the unique equilibrium involves a T bubble.[43] There also exists a unique trading equilibrium without a bubble when τ_c is low while τ and $\frac{c}{\eta}$ are high. See Appendix D and E, respectively. Figure 8 depicts the parameter space partitioned by the existence and type of bubble. The space of NT bubble is enclosed by dark (red) surfaces and the space of T bubble between light (blue) and dark (red) surfaces. Outside light (blue) surfaces no bubble exists.

4.6 The outside option R and the interest rate policy

The effect of R is similar to that of c. When $R > 0$, the main change to Proposition 4.1 is that $d_{NT} \equiv \sqrt{\frac{4c+2R}{\eta}(1-2\tau+\tau_c)(1-\tau)}$. See other changes in Appendix F.

In an NT bubble, it can be verified that the size of the bubble also decreases in the outside option, i.e. $\frac{\partial B}{\partial R} < 0$. This result means that high returns from outside investment opportunities help deflate a bubble.

[43] It may feel counterintuitive that when τ and c are very low break-even traders always incur losses, while when τ and c are moderate break-even traders can possibly make a profit. This is because, when τ and c are very low, only a small gap between buy and sale prices is required for break-even and the stop-buy price is close to the selling price. With such small profit margin, however, a break-even trader must incur a loss if caught in the crash, even if θ turns out to be high ($\theta = v + \frac{\eta}{2}$), which makes her an NT trader.



(a) View 1 (b) View 2 (c) View 3

Figure 8: In the $\frac{c}{\eta}$ - τ - τ_c space $(R = 0)$

Conversely a deterioration of these returns reduce the opportunity costs of riding the bubble and thus can induce a larger bubble. The potential housing bubbles that we have observed since 2009 in major cities outside the United States are partially the result of arbitrageurs switching their investments from US real estate markets plagued by the subprime mortgage crisis.

Similar to the transaction cost c, the outside option R also has a large marginal effect on the bubble when R and c are small: $\lim\limits_{R \to 0, c \to 0} \frac{\partial B}{\partial R} = -\infty$, though this marginal effect decreases in both R and c. If we interpret R as an interest rate (on treasury bill, for instance), then when a central bank sets a very low interest rate, this could inflate bubbles significantly in certain markets, as suggested by the Federal Reserve's low interest rate policy after 2001 and the rise of the housing bubble in the United States until 2008.[44] Thus central banks often face a dilemma: low interest rates help fight unemployment and recessions (in the case of the United States, it was the recession after the tech bubble), but also sow the seeds for the next round of recession. The bank of Canada lowered the rates after the 2008 financial crisis and has maintained an overnight rate at 1% since 2010. In January 2015, the Bank surprisingly further lowered the rate to 0.75%, partially to fight the adverse effect of the oil price plunge in 2014. Our model warns that further lowering the interest rate, when it is at an already very low level, will have a disproportionately inflating effect on housing prices. This could be especially dangerous when there is strong suspicion that a large housing bubble already exists in Canada at this time.

[44] Although there is no agreement that Federal Reserve's low interest rate policy during 2001-2005 indeed caused the housing bubble that burst in 2008, the timing of the housing price run-up and bursting roughly coincide with the declining and rising of the federal funds rate.

5 Empirical discussion

5.1 Effects of τ, c and R: measuring and comparing historical bubbles

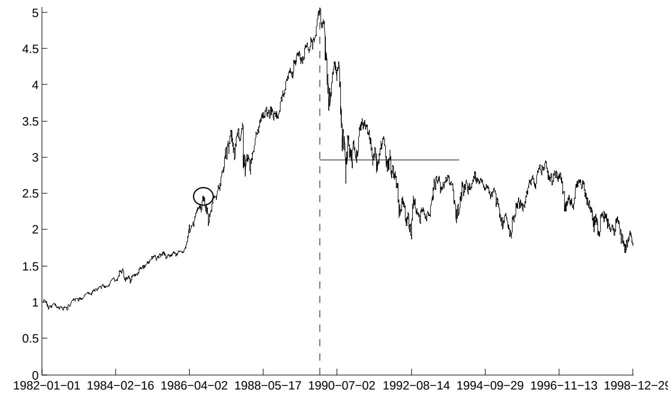
In this section we discuss how to empirically test the results of our model. In our model the tax rate τ, tax credit rate τ_c, transaction cost c and outside option R are potential policy tools, while the belief dispersion η is more intrinsic in a bubble. Ideally if we have data on many historical asset bubbles, we would like to test whether these variables have the relationship with B as described in Proposition 4.1. If a nonlinear test is difficult, then at least we should be able to test whether we have $\frac{\partial B}{\partial \tau} < 0$, $\frac{\partial B}{\partial \tau_c} > 0$, $\frac{\partial B}{\partial c} < 0$ and $\frac{\partial B}{\partial R} < 0$. But given that assent bubbles are not high frequency events and that the cause and the environment for each bubble can be different, we collect data on only 4 bubbles and our discussion is mainly restricted to the data collect. We first explain how to identify the size and parameters of a historical bubble. Then we explain the normalization of the bubble sizes by the belief dispersion so that we can compare the effects of controllable parameters. In the end we explore and apply our method to four historical bubbles and try to fit the empirical results into the theoretical results.

The four historical events of (purported) bubbles we will explore are the 1929 stock market crash, Japan's asset bubble in the 1980s, the Internet Bubble in the late 1990s and the Bitcoin Bubble in 2013.[45] For the three former events, we use major stock indexes as the asset price in our model. To identify the size of a bubble, we define the run-up as a period of time up to the moment when the price hits its peak. The actual size of a bubble is measured from the peak to the average price within a certain period of time after the peak. This is because the price path after the peak is usually quite noisy and usually involves a series of crashes and small bouncebacks so that the initial crash following the peak does not necessarily reflect the asset fundamental value. On the other hand it usually takes time for the price to reach the bottom and rational traders have opportunities to sell before the bottom. In addition at the bottom there often exists a downward price overshooting, which suggests that the market has overacted. Therefore we use an average price to balance these considerations and we set the length of this period as half of the length of the run-up. The actual price paths of these events are presented in Figure 9, where in each panel the vertical dashed line is the peak that delimits the run-up and the crash and the horizontal line is the average post-crash price we adopt.

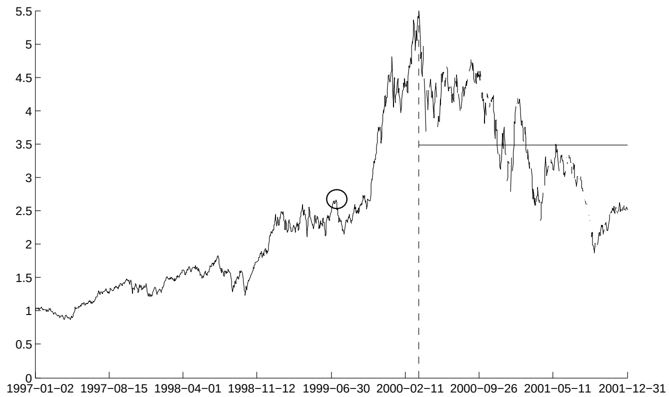
For each bubble we identify parameters τ, τ_c, c, R and η. We also need to determine whether a bubble is an NT or T bubble based on its parameter values (it turns out that all of them are NT bubbles). For

[45] In this paper we do not attempt to judge whether these events are indeed bubbles or not.

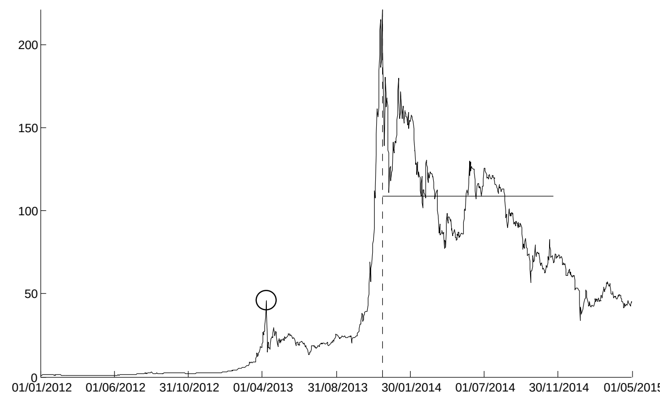
(a) 1929 Stock Market Crash: S&P Stock Index
Source: CRSP



(b) Japan's Asset Bubble: Nikkei 225 Stock Index
Source: Federal Reserve Bank of St. Louis



(c) Tech Bubble: Technology Stock Index
Source: calculated by data from CRSP



(d) Bitcoin Bubble 2013: Bitcoin Price Index
Source: blockchain.info, bitcoin.org

Figure 9: Price index paths of historical bubble events

the capital gains tax rate τ, we try to identify the main contributors of a bubble and then find out the appropriate tax rate that applies to them. The tax credit rate τ_c is more complicated in that the most relevant restriction on credit is time, and the perceived equivalent credit rate depends on a trader's past gains and her expectation for future gains that will offset this loss before the credit expires.[46] In most cases the perceived equivalent credit rate should be imperfect because of the possibility of credit expiration.[47] But the empirical study on this effective equivalent credit rate is essentially nonexistent; therefore we will calculate under two extreme assumptions, no credit and perfect credit, and compare the results. If we have a larger sample of bubbles, we can test whether longer expiration dates or higher ceilings for credits result in larger bubbles. The trading cost usually includes three components: broker's commission, bid-ask spread and the market impact cost. For simplicity we only consider broker's commission. Since c in our

[46]Cooper and Knittel (2006) examine the fraction of tax credits actually claimed by US firms between 1993 and 2003. They show that about 25-30% are never claimed because of credit expiration or failure to generate profit/bankruptcy. Unfortunately their data does not include capital loss, only operation losses.

[47]Unless the investor has gains in previous years (within the allowed period) that can entirely offset this loss. There is also an interest loss if credits are carried forward.

model is a fixed cost, we set c equal to the commission rate times the asset price at the peak of the bubble. This is because the cost and the market entry decision near the peak are more relevant to the bursting. For the outside option, R, we use the return from the treasury bond for an economy-wide bubble, and the return from major stock indexes for a single asset bubble, multiplied by the asset price at the peak.[48]

The belief dispersion η in a real market is usually very difficult to identify. We do not have data on investors' beliefs about the asset fundamentals in these historical events, so we try to infer them from the price dynamics as follows. Theoretically all major price events are caused by either exogenous shocks or trading strategies governed by investors' beliefs (or both). In a real market when major pessimistic traders start to sell, the impact can be felt. In the model rational traders first start selling when the price rises to $P_s^*(\theta - \frac{\eta}{2})$, which is the selling strategy of the lowest type, $\theta - \frac{\eta}{2}$.[49] Thus we use the largest percentage price drop (in the data, before the peak) which does not coincide with major exogenous shocks as a proxy for the initial attack of rational traders. This point is indicated by a circle in each panel of Figure 9. The drop is measured in a period with a length of $\frac{1}{20}$ of the run-up. Alternatively we can use the price where the trading volume suddenly drops as $P_b^*(\theta - \frac{\eta}{2})$, the price at which the lowest type first stops buying. In either case, we can infer the belief dispersion by inverting the function $P_s^{*-1}(\cdot)$ or $P_b^{*-1}(\cdot)$.[50] Note that in our model these two prices do not depend on whether traders' signals are symmetrical about θ or not.[51]

For all events of bubbles, their parameters as well as actual bubble sizes are listed in Table 1. B under the Data column is the bubble size measured from data. η in both the Perfect Credit and No Credit (NT Bubble) columns are inferred belief dispersions. $\frac{B}{\eta}$ under the Theory column is the theoretical B calculated from the inferred η, then divided by the η. $\frac{B}{\eta}$ under the Data column is the measured B divided by the inferred η.

To identify the effects of policy parameters on bubbles, we need to eliminate other factors' contributions to the variation in the bubble size. Our model (and AB2003) suggests that the belief dispersion η is intrinsic in bubbles and is likely to be the primary explanatory variable of the wild variation in their size. Hence we normalize bubble size by belief dispersion and use $\frac{B}{\eta}$ to identify effects of policy parameters on bubbles.

Before we present the results, we briefly review some aspects of these historical events.

[48]Ideally R is calculated from the intended holding period of the highest type of traders when they are indifferent between the outside option and the asset with bubble. Considering the investor sentiment around the peak and the length of the run-up, we use a 1-month to 6-month return from the outside option before the bursting.

[49]In our model traders do not know about this because we assume that they have no knowledge of the price path at all.

[50]This method has an interesting implication that, ex post, the earlier the price drop emerges, the larger the belief dispersion, and hence the larger the bubble there will be.

[51]Traders' strategies will offset any shift in the signal distribution about θ, so that the two price levels are invariant to this shift. But the two prices do depend on the shape of the distribution (which is uniform in our model).

	Parameters			Data	Perfect Credit					No Credit(NT Bubble)				
	c	R	τ	B	η	$\frac{c}{\eta}$	$\frac{R}{\eta}$	Theory $\frac{B}{\eta}$	Data $\frac{B}{\eta}$	η	$\frac{c}{\eta}$	$\frac{R}{\eta}$	Theory $\frac{B}{\eta}$	Data $\frac{B}{\eta}$
1929	0.0179	0.0471	0.12	3.82	5.25	0.00341	0.00897	0.823	0.728	5.33	0.00336	0.00884	0.735	0.717
Japan	0.0355	0.0730	0.375	2.33	3.62	0.00979	0.0201	0.718	0.644	4.37	0.00811	0.0167	0.518	0.534
Tech	0.00557	0.0144	0.35	2.02	3.26	0.0017	0.0044	0.874	0.622	3.47	0.0016	0.0042	0.595	0.583
Bitcoin	6.28	4.58	0.25	130	272	0.0231	0.0168	0.725	0.479	299	0.0209	0.0153	0.532	0.436

Table 1: Summary of parameters and results of historical bubbles

1929 Stock Market Crash The 1929 Stock Market Crash in the United States was rooted in the great economic growth in the 1920s, technology innovations (large-scale use of automobiles, telephones, motion pictures, electricity, etc.) and the differentiated opinion about how long this trend can sustain. We measure the bubble in the S&P index and define the run-up as an 8-year period from August 24, 1921, to September 3, 1929. The index at the beginning is normalized to 1 and 5.97 at the peak. The average index within the 4-year period after the peak is 2.14 and the actual size of the bubble is therefore measured as 5.97-2.14=3.83. The largest price drop (within a period of 90 trading days) started with an index value of 1.64 on March 22, 1923 with a percentage drop of 17.3%. The broker commission rates around 1929 were about 0.3%,[52] which gives $c = 0.3\% \times 5.97 = 0.0179$. The 3-month return from a 3-month treasury bond at June 1929 was 3.16%, which gives $R = \frac{3.16\%}{4} \times 5.97 = 0.047$. We treat the main contributors of the bubble as corporations and hence use the corporate tax rate 12%.[53] The capital loss carryover was first introduced in the Revenue Code 1939, and was restricted to individuals only, and we were not able to find any information about capital loss carryover around 1929. Under the assumption of a perfect tax credit and no credit, we calculate η from $B = 3.82$ or $P_s^*(\theta - \frac{\eta}{2}) = 1.64$, respectively. Then we calculate the theoretical B by using this inferred η. Then we normalize both actual and theoretical bubble sizes by the inferred η.

The calculations for all other events are the same as explained above.

Japan's Asset Bubble in the 1980s Japan experienced an "economic miracle" that lasted more than three decades up to the end of 1989. In the 1970s and 1980s Japanese automobiles gained significant market shares in the United States and consumer electronics products started to dominate global markets. In the 1980s, the rapid accumulation of wealth by both corporations and households, as well as the uncontrolled money supply and credit expansion, greatly encouraged speculative trading and inflated real estate and stock market prices. Evidences show that corporations and investors' opinions and expectations about

[52]See Jones (2002).

[53]At that time the tax codes did not differentiate operation gains and capital gains.

stock prices were highly differentiated[54]. The crashes in stock and real estate markets contributed to what many call the Lost Decade. Although the inflation in the stock market and real estate are intertwined, the former burst a year ahead of the latter and so we choose to study the stock market to rule out the potential causality. We define the run-up from January 1, 1982, to Dec 29, 1989, and measure the bubble in Nikkei 225 index. Normalizing the index at the beginning of the run-up to 1, the index reached a record high of 5.07 at the peak. The average index value within a period of 4 years after the peak is 2.73. For the largest price drop within a period of 180 trading days, we ruled out the episode coinciding the 1987 stock market crash in the United States and adopt the next largest one, which started with an index value of 2.47 on August 20, 1986, and has a 19.7% drop. We use a commission rate of 0.7% for security transactions,[55] and adopt a corporate tax rate 37.5%, [56] as the main contributors of stock market bubble were corporations.[57] R is calculated from an average of 3-month treasury bond yields between September and December of 1989, which is 5.76%. Unfortunately we were not able to find information about capital loss carryover around the late 1980s.

Tech Bubble in the late 1990s In the 1980s and 1990s, the growth in computer ownership and usage and the emergence of the internet ignited unprecedented anticipation and imagination about how information technology could change our life. Investment decisions were not based on current profit or sales, but purely on the anticipation of further rises and many technology companies in the United States were significantly overvalued. This bubble burst in the early 2000 and the technology sector in the United States suffered significant losses. We define the run-up as from January 2, 1997, to March 24, 2000, and replicate the technology stock index in Griffin et al. (2011)[58], on which we measure the bubble. At the peak the index is 5.5 times that at the beginning. The average index value within a period of 18 months after the peak is 3.48. For the largest price drop within 60 days in the run-up, we exclude the one in January 2000 arising from tax deferral concerns and the episodes coinciding with the Russian sovereign debt default and the failure of Long-Term Capital Management. Instead, we use the next largest one, which started with an index value 2.66 on July 16, 1999, with an 18.8% drop. As argued in Griffin et al. (2011), the main contributors to the Tech bubble were institutional traders. We use the short-term capital

[54]See Shiller et al. (1996).
[55]See McCauley and Zimmer (1991).
[56]In 1989 the corporate tax did not differentiate ordinary corporate income and capital gains in Japan.
[57]See Stone and Ziemba (1993).
[58]We use Nasdaq daily trading data from CRSP in the technology sector (firms with SIC code 737). A SIC code starting with 737 means the firm engages in computer programming, data processing and other computer-related services. We restrict ourselves to ordinary common shares (CRSP share code 10 or 11).

gains tax rate at the time for the highest tax bracket, 35%, given the fact that there were (unsuccessful) attacks around April 1999 and many institutional investors may have repurchased the asset after that attack. R is calculated from the 3-month treasury bond yield in March 2000, which is 5.85%. As stated in the introduction, corporate capital losses in the United States can be carried 3 years back and 5 years forward without a dollar limit. This is somewhere in between no tax credit and a perfect tax credit.

Bitcoin Bubble in 2013 Bitcoin is an online payment system released in 2009 on which users can transact directly without any financial intermediary. The system is a peer-to-peer one, meaning that it works without a central administration and no one can control it.[59] As a virtual currency and a form of payment for products and services, merchants have an incentive to accept bitcoin because fees are lower than credit cards. One difference between bitcoin and previous assets is that bitcoin transactions are decentralized without a central market. The price for the bitcoin grew slowly before 2012. Since 2012 the price rose considerably from around \$5 (US dollar) and soared in 2013, hitting its record high at \$1151 on December 4. After that the price declined sharply and was around \$50 in May 2015. The market capitalization at the height of the bubble was close to 14 billion US dollars. The gains from investing on virtual currencies such as bitcoin in the 2013 tax year became taxable in the United States.[60] We define the run-up as between January 1, 2012, and December 4 2013, and scale the price on January 1, 2012, which was US\$5.2, to 1. The nominal transaction costs for bitcoin are very low, but we need to take into account the impact of newly created bitcoin, which is similar to excess money supply that can cause inflation. As such, we use the miners' revenue as a percentage of the transaction volume at the time for the transaction cost, which is 2.84%.[61] The largest price drop during the run-up within 35 trading days started with an index value 45.8 on April 9, 2013, with a 68% drop. We conjecture that the majority of investors of bitcoin are individuals and they are tech-savvy young people with moderate income, and that American investor contributed the most to the bubble. Hence we use the 25% tax that applies to an annual single income bracket between \$36,901 and \$89,350. R is calculated from the average of 1-month

[59] Any online computer installed with this open-source software becomes a node of this system. There are many online merchants where one can purchase bitcoin with local currency and use it. Transactions are executed and verified by randomly chosen nodes and recorded in a public distributed ledger called the block chain. Since a user does not know where this information is processed and stored and all data is encrypted, it is impossible for a user to control the system or tamper the ledger.

[60] See Internal Revenue Service Notice 2014-21.

[61] Users who install the bitcoin software also facilitate the distributed transactions and are rewarded by newly created bitcoin (this is called mining), though this reward is declining as more and more nodes are online. There is an upper bound of 21 million for the total bitcoin that can be circulated in the system. All data of bitcoin, as well as the definition of miners revenue, is from blockchain.info.

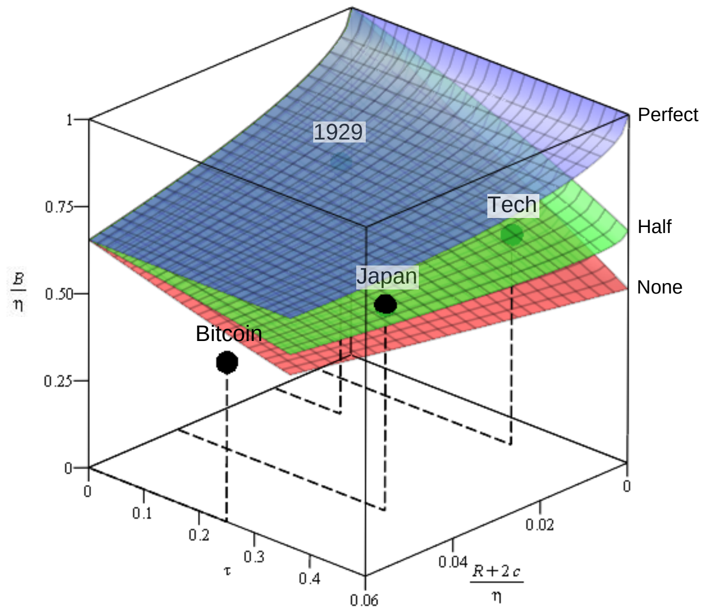
S&P index return within one year before the crash, which is 2.07%. Tax credit for individuals does not expire, but has an \$800 ceiling each year.

Now we fit the empirical results into the theoretical results graphically. From Table 1 we observe that $\frac{c}{\eta}$, $\frac{R}{\eta}$ and Data $\frac{B}{\eta}$ under the two assumptions (perfect credit and no credit) differ only slightly.[62] Therefore we merge the two groups into one by using their averages and plot only one point for each bubble event in Figure 10. To compare, we plot in Figure 10 theoretical results under different assumptions about tax credit: the upper surface (blue) is the bubble size under perfect tax credit, the middle surface (green) is under half tax credit ($\tau_c = 0.5\tau$) and the lower surface (red) is without tax credit. In addition, we treat c and R as a single parameter $2c + R$ when plotting the theoretical surfaces to reduce a dimension so that they can be depicted in a three-dimensional diagram.[63]

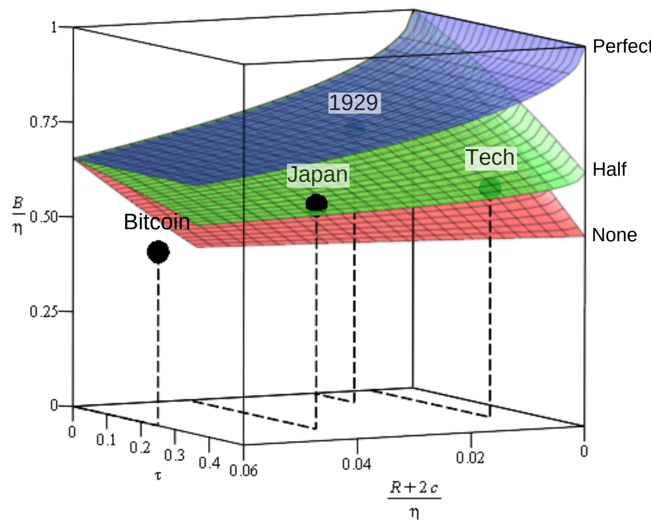
From Figure 10 we observe that three of the four historical bubbles roughly fall in between the two extreme assumptions: perfect and no tax credit, with the Japan bubble close to the half tax credit, the 1929 and the Tech bubbles close to no credit and the Bitcoin falling below no credit. No bubble agrees with the perfect credit. The result for the 1929 seems consistent with the possibility that there was no tax credit at the time. The difference between Japan and Tech bubbles might be accounted for by the lenience of the tax credit policy (if it existed) in Japan since its tax code did not differentiate operation and capital gains at the time while capital losses in the United States cannot offset operation gains, only capital gains. The Bitcoin bubble is smaller than the lowest theoretical result, which may be related to the fact that it is a single asset bubble and there are many outside options, or its data is more peculiar than the composite indexes used in other events. It might also be due to the fact that its trading is decentralized. But given the small sample size and the lack of information about tax credit policies, these results should be interpreted as our theoretical results, $\frac{B}{\tau} < 0$, $\frac{B}{c} < 0$ and $\frac{B}{R} < 0$ do not seem to seriously deviate from empirical evidences, while the evidence on the effect of the tax credit is very weak and there is no evidence on the large marginal effect of c and R so far. Further empirical study to include more sample of bubbles is needed, especially housing bubbles.

[62] This is because they are calculated under only slightly different η.

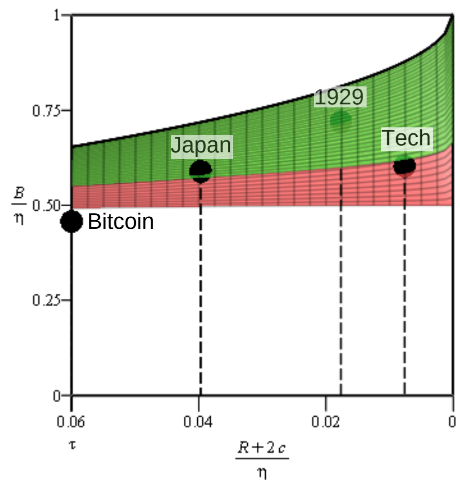
[63] We drop the term $-(\tau - \tau_c)\frac{2c}{\eta}$ in h_{NT} (hence in B) in Proposition 4.1. When τ and τ_c are close and $\frac{c}{\eta} << 1$, this term can be ignored compared to term d_{NT}. Indeed this is the case in all events we study here. Note that the large marginal effects of c and R are preserved since they both appear in d_{NT}.



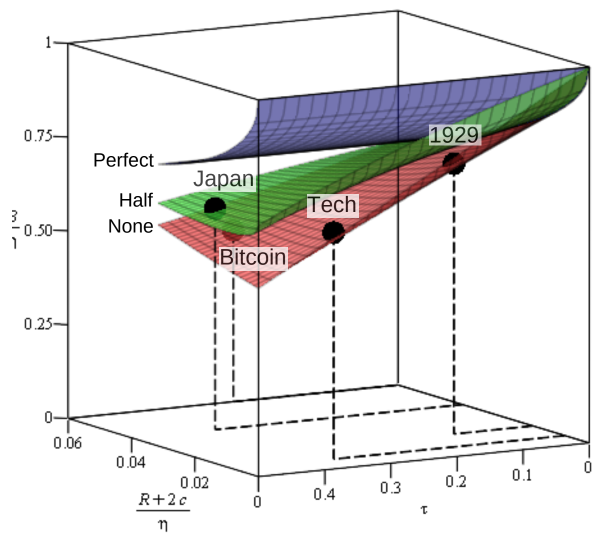
(a) View 1



(b) View 2



(c) View 3



(d) View 4

Figure 10: Actual and theoretical bubble sizes

5.2 Other empirical implications

Our model also generates other testable results. First, the model suggests that the average margin between traders' buying and selling prices are positively correlated with $\frac{c}{\eta}$, $\frac{R}{\eta}$ and τ and negatively with τ_c. These correlations may be tested by comparing average margins before and after policy changes affecting brokers' commission rate, bid-ask spread, interest rate, tax rates and capital loss carryovers, or may be tested across different bubble events.

Second, our model indicates that traders' selling prices are negatively correlated with their purchase prices, i.e. the earlier a trader bought, the later she sells. This can be seen from (8) where a non-break-even

NT traders' selling strategy P_s^* decreases in her purchase price P_p.[64] While an NT bubble also involves T traders, since their purchase prices are even lower and selling prices are the highest (uncorrelated with their purchase prices), the overall correlation among the population is negative. In addition, across different bubble events, the higher the tax rate (and the weaker the tax credit policy), the stronger this correlation will be.

6 Conclusion

In this paper we study the effects of capital gains tax and the transaction cost on asset bubbles. Our model incorporates purchases into the framework of Abreu and Brunnermeier (2003). In the unique equilibrium, we find that the capital gains tax helps deflate bubbles but the capital loss credit tends to offset this deflating effect. Under a perfect tax credit, the bubble becomes immune to the tax. Therefore dealing with bubbles with the capital gains tax also requires tightening the policies on tax credit. We also find that a small transaction cost and outside option have very large marginal deflating effects on bubbles. This implies that the low interest rate policies in the United States between 2001 and 2008 may have contributed considerably to the housing bubble, and that the Bank of Canada's recent move to a lower interest rate from 1% to 0.75% may induce a larger housing bubble in Canada. The model also has an appealing dynamic in which the stop-buy decision and selling decision spread smoothly and continuously from low-belief traders to high-belief traders and high types are still buying when low types have already sold.

To demonstrate how to empirically test these results, we explore several historical bubbles and show that we can use the belief dispersion to normalize bubbles. By this normalization, we eliminate the fundamental variation in bubbles so that we can compare the effects of policy variables. We also show how to infer belief dispersions from the actual index/price histories when there is no explicit data on beliefs. It would be desirable to include more bubble events into the data set so that empirical tests could generate statistically significant results on the effects of the capital gains tax, tax credit, interest rates and transaction costs.

Our model can be extended, for example, by interpreting the compensation factor, τ_c, as government bailout to show that past government assistance may increase the expectation for future bailout and hence

[64]This is because the marginal purchase cost of the pre- and post-crash sales in the first-order condition are scaled by different factors: when purchase cost $P_p + 2c$ increases by δ, the net purchase cost in the pre-crash sale increases by $(1-\tau)\delta$, whereas the purchase cost of the post-crash sale increases by $(1-\tau_c)\delta$ due to the tax credit, with the latter larger than the former. This implies that the tax lowers the pre-crash purchase cost relative to the post-crash cost and makes the former relatively more attractive. In particular the break-even traders who have the highest purchase prices (if they have purchased) sell the lowest (earliest) at $P_s^*(v, P_b^*(v))$.

induce larger bubbles.

References

Abreu, Dilip, and Markus K. Brunnermeier (2003) 'Bubbles and crashes.' *Econometrica* 71(1), 173–204

Brunnermeier, Markus K. (2001) *Asset Pricing under Asymmetric Information: Bubbles, Crashes, Technical Analysis, and Herding* OUP Catalogue (Oxford University Press)

_ (2008) 'bubbles.' In *The New Palgrave Dictionary of Economics,* ed. Steven N. Durlauf and Lawrence E. Blume (Basingstoke: Palgrave Macmillan)

Brunnermeier, Markus K., and Lasse Heje Pedersen (2005) 'Predatory trading.' *Journal of Finance* 60(4), 1825–1863

Brunnermeier, Markus K., and Martin Oehmke (2012) 'Bubbles, financial crises, and systemic risk.' NBER Working Papers 18398, National Bureau of Economic Research, Inc, September

Brunnermeier, Markus K., and Stefan Nagel (2004) 'Hedge funds and the technology bubble.' *Journal of Finance* 59(5), 2013–2040

Carlin, Bruce Ian, Miguel Sousa Lobo, and S. Viswanathan (2007) 'Episodic liquidity crises: Cooperative and predatory trading.' *Journal of Finance* 62(5), 2235–2274

Constantinides, George M (1983) 'Capital Market Equilibrium with Personal Tax.' *Econometrica* 51(3), 611–36

Cooper, Michael, and Matthew Knittel (2006) 'Partial Loss Refundability: How Are Corporate Tax Losses Used?' *National Tax Journal* 59(3), 651–63

De Long, J. Bradford, Andrei Shleifer, Lawrence H. Summers, and Robert J. Waldmann (1990) 'Positive feedback investment strategies and destabilizing rational speculation.' *Journal of Finance* 45(2), 379–95

Doblas-Madrid, Antonio (2012) 'A robust model of bubbles with multidimensional uncertainty.' *Econometrica* 80(5), 1845–1893

Griffin, John M., Jeffrey H. Harris, Tao Shu, and Selim Topaloglu (2011) 'Who drove and burst the tech bubble?' *Journal of Finance* 66(4), 1251–1290

Hu, Shing-yang (1998) 'The effects of the stock transaction tax on the stock market - Experiences from Asian markets.' *Pacific-Basin Finance Journal* 6(3-4), 347–364

Jones, Charles M (2002) 'A century of stock market liquidity and trading costs.' *Available at SSRN 313681*

Kyle, Albert S (1985) 'Continuous Auctions and Insider Trading.' *Econometrica* 53(6), 1315–35

Lanne, Markku, and Timo Vesala (2010) 'The effect of a transaction tax on exchange rate volatility.' *International Journal of Finance & Economics* 15(2), 123–133

Li, Mei, and Frank Milne (2014) 'The role of a large trader in a dynamic currency attack model.' *Journal of Financial Intermediation* 23(4), 590–620

McCauley, Robert N., and Steve A. Zimmer (1991) 'The cost of capital for securities firms in the United States and Japan.' *Quarterly Review* (Aut), 14–27

Scheinkman, Jose A., and Wei Xiong (2003) 'Overconfidence and speculative bubbles.' *Journal of Political Economy* 111(6), 1183–1219

Shiller, Robert J, Fumiko Kon-Ya, and Yoshiro Tsutsui (1996) 'Why Did the Nikkei Crash? Expanding the Scope of Expectations Data Collection.' *The Review of Economics and Statistics* 78(1), 156–64

Stone, Douglas, and William T. Ziemba (1993) 'Land and Stock Prices in Japan.' *Journal of Economic Perspectives* 7(3), 149–165

Temin, Peter, and Hans-Joachim Voth (2004) 'Riding the south sea bubble.' *American Economic Review* 94(5), 1654–1668

Umlauf, Steven R. (1993) 'Transaction taxes and the behavior of the Swedish stock market.' *Journal of Financial Economics* 33(2), 227–240

Westerhoff, Frank H., and Roberto Dieci (2006) 'The effectiveness of Keynes-Tobin transaction taxes when heterogeneous agents can trade in different markets: A behavioral finance approach.' *Journal of Economic Dynamics and Control* 30(2), 293–322

Xiong, Wei (2013) 'Bubbles, Crises, and Heterogeneous Beliefs.' NBER Working Papers 18905, National Bureau of Economic Research, Inc, March

Appendix A Proof of Lemma 3.1

We first prove the following lemma.

Lemma A.1. $\lim_{v\to\infty} P_s^*(v, P_p) = \infty$, $\forall P_p$.

Proof. Assume there exists $\overline{P}$ such that $\overline{P} > P_s^*(v, P_p), \forall v$. For a trader with signal $v > \overline{P} + \frac{\eta}{2}$, she knows for sure that $\theta > \overline{P}$. Then selling at price $P_s^*(v, P_p) < \overline{P} < \theta$ cannot be an equilibrium strategy, because she would rather sell at price v, for instance, if the bubble has not burst, or at θ, if the it has burst. A contradiction. □

Lemma A.2. *There exists v' (and P_p) such that $P_s^*(v', P_p) = P_b^*(v)$, $\forall v \geq \frac{\eta}{2}$.*

Proof. The lowest possible θ is zero, hence the bubble cannot burst below $P_b^*(\frac{\eta}{2})$, and no trader should sell below $P_b^*(\frac{\eta}{2})$. For trader $v = -\frac{\eta}{2}$, she know for sure that $\theta = 0$ and the bubble will burst at $P_b^*(\frac{\eta}{2})$. Hence she will sell exactly at $P_b^*(\frac{\eta}{2})$. Because both $P_b^*(\cdot)$ and $P_s^*(\cdot, \cdot)$ are continuous, and $P_s^*(\cdot, \cdot)$ does no have an upper bound, the lemma holds. □

Corollary 3.1 implies that $P_b^*(\cdot)$ is weakly increasing. Suppose that there exists $\frac{\eta}{2} \leq \underline{v} < \overline{v}$ such that $P_b^*(\underline{v}) = P_b^*(\overline{v})$. Then the probability of bursting at $P_b^*(\underline{v})$ is strictly positive. By Lemma A.2, there exists v such that $P_s^*(v, P_p) = P_b^*(\underline{v}) + \varepsilon$, where $\varepsilon > 0$. Consider trader v. When ε is small enough, the extra benefit, ε, from selling at $P_b^*(\underline{v}) + \varepsilon$ instead of at $P_b^*(\underline{v})$ will be smaller than the loss from being caught in the crash at $P_b^*(\underline{v})$. Then trader v is strictly better off by lowering her selling price from $P_b^*(\underline{v}) + \varepsilon$ to $P_b^*(\underline{v})$. A contradiction.

Appendix B Proof of Proposition 3.1

Suppose trader v who does not hold the asset stops buying for the first time at $P_b^*(v)$ and then switches back to *buy*. Since price is continuously and strictly rising and $P_b^*(\cdot)$ is continuous, there must be trader $v+\varepsilon$ who does not hold the asset and has stopped buying before v could restart buying, by Assumption 3.2. By Corollary 3.1, trader v cannot restart buying before trader $v+\varepsilon$ stops and then restarts buying, and trader $v+\varepsilon$ cannot do so before trader $v+2\varepsilon$ does so. Proceeding this way, trader v is not buying until trader $\theta+\frac{\eta}{2}$ stops and then restarts buying. That trader $\theta+\frac{\eta}{2}$ stops buying is exactly what triggers the burst.

If a trader v has previously sold the asset at $P_s^*(v,P_p)$ and now tries to buy again, it contradicts the belief that all lower types have stopped buying, since $P_b^*(v) \le P_s^*(v,P_p)$.

Appendix C Proof of Proposition 4.1 and D.1

Part 1: Equilibrium verification for $v > \frac{\eta}{2}$

Suppose that all other traders use strategies prescribed in Proposition 4.1 or D.1.

Sale decision: Since $\theta = P_b^{*-1}(P_s) - \frac{\eta}{2} = P_s - \theta - D_{NT \text{ or } T}$, then $\Phi(\theta|v) = \frac{\theta-(v-\frac{\eta}{2})}{\eta}$ and $\phi(\theta|v) = \frac{1}{\theta}$. Substitute into Equation (6), we have $\frac{d\omega}{dP_s} = \frac{1}{\eta}(-P_s+\frac{\eta}{2}+v)$, and SOC is $\frac{d^2\omega}{dP_s^2} = -1 < 0$. Set $\frac{d\omega}{dP_s} = 0$, we can solve for $P_s^*(v,P_p)$. Therefore, given all traders stop buying at $P_b^*(v)$, selling at $P_s^*(v)$ is an equilibrium.

Purchase decision: Because $\omega(P_p,P_s)$ (in both equation 4 and 5) is decreasing in P_p, $P_b^*(v) = v + D_{NT \text{ or } T}$ is uniquely pinned down by (3). Under the strategy $P_b^*(\cdot)$, we can use the condition that $P_b+2c \lesseqgtr P_b^{*-1}(P_s) - \frac{\eta}{2}$ and $B>0$ to derive the restrictions for parameters τ_c, τ, $\frac{R}{\eta}$ and $\frac{c}{\eta}$. Therefore $B>0$ when $\frac{c}{\eta} < \frac{1}{4}$.

Part 2: Equilibrium uniqueness for $v > \frac{\eta}{2}$

We prove the uniqueness by four lemmas. Let $p_b^*(v) \equiv P_b^*(v) - v$. Suppose that $B>0$ upon burst.

Lemma C.1. *Any equilibrium strategy must be that $p_b^*(v)$ is bounded.*

Proof. The continuity of $P^*(\cdot)$ implies that $P^*(\cdot)$ is finite for finite v. Recall that $P^*(\cdot)$ is an increasing function. If $lim_{v\to\infty} p^*(v) = \infty$, then $\forall \overline{B} > 0$, there exists θ such that $P_b^*(\theta+\frac{\eta}{2}) - \theta > \overline{B}$. Hence $P^*(v)$ cannot be an equilibrium strategy. □

Lemma C.2. *In any equilibrium, $FOC = 0$ holds for all traders in $(\frac{\eta}{2}, \infty)$.*

Proof. Assume in an equilibrium strategy $P_b^*(\cdot)$ and $P_s^*(\cdot)$, there exists a trader v such that her $\frac{\partial\omega}{\partial P_s} \neq 0$. If her $\frac{\partial\omega}{\partial P_s} < 0$, that means she can increase her expected payoff by decreasing her selling price p. The only lower boundary for price is zero. But selling at zero is never optimal because $\theta > 0$ for sure. Hence for all trader $v > \frac{\eta}{2}$, selling at the corner solution zero cannot be optimal. Therefore in any equilibrium a trader v's $\frac{\partial\omega}{\partial P_s}$ cannot be strictly negative. If her $\frac{\partial\omega}{\partial P_s} > 0$, that means she can increase her expected payoff by increasing her selling price p. But there is no upper boundary for price, so $\frac{\partial\omega}{\partial P_s} > 0$ itself implies this strategy is not optimal. Therefore in any equilibrium a trader v's FOC cannot be strictly positive. In addition, those P_s at which $\omega(P_p,P_s)$ is non-differentiable ($\frac{\partial\omega}{\partial P_s}$ does not exist) cannot be in the equilibrium for the following reason. $\frac{\partial\omega}{\partial P_s}$ does not exists at two points: $P_b^{*-1}(P_s) - \frac{\eta}{2} = v \pm \frac{\eta}{2}$. But P_s at neither point can be equilibrium strategy, because $P_b^{*-1}(P_s) - \frac{\eta}{2} = v + \frac{\eta}{2} \Longrightarrow P_s = P_b^*(v+\eta)$, which is not rational when bubble size is strictly positive, and $P_b^{*-1}(P_s) - \frac{\eta}{2} = v - \frac{\eta}{2} \Longrightarrow P_s = P_b^*(v)$, which is not rational when transaction cost $c > 0$. Hence FOC$=0$ holds for all traders $v > \frac{\eta}{2}$ in equilibrium. □

From $\omega(P_b^*(v), P_s^*(v)) = (1-\tau)R$ we have $\frac{d\omega(P_b^*(v),P_s^*(v),v)}{dv} = \frac{\partial\omega}{\partial P_b}\frac{dP_b}{dv} + \frac{\partial\omega}{\partial P_s}\frac{dP_s}{dv} + \frac{\partial\omega}{\partial v} = 0$. Since $\frac{\partial\omega}{\partial P_s} = 0$ in EQ, we have $\frac{\partial\omega}{\partial P_b}P_b' + \frac{\partial\omega}{\partial v} = 0 \Longrightarrow -P_b' - (v-\frac{\eta}{2})\phi(v-\frac{\eta}{2}|v) + P_s\phi(v+\frac{\eta}{2}|v) = 0$. For uniform distribution, this means

$$P_s^*(v) = \eta P_b^{*'}(v) + v - \frac{\eta}{2} \tag{9}$$

Substitute (9) into FOC: $\frac{\partial \omega(P_b, P_s)}{\partial P_s} = 0$ (partially) and for uniform distribution, we have

$$P_b^{*'}(P_b^{*-1}(P_s^*(v))) - 1 = \frac{\eta}{\eta + v - P_b^{*-1}(P_s^*(v))}[P_b^{*'}(v) - 1] \tag{10}$$

From $P_s^*(v) > P_b^*(v) \implies P_b^{*-1}(P_s^*(v)) > v$. From $P_s^*(v) \leq P_b^*(\theta + \frac{\eta}{2}) < P_b^*(v + \frac{\eta}{2} + \frac{\eta}{2}) = P_b^*(v+\eta) \implies P_b^{*-1}(P_s^*(v)) < v + \eta$. Hence, $0 < \eta + v - P_b^{*-1}(P_s^*(v)) < \eta \implies \frac{\eta}{\eta + v - P_b^{*-1}(P_s^*(v))} > 1$. Therefore, from (10) we know $\begin{cases} P_b^{*'}(P_b^{*-1}(P_s^*(v))) > P_b^{*'}(v), \text{ if } P_b^{*'}(v) > 1 \\ P_b^{*'}(P_b^{*-1}(P_s^*(v))) < P_b^{*'}(v), \text{ if } P_b^{*'}(v) < 1 \end{cases}$.

For $\forall v_1$, its "images" v_2, v_3, ... are defined as $v_2 \equiv P_b^{*-1}(P_s^*(v_1))$, $v_3 \equiv P_b^{*-1}(P_s^*(v_2))$, etc. Let $x_i \equiv \frac{\eta}{\eta + v_i - P_b^{*-1}(P_s^*(v_i))} = \frac{\eta}{\eta + v_i - v_{i+1}}$ and $X_i \equiv \Pi_{j=1}^{i} x_j$. Then given any v_1, we have $P_b^{*'}(v_i) - 1 = x_{i-1}[P_b^{*'}(v_{i-1}) - 1] = X_i[P_b^{*'}(v_1) - 1]$.

Lemma C.3. $lim_{i\to\infty} X_i = \infty$

Proof. Suppose $lim_{i\to\infty} X_i$ converges to a finite value. This implies that $lim_{i\to\infty} x_i = 1$, which in turn implies that $v_{i+1} \to v_i$, i.e. $P_b^{*-1}(P_s^*(v_i)) \to v_i$. Recall that $P_s^*(v_i) - P_b^*(v_i) \geq 2c$. This means that the slope of the line segment $(v_{i+1}, P_b^*(v_{i+1})) - (v_i, P_b^*(v_i))$ can be arbitrarily large, and this slope goes to infinity as $i \to \infty$. Then $P_b^*(\cdot)$ must diverge upward from 45° line, i.e. $P_b^*(v) - v$ is not bounded. A contradiction. □

Lemma C.4. $P_b^{*'}(v) = 1$, $\forall v > \frac{1}{2}$

Proof. Suppose $\exists v_1$ such that $P_b^{*'}(v_1) < 1$. Since $x_i > 1$, $\forall i$, if $X_i \to \infty$, then there must exist v_i such that $P_b^{*'}(v_i) - 1 < -1 \implies P_b^{*'}(v_i) < 0$, which violates the assumption that $P_b^{*'}(\cdot) > 0$. Suppose $\exists v_1$ such that $P_b^{*'}(v_1) > 1$, we have $lim_{i\to\infty} P_b^{*'}(v_i) = \infty$. We know that in equilibrium $P_s^*(v_i) = \eta P_b^{*'}(v_i) + v_i - \frac{\eta}{2} \leq P_b^*(v_i + \eta)$. Since $P_b^{*'}(v_i)$ will grow unboundedly, we see that $\eta P_b^{*'}(v_i) - \frac{3}{2}\eta \leq P_b^*(v_i + \eta) - (v_i + \eta)$, which implies that $P_b^*(v_i + \eta) - (v_i + \eta)$ will also grow unboundedly, which contradicts Lemma C.1 that $p_b^*(v_i) = P_b^*(v_i) - (v_i)$ must be bounded. □

Let $P_b^*(v) = v + g$, where g is a constant. We can solve for $P_s^*(\cdot)$ through standard procedure. Because $\omega(P_p, P_s)$ is decreasing in P_b in both NT and T equilibria, $P_b^*(v) = v + D_{NT \text{ or } T}$ is uniquely pinned down by (3), i.e. $g = D_{NT \text{ or } T}$. The belief that $B \leq 0$ will lead to an entirely different strategy profile (Proposition E.1), which cannot be equilibrium strategy when parameters satisfy Proposition 4.1 or D.1.

Appendix D Equilibrium with the T bubble

Proposition D.1. *(**Equilibrium with the T bubble**) When* $\begin{cases} \tau_c \leq \tau_c^1 \text{ and } \tau^{TNT1} < \tau < \tau^B, \text{ or} \\ \tau_c^B \leq \tau_c \leq \tau_c^1 \text{ and } \tau^B < \tau \end{cases}$ *and* $\frac{R}{\eta} < \frac{1}{8}(1 - \frac{2c}{\eta})^2 - \frac{c}{\eta}$ *and* $\frac{c}{\eta} < \frac{3 - 2\sqrt{2}}{2}$*, or when* $\begin{cases} \tau_c \leq \tau \text{ and } \tau < \tau^B, \text{ or} \\ \tau_c^B \leq \tau_c \leq \tau \text{ and } \tau^B < \tau \end{cases}$ *and* $\frac{1}{8}(1 - \frac{2c}{\eta})^2 - \frac{c}{\eta} < \frac{R}{\eta} < \frac{1}{2}(1 - \frac{4c}{\eta})$ *and* $\frac{c}{\eta} < \frac{1}{4}$*, there exists a unique trading equilibrium in which bubble size is* $B = \frac{\eta}{2} + D_T > 0$*, the bubble bursts at* $\theta + B$*, all the trader are* T *traders and a trader* $v > \frac{\eta}{2}$ *will*

$$\begin{cases} \text{buy, if price } < P_b^*(v) = v + D_T; \\ \text{sell, if price } \geq P_s^*(v, P_p) = v + \frac{\eta}{2}; \\ \text{hold, if } P_b^*(v) \leq \text{ price } < P_s^*(v, P_p). \end{cases}, \text{ where } D_T \equiv \frac{1 + 4\tau\frac{c}{\eta} - \tau_c(1 + 4\frac{c}{\eta}) - 2d_T}{2(1 - 2\tau + \tau_c)}\eta,$$

$d_T \equiv \sqrt{(1-\tau)\left[(\tau-\tau_c)(1-\frac{2c}{\eta})^2+\frac{2R}{\eta}(1-2\tau+\tau_c)+\frac{4c}{\eta}(1-\tau_c)\right]}$, $\tau_c^B \equiv \tau - \frac{\eta^2}{4c^2}(1-\tau)(1-\frac{2R}{\eta}-\frac{4c}{\eta})$ *and* $\tau^B \equiv \frac{1-\frac{2R}{\eta}-\frac{4c}{\eta}}{(1-\frac{2c}{\eta})^2-\frac{2R}{\eta}}$.

The equilibrium strategies with a T bubble in Proposition D.1 is depicted in Figure 11. Now all traders

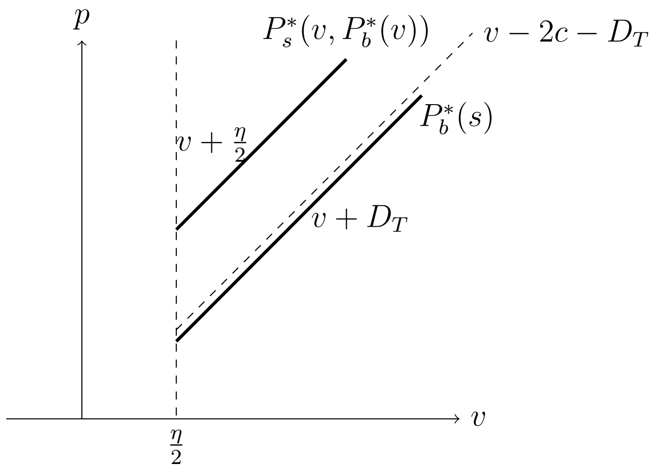


Figure 11: Equilibrium strategies without tax credit and with positive bubble in case T

(including break-even traders) are T traders[65] and they all sell at $v + \frac{\eta}{2}$ (undistorted), irrespective of their purchase prices. There is no NT trader in this case. It can be verified that the conclusions on an NT bubble (i.e. $\frac{\partial B}{\partial \tau_c} > 0$, $\frac{\partial B}{\partial \tau} < 0$, $\frac{\partial B}{\partial c} < 0$ and $\frac{\partial B}{\partial R} < 0$) extend to T bubble, and that an NT bubble is always larger than a T bubble. See Figure 5 and 6.

Appendix E Equilibrium without bubble

The equilibrium where there is no bubble (bubble size < 0) corresponds to a corner solution of (2).

Proposition E.1. *(**Equilibrium without bubble**) When $\tau_c \leq \tau_c^B$ and $\frac{1}{8}(1-\frac{2c}{\eta})^2 - \frac{c}{\eta} < \frac{R}{\eta} < \frac{1}{2}(1-\frac{4c}{\eta})$ and $\frac{c}{\eta} < \frac{1}{4}$, or when $\frac{R}{\eta} \geq \frac{1}{2}(1-\frac{4c}{\eta})$, or $\frac{c}{\eta} \geq \frac{1}{4}$, there exists a "unique" trading equilibrium in which bubble size $B = \eta + D_N \leq 0$, and a trader*

- *$v \leq -D_N - \frac{\eta}{2}$ will never buy the asset;*
- $$v > max(\tfrac{\eta}{2}, -D_N - \tfrac{\eta}{2}) \text{ will } \begin{cases} \text{buy, if price } < P_b^N(v) = \begin{cases} v + \frac{\eta}{2} + D_N, \text{ if } 2\frac{R}{\eta} \leq 1; \\ v - 2c - R, \text{ if } 2\frac{R}{\eta} > 1 \end{cases} \\ \text{hold, if } P_b^N(v) \leq \text{ price } < v + \frac{\eta}{2}. \\ \text{sell, if price } \geq v + \frac{\eta}{2} \end{cases}$$

where $D_N \equiv -\frac{\eta}{\tau} - 2c + \frac{\eta}{\tau}\sqrt{(1-\tau)(1-2\tau\frac{R}{\eta})}$

[65] Their the stop-buy strategy are all below the dividing line $v - 2c - D_T$, because the tax and transaction cost are higher now.

Proof. We first verify that this is an equilibrium. When everyone else stops buying at $P_b^N(v)$,[66] bubble bursts at $p_T = \theta + \frac{\eta}{2} - 2c - R$, which is smaller than θ when $\frac{2c+R}{\eta} \geq \frac{1}{2}$. As the bubble size is non-positive, it is optimal for traders to sell after the burst, i.e. they will try selling as late as possible. For trader v, the largest possible θ is $v + \frac{\eta}{2}$, so selling at any price $\geq v + \frac{\eta}{2}$ is justified and is equivalent. We assume that in this case traders sell at $v + \frac{\eta}{2}$. Then everyone simply gets the after burst price θ, which means their expected sale revenue is $E[\theta|v]$. Then the best response in purchase stage is to stop buying at $P_b^*(v) = E[\theta|v] - 2c - R = v - 2c - R$. Then for traders with $v \leq 2c + R$, their stop-buy price is zero or negative, so they will never buy the asset.

Uniqueness: If $B < 0$, then selling below $P_b^*(v + \frac{\eta}{2} + \frac{\eta}{2}) = v + \eta + g$ is strictly dominated by at or above $v + \eta + g$, and the latter means that the trader will get θ for sure. Knowing that the expected sale price is $E[\theta|v] = v$, a trader will stop buying at $v - 2c - R$ to maintain a non-negative expected payoff. Then $B \leq 0$ requires that $\frac{2c+R}{\eta} \geq \frac{1}{2}$. The belief that $B > 0$ will lead to an entirely different strategy profile (Proposition 4.1 or D.1), which cannot be equilibrium strategy when $\frac{2c+R}{\eta} > \frac{1}{2}$. □

In this equilibrium, when the highest type stops buying, the price is still lower than θ. When the "bubble" bursts, the price jumps *up* to θ. This is because the fixed transaction cost and the return from outside option are too large compared to η, and the stop-buy strategy is pushed so low that the bubble becomes negative. With a negative "bubble", if a trader sells before the "crash", she forgoes the price appreciation that would have certainly realized had she waited till the "bubble" bursts. As a result, everyone has an incentive to hold the asset until the uncertainty is resolved. Obviously the strategy profile in Proposition 4.1 or D.1 is no longer an equilibrium. Knowing that she will be selling at θ after the "crash" and the expected sale price is $E[\theta|v]$, a trader v should buy the asset if the current price is lower than $E[\theta|v] - 2c - R$.

This equilibrium is unique in the sense that the stop-buy strategy and "bubble" size are unique, but selling strategy is not unique. For trader v, since the bubble will burst below $v + \frac{\eta}{2}$ for sure and the price will be fixed at θ thereafter, price will never rise above $v + \frac{\eta}{2}$. So all strategies selling at any price above $v + \frac{\eta}{2}$ are optimal, although they will never be implemented.

Appendix F Equilibrium with NT bubble and $R > 0$

With $R > 0$, Proposition 4.1 also has to be modified as follows. $\tau^{TNT1} \equiv \frac{1}{2} - \frac{\frac{4c+2R}{\eta}}{(1-\frac{2c}{\eta})^2}$ and $\tau_c^1 \equiv 2\tau - 1 + 8(1-\tau)\frac{\frac{2c+R}{\eta}}{(1+\frac{2c}{\eta})^2}$.

[66] The stop-buy price $P_b^N(v)$ is pinned down by $\int_{\theta=v-\frac{\eta}{2}}^{P_b^N+2c}(\theta - P_b^N - 2c)\phi(\theta|v)d\theta + (1-\tau)\int_{\theta=P_b^N+2c}^{v+\frac{\eta}{2}}(\theta - P_b^N - 2c)\phi(\theta|v)d\theta = (1-\tau)R$ and we have $P_b^N(v) = v + \frac{\eta}{2} - \frac{\eta}{\tau} - 2c \pm \frac{\eta}{\tau}\sqrt{(1-\tau)(1-2\tau\frac{R}{\eta})}$. Since stopping below the lowest possible θ is not optimal, it must be that $P_b^N(v) + 2c \geq v - \frac{\eta}{2}$, we then have $P_b^N(v) = v + \frac{\eta}{2} - \frac{\eta}{\tau} - 2c + \frac{\eta}{\tau}\sqrt{(1-\tau)(1-2\tau\frac{R}{\eta})}$. Furthermore the square root requires that $2\frac{R}{\eta} \leq 1$. If $2\frac{R}{\eta} > 1$, then $P_b^N(v) = v - 2c - R$